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A comparison of two metamodeling techniques for analog ICs: contemporary Kriging metamodels vs. classical RSMs

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Abstract. Metamodels are widely used in design optimization of analog ICs. Optimization of these ICs to meet required performance specifications is computationally intensive when circuit simulators are involved. Metamodels can be used to replace simulators to determine IC performance specifications and to increase optimization efficiency while maintaining accuracy. Metamodel accuracy is largely dependent on the type of metamodel, and the computer experimental design used to generate it. In this work, two popular metamodeling types are investigated: the classical Response Surface Models (RSMs), and Kriging metamodels which have recently gained popularity. Kriging metamodels are generated from Latin hypercube (LHC) designs in many cases. The techniques of this work are compared using analog ICs examples. The objective of this work is to renew interest in the classical RSMs and demonstrate that better metamodel accuracy is achievable at the same cost as Kriging metamodels, provided that minimum bias designs (MBDs) are used in RSMs derivation.

Keywords. Metamodels; experimental designs; Latin hypercube; minimum bias designs; Kriging metamodels; response surface models; root-mean-square-error; analog ICs.

1. Introduction

Traditional simulation methods used in computer-aided design (CAD) of analog integrated circuits (ICs) are time consuming, especially if the number of design variables (e.g., transistor widths) is large [1]. The number of transistors per unit area is always on the rise; hence, the design and manufacture of these ICs take longer and longer times. When computer simulations are involved, the complexity of the circuit determines the time needed for design. It may take from a few minutes to several hours or days [2]. Metamodels (also called surrogate models) of circuit performance specifications are widely used instead of simulators to reduce the time for circuit design optimization, and hence allow faster production of analog ICs.

A metamodel is an 'approximation' of another model which describes a physical/electrical quantity [1], e.g., to characterize the performance specifications of a circuit in terms of its design variables. To generate a metamodel, samples of points in the design variables space are used to determine the parameters of a function that estimates the relation between the design variables and performance specifications of the circuit under investigation [2]. These metamodels are then used in the design process to predict circuit performance specifications at other points in the design variables space [3], rather than using an otherwise expensive computer simulation code.

Different types of metamodels are used in the literature to model analog ICs performance specifications, including among others Kriging metamodels [1, 4–8], Artificial Neural Networks (ANN) [6], Response Surface Models (RSMs) [9, 10], Multivariate Adaptive Regression Splines (MARS) [9, 11], Radial Bases Functions (RBF) [11, 12], Rational Functions [4], and Support Vector Machines (SVM) [2].

Metamodel parameters must be calibrated to achieve acceptable accuracy levels. Accuracy is largely dependent on the computer experimental designs (combinations of design variables values) used to fit metamodel parameters. Different experimental designs are used in the literature including Latin hypercubes (LHCs) [1, 4], factorial designs [5], Box-Behnken designs [5], and minimum bias designs (MBDs) [6].

More research in analog IC design use LHCs in Kriging metamodeling [7, 8, 12–15]. It is widely believed that this results in fewer experimental design points and better accuracy levels by comparison to many other metamodeling techniques. In this work, we investigate this increasing popularity of using Kriging metamodels in analog IC design in terms of accuracy, by comparison to the traditional RSM metamodels. Unlike other work that derives RSM metamodels using LHCs or classical experimental

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designs, MBDs are used in this work to generate more accurate RSMs *using the same number of experimental design points* as the LHCs.

This paper is organized as follows. Section 2 highlights the underlying principles for Kriging and RSM metamodels. Section 3 is a background for LHCs and MBDs–the experimental design methods used in this work. Techniques to generate these experimental designs are also summarized in section 3. These principles are applied in the analog ICs examples of section 4. Conclusions and recommendations for future research are presented in section 5.

2. Kriging and response surface models

Different types of metamodels are used in analog IC design as mentioned in Section 1. Due to the increasing popularity of Kriging metamodels as mentioned previously, the discussion in this section is limited for the cases involving Kriging metamodels and RSMs.

2.1 *Kriging metamodels*

Kriging metamodels were originally developed in geostatistics by Krige *et al* [16]. Kriging is an interpolation algorithm for spatial data to find a value at an unmeasured location from observed data at surrounding locations [1, 8]. Likewise, a Kriging metamodel estimates the value of the function (the 'original' model) at a given point by calculating a weighted average of the known values of the function in the surrounding of the point [17]. The Kriging metamodel $\hat{Y}(\boldsymbol{x})$ of a response $Y(\boldsymbol{x})$ is expressed as:

$$\hat{Y}(x) = Y(x) + z(x) \quad (1)$$

where $Y(\boldsymbol{x})$ is a polynomial of the design variables $\boldsymbol{x}$ that interpolates the design points. $z(\boldsymbol{x})$ is a Gaussian function that represents the stochastic process (realization of a random process) with zero mean and variance σ^2 [17]. The goal is to determine weights λ_i that minimize the covariance.

$$\mathrm{COV}(x) = \sigma^2 R(x_i, x_j) \quad (2)$$

where $R(x_i, x_j)$ is the correlation matrix, and x_i, x_j are the design variables.

2.2 *Response surface models (RSMs)*

A Response Surface Model (RSM) is a polynomial of a response as a function of the various inputs (design variables). It is a parametric regression model, which means that it uses experimental design points to estimate unknown parameters of the polynomial [18, 19].

As it is well known, the behavior of the RSM depends on the order of polynomial; so, the chosen order of the RSM polynomial is important for the accuracy of the metamodel [8].

A second-order RSM $\hat{Y}(\boldsymbol{x})$ in k-design variables is expressed as:

$$\hat{Y}(\boldsymbol{x}) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{i \geq j}^{k} \beta_{ij} x_i x_j \quad (3)$$

β_o, β_i, and β_{ij}, are the coefficients (parameters) of the RSM, and x_i refers to one of the k-design variables.

3. Experimental design methods

When selecting a metamodeling technique, probably the most important issue that has to be taken into account is the experimental design (the sampling method) of the design variables space. An experimental design involves selecting the right variations in input (design variables) to build a metamodel of the response (performance specifications) as a function of these design variables [8].

In the design process, using all area of the design variables space requires high cost of performing many experiments and it takes a long time. Instead, an experimental design is used to determine the location of a set of sample points in the design variables space that covers the information gained from the response that is necessary to fit the metamodel [13, 18].

The different types of experimental designs can be classified into two categories: 'classic' experimental designs, and 'modern' experimental designs. Classic experimental designs such as factorial designs [5], Box-Behnken designs [5] and composite central designs [5] are traditionally used in RSMs, while modern experimental designs such as Latin hypercubes (LHCs) [1, 18, 19] and Orthogonal Array Designs (OADs) are mostly used in Kriging metamodels. In this research, two experimental designs will be investigated: the popular contemporary Latin hypercubes (LHCs) and the almost totally abandoned classical minimum bias designs (MBDs).

3.1 *Latin hypercube sampling*

Latin hypercube (LHC) sampling is a type of stratified sampling (sampling from a population). It works by controlling the way that random samples are generated for a probability distribution [18, 19]. An LHC sample is generated by dividing the design variables space into subintervals and choosing randomly a sample in these subintervals, ensuring that every design variable is used exactly once, where each sample covers one of possible probabilities of all design variables. Figures 1(a) and 1(b) show two sampling trials having nine points each of a

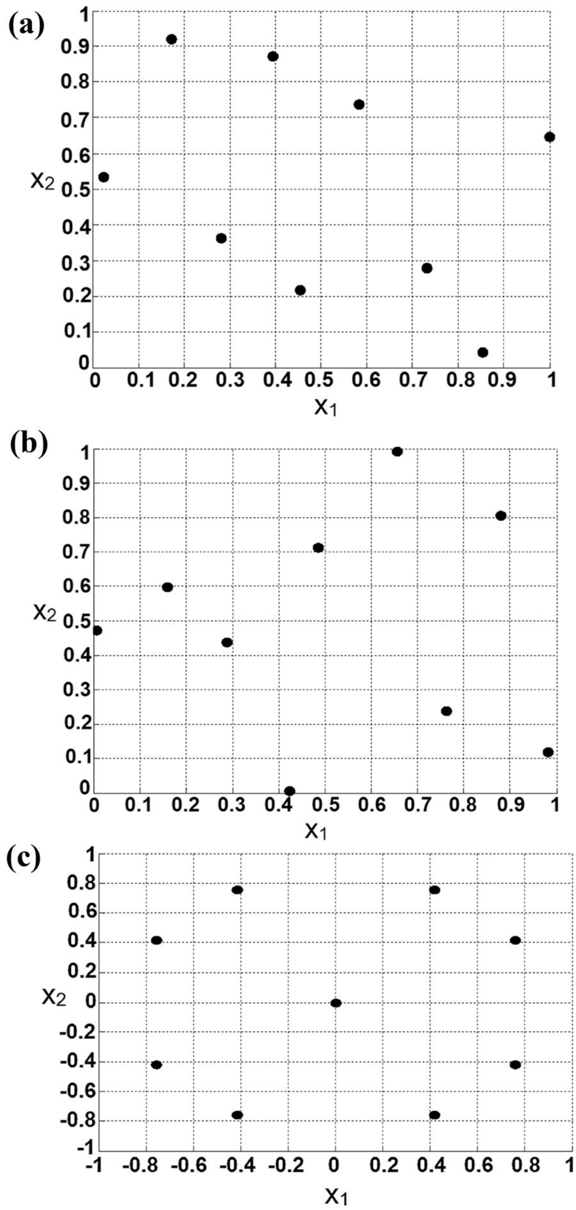

Figure 1. Two-dimensional experimental designs having 9 points each (a) LHC (trial 1), (b) LHC (trial 2) and (c) second-order MBD.

two-dimensional LHC; one square in each row and column contains one sample chosen randomly in the space covered by that square [3].

In this work, LHC sampling is generated using the command "*lhsdesign*" in MATLAB software. This command requires a prior knowledge about the number of sample points to be generated and the number of design variables. It generates sample points with coordinates of values between 0 and 1; this is called the "coded" variable. The coded variables are then converted to their true "natural" values of the design variables. For example, a resistor in an electronic circuit with values from 200 Ω to 1000 Ω is linearly mapped to values from 0 to 1 in LHC such that 0 in the LHC is the corresponding value for 200 Ω and 1 map to 1000 Ω. Generally, the natural values for a variable x are computed as follows:

$$x_n = x_c(x_{nmax} - x_{nmin}) + x_{cmin} \tag{4}$$

where x_n and x_c are the natural and coded values of the variable, respectively.

3.2 *Minimum bias designs (MBDs)*

When finding an interpolation function using a chosen set of points from the design variables space, the resulting errors are attributed to two factors: variance error which is primarily caused by data measurement, and bias error which is caused by the choice of the interpolating function (e.g., choosing a first-order polynomial for a metamodel while the actual model follows more closely a second-order behavior) [6]. In deterministic computer codes, bias is the only source of error since there is no variance in data measurement. Thus, in an MBD the sample points are located in the design variables space such that bias error caused by the inadequacy of the metamodel function to represent the 'true' function is minimized.

Box and Draper introduced the minimum bias (also called all bias) criterion to generate MBDs [17], with more recent treatment in [6]. In this paper, MBDs are used in conjunction with metamodeling of data obtained from deterministic computer codes (the circuit simulators) since MBDs are statistical design of experiments used to select optimal points that minimize bias error (the sole error) that arise in approximations via metamodels–unlike classical designs which minimize variance error assuming no bias in the metamodel (i.e., the metamodel perfectly matches the complexity of the underlying response). More information and MBDs tables can be found in [6]. Figure 1(c) is generated using these tables, for coded design variables between −1 and +1, as is customarily the case for classical experimental designs.

4. Experimental results

As mentioned previously, this research focuses on investigating the *differences* in terms of predication accuracy between the two metamodeling techniques: Kriging using LHCs and RSM using MBDs. The objective is to find which technique is most suitable—in terms of accuracy using *comparable experimental design sample sizes*–for analog IC design. The statistic used in this work to measure accuracy is the root-mean-square-error (RMSE), which is very widely used in metamodel validation [19]. It is emphasized that the objective of using RMSE here is for comparison purposes only; we are not claiming here that an approach is 'good', and the other approach is 'bad' vis-àvis metamodeling activities, but rather we are claiming that one approach is 'better than' or 'worse than' the other. Hamad *et al* devise in references [20–22] both objective and subjective validation measures which can be applied to a particular metamodeling approach to test its 'goodness-of-fit' by comparison to the underlying response.

As shown in figure 2, metamodeling procedure [23] starts by selecting the design variables as well as their ranges; these are usually provided by the user. In the second phase, LHCs and

MBDs are generated as described above, and used to fit Kriging and RSM metamodels, respectively. Finally, metamodel validation phase is performed by calculating RMSEs in order to compare the accuracy of the results of the two metamodeling activities. This methodology is applied to compare Kriging and RSM metamodels for performance specifications of a MOSFET amplifier with four design variables, and an operational amplifier IC having eight design variables [24].

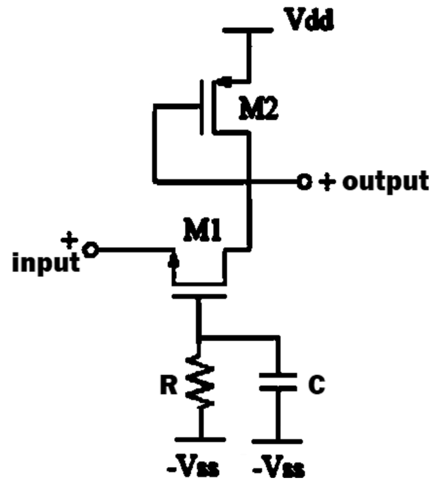

Figure 3. MOSFET Amplifier.

4.1 *MOSFET amplifier*

Performance specifications of the MOSFET amplifier shown in figure 3 are modeled using Kriging and second-order RSM metamodels. Metamodels for the voltage gain are derived as functions of the four design variables W1 and W2, the width of the two MOSFETS M1 and M2, respectively: in addition to resistance R and capacitance C. The design variables ranges are given in table 1.

Table 1. Design variables ranges for MOSFET amplifier.

Design variable	Min. value	Max. value	Unit
W1	2	200	μm
W2	2	200	μm
R	100	200	kΩ
C	10	20	nF

The circuit is modeled using a second-order RSM in four dimensions. The second- order MBD having 65 sample points in [6] is used to determine the parameters of the RSM metamodel for voltage gain A. A LHC experimental design is then generated via "*lhdesign*" in MATLAB (having the *same size* as the MBD) to fit the Kriging metamodel. LHC sampling is repeated to generate 10 different sampling trials with their corresponding Kriging metamodels. RMSEs for these metamodels are given in table 2, along with NRMSEs errors normalized to the RSME for RSM metamodel.

Normalized errors NRMSEs in table 2 are plotted in figure 4. As shown, RMSE for the Kriging metamodel for

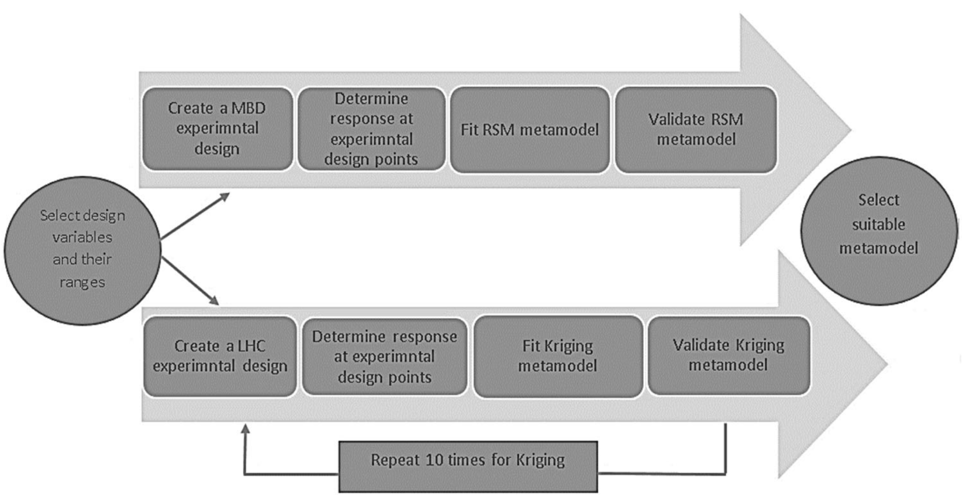

Figure 2. Methodology used in this work for metamodeling of analog ICs.

Table 2. Errors of RSM and Kriging metamodels for *A*.

RMSE for RSM (using MBD with 65 design points)	RMSEs for Kriging metamodel (using 10 LHC trials each having 65 design points)		
	Trial number	RMSE	NRMSE
0.338	1	0.528	1.5621
	2	0.529	1.5651
	3	0.531	1.5710
	4	0.536	1.5858
	5	0.530	1.5680
	6	0.529	1.5651
	7	0.531	1.5710
	8	0.529	1.5651
	9	0.528	1.5621
	10	0.528	1.5621

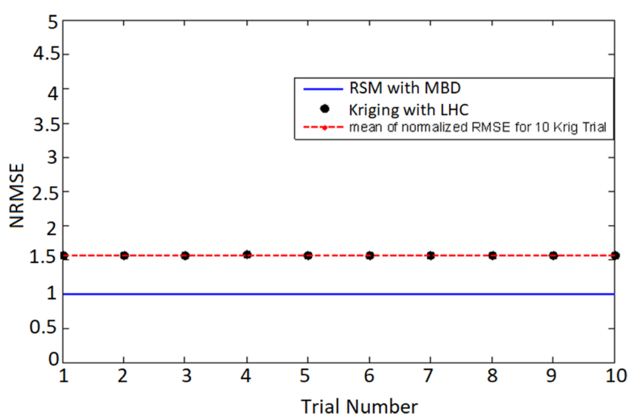

Figure 4. NRMSEs for gain *A* of the MOSFET amplifier example.

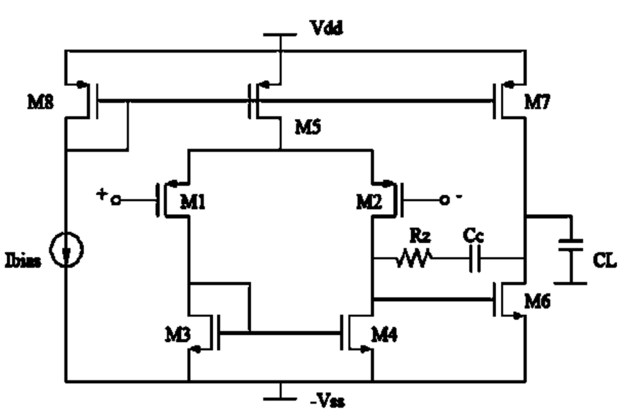

Figure 5. Operational Amplifier.

Table 3. Design variables and their ranges for the operational amplifier example.

Design variable	Min. value	Max. value	Unit
W1	10	100	μm
W3	10	100	μm
W5	50	100	μm
W6	200	300	μm
W7	100	200	μm
W8	10	100	μm
I_{bias}.	5	25	μA

the voltage gain *A* is higher than the RSM metamodel by 156%. It is emphasized here that this improvement in accuracy comes at no extra cost in terms of experimental design size.

4.2 *Operational amplifier*

Performance specifications of the operational amplifier shown in figure 5 are modeled using Kriging and second-order RSM metamodels. Specifications modeled are voltage gain *A*, common-mode-rejection ratio *CMRR*, and power dissipation *P*. There are seven design variables: widths W1, W3, W5, W6, W7, and W8 for the corresponding MOS-FETs in figure 5, in addition to I_{bias}. Design variables ranges are given in table 3.

Firstly, the circuit is modeled using second-order RSMs in seven dimensions. The second-order MBD having 552 sample points in [6] is used to generate RSMs for *A*, *CMRR* and *P*. An LHC experimental design is then generated (via "*lhdesign*" in MATLAB) having the same sample size as the MBD to fit the Kriging metamodels. The Kriging metamodeling activity is repeated using 10 LHC sampling trials.

RMSEs and NRMSEs for *A* metamodels are given in table 4. Normalized errors NRMSEs are plotted in figure 6.

From these results, RMSE for the Kriging metamodel for voltage gain A is higher than the RSM metamodel by 515%.

Errors in *CMRR* metamodels are given in table 5. Normalized errors NRMSEs are plotted in figure 7.

From these results, RMSE for the Kriging metamodel for the common-mode-rejection-ratio *CMRR* is higher than the RSM metamodel by 449%; no extra cost is incurred in terms of experimental design sizes,

RMSEs in power *P* metamodels of the operational amplifier are given in table 6. The normalized RMSEs are plotted in figure 8.

From these results, RMSE for the Kriging metamodel for power *P* is higher than the RSM metamodel by 307%.

Table 4. Errors in RSM and Kriging metamodels for *A*.

RMSE for RSM (using MBD with 552 design points)	RMSEs for Kriging metamodel (using 10 LHC trials each having 552 design points)		
	Trial number	RMSE	NRMSE
148212	1	764042	5.1551
	2	764201	5.1561
	3	763987	5.1547
	4	764061	5.1552
	5	764158	5.1558
	6	764003	5.1548
	7	764261	5.1565
	8	764297	5.1568
	9	764351	5.1571
	10	764146	5.1558

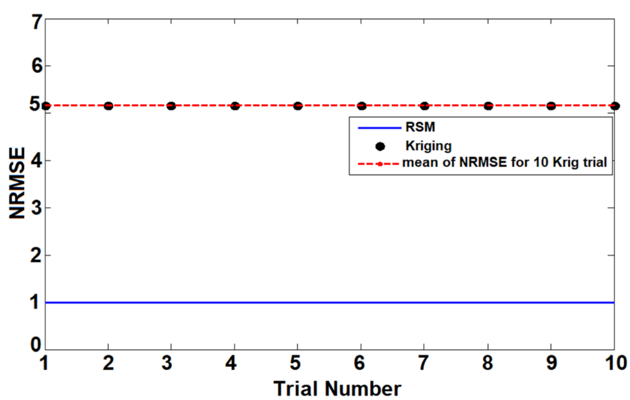

Figure 6. NRMSEs for *A* metamodels.

Table 5. Errors in RSM and Kriging models for *CMRR*.

RMSE for RSM (using MBD with 552 design points)	RMSEs for Kriging metamodel (using 10 LHC trials each having 552 design points)		
	Trial number	RMSE	NRMSE
304125	1	1366395	4.4929
	2	1366416	4.4929
	3	1366175	4.4921
	4	1366311	4.4926
	5	1366370	4.4928
	6	1366420	4.4930
	7	1366196	4.4922
	8	1366234	4.4923
	9	1366402	4.4929
	10	1366323	4.4926

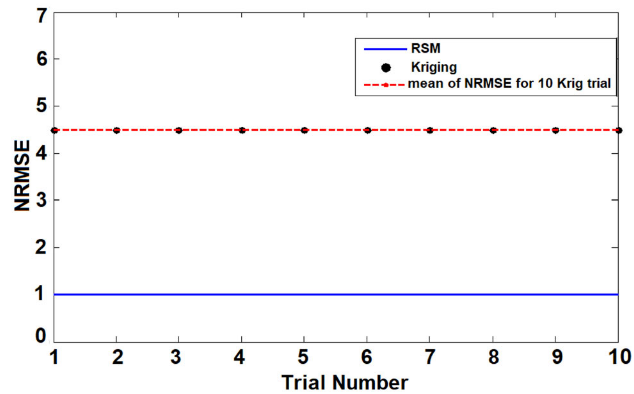

Figure 7. NRMSEs for *CMRR* metamodels.

Table 6. RMSE in RSM and Kriging metamodels for *P*.

RMSE for RSM (using MBD with 552 design points)	RMSEs for Kriging metamodel (using 10 LHC trials each having 552 design points)		
	Trial number	RMSE	NRMSE
126.699	1	389.023	3.0705
	2	388.892	3.0694
	3	389.261	3.0723
	4	389.025	3.0705
	5	388.997	3.0702
	6	389.318	3.0728
	7	389.230	3.0721
	8	389.001	3.0703
	9	389.225	3.0720
	10	389.542	3.0745

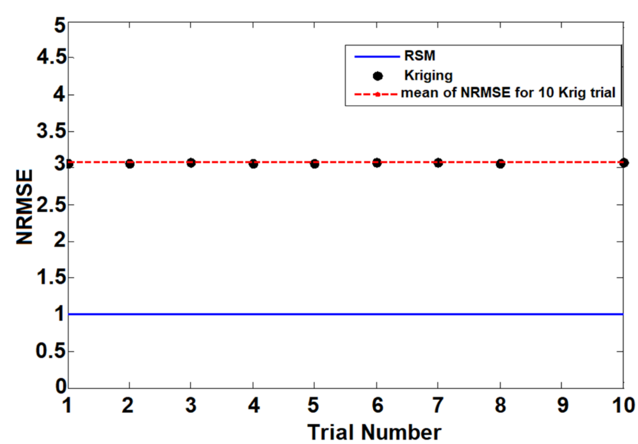

Figure 8. NRMSEs for *P* metamodels.

Again, the same experimental design size is the same as always.

5. Conclusions

Metamodels in analog IC design are generated from simulation results for an experimental design such as an MBD or an LHC. The aim is to reduce computational efforts in design optimization while still revealing the accurate information obtained from simulation.

In this paper, two types of metamodeling techniques– Kriging metamodels derived using LHC sampling, and response surface models (RSMs) based on minimum bias designs (MBDs) are studied. The former metamodeling technique (i.e., Kriging metamodeling with LHC designs) is more recent and gaining popularity in the literature, as compared to the classical RSM metamodels based on MBDs.

Both metamodeling techniques are applied in this work to different analog integrated circuits. The results show that using MBDs to obtain RSM metamodels improves accuracy in relation to Kriging metamodels with LHCs, *using the same size for experimental designs.* The results of our study show clearly that MBDs combined with RSM for metamodeling of analog ICs outperforms LHCs combined with Kriging in terms of prediction accuracy. Although using a few examples may not be sufficient to draw solid conclusions, but the prime objective of this work is to renew interest in the classical RSMs, *provided that these models are derived using MBDs*, not classical designs, nor LHCs.

Metamodeling using RSMs based on MBDs may prove to be a powerful metamodeling technique. This is so because *sampling at the outset* in MBDs is performed in such a way as to minimize bias error, i.e., the inadequacy of the metamodel in representing the 'true' model. It should be emphasized here that bias error is the sole error present in deterministic simulation results. Here, variance errors incurred in measured data (the basis for classical RSM derived from classical experimental designs) are irrelevant.

An obvious direction for future work is to embed codes (like LHC design codes) to generate MBDs in popular software tools such as MATLAB, to facilitate and expedite the generation of MBDs.

List of symbols

RSMs	Response Surface Models
LHC	Latin Hypercube
MBDs	Minimum Bias Designs
CAD	Computer-Aided Design
ICs	Integrated Circuits
MSE	Mean-Square Error
ANN	Artificial Neural Networks Response
MARS	Multivariate Adaptive Regression Splines
RBF	Radial Bases Functions
SVM	Support Vector Machines
OADs	Orthogonal Array Designs
RMSE	Root-Mean-Square-Error
CMRR	Common Mode Rejection Ratio
NRMSEs	Normalized Errors

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