Sādhanā (2021)46:104
https://doi.org/10.1007/s12046-021-01632-1

Optimal design of FOPID Controller for the control of CSTR by using a novel hybrid metaheuristic algorithm

NEHA KHANDUJA* and BHARAT BHUSHAN

Department of Electrical Engineering, Delhi Technological University, Delhi, India
e-mail: nehakhanduja.dce@gmail.com

MS received 20 October 2020; revised 2 April 2021; accepted 12 April 2021

Abstract. The escalating complexity in the process control industry emanates the demand for novel and advanced control techniques, which results in enhanced performance indices. A hybrid optimal control method i.e., FOPID control using chaotic state of matter search with elite opposition-based learning for controlling CSTR is proposed in this paper. Fractional order PID is a generalized form of PID Controller. It uses fractional calculus, resulting in a more flexible and better response accompanying rigorous adoption for substantially closed-loop system stability. Hybridization of SMS with chaotic maps and elite oppositional-based learning results in enhanced exploration capability along with randomization. In this paper, the results show that the CSMSEOBL tuned FOPID controller provides superior and optimum performance when compared to other metaheuristic algorithms.

Keywords. State of matter search algorithms; chaotic maps; elite opposition based learning; continuously stirred tank reactor.

1. Introduction

A literature review of process control depicts that in contempt of many new advanced, adaptive, and optimal control methodologies, the use of PID controllers has been stagnated, especially in the areas where reference tracking and disturbance rejection are the major tasks. Some key features which make PID popular are robust performance, self-explanatory, diversified application areas, simple implementation validation, and many more [1]. Although a simple PID controller provides the least impenetrable, most productive, and effortless tuning of controller parameters for the practical process. But with advantages, PID controllers have limitations also like the less optimal solution for a system loaded with non-linearity, time delay, high order disturbances, noise, etc. These limitations lead to introducing new and advanced tuning methods like Fuzzy Logic, Neural Network, Adaptive Control, Internal Model Control, etc. which ameliorate the capability and performance of the traditional PID controller [2] along with enhanced flexibility of conventional PID controller.

FOPID as an alternative can be adopted with five parameters to tune, whereas a conventional PID Controller has only three parameters. Although it increases the complexity of parameter tuning to some extent at the same time, resulting in comparatively fine-tuning [3]. FOPID is an advanced form of PID controller which is proposed by Podlubny [4] as $PI^{\lambda}D^{\mu}$ controller, where λ and μ are non-integer order of integral and differential term, respectively. A literature review shows that FOPID gives better performance as compared to conventional PID Controller [5]. In continuation of this, presently many metaheuristic algorithms are in great demand for control tuning parameters of the PID controller [6]. The rising complexities in the research area result in limiting the mathematical methods of finding optimal solutions and this necessity results in the investigation of metaheuristic optimization algorithms. Major limitations with traditional methods of optimization are time-consuming, tedious, less efficient, and less accurate [7]. The imperative feature of the metaheuristic algorithm which makes it prominent among researchers is its adaptability and versatility. It can adapt to the problem and determine the optimal solution of different types of problems, whether it is related to mathematics, engineering, process industry, etc. [8]. Other features of the metaheuristic algorithm which makes it popular are:

- Can be easily integrated with the already existing implementation
- Wide applicability area
- Gradient information is not required.
- Decision-making is easy [9].

A complex problem can be solved in a reasonable time and may give an acceptable solution by using metaheuristic which is based on trial and error. The main objective is to produce an attainable solution in a reasonable time frame.

*For correspondence
Published online: 21 May 2021

Whenever a metaheuristic is chosen for a problem, it never guarantees the best solution and even, we are not known if it will give the optimal solution or not. The main point of selecting an algorithm is to give acceptable or accurate solutions most of the time with minimum deviation. Exploration (diversification) and exploitation (intensification) are two key components of any metaheuristic algorithm with exploration searches in undetected areas while exploitation searches in other promising territories in the sample space. Therefore, the success of an algorithm depends on a good balance between exploration and exploitation which leads to the assurance of convergence to optimality [10].

Most of the chemical processes such as continuous stirred tank reactor (CSTR), biochemical reactor, and conical tank systems persist dynamic and highly nonlinear behaviour as they consist of multiple process variables to be manipulated. Many advanced controlling and optimization methods are proposed to control such types of MIMO (Multi-input Multi-output) systems. Extensive Literature review shows that evolutionary techniques like PSO (Particle Swarm Optimization) [11, 12], IWO (Invasive Weed Optimization) [13], FS (Stochastic Fractal Search) [14] FFA (Firefly Algorithm) [15], GWO (Grey Wolf Optimizer) [16], CSO (Cat Swarm Optimization) [17] TLBO (Teacher-Learner based Optimization) [18, 19], SMS (State of Matter Search) [20], CKH (Chaotic Krill Herd) [21], RDO (Red Deer Optimization Algorithm) [22], SOA (Sailfish Optimization Algorithm) [23], and many more have proved their superiority as compared to traditional controllers like Z-N tuned PID, refined Ziegler-Nichols rule [24], intelligent controllers Fuzzy-PID [25], Neural-PID [26], Model-based controllers MRAC (Model reference adaptive control) [27] and Internal model control (IMC) [28].

The proposed methodology is used for concentration and temperature control of continuously stirred tank reactors (CSTR). A vast literature is available for controlling methodologies of CSTR but as it is highly nonlinear and its complex dynamics properties make it a complex problem. Therefore, it is a tedious task to control CSTR by the conventional controller [29]. Nowadays optimization-based control is preferred over the conventional or intelligent controller and to achieve this a hybrid CSMSEOBL methodology is proposed. It is a modified form of SMS algorithm (state of matter search) in which, Chaotic Maps and Elite opposition-based learning (EOBL) are embedded with SMS to enhance the efficiency and efficacy of the SMS algorithm. The basic principle of the SMS algorithm lies in the heart of the thermal energy motion system. The whole algorithm is divided into three states of matter solid, liquid, and gas and each state persist of a different diversification-intensification ratio. The algorithm starts with the gas state and modifying the diversification-intensification ratio and ends at a solid state [20]. The chaotic concept is used for the systems which have high sensitivity towards the initial condition, and also it increases the randomness because the range of random numbers is limited. The chaotic theory has been used with many evolutionary algorithms like PSO, Krill herd, BFO, etc. [21]. This concept of chaotic SMS algorithms is used to define some random variables to stimulate the convergence of SMS. Further, chaotic SMS is merged with elite oppositional-based learning. The concept of OBL was introduced by Tizhoosh in 2005 which increases the exploration capability of the existing algorithm by combining two main properties of OBL which are a global search and good convergence rate [2]. EOBL is the superior form of OBL which gives better global search and a higher convergence rate [30]. A fractional-order PID control of CSTR using a hybrid metaheuristic algorithm CSMSEOBL is implemented on MATLAB and results obtained from this hybrid algorithm prove the excellence of the proposed methodology.

The rest of the paper is organized as follows. Section 2 describes a non-linear problem of CSTR, section 3 elaborates the FOPID Controller, the considered Metaheuristic optimization techniques have been described in section 4. Results and discussion are illustrated in section 5, and the conclusions along with future scope are detailed in section 6.

2. Continuously stirred tank reactor (CSTR)

A Continuous Stirred Tank Reactor (CSTR) is one of the most significant unit tasks in the Chemical process industries. It shows a profoundly nonlinear nature and for the most part, has wide working ranges. Chemical responses in a reactor are either exothermic or endothermic and consequently necessitate that heat either be evacuated or added to the reactor to keep up a steady temperature [31]. A jacket encompassing the reactor additionally has fed and leaves streams. The jacket is thought to be entirely blended and at a lower temperature than the reactor, energy at that point goes through the reactor walls into the jacket, to evacuate the heat produced by the chemical reaction. Consider for uniform volume, exact blending, and uniform values of the parameter inside the reactor, the mass-energy balance condition is given by

$$f_1(C_A, T) = \frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - r \tag{1}$$

$$f_2(C_A, T) = \frac{dT}{dt} = \frac{F}{V}(T_f - T) + \left(\frac{-\Delta H}{\rho C_p}\right)r - \frac{UA}{V\rho C_p}(T - T_j) \tag{2}$$

where C_A stands for concentration, r stands for Arrhenius expression for a chemical reaction is given by

$$r = k_0 \exp\left(\frac{-\Delta E}{RT}\right) C_A$$

Figure 1 shows an irreversible and exothermic compound response that happens inside the reactor where a solitary coolant stream cools a consistent volume reactor [32].

The description of CSTR parameters is given in table 1 [33]. The main objective is to control the reactor temperature and concentration by controlling the cooling rate.

3. FOPID controller

The most common form of PID controller combines three kinds of corrective measures to the error signal, which is the representation of closeness or distance of the desired output from the actual one. In general, these three corrective measures are termed proportional, integral, and derivative. The general form of a PID Controller is given by [34].

$$u(t) = k_P e(t) + \frac{1}{k_i} \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt}) \tag{3}$$

Podlubny [35] proposed FOPID Controller in 1999 as an extended form of PID controller which has a comparatively wider range for controlling. The FOPID Controller is shown in figure 2 and represented a

$$u(t) = k_p e(t) + k_I D^{-\gamma} e(t) + k_D D^{\mu} e(t) \tag{4}$$

where γ and μ are real numbers with $\gamma > 0, \mu > 0$ [5], D is a fractional calculus operator which is defined by Riemann–Liouville as (n is general non-integer order and $\Gamma(n)$ is Euler's gamma function)

$$D^{-n} f(t) = \frac{1}{\Gamma(n)} \int_0^t f(y)(t-y)^{n-1} dy \tag{5}$$

The FOPID controller also takes current error, accumulated error, and predicted error into account same as classical PID controller but fractional operators are non-local in FOPID which gives a modified definition to the integral as well as derivative action [34]. For the analysis purpose,

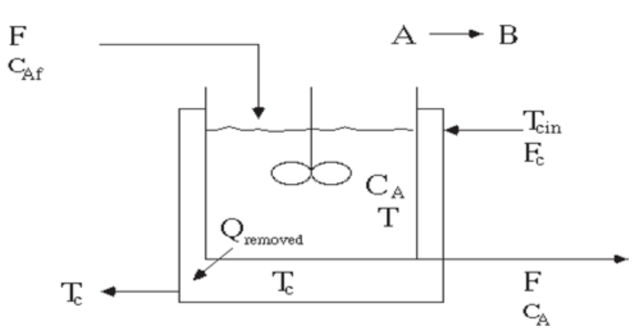

Figure 1. Schematic representation of Jacketed CSTR system.

fractional calculus equations must be transferred into algebraic equations. The Laplace transform of the equation for $D^{-n} f(t)$ can be expressed as

$$\int_0^\infty e^{-st} f(t) dt = s^{\alpha} F(s) \tag{6}$$

Here, it is assumed that all initial conditions are zero [36]. Case I, if $\gamma = 1$ and $\mu = 1$ results in PID controller. Case II, if $\gamma = 1$ and $\mu = 0$, results in PI Controller. Case III, if $\gamma = 0$ and $\mu = 1$, resulting in PD controller. Case IV, if $\gamma = 0$ and $\mu = 0$ results in gain controller only. The transfer function of FOPID Control [37] is represented as,

$$G_C(S) = \frac{U(S)}{E(S)} \left(k_p + k_I \frac{1}{S^{\gamma}} + k_D S^{\mu}\right) \tag{7}$$

Use of FOPID Controller results not only in enhanced performance of the control system, better adaptability but fine control of the dynamical system as well as very fewer variations in parameters of a control system [38].

- Fractional-order linear matrix diversity framework
- The powerful interim check technique
- Fractional-order Lyapunov disparity technique [39].

4. Metaheuristic optimization algorithms

Figure 3 gives the flow of the proposed work with parameters of the FOPID Controller which are optimized by the metaheuristic optimization and controlled parameters are fed into the process.

4.1 *Particle swarm optimization (PSO)*

PSO is a swarm intelligence-based optimization algorithm that was proposed by Kennedy and Eberhart in 1995. It simulates the concept of cooperation, communication, and social behavior in fish and bird schooling. Literature [40] reveals that extensive research has been done on PSO to demonstrate its efficiency in solving real-valued complex, non-linear, non-differentiable optimization problems. However, since the search space dimension can be sufficiently increased, PSO is sensitive to the trend of falling into local optima. To solve this limitation with traditional PSO some improved and hybridized version of PSO has been introduced from time to time to enhance its convergence performance [41]. It is a population-based optimization technique that gives rise to high-quality results within a more concise time and shows stable converge characteristics [33].

PSO is an iterative process. On each iteration of the PSO's main processing loop, each particle's current velocity is first updated based on the particle's current

Table 1. CSTR Parameters.

Reactor Parameter	Description	Values
F/V(hr-1)	Flow rate*reactor volume of the tank	1
K_o (hr-1)	Exponential factor	$10e^{15}$
-ΔH (kcal/kmol)	Heat of reaction	6000
E(kcal/kmol)	Activation energy	12189
ρC_P (BTU/ ft^3)	Density*heat capacity	500
Tf (°K)	Feed temperature	315
C_{Af}(lbmol/ft^3)	The concentration of feed stream	1
$\frac{UA}{V}$	Overall heat transfer coefficient/reactor volume	1451
Tj(K)	Coolant Temperature	300

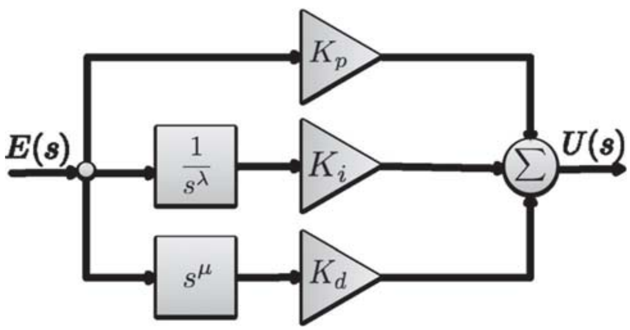

Figure 2. FOPID Controller.

velocity, the particle's local information, and global swarm information. Then, each particle's position is updated using the particle's new velocity. In math terms the two update equations are:

$$\boldsymbol{v}(\boldsymbol{t}+1) = (\boldsymbol{w} * \boldsymbol{v}(\boldsymbol{t})) + (\boldsymbol{c}_1 * \boldsymbol{r}_1 * (\boldsymbol{p}(\boldsymbol{t}) - \boldsymbol{x}(\boldsymbol{t})) + (\boldsymbol{c}_1 * \boldsymbol{r}_2 * (\boldsymbol{g}(\boldsymbol{t}) - \boldsymbol{x}(\boldsymbol{t})) \tag{8}$$

$$\boldsymbol{x}(\boldsymbol{t}+1) = \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{v}(\boldsymbol{t}+1) \tag{9}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are random numbers with a value between [0, 1], $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ are two acceleration constants, $\boldsymbol{w}$ is inertia weight,$\boldsymbol{x}(.)$ is the position of the particle, $\boldsymbol{p}(\boldsymbol{t})$ is the personal best position of the particle, $\boldsymbol{g}(\boldsymbol{t})$ is the global best position of the group. The term $\boldsymbol{v}(\boldsymbol{t}+1)$ means the velocity at time $(\mathbf{t}+1)$. Once the new velocity, $\boldsymbol{v}(\boldsymbol{t}+1)$, has been determined, it is used to compute the new particle position $\boldsymbol{x}(\boldsymbol{t}+1)$ [42].

4.2 *Cuckoo search algorithm (CS)*

Yang and Deb [43] developed novel meta-heuristic calculations cuckoo search in 2009. The CS depends on the brood parasitism of some cuckoo species. Moreover, the calculation is upgraded by the purported Lévy flights, as opposed to basic isotropic irregular strolls. Cuckoos are interesting flying creatures, not just as a result of the excellent sounds they can make yet additionally on account of their forceful generation methodology. A few animal types, for example, the ani and guira cuckoos lay their eggs in shared homes, however, they may evacuate others' eggs

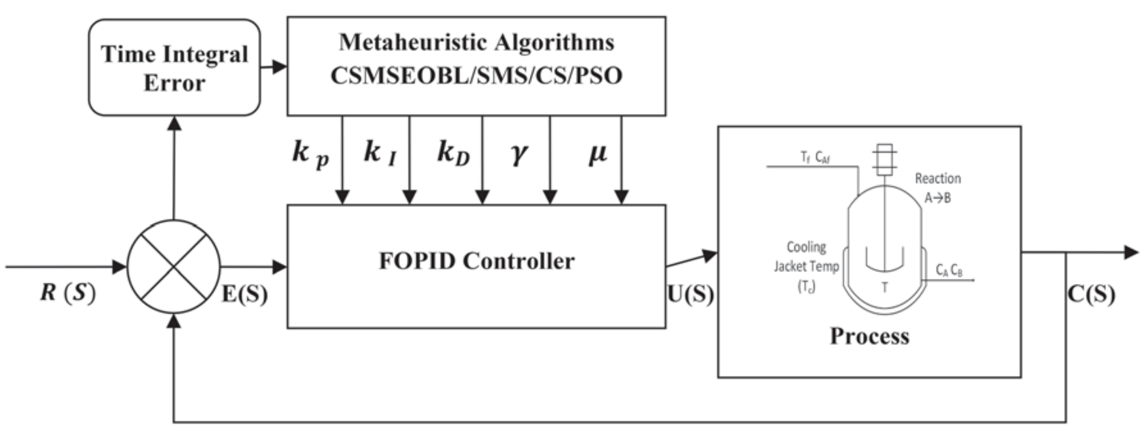

Figure 3. Optimized FOPID for Control of CSTR.

to expand the bring forth likelihood of their eggs. A lot of animal varieties connect with the committed brood parasitism by laying their eggs in the homes of other host winged animals [30]**.** In cuckoo search calculation cuckoo egg speaks to a potential answer for the structure issue which has an objective function. The calculation utilizes three glorified guidelines:

- Each cuckoo lays each egg in turn and dumps it in a haphazardly chosen home.
- The best home with great eggs will be extended to the next generation.
- The quantity of accessible host homes is fixed and a host winged animal can find an outsider egg with a probability of $P_a \in [0, 1]$ [44].

4.3 *State of matter search algorithm (SMS)*

SMS algorithm is a nature-inspired algorithm that lies in the category of evolutionary algorithm and can be used to solve MIMO type global optimization problems. It is based on a thermal energy motion mechanism. Three states of matter i.e., solid, liquid, and gas are simulated in this algorithm and each state has a different exploration-exploitation ratio. The algorithm begins with the gas state which is purely exploration, then after reforming the exploration and exploitation ratio it reaches a liquid state in which a moderate transition takes place between exploration and exploitation, and this reforming is continued till solid-state i.e., pure exploitation is reached. This entire process results in the enhancement of population diversity and simultaneously escapes the particles to concentrate within local minima [20]. The complete SMS Algorithm can be a divided into four stages:

Stage 1: Initialization state and general procedure:

- Find the best element from population $\boldsymbol{P}$

$$P^{best} \in \{P\} \middle| f\left(P^{best}\right) = \max\{f(P_1), f(P_2), \ldots\ldots\ldots f(P_{N_p})\} \tag{10}$$

- Calculate initial velocity magnitude

$$v_{st} = \frac{\sum_{j=1}^{n}(b_j^h - b_j^l)}{n} * \beta \tag{11}$$

where, b_j^h is the upper bound of j parameter, b_j^l is lower bound of j parameter, β is a factor ranging [0,1]

- Update the direction vector to control the movement of the particle

$$d_i^{k+1} = d_i^k\left(1 - \frac{k}{gen}\right)0.5 + a_i \tag{12}$$

$$a_i = \frac{(p_{best} - p_i)}{\|p_{best} - p_i\|}$$

where, a_i attraction unitary vector,p_{best} is the best molecule in population P, p_i is molecule i of population P, k is current iteration number; gen-total number of iterations.

- Calculate velocity,v_i of each molecule

$$v_i = d_i * v_{st} \tag{13}$$

- Calculate collision radius, r and $0 \le \alpha \le 1$

$$r = \frac{\sum_{j=1}^{n}(b_j^h - b_j^l)}{n} * \alpha \tag{14}$$

- Then update the Position of each molecule, which is given by (H is a threshold limit)

$$p_{i,j}^{k+1} = \begin{cases} p_{i,j}^k + v_{i,j} * rand(0,1) * \rho * (b_j^h - b_j^l); & if\ rand \le H \\ p_{i,j}^k; & if\ rand > H \end{cases} \tag{15}$$

Stage 2: Gas state

- Set the parameters for the gas state:$\boldsymbol{\rho} \in [\mathbf{0.8}, \mathbf{1}]$, $\boldsymbol{\beta} = \mathbf{0.8}$, $\boldsymbol{\alpha} = \mathbf{0.8}$ & $\boldsymbol{H} = \mathbf{0.9}$**.**
- Apply the general procedure as described in Stage 1.
- If the no. of iteration=**50%** of total no. of iterations then the process shifted to liquid state otherwise the general procedure is repeated.

Stage 3: Liquid State

- Set the parameters for the liquid state:$\boldsymbol{\rho} \in [\mathbf{0.3}, \mathbf{0.6}]$, $\boldsymbol{\beta} = \mathbf{0.4}$, $\boldsymbol{\alpha} = \mathbf{0.2}$ & $\boldsymbol{H} = \mathbf{0.2}$**.**
- Apply the general procedure as described in Stage 1.
- If no. of iteration=**90%** of total no. of iterations then process shifted to solid-state otherwise the general procedure is repeated.

Stage 4: Solid State

- Set the parameters for solid-state:$\boldsymbol{\rho} \in [\mathbf{0.0}, \mathbf{0.1}]$, $\boldsymbol{\beta} = \mathbf{0.1}$, $\boldsymbol{\alpha} = \mathbf{0}$ & $\boldsymbol{H} = \mathbf{0}$**.**
- Apply the general procedure as described in Stage 1.
- If the total no. of iteration=**100%** then the process is finished otherwise the general procedure is repeated.

4.4 *CSMS-EOBL algorithm*

A hybrid metaheuristic approach is used to enhance the balance between exploration and exploitation capability of the existing algorithm along with accelerated convergence rate. The benefits of all three algorithms are combined to form this hybrid algorithm. Chaotic Maps are used to calculate the random variable of the SMS algorithm and increase the exploitation capability. Further, the inclusion of EOBL enhances the exploration capability of the SMS Algorithm.

Table 2. Chaotic maps [45].

Name	Chaotic Map	Range
Chebyshev	$x_{k+1} = \cos(k\cos^{-1}(x_k))$	(0,1)
Circle	$x_{i+1} = mod\left(x_i + b - \left(\frac{a}{2\pi}\right)\sin(2\pi x_k), 1\right) a = 0.5, b = 0.2$	(0,1)
Gauss	$x_{i+1} = \begin{cases} 1, & x_i = 0 \\ \frac{1}{mod(x_i, 1)}, & otherwise \end{cases}$	(0,1)
Iterative	$x_{k+1} = \sin\left(\frac{a\pi}{x_k}\right), a \in (0.1)$	(0,1)
Logistic	$x_{i+1} = ax_i(1 - x_i), a = 4$	(0,1)
Piecewise	$\frac{x_k}{P}; 0 \le x_k < P \frac{x_{k-} - P}{0.5 - P};$ $P \le x_k < 0.5 \frac{1 - P - x_{k-}}{0.5 - P};$ $0.5 \le x_k < 1 - P \frac{1 - x_{k-}}{P};$ $1 - P \le x_k < 1$	(0,1)
Sine	$\frac{a}{4} sin(\pi x_k);\ 1 < a < 4$	(0,1)
Singer	$x_{i+1} = \mu(7.86x_i - 23.31x_i^2 + 28.75x_i^3 - 13.302875x_i^4)$ $\mu = 1.07$	(0,1)
Sinusoidal	$x_{i+1} = ax_i^2 \sin(\pi x_i), a = 2.3$	(0,1)
Tent	$x_{i+1} = \begin{cases} \frac{x_i}{0.7} & x_i < 0.7 \\ \frac{10(1 - x_i)}{3} & x_i \ge 7 \end{cases}$	(0,1)

4.4.1 *Chaotic theory and maps* Chaos is a deterministic concept that shows irregular motions and can be used in numerous applications like non-linear control, automobile, industrial applications, etc. It is a randomly based optimization algorithm that uses chaotic variables instead of random variables. The concept of chaos possesses three important properties of non-recurrence, randomness, and dynamic [21]. These features of chaos ensure that various solutions produced by the algorithm can search on the complex multimodal landscape at a higher speed with various movement patterns. Hundreds of metaheuristic algorithms have been designed to achieve a good balance between exploration and exploitation, according to the literature on metaheuristic algorithms. In this thread, chaotic theory or COA (chaotic-based optimization algorithm) can be understood as a system that is nonlinear, highly sensitive to initial conditions, and possesses the properties of randomness and non-

Table 3. Parameter Setting of Metaheuristic Algorithms.

Algorithm and Parameters	Parameter Value	Algorithm and Parameters	Parameter Value
PSO		**CS**	
Population	50	Population	50
Iteration	25	Iteration	25
Weight Function	[0.2,0.9]	Pa	0.25
Acceleration constants	2	Beta	1.5
The dimension of search space	5	The dimension of search space	5
Iteration	25	Iteration	25
SMS		**CSMS-EOBL**	
Vector Adjustment, ρ	1	Vector Adjustment, ρ	1
Beta	[0.8, 0.4, 0.1]	Beta	[0.8, 0.4, 0.1]
Alpha	[0.8, 0.2, 0]	Alpha	[0.8, 0.2, 0]
Threshold Probability, H	[0.9, 0.2, 0]	Threshold Probability, H	[0.9, 0.2, 0]
Phase Percent	[0.5, 0.1, -0.1]	Phase Percent	[0.5, 0.1, -0.1]
Adjustment Parameters	[0.85 0.35 0.05]	Adjustment Parameters	[0.85 0.35 0.05]
Iteration	25	Iteration	25

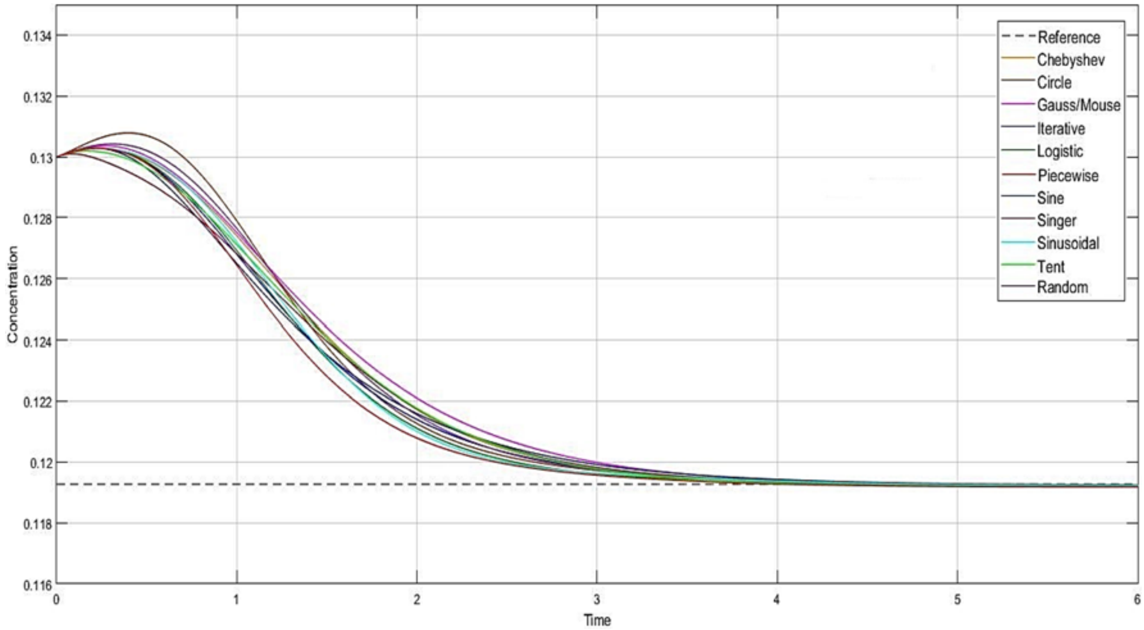

Figure 4. Simulation result for concentration control of CSTR with CSMSEOBL algorithm for different types of Chaotic Maps.

recurrence [45]. Whenever any metaheuristic optimization algorithm is embedded with chaotic maps it has three steps: Initialization, operator, and random generator (table 2).

4.4.2 *Elite opposition based learning algorithm (EOBL)* Tizhoosh proposed opposition-based learning (OBL) in 2005 [46] and from the day of its introduction this learning has been embedded with many metaheuristic optimization algorithms to enhance the exploration capability along with the accelerated convergence rate of the existing metaheuristic. The main concept of OBL lies in the fact that the current population and its opposite population are considered at the same point in time. In 2013, Wang *et al* proposed a modified form of OBL Strategy called Elite Opposition Based Learning (EOBL). It uses dynamic bounds instead of fixed bounds which make the search space shortening also, elite oppositional numbers are defined at the center point of search space which results in better convergence and exploration capability [47]. As EOBL is a modified form of OBL so the first OBL is explained.

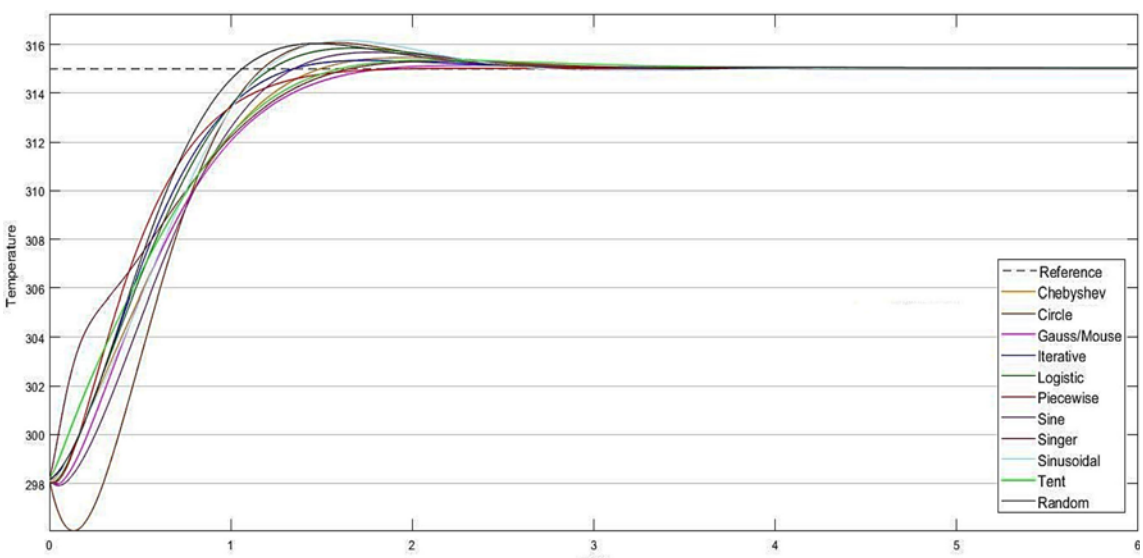

Figure 5. Simulation result for temperature control of CSTR with CSMSEOBL algorithm for different types of Chaotic Maps.

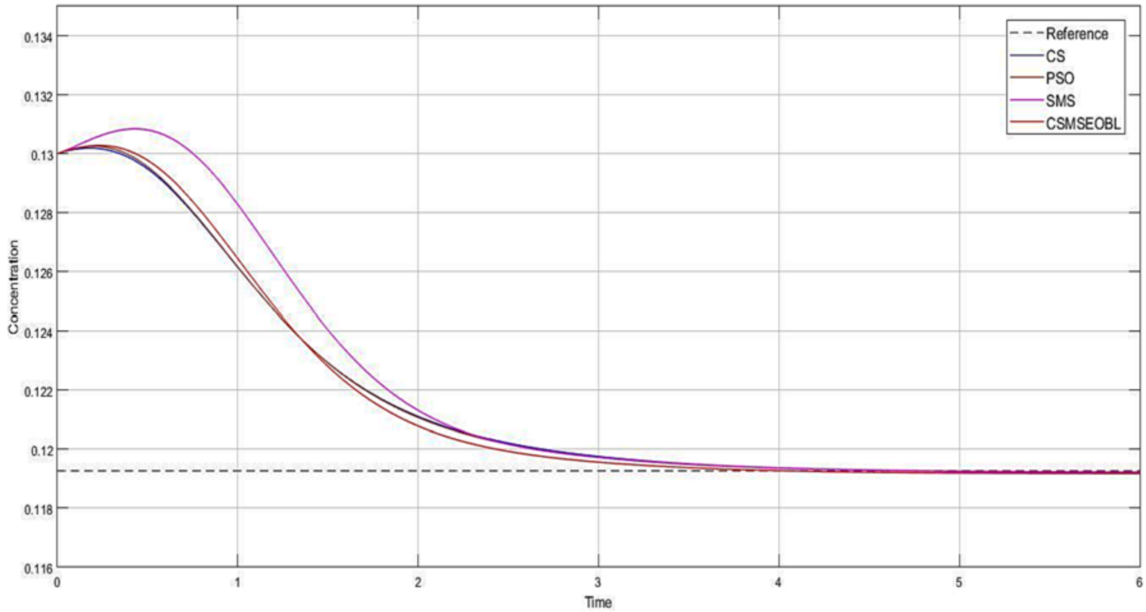

Figure 6. Comparison of concentration control of the CSTR system among different metaheuristic algorithms.

Let $\boldsymbol{x} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots\ldots\ldots \boldsymbol{x}_j\}$ is a point in the existing population and $\boldsymbol{j}$ is the dimension of search space, $\boldsymbol{x_j} \in [\boldsymbol{a_j}, \boldsymbol{b_j}]$, where, $\boldsymbol{a_j} = \mathbf{min}\{\boldsymbol{x_{ij}}\}$ and $\boldsymbol{b_j} = \mathbf{max}\{\boldsymbol{x_{ij}}\}$**.** The opposite point of $\boldsymbol{x}$ can be defined as follows:

$$\widetilde{x_j} = a_j + b_j - x_j \tag{16}$$

The further elite individual in the current population is defined as $\boldsymbol{x_e} = \{\boldsymbol{x}_{e1}, \boldsymbol{x}_{e2}, \ldots\ldots \boldsymbol{x}_{ej}\}$, and then the elite oppositional solution is given by

$$\tilde{x}_{i,j} = \rho * (da_j + db_j) - x_{e,j} \tag{17}$$

Where, $i = [1, 2, \ldots.., P]$, P is the size of the population, ρ is the generalized coefficient, $[da_j, db_j]$ are dynamic bounds and can be calculated as:

$$da_j = \min(x_{i,j}), \quad db_j = \max(x_{i,j}) \tag{18}$$

In EOBL, dynamic bounds are used instead of fixed bounds to secure the search space from shortening. If $\tilde{x}_{i,j}$

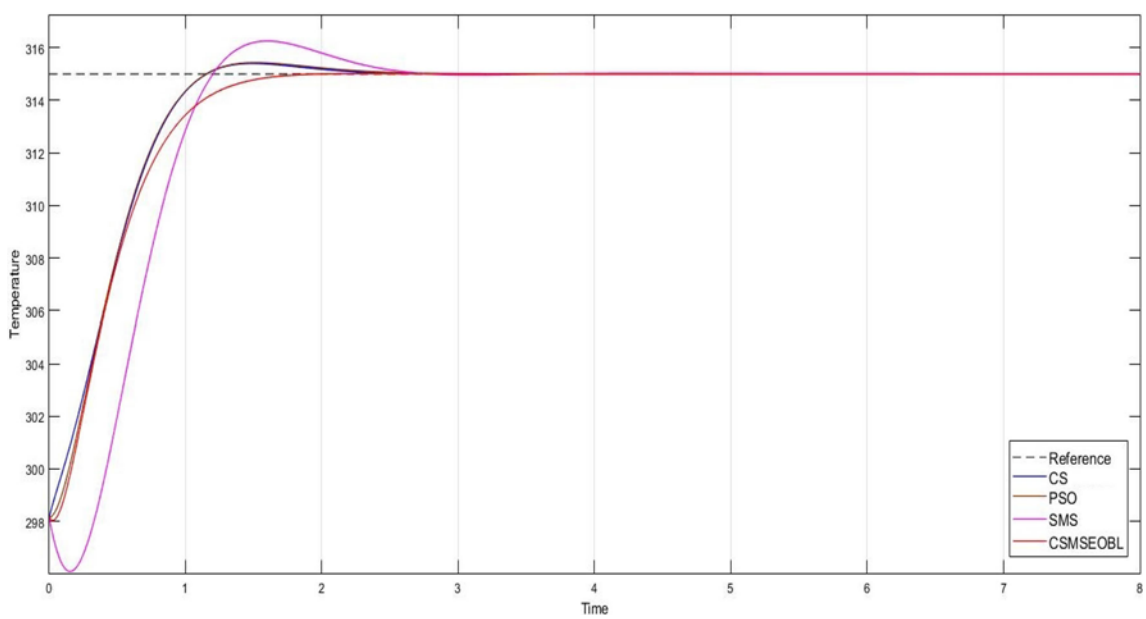

Figure 7. Comparison of Temperature control of the CSTR system among different metaheuristic algorithms.

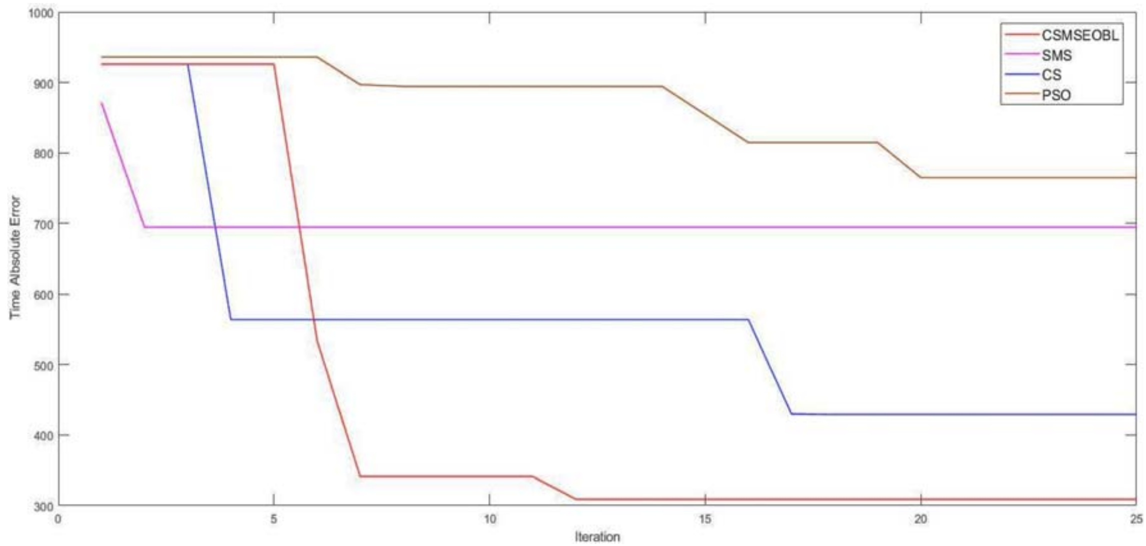

Figure 8. Variation of ITAE for different metaheuristic algorithms.

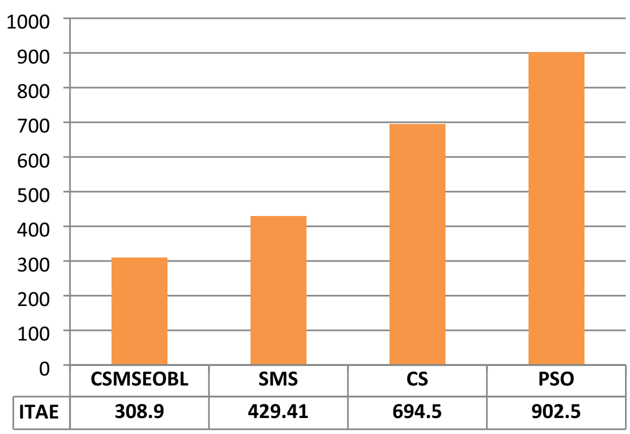

Figure 9. Comparison of objective function ITAE for different metaheuristic algorithms.

crosses it's dynamic bound it can be reset by using the following equation:

$$\tilde{x}_{i,j} = rand\left(da_j, db_j\right) \tag{19}$$

The main benefit of EOBL is that the elite population and current population can be evaluated at the same time which further results in a diversified population and enhanced global search ability [48].

Pseudocode for CSMSEOBL

Input: Define fitness function f(x), where, X = (x_1, x_2,., x_D)

Output: The optimal solution x*

Step 1. Initialize the gas state parameter of the algorithm with the dynamic boundary of the search space.

Step 2. While stop criterion is not satisfied do

Step 3. The current population is updated by applying the EOBL strategy.

Step 4. For each xεP do

Step 5. All random variables are updated by employing chaotic maps

Table 4. Comparative analysis of controller parameters and time response specifications.

	FOPID Controller Parameter								
	K_p	K_I	K_D	γ	μ	Rise time	Peak time	Overshoot	Settling time
SMS	12.1	32.5	1	1.006	0.1000	1.22	1.53	.38	2.27
CS	21.7	50	0.2	1.002	0.7850	1.13	1.36	.12	1.86
CSMSEOBL	15.8	43.3	1.9	.9999	0.1386	2.04	1.68	0	1.43
PSO [50]	.2510	.0243	.499	.5968	.0706	3.65	4.76	7	14

Step 6. For gas, the state calculates initial velocity and collision radius.
Step 7. The new molecules are computed by using direction vector
Step 8. By employing a collision operator, solve for the collision.
Step 9. The new random position is generated by using collision operator
Step 10. If the total no. of iterations completed$\leq$50% of the total number of iterations
Step 11. Go to the liquid state and repeat Steps 6, 7, 8, and 9.
Else
Step 12. Check if the total no. of iterations completed $\leq$90% of the total number of iterations
Step 13. Go to solid-state and repeat Steps 6, 7, 8, 9
Step 14. If 100% of the total iterations completed
Step 15. Update x with x*
End if
End for
End while

5. Simulation results

To confirm the practicality and viability of the proposed hybrid CSMSEOBL approach, a progression of comparative experiments has been performed on CSTR against the accompanying three states of the art metaheuristic optimization techniques: PSO, CS, SMS, and CSMS-EOBL. MATLAB 2018 is used for simulation and Intel (R) Core (TM) 2 Duo CPU T6400@ 2.00 GHz 1.20 GHz, 1.99 GB of RAM. The performance is verified for Control Temperature and Concentration of CSTR by running CSMSEOBL based FOPID, SMS based FOPID, CS-based FOPID, and PSO based FOPID controller, and results are compared. Parameter setting for all mentioned algorithms have been shown in Table 3.

For any optimization process convergence of metaheuristic algorithm towards the global optima of the tuned parameters of FOPID, the problem is defined with an objective function or fitness function. In this paper, to get the finest transient response as well as minimum steady-state error along with the least overshoot, Integral time absolute error (ITAE) is utilized as the objective functions. Since ITAE the most aggressive controller setting criteria that avoid peaks and give controllers with a greater load disturbance rejection and lessen the overshoot of the system while retaining the robustness of the system. ITAE is defined as

$$J_{ITAE} = \int_{0}^{T} t|e(t)|dt$$

The CSMSEOBL is used to optimize the parameters of FOPID for concentration and temperature control of CSTR and also to show the comparative study among Cuckoo Search, State of Matter Search and Particle Swarm Optimization algorithm is also implemented on CSTR. MATLAB Simulink environment is utilized for evaluating the results.

Further, different types of chaotic maps are used to enhance the randomness of the SMS algorithm. Figures 4 and 5 show the variation of concentration and temperature for different types of chaotic maps, respectively.

Further, to prove the superiority of the proposed algorithm, comparative result analysis with the best solution obtained from figures 4 and 5 are done with the existing algorithms i.e., SMS, Cuckoo Search (CS), and particle swarm optimization (PSO) are shown in figures 6 and 7, respectively. The setpoint for Concentration is taken as .119 (lb. mol/ft^3) and the temperature is at 315K.

The fitness (or objective) function is optimized by using different metaheuristic algorithms, and the dynamic performance of the CSTR is strengthened by optimizing various performance indices like rise time, settling time, and overshoot using a mathematical formulation of the objective function ITAE (Integral Time Absolute Error). ITAE decreases not only the initial extent of error but also decreases the error which develops in later responses [49]. Variation of ITAE for different metaheuristic algorithms has been shown in figure 8. The comparative analysis of considered metaheuristic algorithms in terms of ITAE is shown in figure 9. The proposed CSMSEOBL algorithm outperformed the other metaheuristic algorithms and it has been shown in table 4.

From table 4 we could conclude that the proposed CSMSEOBL shows the promising approach for concentration and temperature control of CSTR because in process control problems main aim is to obtain the least settling time and minimum overshoot. Even though the rise time and the peak time are large for CSMSEOBL-FOPID as compared to SMS-FOPID, CS-FOPID, and PSO-FOPID but the cost function is minimized along with minimum overshoot and least settling time.

6. Conclusion

This paper fixes the limitations of the standard SMS Algorithm by hybridizing it with chaotic maps and Elite Opposition Based Learning. Further, this hybrid algorithm CSMSEOBL is used to find optimal parameters of the FOPID Controller for the temperature and concentration control of a Continuously Stirred Tank Reactor (CSTR). Major findings of the work are as follows:

- CSMSEOBL gives better exploration and exploitation capability.

- The use of CSMSEOBL on a non-linear control problem results in faster convergence.
- CSMSEOBL shows promising results in terms of overshoot, settling time, and ITAE for optimizing the performance.
- The proposed controller is validated for concentration and temperature control of CSTR.

This study of hybrid metaheuristic can be further extended to reform the transient performance using multiple models, adaptive control strategy, and other latest metaheuristic algorithms.

References

[1] Rajinikanth V and Latha K 2012 Controller parameter optimization for nonlinear systems using enhanced bacteria foraging algorithm. *Applied Computational Intelligence and Soft Computing* 2012: 1–12

[2] Nasrabadi M S, Sharafi Y and Tayari M 2016 A parallel grey wolf optimizer combined with opposition-based learning. In: *1st Conference on Swarm Intelligence and Evolutionary Computation (CSIEC)*, 18–23

[3] Sundaravadivu K, Arun B and Saravanan K 2011 Design of fractional order PID controller for liquid level control of the spherical tank. In: *IEEE International Conference on Control System, Computing and Engineering*, 291–295

[4] Podlubny I 1994 Fractional-order systems and fractional-order controllers. Institute of Experimental Physics, Slovak Academy of Sciences. *Kosice* 12: 1–18

[5] Nagarajan M and Asokan A 2014 Design and implementation of fractional order controller for CSTR process. *Int. J. Comput. Appl.* 975: 8887

[6] Yadav P, Kumar R, Panda S K and Chang C S 2012 An intelligent tuned harmony search algorithm for optimization. *Inf. Sci.* 196: 47–72

[7] Shayanfar H and Gharehchopogh F S 2018 Farmland fertility: a new metaheuristic algorithm for solving continuous optimization problems. *Appl. Soft Comput.* 71: 728–746

[8] Catalbas M C and Gulten A 2018 Circular structures of pufferfish: a new metaheuristic optimization algorithm. In: *2018 Third International Conference on Electrical and Biomedical Engineering, Clean Energy and Green Computing (EBECEGC) IEEE*, 1–5

[9] El-Henawy I and Ahmed N 2018 Meta-heuristics algorithms: a survey. *Int. J. Comput. Appl.* 179: 45–54

[10] Gandomi A H, Yang X S, Talatahari S and Alavi A H 2013 Metaheuristic algorithms in modeling and optimization. *Metaheuristic Applications in Structures and Infrastructures*. 1–24

[11] Banks A, Vincent J and Anyakoha C 2007 A review of particle swarm optimization. Part I: background and development. *Nat. Comput.* 6: 467–484

[12] Kennedy J and Eberhart R 1995 Particle swarm optimization. In: *Proceedings of ICNN'95 - International Conference on Neural Networks*, pp. 1942–1948

[13] Goyal R, Parmar G and Sikander A 2019 A new approach for simplification and control of linear time-invariant systems. *Microsyst. Technol.* 25: 599–607

[14] Bhatt R, Parmar G, Gupta R and Sikander A 2019 Application of stochastic fractal search in approximation and control of LTI systems. *Microsyst. Technol.* 25: 105–114

[15] Naidu K, Mokhlis H and Bakar A H A 2013 Application of Firefly Algorithm (FA) based optimization in load frequency control for interconnected reheat thermal power system. In: *2013 IEEE Jordan Conference on Applied Electrical Engineering and Computing Technologies (AEECT)*, pp. 1–5. IEEE

[16] Mirjalili S, Mirjalili S M and Lewis A 2014 Grey wolf optimizer. *Adv. Eng. Softw.* 69: 46–61

[17] Ahmed A M, Rashid T A, and Saeed S A M 2020 Cat swarm optimization algorithm: a survey and performance evaluation. *Computational Intelligence and Neuroscience*

[18] Rao R 2016 Review of applications of TLBO algorithm and a tutorial for beginners to solve the unconstrained and constrained optimization problems. *Decis. Sci. Lett/* 5: 1–30

[19] Sahu B K, Pati S, Mohanty P K and Panda S 2015 Teaching–learning-based optimization algorithm based fuzzy-PID controller for automatic generation control of multi-area power system. *Appl. Soft Comput.* 27: 240–249

[20] Cuevas E, Echavarría A and Ramírez-Ortegón M A 2014 An optimization algorithm inspired by the States of Matter that improves the balance between exploration and exploitation. *Appl. Intell.* 40: 256–272

[21] Wang G G, Guo L, Gandomi A H, Hao G S and Wang H 2014 Chaotic krill herd algorithm. *Inf. Sci.* 274: 17–34

[22] Fathollahi-Fard A M, Hajiaghaei-Keshteli M and Tavakkoli-Moghaddam R 2020 Red deer algorithm (RDA): a new nature-inspired meta-heuristic. *Soft Comput.*. 1–29

[23] Shadravan S, Naji H R and Bardsiri V K 2019 The Sailfish Optimizer: a novel nature-inspired metaheuristic algorithm for solving constrained engineering optimization problems. *Eng. Appl. Artif. Intell.* 80: 20–34

[24] Liu G P, Daley S and Duan G R 2002 Application of optimal-tuning PID control to industrial hydraulic systems. *IFAC Proc.* 35: 179–184

[25] Yesil E, Guzelkaya M and Eksin I 2003 Fuzzy PID controllers: an overview. In: *Third Triennial ETAI International Conference on Applied Automatic Systems, Skopje, Macedonia*, pp. 105–112

[26] Ayomoh M K O and Ajala M T 2012 Neural network modeling of a tuned PID controller. *Eur. J. Sci. Res.* 71: 283–297

[27] Khanduja N and Sharma S 2014 Performance analysis of CSTR using adaptive control. *Int. J. Soft Comput. Eng. (IJSCE)* 4: 80–84

[28] Khanduja N and Bhushan B 2019 Control system design and performance analysis of PID and IMC controllers for continuous stirred tank reactor (CSTR). *J. Control Instrum.* 10: 16–22

[29] Devadhas G and Pushpakumar S 2011 An intelligent design of PID controller for a continuous stirred tank reactor. *World Appl. Sci. J.* 14: 698–703

[30] Huang K, Zhou Y, Wu X and Luo Q 2016 A cuckoo search algorithm with elite opposition-based strategy. *J. Intell. Syst.* 25: 567–593

[31] Chaudhari Y 2013 Design and implementation of intelligent controller for a continuous stirred tank reactor system using genetic algorithm. *Int. J. Adv. Eng. Technol.* 6: 325

[32] Goud H and Swarnkar P 2019 Investigations on metaheuristic algorithm for designing adaptive PID controller for continuous stirred tank reactor. *MAPAN* 34: 113–119
[33] Khanduja N 2015 CSTR control by using model reference adaptive control and PSO. *Int. J. Mech. Mechatron. Eng.* 8: 2144–2149
[34] Tejado I, Vinagre B M, Traver J E, Prieto-Arranz J and Nuevo-Gallardo C 2019 Back to basics: meaning of the parameters of fractional order PID controllers. *Mathematics* 7: 530
[35] Podlubny I 1999 Fractional order systems and PIλDμ-controllers. *IEEE Trans. Autom. Control* 44: 208–214
[36] Zhang C, Peng T, Li C, Fu W, Xia X. and Xue X 2019 Multiobjective optimization of a fractional-order PID controller for pumped turbine governing system using an improved NSGA-III algorithm under multi-working conditions. *Complexity* 2019: 1–18
[37] Poovarasan J, Kayalvizhi R and Pongiannan R K 2014 Design of fractional order PID controller for a CSTR process. *Int. Ref. J. Eng. Sci.* 3: 8–14
[38] Agarwal J, Parmar G, Gupta R and Sikander A 2018 Analysis of grey wolf optimizer based fractional order PID controller in speed control of DC motor. *Microsyst. Technol.* 24: 4997–5006
[39] Soukkou A, Belhour M C and Leulmi S 2016 Review, design, optimization and stability analysis of fractional-order PID controller. *Int. J. Intell. Syst. Appl.* 8: 73
[40] Sengupta S, Basak S and Peters R A 2019 Particle swarm optimization: a survey of historical and recent developments with hybridization perspectives. *Mach. Learn. Knowl. Extract.* 1: 157–191
[41] Sun L, Song X and Chen T 2019 An improved convergence particle swarm optimization algorithm with a random sampling of control parameters. *J. Control Sci. Eng.* 2019: 1–11
[42] Khanduja N and Bhushan B 2019 CSTR control using IMC-PID, PSO-PID, and hybrid BBO-FF-PID Controller. In: *Applications of Artificial Intelligence techniques in Engineering*, pp. 519–526
[43] Yang X S and Deb S 2010 Engineering optimization by cuckoo search. *Int. J. Math. Model. Numer. Optim.* 1: 330–343
[44] Kumar S R and Ganapathy S 2013 Cuckoo search optimization algorithm based load frequency control of interconnected power systems with GDB nonlinearity and SMES units. *Int. J. Eng. Invent.* 2: 23–28
[45] Tang R, Fong S and Dey N 2018 Metaheuristics and chaos theory. *Chaos Theory*. 182–196
[46] Tizhoosh H R 2005 Opposition-based learning: a new scheme for machine intelligence. In: *International Conference on Computational Intelligence for Modeling, Control and Automation and International Conference on Intelligent Agents, Web Technologies, and Internet Commerce* 1: 695–701
[47] Wu X, Zhou Y and Lu Y 2017 Elite opposition-based water wave optimization algorithm for global optimization. *Math. Probl. Eng.* 2017: 1–25
[48] Ai B, Dong M G and Jang C X 2016 Simple PSO algorithm with opposition-based learning average elite strategy. *Int. J. Hybrid Inf. Technol.* 9: 187–196
[49] Singh A and Sharma V 2013 Concentration control of CSTR through fractional order PID controller by using soft techniques. Fourth International Conference on Computing, Communications and Networking Technologies (ICCCNT). 1–6.
[50] Gupta R 2016 PSO based optimal design of fractional order controller for industrial application. *Int. J. Electr. Comput. Eng.* 9: 665–672