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A novel method for transmission system cost allocation with better accuracy and fairness

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Abstract. This paper proposes an integrated method for the accurate and fair cost allocation of the transmission system among the users (generators and loads) of the network. It mitigates the existing research gaps and implementation issues in the marginal participation approach for network cost allocation. Major challenges in the marginal participation approach are 1) fair selection of economic slack busses 2) inaccuracy due to use of DC power flow 3) treatment of the counter flows and 4) enforcement of cost-causality. While the prior art has addressed the sub-problems individually, an integrated approach has been missing. This work fills this research gap. Besides, it for the first time introduces the use of linearized AC power flow for calculating the marginal flows. This provides a major improvement over the DC power flow model as nominal voltage and reactive power variables can be modeled without compromising linearity. Economic slack bus selection by min-max fairness approach, modelling of counter flows without resulting in negative cost-shares (payoffs), enhancement of cost-causality by segregating usage cost, reliability cost, and residual costs are other salient contributions of the proposed work. These features result in a rigorously fair, accurate, and yet tractable method computationally. Case-studies on many systems including the IEEE-118 bus system demonstrate the claims made.

Keywords. Cost-causality; marginal line power flow; min-max fairness; linearized AC power flow; reactive power; transmission system cost allocation.

1. Introduction

The fair allocation of the yearly cost of the transmission system among the load and the generation entities is a tricky and complex problem. Many methodologies have been proposed to address this problem and yet it has not been solved satisfactorily. In the Marginal Participation (MP) approach, the 'extent of use' of the network by an entity is determined by first evaluating the marginal line flows for unit increment in load' s active power withdrawal or generator' s injection. Extrapolating the flows in proportion to MWs associated with the load or injection will lead to the actual power flow. Subsequently, an entity's 'extent of use', which may be a load or a generator, is monetized by the MP approach as explained in [1].

In [1–3], the fundamental principles of transmission pricing are discussed. The various existing tariff methodologies can be evaluated regarding the fulfillment of these guidelines or principles. One of these guidelines signifies the need for achieving economic efficiency in power market operation and giving the economic signals about the investment in generation, demand, and transmission systems. Economic efficiency is said to be achieved by maximizing the global surplus of the market agents in the power system operation, investment for the transmission system expansion, and the choice of location for the generation and demand. For these, the transmission pricing must be able to send appropriate economic signals for the network.

In the MW-Mile methods like [4–6], circuit-theory based methods like [7, 8] and the MP methods like [9–12] from the beginning till the recent times, the treatment of the cost component associated with the counter flows is that of a *zero counterflow (zcf)* or an *absolute counterflow (abs)*. The negative sign is masked either by taking the absolute value or forcing it to zero to eliminate negative cost-shares or payoffs. Payoffs imply an extra burden on remaining entities as the total cost now recovered from rate takers is the sum of network cost and the payoffs. The principle of economic efficiency requires simulation of the day-to-day power system network operation, which is not done by the *absolute* or the *zcf* approach. It can lead to misleading economic signals.

The approach *reverse (rev)* of retaining a negative cost to counter-flows are explored in [13–20]. Counterflows should

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be incentivized because if the entities which create the counter flow increase their volume (MW), then it will help decongest the lines. Moreover, it can provide relief to the high price takers. The linearity of the sensitivity matrix is preserved and the superposition principle for the contribution of entities in the marginal line flows is applicable. This issue is recognized in [14] and an algorithm to redistribute an extra income due to the payoffs are provided as a solution to the over-recovery. In the proposed research work, one of the objectives is to overcome the problem of payoffs encountered in the *rev* approach. If the tariff mechanism is correct, payoffs should not occur even when *rev* approach is practised. Neither the masking of negative signs before the cost-allocation nor using the redistribution algorithm post-facto is required.

The principle of cost-causality introduced in [21] states that the cost should be borne by the entity responsible for it i.e., the beneficiary pays. While this principle or guideline is straightforward to understand; imbibing it in the allocation methods has always been a challenge. For example, almost all of the MP approaches (and many others as well) use line cost rate as equal to the ratio of the line cost to line-power flow as obtained from the power flow solution. It appears that by this choice of line cost rate, cost-causality follows. But, this also allocates the cost for the unused capacity, which is significantly high given the redundancy in the network, the economy of scale, the indivisibility of the projects, and other factors. The cost transfers to a few beneficiaries based on the power flow rather than socializing it across the entities which are connected to the grid. The problem can be resolved by altering the line cost rate definition to line cost per capacity base. Reference [12] outlines a method to estimate the invalid or wrongly planned (inefficient) capacity. Given the uncertainties in the long-term planning, the indivisibility of the transmission project, and the principle of economy of scale [2, 21, 22], this invalid capacity is unavoidable. The non-recovery (as a penalty) for it as suggested in [12] is inappropriate as per the guideline of cost-recovery given in [2, 21]. In [19], the proposed tariff comprises of two components, a first component for the cost allocation of the utilized capacity of the network and a second component for the recovery of the cost of the remaining transmission capacity of the network. In India, vide the new regulations as per [23] introduced recently by Central Electricity Regulatory Commission, the line cost rate has been changed to line power flow divided by the line capacity (Surge Impedance Loading -SIL). It is a two-part tariff mechanism comprising of used capacity cost to be shared by the hybrid method of Marginal Participation-Average Participation (MP-AP) and the unused capacity cost to be socialized among the participating entities. To further improve cost-causality, a new reliability component is estimated in [18–20]. In our present work, the estimation of reliability cost component is done as per [20] but a new linearized AC power flow is integrated with the other desirable features.

A review conducted in [24] brings out the need to model reactive power flows while calculating the Point of Connection (PoC) tariffs. This is because the MW flows also depend upon the voltage profile and hence on the reactive power injections. In India, as per the previous mandate of the Central Electricity Regulatory Commission [25] and also the new [23], a Marginal Participation-Average Participation (MP-AP) hybrid method is being used for the 'extent of use' based cost-allocation. The approach is the same as in [11], but the sensitivities have to be computed from AC power flow. The CERC's concern was that the deviation in the voltage from 1 p.u. may not be small enough to be ignored. It was felt that the assumptions in DC power flow will compromise the accuracy of the computed marginal flows and hence the nodal prices. However, calculating the marginal flows from the sensitivities obtained by the linearization around the AC power flow solution point is also questionable. The approach does not seem to be on a sound theoretical footing because in the MP approach the flow-share of an entity in a line flow is obtained by linear extrapolation. Unlike DC power flow, where the sensitivities are constant, extrapolation of a marginal flow around linearized AC power flow solution will not lead to the original (AC power-flow based) line flows. One of the motivations of the proposed work is to contribute to the policy framework for transmission pricing at the national level in India.

Improvements in network modelling for transmission system expansion planning have been reported in [26–28]. In [29], the reactive power, off-nominal bus voltage magnitudes, and network losses have also been modeled in a linearized AC framework (LAC). Unlike linearizing around the AC power flow solution as done in [23, 25], here the sensitivities are constant and do not depend on the operating point. Hence, the superposition principle can be applied to the network. In this paper, the equations for the computation of the marginal line flows are derived using the LAC framework. In [30], the validity of linear approximation for power flow solution for distribution system is proven theoretically. The focus in it is on the distribution system and variation in voltage magnitudes and angles at PV buses during a change in certain operating conditions like overload, etc. The focus of our research work is to develop a transmission system pricing model based on a more inclusive and realistic network model as proposed in the manuscript. The voltage magnitudes and angles are specifically calculated at load buses by linearized AC power flow (LAC) analysis and compared with that for ACLF. Methodology to compute marginal line power flow is developed. The comparison of line power flows to confirm improved accuracy with the proposed LAC model for real-life transmission system networks is given in section 3.3.

In the proposed integrated method, it is shown that the existing DC models create unacceptable inaccuracies in the division of the line flows when dealing with the lines with different *x*/*r* ratios. Then, a composite, min-max fair, MP approach with the linearized AC (LAC) framework is formulated. It is termed the LAC-PoC model. The significant improvement in accuracy is demonstrated by case studies using the proposed LAC network model for prevalent MP variant methods.

In this paper, through many comparative evaluations over many systems, it is shown that a rigorous, fair, cost-causal and more accurate allocation of network costs can be achieved by integrating the following features:

1. Min-max fair selection of the economic slack bus
2. Capacity based line cost rate
3. Full incentivizing of the counter flows
4. Segregation of reliability costs and their bundling with the 'extent of use' cost and allocating the combined cost using the min-max fairness policy
5. Modelling of the reactive power variables, the line resistance, and the voltage deviations from unity on load buses using the linearized AC power flow.

The organization of the paper is as follows. The research gap analysis and justification of the proposed approach are elaborated in section 2. In section 3, the formulation of the proposed Point of Connection Tariff (LAC-PoC) model based on the linearized AC power flow is developed. Moreover, the comparative results for improvement in the accuracy in the line power flow for a five bus test-case and the real-life IEEE test-cases are explained. In section 4, the overview of a proposed integrated min-max fair MP approach is given. Section 5 depicts the testing and validation for IEEE 14, 30, 57, and 118 bus networks. The conclusions are summarized in the section 6.

2. Research Gap analysis for the inclusion of desirable attributes in the existing methods

Table 1 gives the analysis of gap in research for the achievement of desirable attributes like the *economic efficiency*, *cost-causality*, *fairness* and *accuracy*. The factors covered under the survey are:

1. treatment to counter-flows,
2. reliability cost component,
3. rigour in fairness,
4. and type of network model which is indicative of the accuracy of marginal flows which are monetized.

The gap analysis confirms that there is a further scope of improving accuracy by more realistic network models while maintaining linearity so that linear programming can be adopted to improve fairness. It is to be noted that linearity is assured by the inclusion of counter-flows in the modelling and the use of a network model which is a linearized version of the AC power flow network model. The desirable attributes are not accomplished in one single method and an integrated effort for their fulfillment is needed. This is the main motivation for developing an integrated method that incorporates all the features in one method.

2.1 *Justification of the proposed approach*

The proposed approach has been evolved to fill the research gaps mentioned in table 1. The justification for the features imbibed into the proposed method is as follows.

1. Fairness in the treatment of the counter flows: It is an important aspect related to both the cost fairness and the cost-causality. The handling of this aspect is varied; researchers have proposed the following three alternatives - 1) no incentivizing, 2) partial incentivizing, and 3) complete incentivizing of the counter flow benefits. In the partial incentivizing schemes, either the entities responsible for the counterflow on lines are exempt from paying the usage cost, or a separate payoff reallocation problem is solved. These schemes cannot be argued to be fair enough to the high PoC taking entities as it lacks realistic cost-function definition and transparency. On the other hand, methods that ignore the direction of the entity's flow component vis-a-vis the net flow are not direction sensitive and do not promote operational efficiency because congestion management is ignored. They also fail to provide correct siting signals for new loads and generators. It is shown in [17] and [20] by the case studies over many systems and methods, that complete incentivizing with the existing MP approaches leads to the problem of payoffs or negative PoC tariffs. The problem is resolved for the first time in [17] and [20] by the simultaneous application of both the min-max fair economic slack bus selection and the line cost rate selection on the capacity base. Proper management of counter flows within the MP framework is one of the main contributions of this work. This feature is incorporated into the proposed work.
2. Fairness in economic slack bus selection: Cost allocation problem can have multiple solutions. This raises the question of fairness in the cost allocation problem. Usually, cooperative game-theoretic methods are used to address such problems. Among many possible approaches, including the Nucleolus, Shapley Value allocation, Aumann Shapley method, min-max fairness, and the combinatorial game, only the Aumann Shapley allocation method, and the min-max fair marginal participation approach can address the requirement of scalability for large networks. Min-max fair MP method, as shown in [16, 17, 20], does the rigorously fair selection of the economic slack bus. It equivalences PoC tariff (Paise/kWh), defined as network usage cost of entity per unit power or energy with the entity's regret. Therefore, it assigns economic slack bus vectors for the

Table 1. Research gap analysis for treatment to counter flows, rigor in fairness, reliability cost component, and network model for various methods.

Method & reference	Counter flows treatment	Rigor in fairness	Reliability component included	Network model
MP Basic [9]	Made zero	No	No	DC power flow
MP Pro-rata [11]	Made zero	No	No	DC power flow
PSP/APF [31–33]	Entity flow always in the same direction to overall flow	No	No	decomposition by transportation model
MP-AP [11]	Made zero	No	No	DC power flow
EBE [5]	Absolute	No	No	DC power flow
Z_{bus} [7]	absolute	No	No	AC power flow
LRMC [34]	Made zero	No	No	DC power flow
LRMC [35]	Retained	No	No	DC power flow
Aumann Shapley [36]	Absolute	Yes	No	DC power flow
Aumann Shapley Circuit approach [37]	Absolute	Yes	No	AC power flow
A more fair ... [15]	All 3 modes	No	No	DC power flow
A new Methodology ... [14]	Retained	No	No	DC power flow
MP min-max [16]	Retained	Yes	No	DC power flow
Improved MW-Mile Method [6]	Absolute	No	No	DC power flow
A structural Cost allocation ... [12]	Yes Made zero for reliability component	No	No	DC power flow
Anewtransmission... [18]	Retained	No	Yes	DC power flow
Efficient... [17]	Retained	Yes	No	DC power flow
Transmissionnetwork... [19]	Retained	No	Yes	DC power flow
Circuittheory.... [8]	Made zero	No	No	AC power flow
Acost − causal... [20]	Retained	Yes	Yes	DC power flow
Proposed	Retained	Yes	Yes	Linearized AC (LAC) model

cost bearing entities in such a way that at the optimal solution, any further benefit (i.e., lowering the extent of use) obtained by assigning a better economical slack bus to a load or generator entity will result in higher PoC tariff of one or more entities who will have to pay same or higher PoC tariff than the said entity. Since such a move is not fair, it is not permitted. As such min-max fair slack bus selection leads to an equilibrium solution wherein, typically, the maximum PoC tariff (as well as other high PoC tariffs) is reduced significantly as compared to the MP-AP (or concerning any other dispersed slack bus selection approaches) [11]. The maximum PoC tariffs are reduced in a lexicographic manner subject to network constraints and the requirement of full cost recovery for the network. Thus, the min-max fair MP method is fairer as compared to other known MP approaches. As an outcome of a rigorous optimization process, clusters of same or nearby prices are formed i.e., the set of cost-sharing entities are grouped such that the entities in a group have the same PoC tariff. Due to all these benefits, the min-max fair economic slack bus selection approach introduced in [16, 17, 20] is retained as an important attribute and also further developed in this work. The tariff obtained by this method using DC power flow is termed as PoC_1.

Algorithm 1 MP min-max programming algorithm for TSU-cost allocation

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**Initialize:** $S_0 = \{1, 2, \cdots, n_E\}$, $M_0 = \{\phi\}$, $z_0^* \approx \infty$, $k = 1$.
**while** $S_{k-1} \neq \{\phi\}$ **do**
    1. Solve the following LP problem ($LP_k$):
        $LP_k \quad : \quad \left\{ \begin{array}{cc} \min & z_k \\ \forall\, d \in \mathscr{R}_+^{n_D} \cap D_0,\ z_k & \end{array} \right.$
        subject to min-max constraints:
            $z_k \geq p_{G_i}(\bar{d}) \ \ \forall\, i \in S_{k-1}$    (1)
        For $j = 1, \cdots, k-1$ set
            $p_{G_i}(\bar{d}) \leq z_j^* \ \ \forall\, i \in M_j$    (2)
    $p_{G_i}(d)$ is a linear function as defined in [16]. Let the solution be given by $z_k^*$.
    2. Compute $M_k = \{i : \mu_i > 0\}$ corresponding to dual variable of constraint set $z_k \geq p_{G_i}(\bar{d})\ \forall\ i\ \in S_{k-1}$. $M_k$ is the set of entities, whose price cannot be improved beyond $z_k^*$, $\beta_h$ and $\alpha_h$ determine the dispersed slack bus of entity $h \in M_k$.
    3. Update: $S_k = S_{k-1} \setminus M_k$
    $k = k + 1$
**end while**
```

Algorithm 1 summarizes the proposed min-max algorithm for the MP method. Constraints (2) ascertain that the PoC tariff of the entities, fixed in the previous steps, is not increased (altered) any further. For compactness, the set of linear constraints modeled for the economic slack bus selection is grouped into a set D_0. Note that the constraint space of the min-max MP problem is closed and bounded. The objective function in each optimization problem is linear. Hence, the solution of the min-max MP problem exists and is unique.

3. Estimation of reliability cost component: Once the 'extent of use' costs have been segregated, the unused capacity costs need further subdivision into reliability costs and the residual costs. The principle of cost-causality can be applied to the component of reliability cost. As explained in [20], in this work, the reliability cost component is estimated by first computing the maximum power flow on the line under N-1 contingency scenarios. Then assigning costs to the difference of the maximum line-power flow magnitude and the base case line-power flow magnitude using the capacity-based line cost rate gives the reliability cost component of the line. The aggregation of reliability costs of all the lines gives the network reliability cost. In the proposed approach, it is then bundled with the 'extent of use' costs and shared using the min-max fairness policy. The residual costs are shared as per the postage stamp allocation approach i.e., in proportion to the MW injected or withdrawn by the entity. Thus, harmony is established between the cost-causality and the socializing of the residual costs, thereby improving the fairness in the cost allocation.
4. Accuracy of the network model: The accuracy of the cost-shares should not be sacrificed while going for simplicity. The requirement of simplicity is sometimes misconstrued as a mandate for gross approximation resulting in oversimplifications. The transmission network cost allocation based on the 'extent of use' requires load-flow studies to find out the marginal line flows. The approximations are inherently there in the DC power flow method where the resistance parameter, shunt line capacitance, shunt capacitors/reactors at load buses, and the voltage deviations at the load buses are not included in the network model. In their absence, the marginal line flows represented by the sensitivity matrix are inaccurate. In AC power flow-based methods like Zbus [7, 8] and Aumann Shapley (circuit approach) [37] this attribute is achieved. However, as summarized in table 1, in all these methods counter flows are either made zero or made absolute. In [19], the counter flows are retained and the reliability component is indirectly included by considering the aspects of transmission expansion planning. The rigour is missing as no optimization or cooperative-game-theory-based method is used. Moreover, the accuracy is compromised due to the use of DC load flow. Efforts are made in the proposed approach to reduce this loss of accuracy while preserving all the other features mentioned above.

The features 1-3 above are incorporated in one method in [20]. The tariff obtained in [20] by min-max fair MP method with the DC power flow and reliability cost segregation is termed as $PoC1_{REL}$. However, this method suffers from a limitation of accuracy due to the use of DC power flow for computing the marginal line flows. In this work, improving accuracy while preserving the desirable attributes and guidelines of the tariff mechanism as per [1–3] is the main contribution. A more accurate version of the min-max fair MP method based upon a recent linearized AC network model given in [29] is proposed. Two load modelling approaches viz. 1) the constant reactive power model and 2) the constant power factor model are considered. Simulations indicate that this improved network model can alter the PoC tariff of the entities up to the extent of 10%. The development of the linearized AC network model (LAC) is explained in the next section.

3. Formulation of the sensitivity matrix of the line-power flow based on Linearized AC Network (LAC) model

A π equivalent model of a transmission line between a node(bus) l and a node m is shown in figure 1. Further, $g_{lm}\&b_{lm}$ represents the series conductance and susceptance respectively of the line, and b_{lm0} represents the shunt susceptance (a positive value). b_{sh_i} represents the net shunt susceptance of the shunt reactor and/or capacitor (if at all) connected at bus i in the network. The AC power flow in a transmission line, lm, from a node l to a node m can be written as follows:

$$P_{lm_{AC}} = V_l^2 g_{lm} - V_l V_m \left[g_{lm}\cos(\delta_{lm}) + b_{lm}\sin(\delta_{lm})\right] \tag{3}$$

$$Q_{lm_{AC}} = -V_l^2(b_{lm} + b_{lm0}) + V_l V_m \left[b_{lm}\cos(\delta_{lm}) - g_{lm}\sin(\delta_{lm})\right] \tag{4}$$

The angle $\delta_{lm} = (\delta_l - \delta_m)$ is the difference of the bus voltage angles at the buses l and m of the transmission line from bus l to bus m. Unlike the DC power flow where voltages are fixed to 1 p.u., in the above expressions, a more accurate voltage magnitude model is used as follows.

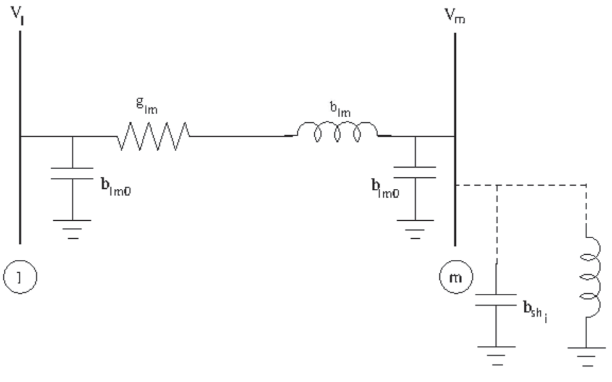

Figure 1. The π equivalent model of a transmission line.

$$V_i = 1 + \Delta V_i \tag{5}$$

As a consequence of linearization, the quadratic terms associated with ΔV_l and ΔV_m are neglected. The linearized form of the resulting power flow equations [29] are as follows:

$$P_{lm_{LAC}} = -b_{lm}(\delta_l - \delta_m) + g_{lm}(\Delta V_l - \Delta V_m) \tag{6}$$

$$Q_{lm_{LAC}} = -g_{lm}(\delta_l - \delta_m) - (b_{lm} + 2b_{lm0})(\Delta V_l) + b_{lm}(\Delta V_m) - b_{lm0} \tag{7}$$

Equations (6) and (7) are the improved versions of the line flow model in DC power flow. Note that the $P_{lm_{LAC}}$ equation (6) includes resistance effect by g_{lm} and also two additional variables for the voltage magnitude deviations. Also, the modification permits is of the reactive power flow of the line.

Now using the equations (6) and (7), the vector of the active and reactive power flows on the line can be represented as follows. The convention used for choosing the line flow is such that $P_{lm_{LAC}} \geq 0$.

$$\underset{2n_l\times 1}{\begin{bmatrix}\mathbf{P_{line}}\\ \mathbf{Q_{line}}\end{bmatrix}} = \underset{2n_l\times(2n-n_G-1)}{[\mathbf{C}]} \underset{(2n-n_G-1)\times 1}{\begin{bmatrix}\delta\\ \mathbf{\Delta V}\end{bmatrix}} - \begin{bmatrix}\underset{n_l\times 1}{\mathbf{0}}\\ \underset{n_l\times 1}{\mathbf{b_{0_{line}}}}\end{bmatrix} \tag{8}$$

The power injected at bus i will be equal to the sum of the line-power flows of all lines connected to that bus i.

$$\mathbf{P_i^{inj}} = \sum_{j\in\Omega_i}\left[-b_{ij}(\delta_i - \delta_j) + g_{ij}(\Delta V_i - \Delta V_j)\right] \tag{9}$$

$$\mathbf{Q_i^{inj}} = [\sum_{j\in\Omega_i}(-g_{ij}(\delta_i - \delta_j) - (b_{ij} + 2b_{ij0})(\Delta V_i) + b_{ij}(\Delta V_j) - b_{ij0})] - b_{sh_i}(1 + 2\Delta V_i) \tag{10}$$

Set Ω_i is the set of all the buses adjacent to the bus i. The last term outside the summation in equation (10) models the net shunt susceptance of the shunt reactor and/or capacitor (if at all) connected at bus i in the network. Rearranging and representing in the linear system solver format,

$$\underset{(2n-n_G-1)\times(2n-n_G-1)}{[\mathbf{D}]} \underset{(2n-n_G-1)\times 1}{\begin{bmatrix}\delta\\ \mathbf{\Delta V}\end{bmatrix}} = \begin{bmatrix}\underset{(n-1)\times 1}{\mathbf{P_G} - \mathbf{P_L}}\\ \underset{(n-n_G)\times 1}{-\mathbf{Q_L}}\end{bmatrix} - \begin{bmatrix}\underset{(n-1)\times 1}{\mathbf{0}}\\ \underset{(n-n_G)\times 1}{\mathbf{k}}\end{bmatrix} \tag{11}$$

Note that, the $[-\mathbf{Q_L}]$ part on the right hand side of the equation (11) has to be compensated by a sub-vector $[\mathbf{k}]$, which comprises of $(-b_{ij0} - b_{sh_i})$ at the load buses. Let n be the number of nodes and n_G be the number of generators in the system. In the above equations, $(n-1)$ active power (MW) injection equations and $(n - n_G)$ reactive power equations are modelled. This is exactly equal to the number of unknowns; $(n-1)$ angles at buses other than the reference bus and $(n-n_G)$ voltage magnitudes at buses other than the generator buses.

Thus, using the LAC method given in equation (11), the unknown values of phase angles δ_i at all the buses and the deviation in the voltage magnitudes ΔV_i at P-Q (load) buses can be obtained. The line power flows can be computed using the equation (3).

3.1 *Influence of Heterogeneity in x/r ratios in the transmission network on the line power flow computation*

Let us consider two parallel transmissions lines wheeling a net current of I Amperes. We are interested in finding out the division of the currents in the two lines when the line currents are divided in an inverse proportion to 1) the line impedance and 2) when only the line reactance is used. The second approach mimics the DC power flow approximation. For line impedances, we consider the following two scenarios.

1. Two identical parallel lines with *x*/*r* ratio of 10.
2. Reactances of the two lines are identical but the resistance of the second line is twice that of the first line.

For the sake of simplicity, shunt capacitance contribution is neglected. For the first scenario, it is clear that an equal division of current will be obtained irrespective of whether the line resistance is considered or not. On the other hand, when the line parameters are as per the second scenario and when the accurate modelling is done, it leads to the current sharing ratio of $1.015\angle -6^\circ : 1$. However, when the line resistance is neglected, it is seen that the ratio is altered to $1 : 1$. This implies that neglecting the resistances in a network having lines with different *x*/*r* ratios could lead to noticeable errors in the distribution of the flows. Hence, it is necessary to model the line resistance, voltage deviations, and reactive power injections in the computation of the line flow sensitivities. We now illustrate this observation on a 5-bus test example. A five bus case-study with heterogeneity in *x*/*r* ratios of the lines is used to demonstrate the impact of the improved network modelling on the line flows.

3.2 *Comparative results for improvement in the accuracy of the line power flows for a five bus test-case*

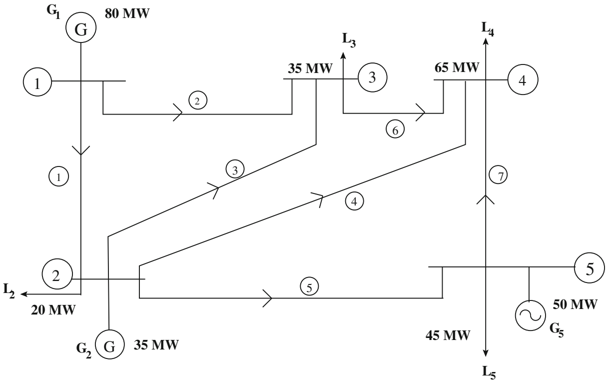

Figure 2. A five bus test case.

A five bus test case [10] is shown in figure 2. There are five buses numbered 1 to 5 in a circle. The lines interconnecting the buses are numbered from 1 to 7 circled by a smaller circle. These lines are enlisted in the same sequence 1 to 7 in table 3. All loads have a power factor of 0.8. The results of AC power flow are compared with that of the DC power flow and LAC models. The resistance parameters of the lines have been altered to create different *x*/*r* ratios $(\sigma(x/r) = 3.95)$ keeping x and b_0 same as in [10]. The results are summarized in tables 2 and 3 (with the maximum percentage errors highlighted in bold).

In table 2, The voltage magnitudes and angles obtained in ACLF are shown in column 2. For the sake of compactness, the magnitudes and angles for DCLF and LAC are not shown. The respective percentage errors are displayed in columns 3 and 4. It can be observed that the maximum error in the voltage magnitude with the DC power flow is 4.75%, while with the LAC model, it is 0.35%. The average error in the voltage magnitude with the DC power flow is 2.26%, while with the LAC model, it is 0.17%. The average error in voltage angle magnitudes is 5.7% for DC power flow and 3.21% for LAC. The maximum angle error reduces from −11.74% to −3.89%.

In table 3, the average error in line-power flow is −9.03% for DC power flow and −3.47% for LAC. The maximum error reduces from −19.59% to −6.02%. Thus, the average and maximum of the percentage errors in the results of LAC power flow are lower than that in the results of DC power flow. Therefore, it can be stated that there is overall accuracy improvement achieved in LAC as compared to DC power flow.

Table 2. Comparison of the voltages and angles at the nodes (AC power flow as reference) for five bus test case with different *x*/*r*.

Node No.	$V\angle\delta$ in p.u.$\angle^\circ$ AC power flow	% Error in V(δ) DC power flow	% Error in V(δ) LAC
2	$1.00\angle -1.91$	0.00 (− **11.74**)	0.00 (− 2.71)
3	$0.96\angle -4.31$	4.27 (− 1.30)	0.33 (− 3.54)
4	$0.95\angle -4.67$	**4.75** (− 1.37)	**0.35** (− **3.89**)
5	$1.00\angle -2.65$	0.00 (− 8.37)	0.00 (− 2.69)

Table 3. Comparison of the line power flows for five bus test case with different x/r ratio for all lines (AC power flow as reference).

Line	r in p.u.	P_{line} in AC	P_{line} in DC	P_{line} in LAC	% Error DC	% Error LAC
1–2	0.02	49.75	48.92	48.92	− 1.66	− 1.66
1–3	0.02	31.08	30.11	30.28	− 3.11	− 2.59
2–3	0.04	25.76	21.36	24.94	− 17.09	− 3.20
2–4	0.02	27.83	25.30	26.79	− 9.08	− 3.74
2–5	0.02	10.51	10.26	10.26	− 2.43	− 2.43
3–4	0.01	21.78	17.51	20.47	− **19.59**	− **6.02**
5–4	0.02	15.40	13.83	14.69	− 10.23	− 4.64

3.3 *Comparison of line power flows for real-life transmission networks*

To confirm the concept of better accuracy using the LAC model, exhaustive case studies are conducted to compute the line power flows for DCLF and the linearized AC power flow model for real-life transmission networks. The results for IEEE 14 Bus, 30 Bus, 57 Bus, and 118 Bus networks (without any modifications) for line power flow obtained using DCLF and Linearized AC power flow as compared with the AC power flow conducted using MATPOWER [38, 39] and OCTAVE [40] are shown in tables 4 and 5. In Table 4, the percentage count of the number of lines w.r.t. total lines in the network for which the LAC is more accurate are given. It can be observed that the line power flows obtained by LAC are more accurate (with less percentage error than DCLF) than that for DCLF for the majority of the lines. Incidentally, by chance, for a few lines, the accuracy of DCLF is higher than LAC. To further confirm the improvement in the accuracy, it can be observed in Table 5 that both, the maximum percentage error and the average percentage error for line power flows are much lower in the LAC model than that in the DCLF. The results validate the theoretical concept about better accuracy with linearized AC power flow model proposed in [29] as well as [30].

Table 4. Percentage count of lines for which percentage error in line power flows for LAC is less than that for DCLF with reference to that for AC power flow.

IEEE network	Percentage (%) count of lines w.r.t total lines in the network
14 Bus	70
30 Bus	82.93
57 Bus	73.75
118 Bus	81.08

Table 5. Comparison of line power flows for DCLF and LAC power flows with reference to that for AC power flow.

IEEE network	Maximum % Error		Average % Error	
	DCLF	LAC	DCLF	LAC
14 Bus	83.42	41.1	20.26	12.71
30 Bus	62.83	40.09	20.23	8.42
57 Bus	147.27	97.34	28.59	19.53
118 Bus	694.45	302.37	35.65	29.01

3.4 *Marginal line flow equations using LAC model*

By using equations (8) and (11), the line-power flow equations can be:

$$\underset{2n_l\times 1}{\begin{bmatrix}\mathbf{P_{line}}\\ \mathbf{Q_{line}}\end{bmatrix}} = \underset{2n_l\times(2n-n_G-1)}{[\mathbf{S}]}\left[\underset{(2n-n_G-1)\times 1}{\begin{bmatrix}\mathbf{P_G}-\mathbf{P_L}\\ -\mathbf{Q_L}\end{bmatrix}} - \begin{bmatrix}\mathbf{0}\\ \mathbf{k}\end{bmatrix}\right] - \underset{2n_l\times 1}{\begin{bmatrix}\mathbf{0}\\ \mathbf{b0_{line}}\end{bmatrix}} \tag{12}$$

where, the sensitivity matrix $[\mathbf{S}]$ is the product of the two matrices $[\mathbf{C}]$ and $[\mathbf{D}]^{-1}$. It depends only on the network parameters and the connectivity, and, is independent of the injection scenario. An additional column for the reference bus whose elements are set to zero is appended to $[\mathbf{S}]$ of equation (12).

As, the active line-power flows are important, only the equation for active line-power flow is considered. Hence,

$$[\mathbf{P_{line}}]_{n_l\times 1} = \underset{n_l\times(2n-n_G)}{\begin{bmatrix}\underset{n_l\times n}{\mathbf{S_p}} & \underset{n_l\times(n-n_G)}{\mathbf{S_q}}\end{bmatrix}} \underset{(2n-n_G)\times 1}{\begin{bmatrix}\mathbf{P_G}-\mathbf{P_L}\\ -\mathbf{Q_L}-\mathbf{k}\end{bmatrix}}. \tag{13}$$

Note that the vector $\mathbf{Q_L}$ is augmented by a vector $\mathbf{k}$. Equation (13) permits is of the reactive power variables during the network pricing. The two models for sensitivity matrices (marginal line flows) to be used for PoC tariff computation (LAC-PoC Model) are proposed as follows.

3.4.1 *Constant reactive power specification load model*

In this case the reactive power withdrawal is kept constant, i.e., $\Delta Q_L = 0$. From equation (13), the incremental line-power flow vector is given by

$$\underset{n_l\times 1}{[\Delta \boldsymbol{P_{line}}]} = \underset{n_l\times n}{[\mathbf{S_p}]}\underset{n\times 1}{[\Delta \boldsymbol{P_G} - \Delta \boldsymbol{P_L}]}. \tag{14}$$

Note that the matrix $[\mathbf{S_p}]$ is different from that used for PoC_1 formulation with the DC power flow model reported in [17]. The PoC tariff computed by this model is designated as PoC_2.

3.4.2 *Constant power factor specification load model*

It is evident from equation (13) that the active line-power flows are also dependent on the reactive power withdrawals at the load buses. $[\mathbf{S_q}]$ gives the linear relation between the active line-power flows and the reactive power withdrawals. Thus, it also captures the effect of the power factor at load buses on the PoC tariffs. Including the perturbation in reactive power with the constant power factor at the load buses, the incremental line-power flow vector is given by the following equation

$$\underset{n_l\times 1}{[\boldsymbol{\Delta P_{line}}]} = [\boldsymbol{S_p}][\boldsymbol{\Delta P_G}] + [\boldsymbol{S_p} + \boldsymbol{S_q}\cdot \boldsymbol{diag}(\tan(\boldsymbol{\phi_L}))][-\boldsymbol{\Delta P_L}]. \tag{15}$$

The PoC tariff computed by this model is designated as PoC_3.

3.5 *Overview of the proposed LAC-PoC approach*

The allocation methodology of the min-max fair MP method using LAC-PoC approach is depicted in figure 3. The line power flows are obtained using linearized AC power flow method. The ‘ extent of use’ and ‘ unused capacity’ costs are separated and allocated as shown in it.

The main steps of the proposed algorithm are as follows.

Step 1: Read network, injections and the cost data.

Step 2: Compute the line cost-rates c_{lm} on the capacity base (S_{lm}).

Step 3: Compute the line-power flows $P_{lm_{LAC}}$ using the LAC framework.

Step 4: Compute the ‘extent of use’ cost $NetEOUCost_{LAC}$ of the network using the LAC framework as follows.

$$NetEOUCost_{LAC} = \sum_{\forall lm} c_{lm} P_{lm_{LAC}} \tag{16}$$

Step 5: Using the model given by Equations (14) for the proposed tariff PoC_2 and (15) for the proposed tariff PoC_3, compute the sensitivities of the line flows to injection/withdrawals of the entities at the network buses.

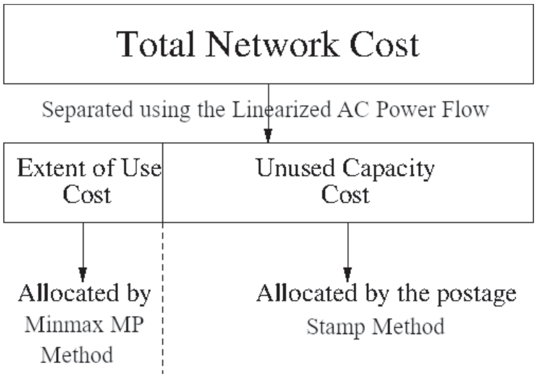

Figure 3. Allocation methodology for proposed PoC_2 and PoC_3.

Step 6: Allocate the ‘extent of use’ cost of the network obtained in equation (16) using min-max fairness algorithm as given in the feature number 2 in section 2.1. The usage component of PoC tariff computed for $NetEOUCost_{LAC}$ for entity E_i is represented as $\underline{pLAC}^*_{E_i}$.

Step 7: Calculate the residual cost RC_{LAC} as

$$RC_{LAC} = (TotNetCost - NetEOUCost_{LAC}). \tag{17}$$

Note that $RC_{LAC} \geq 0$. Use the pro-rata method to allocate RC_{LAC}. Let the constant postage stamp rate for allocating RC_{LAC} be K_{LAC}.

$$K_{LAC} = \frac{RC_{LAC}}{TotNetMW} \tag{18}$$

where, $TotNetMW$ is the total of network injections and withdrawals in MW.

Step 8: Evaluate the min-max fair composite PoC tariff, $pLAC^*_{E_i}$, where $E_i \in \{S_E\}$ is an entity in the set of entities $\{S_E\}$ as

$$pLAC^*_{E_i} = \underline{pLAC}^*_{E_i} + K_{LAC} \tag{19}$$

4. Proposed Integrated approach with the linearized AC network (LAC) and reliability models

The PoC tariffs proposed until now are as follows.

1. PoC_2 - min-max fair composite PoC tariff with the proposed constant reactive power load model, and
2. PoC_3 - min-max fair composite PoC tariff with the proposed constant power factor load model.

Next, integration of the reactive power variables and reliability cost component is done. The final proposed integrated PoC tariffs are designated as follows.

1. $PoC_{2_{REL}}$ - min-max fair composite PoC tariff with the proposed constant reactive power load model and the reliability cost is
2. $PoC_{3_{REL}}$ - min-max fair composite PoC tariff with the proposed constant power factor load model and the reliability cost is.

An integrated approach as given below can be used for computing both $PoC_{2_{REL}}$ and $PoC_{3_{REL}}$. The only difference between them arises in step 5 of the approach. The modelling of the reliability cost component is an add-on feature to the basic LAC-PoC model of PoC_2 and PoC_3. In the PoC_2 tariff mechanism, the reliability cost was masked with the unused capacity cost. However, it can be estimated using the N-1 contingency studies for the network. For a line, the maximum power flowing through it for the outage of all other lines (one by one) in the network can be computed using the power flow studies or using the line outage distribution factor (LODF) method as per [12, 41]. The difference between the maximum power, thus found, and the base case line flow in that line is the reliability capacity of that line. The segregation of transmission system cost is outlined by a block diagram given in figure 4. The line power flows are obtained using the linearized AC power flow method. The ‘ extent of use’ and ‘ unused capacity’ costs are separated. The reliability cost component is further demarcated from ‘ unused capacity cost’ and is monetized using line cost rate as shown in figure 4. The combined cost of network’s ‘ extent of use’ and reliability costs are allocated using a min-max fair algorithm and the residual cost is allocated pro-rata.

The step by step procedure for the proposed method is as follows.

Step 1: Input the data of the network configuration, load/generation capacities, and line cost.

Step 2: Compute the capacity based line cost-rates c_{lm}.

Step 3: Determine the power flows $P_{lm_{LAC}}$ on the lines.

Step 4: Evaluate $NetEOUCost_{LAC}$, the ‘ extent of use’ cost of the network, using the LAC model as follows.

$$NetEOUCost_{LAC} = \sum_{\forall lm} c_{lm} P_{lm_{LAC}} \qquad (20)$$

Step 5: Based upon the LAC power flow model, work out the marginal line-power flows resulting from the injection/withdrawal by the generation/load entities at the buses of the network using the equations (14) and (15).

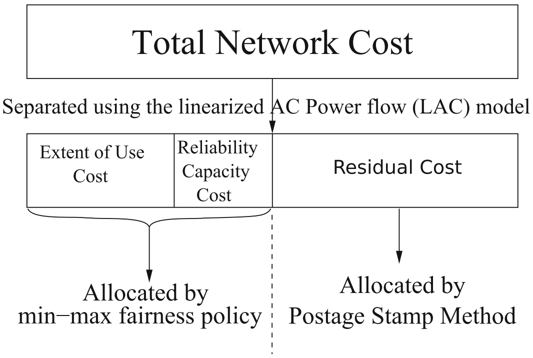

Figure 4. Segregation of the cost for the proposed $PoC2_{REL}$ and $PoC3_{REL}$.

Step 6: The new power flow $P_{LAC\,lm,rs}$ is computed by adding the surplus line-power flow due to an outage between buses r and s, as explained in [12].

$$P_{LAC\,lm,rs} = P_{lm_{LAC}} + \Delta P_{LAC\,lm,rs} \qquad (21)$$

Likewise, there can be a total of n_k such contingencies.

Step 7: For the line lm, the maximum line-power flow considering all the other contingencies will give the line capacity ($P_{REL_{lm_{LAC}}}$), required from the reliability perspective. The extra capacity for ensuring the reliability of the network through the line lm is computed as,

$$P_{REL_{lm_{LAC}}} = \max_{k\in\{\mathcal{C}\}}((|P^{k}_{lm_{LAC}}| - |P_{lm_{LAC}}|), 0). \qquad (22)$$

Step 8: The network’s reliability capacity cost is calculated as

$$NetRelCapCost_{LAC} = \sum_{\forall lm} c_{lm} P_{REL_{lm_{LAC}}} \qquad (23)$$

Step 9: Perform summation of the network’s reliability capacity cost ($NetRelCapCost_{LAC}$) and the network’s ‘ extent of use’ cost ($NetEOUCost_{LAC}$) to obtain the network’s combined cost ($NetCombCost_{LAC}$) as follows:

$$\begin{aligned} NetCombCost_{LAC} &= NetEOUCost_{LAC} \\ &+ NetRelCapCost_{LAC} \end{aligned} \qquad (24)$$

Allocate $NetCombCost_{LAC}$ using the composite min-max fair MP approach described in feature number 2 in section 2.1. Let $\underline{pINT}_{E_i}$ be the PoC tariff component for the allocation of $NetCombCost_{LAC}$ for entity E_i.

Step 10: The residual cost RC_{INT} that is to be recovered is given by

$$RC_{INT} = (TotNetCost - NetCombCost_{LAC}) \qquad (25)$$

where, $RC_{INT} \geq 0$. It is to be allocated on pro-rata basis. Let K_{INT} be the constant postage stamp rate for allocating RC_{INT}. Then,

$$K_{INT} = \frac{RC_{INT}}{TotNetMW}, \qquad (26)$$

Step 11: The proposed integrated (min-max fair MP with reliability cost segregation and linearized AC network model) PoC tariff can be obtained as

$$pINT_{E_i} = \underline{pINT}_{E_i} + K_{INT}. \qquad (27)$$

The first component $\underline{pINT}_{E_i}$ is obtained by using the min-max algorithm to allocate $NetCombCost_{LAC}$ as given in the feature number 2 in section 2.1.

The PoC tariffs obtained as above are designated as $PoC_{2_{REL}}$ for the constant reactive power load model and $PoC_{3_{REL}}$ for the constant power factor load model.

5. Results

The proposed integrated method using the linearized AC framework (LAC-PoC) has been programmed in *MATLAB*. The linear programming is performed using a version 6.0.4 of *Gurobi*.

5.1 *Improvement in the accuracy due to the improved network is for the existing MP methods*

The improvements of the proposed LAC-PoC modelling in the network can potentially also improve the accuracy of other variants of the MP method. The comparison of the PoC tariffs for the four IEEE test-cases (14, 30, 57, and 118-bus) obtained with the improved network sensitivities using LAC-PoC vis-a-vis conventional DC power flow network model is presented for the following methods:

1. MP Original (Basic) method [9],
2. MP Pro-rata method [11], and
3. EBE method [5].

The PoC tariffs obtained from these methods implemented with the marginal line flows obtained using a) DC power flow model, b) constant reactive power load model, and c) constant power factor load model. The corresponding PoC tariffs are designated as PoC_a, PoC_b and PoC_c respectively. The summary of the maximum percentage differences is given in table 6. The maximum value of the percentage differences among the four test cases is highlighted in bold.

Table 6 (columns 2 to 4) demonstrate the improvement in accuracy after using the LAC-PoC based constant reactive power model for the four IEEE networks. The model accuracy improvement can impact the PoC tariffs up to 14.5%. Columns $5-7$ of table 6 show the corresponding improvement in the accuracy with the constant power factor model. The impact on the accuracy is up to 13.5% for the IEEE 57-bus test system. It can be concluded that the proposed network is improvements are essential and can impact the accuracy across the spectrum of the cost allocation methods.

5.2 *Accuracy improvement due to LAC-PoC approach for existing min-max fair methods*

The PoC tariff PoC_1 given in [17] has been compared with various existing methods and proved to be better at getting the lowest maximum PoC tariff, the lowest variance among PoC tariffs, rigour in fairness, and no negative PoC tariffs even-though the counter flows are retained. In this new formulation of PoC_2 and PoC_3, all other features except the linearized network model (LAC) are kept same as for PoC_1 proposed in [17]. Hence, the nodal price PoC_1 is taken as a reference for quantifying the improvements due to a more accurate network model in the LAC framework. Testing and validation are done in the case studies of IEEE 14, 30, 57, and 118-bus networks. The PoC tariff has the two cost components, namely, 1) the 'extent of use' and 2) the

Table 7. Comparison of PoC_1, PoC_2 and PoC_3 tariffs for IEEE networks.

IEEE network	Max. of $\lvert\%Diff_{2-1}\rvert$	Max. of $\lvert\%Diff_{3-1}\rvert$	Max. of $\lvert\%Diff_{3-2}\rvert$
14-bus	**9.66**	**6.44**	**2.93**
30-bus	4.9	2.9	1.9
57-bus	5.71	5.52	1.93
118-bus	3.48	3.46	0.13

Table 6. Summary of the maximum percentage difference between the PoC tariffs - PoC_b and PoC_c with PoC_a as reference for MP Basic, MP Prorata and EBE methods.

IEEE Network	Maximum of $\lvert\%\text{Diff}_{b-a}\rvert$			Maximum of $\lvert\%\text{Diff}_{c-a}\rvert$		
	MP Basic	MP Prorata	EBE	MP Basic	MP Prorata	EBE
14-bus	5.21	3.34	3.12	5.47	1.83	4.05
30-bus	3.4	2.98	3.33	3.34	2.81	2.37
57-bus	**13.45**	**11.66**	**14.54**	**12.8**	**11.31**	**13.55**
118-bus	4.25	1.7	1.3	4.2	1.8	1.25

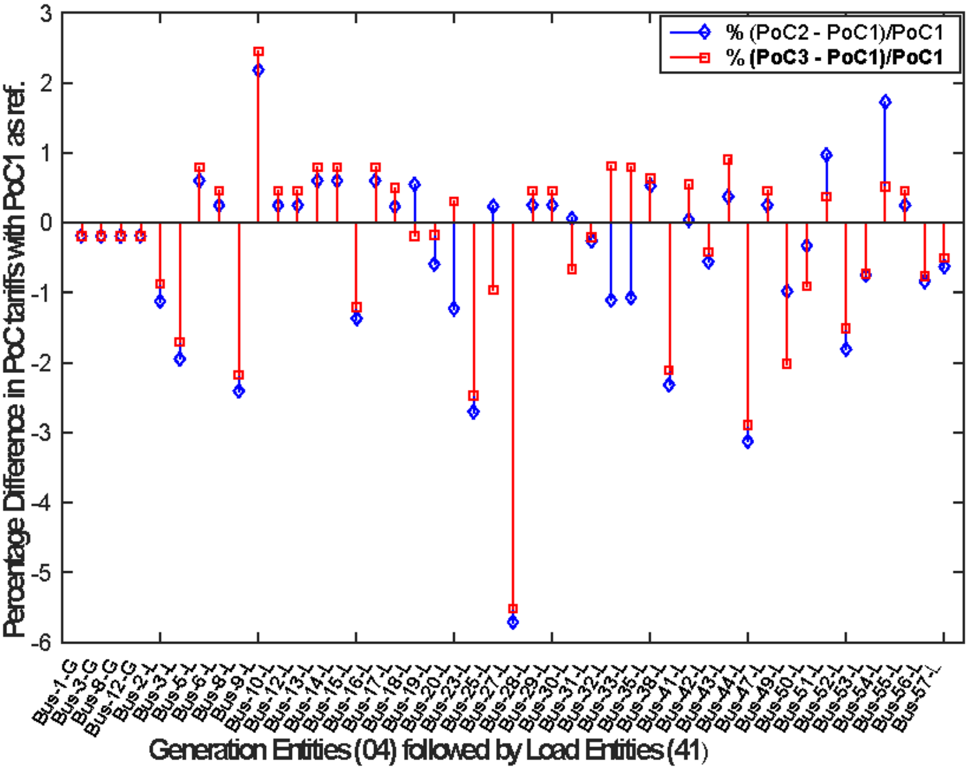

Figure 5. Percentage difference of tariffs PoC_2 and PoC_3 with PoC_1 as the reference for IEEE 57-bus system.

unused capacity. As the accuracy improvement affects the usage component, an exclusive comparison is presented for the 'extent of use' component.

The maximum of absolute values of the percentage difference in PoC tariff values for PoC_2 and PoC_3 with tariff PoC_1 as reference ($|\%Diff_{2-1}|$ and $|\%Diff_{3-1}|$) are tabulated for the four IEEE test cases in table 7 (see columns 2 and 3). The highest values are highlighted in bold. It shows that the price variation for some entities could be as much as 10%. This brings out the importance of is voltage deviations from unity.

For the sake of brevity, a stem chart showing the percentage difference between PoC_2 and PoC_1 (with PoC_1 as reference) and also percentage difference between PoC_3 and PoC_1 (with PoC_1 as reference) for the IEEE 57-bus network cases is shown in figure 5. It can be observed that there is a substantial difference in PoC tariffs obtained using the same method with the proposed linearized AC power flow models as compared to the same method with the DC power flow model.

5.3 *Comparison of the two reactive power models*

The maximum of absolute values of the percentage difference in the PoC tariff values for PoC_3 with PoC_2 as reference ($|\%Diff_{3-2}|$) for various test systems is tabulated in column 4 of table 7. The highest values are highlighted in bold. In the IEEE 14-bus network, the maximum impact is of the order of 3%, while for the IEEE 57-bus network, it is only 2%. Thus, in practice, any of the two models can be

Table 8. Comparison of the Minimum PoC Tariffs of the existing methods with the proposed PoC_2 and proposed integrated $PoC_{2_{REL}}$ for IEEE Networks.

IEEE Network	MP Minmax as per [16]	EBE as per [5]	Proposed LAC-PoC PoC_2	Proposed Integrated $PoC_{2_{REL}}$
14	− 15.31	0.99	7.34	6.76
30	− 32.54	0.83	4.12	3.41
57	− 193.54	3.37	7.77	6.98
118	− 3.87	2.12	3.17	2.89

Table 9. Comparison of the Maximum PoC Tariffs of the existing methods with the proposed PoC_2 and proposed integrated $PoC_{2_{REL}}$ for IEEE Networks.

IEEE Network	MP Minmax as per [16]	EBE as per [5]	Proposed LAC-PoC PoC_2	Proposed Integrated $PoC_{2_{REL}}$
14	48.77	32.35	13.02	13.92
30	52.46	31.22	13.32	15.04
57	395.89	130.92	43.62	55.81
118	42.77	64.65	4.92	5.25

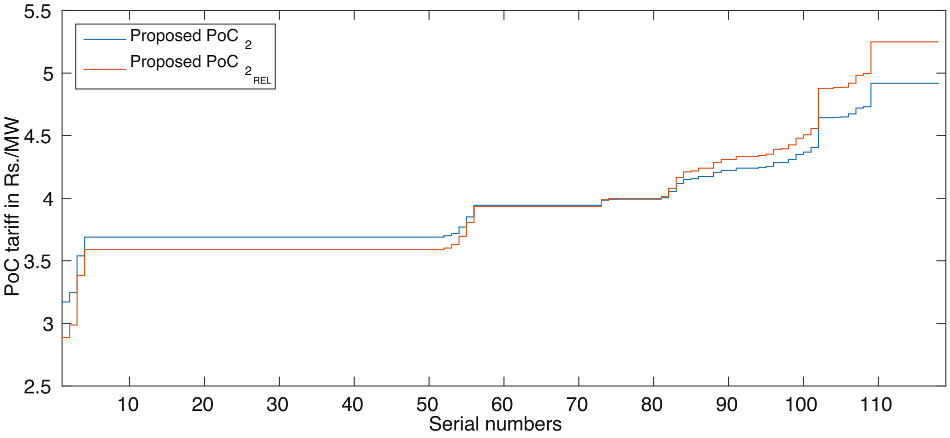

Figure 6. Comparison of the proposed tariffs PoC_2 and $PoC_{2_{REL}}$ for the IEEE 118-bus system.

used. It is seen that it is more important to model sensitivities in the LAC-PoC framework where voltage deviations from 1 p.u. are recognized. Which of the two reactive power models is to be used is of secondary importance and also system dependent.

5.4 *Comparison of the LAC-PoC version PoC_2 and integrated $PoC_{2_{REL}}$ tariffs with existing methods*

The proposed composite min-max fair MP LAC-PoC method PoC_2 and the integrated method $PoC_{2_{REL}}$ are compared with the two existing methods in the literature in tables 8 and 9. It can be observed from table 8 that the advantages like the inclusion of counter flows with no resulting negative PoC values as explained in the section 2.1 is preserved in the proposed PoC_2. In table 9, it can be observed that the feature of lowest maximum PoC tariff as compared to existing methods as described in the section 2.1 is continued in the proposed PoC_2. The proposed PoC_2 has the new added feature of improved accuracy. In an integrated approach of $PoC_{2_{REL}}$, the feature of cost-causality is enhanced by further separating the unused capacity cost into the reliability cost component and the residual cost.

For the sake of brevity, only the result of an IEEE test case of 118 buses is demonstrated graphically. Figure 6 shows the tariffs PoC_2 and $PoC_{2_{REL}}$ in ascending order to observe the clustering effect for a 118-bus case. The clustering of PoC tariffs is observed due to the min-max fairness policy for dispersed slack bus selection. This will help in promoting the usage of the lowest marginal cost resources in the system. As seen in these figures, the minimum PoC tariff decreases while the maximum PoC tariff increases when the reliability component is modeled in the $PoC_{2_{REL}}$ tariff. The entities having a lower contribution in reliability cost component get benefited by a reduction in their 'extent of use' component. Their PoC tariffs get reduced. The entities having more contribution in a reliability cost component will have higher 'extent of use' increasing their PoC tariffs and hence the maximum POC tariff increases in $PoC_{2_{REL}}$.

6. Conclusions

The gaps identified in the existing MP methods in the literature are mitigated by an integrated approach. Real-life transmission systems have different voltage levels and hence the x/r ratio of the transmission lines will vary. Under this situation, the linearized AC power flow (LAC) model is more accurate than the DC power flow model and therefore should be preferred. The marginal line-power flows change considerably when the line resistance and the deviation in voltages from 1 p.u. at load buses are modeled. Moreover, with the LAC model, the linearity is retained and this permits superposition, used in the marginal participation method. Thus, the linearized AC network model renders better accuracy without sacrificing linearity. The two new LAC-PoC tariff formulations are derived, 1) with constant reactive power at load buses and 2) with constant power factor at load buses. The proposed PoC tariffs PoC_2 and PoC_3 calculated using the two LAC-PoC formulations can differ significantly at some buses from PoC_1 which uses the DC power flow model. The genericness and versatility of the proposed reactive power modelling have also been demonstrated for the other variants of the MP method. The final PoC tariff, $PoC_{2_{REL}}$ and $PoC_{3_{REL}}$ integrate the improvements in the MP approach, i.e., 1) improved economic efficiency and congestion management by incentivizing the counter-flows, 2) Implementation of min-max fairness policy, 3) segregation of the 'extent of use' costs, reliability costs and residual costs, and 4) modelling of reactive power variables. The advantages like better accuracy, more cost-causality, lowest maximum PoC tariffs (regret), and no Payoffs are achieved. The comparative evaluation of a proposed integrated method with the existing methods on the IEEE 14-bus, 30-bus, 57-bus, and 118-bus test systems supports the benefits claimed.

List of symbols

ΔV_l	Deviation in voltage magnitude from 1 p.u. at bus *l*
ΔV_m	Deviation in voltage magnitude from 1 p.u. at bus *m*
δ_l	Phase angle of voltage at bus *l*
δ_m	Phase angle of voltage at bus *m*
$\mathbf{P_G}$	Vector of active power injection at buses
$\mathbf{P_L}$	Vector of active power withdrawal at buses
$\mathbf{P_{line}}$	Active line-power flow vector
$\mathbf{Q_L}$	Vector of reactive power withdrawal at load buses
$\mathbf{Q_{line}}$	Reactive line-power flow vector
$\mathbf{S_P}$	Sensitivity matrix for active line-power flows
$\mathbf{S_p}$	Sensitivity matrix for active line-power flows for change in active power injection at all buses
$\mathbf{S_Q}$	Sensitivity matrix for reactive line-power flows
$\mathbf{S_q}$	Sensitivity matrix for active line-power flows for change in reactive power injection at (P-Q) buses
$\mathbf{S}$	Sensitivity matrix for active and reactive line-power flows
Ω_i	The set of buses adjacent to bus *i*
ϕ_L	The angle between the voltage and current at the load busses
b_{lm0}	Shunt susceptance (as per π model) - half of total shunt susceptance of line *lm*, a positive value
b_{lm}	Series susceptance of line *lm*, a negative value
b_{sh_i}	Net shunt susceptance at bus *i*, a positive value for shunt capacitor and a negative value for shunt reactor
c_{lm}	Capacity based cost-rate for line *lm*
g_{lm}	Series conductance of line *lm*, a positive value
M_k	Set of entities whose price have been fixed in k^{th} LP $M_0 = \{\phi\}$
n	Number of buses
n	Number of the busses in the system
n_D	$(2 \times n_G \times n_L + 1)$
n_E	Total number of entities $(n_L + n_G)$
n_G	Number of the generator busses in the system
n_L	Number of the load busses in the system
n_l	Number of lines
$NetCombCost_{LAC}$	Network combined cost for ‘extent of use’ and reliability capacity using LAC framework
$NetEOUCost_{LAC}$	‘Extent of use’ cost of the network using LAC framework
$NetRelCapCost_{LAC}$	Network reliability capacity cost obtained using LAC framework
P_i^{inj}	Active power injected at bus *i*
$P_{lm_{LAC}}$	Active power flow in line *lm* using LAC framework
P_{lm}	Active power flow on line from bus *l* to bus *m* using DC power flow
$P_{REL_{lm_{LAC}}}$	Reliability capacity in line *lm* using LAC framework
Q_i^{inj}	Reactive power injected at bus *i*
$Q_{lm_{LAC}}$	Reactive power flow on line from bus *l* to bus *m* using LAC framework
RC_{INT}	Residual capacity cost with reliability is in LAC framework
RC_{LAC}	Residual capacity cost in LAC framework
S_k	$S_k = S_{k-1} \setminus M_k \ (S_0 = \{N\})$
$TotNetCost$	Total network cost to be recovered
$TotNetMW$	Total network MW capacity of generator injection and load withdrawals of all participating entities
V_l	Bus voltage magnitude in p.u. at bus *l*
V_m	Bus voltage magnitude in p.u. at bus *m*
z_k^*	Optimal value of z_k
z_k	Dummy variable in k^{th} LP problem
$\mathbf{d}$	Vector of unknown variables in **LP** $\mathbf{d} \in R_+^{n_D}$
EOU	Extent of use
LAC	Linearized AC power flow
LP	Linear programming

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