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Stochastic modelling and performance analysis of cooperative HARQ in multi-cluster underwater acoustic sensor networks

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Abstract. Cooperative hybrid automatic repeat request (C-HARQ) scheme based on multi-hop relaying has been adopted as an efficient strategy in underwater acoustic sensor networks (UASNs) to extend network coverage and to enhance the performance by utilizing spatial diversity gain. In this letter, we develop a stochastic model for multi-cluster transmissions in UASNs, considering the underwater-specific characteristics such as frequency-dependent signal attenuation, acoustic spreading, multi-path fading and underwater noises. From this generalized stochastic model, we derive accurate analytical expressions to analyse the end-to-end packet error rate (PER) and energy efficiency. Analytical results demonstrate that the C-HARQ scheme can significantly improve the performance of UASNs, especially with the increase in number of relay nodes. Analytical results are corroborated with extensive simulation studies.

Keywords. Cooperative HARQ; multi-hop relaying; underwater acoustic sensor networks.

1. Introduction

Underwater acoustic sensor networks (UASNs) have recently received significant recognition from both industry and researchers due to its capability to support diverse applications, such as ocean exploration, military surveillance, environmental monitoring and much more [1, 2]. UASNs have unique channel characteristics, namely frequency-dependent path loss, time-varying multi-path fading and limited transmission bandwidth [1]. In addition, the propagation characteristics of acoustic waves change with the depth of the sensor nodes. All these factors have a significant impact on network connectivity and coverage. As a result, transmission based on multi-hop relaying is preferred in UASNs to build a network with improved coverage and connectivity [3]. Initially, the integration of cluster-based communication protocols using cooperative communication (CC) was proposed for ad-hoc networks [4]. Subsequently, Lee and Hossain [5] presented a mathematical model for computing the end-to-end throughput and delay for cluster-based CC in multi-hop wireless networks. In general, CC uses multiple relay nodes for user cooperation in the case of packet retransmissions [6]. Jamshidi [7] and Chen *et al* [8] combined the CC scheme with automatic repeat request (ARQ), referred to as cooperative ARQ (C-ARQ), proposed for UASNs. In C-ARQ, the destination node uses a feedback channel for the packet retransmissions. In addition, the idea of combining CC with hybrid automatic repeat request (C-HARQ) protocol further improves the reliability and energy efficiency (EE) of UASNs [9, 10]. In C-HARQ, both forward and backward error correction techniques are used in conjunction with the CC scheme to improve EE and reliability. As an extension to the afore-mentioned research works on multi-hop UASNs, we present a stochastic model for the calculation of packet error rate (PER) and EE of the C-HARQ-based multi-cluster UASNs.

Major contributions of this letter are as follows: 1. We develop a stochastic model for C-HARQ-based multi-cluster UASNs by considering the underwater channel characteristics, namely frequency-dependent path loss, acoustic spreading and multi-path fading effects. 2. From this generalized stochastic model, we derive accurate analytical expressions for PER and EE. 3. The results show that there exists a threshold distance between the transceiving nodes in both shallow and deep water scenarios, which acts as a deciding factor for selection of the optimal transmission scheme. It is also observed from the results that an increase in the number of relay nodes improves the PER performance of C-HARQ in both shallow and deep water scenarios.

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2. Physical layer modelling of UASNs

In this section, we present mathematical model of the underwater physical layer. The signal-to-noise ratio (SNR) of an underwater link depends on source level (SL), transmission loss (TL), directivity index (DI) and ambient noise level (NL) [1, 2, 10]. Here, DI is considered to be 0 dB for an omnidirectional antenna. The source signal strength related to the reference intensity is given by $SL = \frac{P_t}{I_{ref} \times area}$, where P_t is transmit power, $I_{ref} = \frac{q^2}{\rho c}$ is the reference intensity, ρ is the density and c is the speed of acoustic signal in the underwater medium. NL comprises turbulence, shipping, waves and thermal noises. NL is given by $NL = \frac{10^5}{f^{1.8}}$. *TL* consists of spreading and absorption losses. These losses depend mainly on the geometry of acoustic signal propagation, frequency and transmission distance. *TL* between two nodes can be calculated by $TL = d^k a(f)^d$, where k is the spreading factor, d is the distance between two nodes and $a(f)$ is the absorption coefficient. In deep water, the geometry of acoustic signals exhibits spherical spreading. However, in shallow water, the geometry of acoustic signals exhibits cylindrical spreading due to the signals being bounded by the floor and surface of the sea. As a result, k is 1 for shallow water and 2 for deep water scenarios. The *SNR* of an underwater link is given by [2]

$$SNR = \frac{P_t f^{1.8}}{\varphi \times 10^5 I_{ref} d^k a(f)^d}, \tag{1}$$

where $\varphi = 4\pi$ for deep water and $\varphi = 2\pi H$ for shallow water; H is the depth in m. We model the underwater channel as a Rayleigh fading channel due to multi-path signal propagation and also assume statistically *i.i.d.* channels. We also consider the M-QAM modulation technique with a modulation level of $m = \log_2 M$ bits/symbol. A closed-form approximation for finding the average symbol error rate (SER) of a Rayleigh fading channel link is given by

$$SER^* \approx 2(1 - 2^{\frac{-m}{2}})\left(1 - \sqrt{\frac{3SNR}{2(2^m - 1) + 3SNR}}\right). \tag{2}$$

The *PER* of a link is given by $PER^* = 1 - (1 - SER^*)^{\frac{X}{m}}$, where X is the packet size consisting of message bits (X_b) and header bits (H_b). In the HARQ scheme, the receiver can detect and correct a few errors present in the data packet with the help of the redundancy check bits generated by R-S codes. In this scheme, the data packet consists of X_b, H_b and check bits (C_b). The decoded average SER of the link is given by [11]

$$SER \approx \frac{1}{X} \sum_{k=t+1}^{X} k \binom{X}{r} (SER^*)^k (1 - SER^*)^{X-k}, \tag{3}$$

where the size of data packet $X = 2^K - 1$ and t is the error-correcting capability of R-S codes. The decoded PER of the link is given by $PER = 1 - (1 - SER)^{\frac{X+C_b}{m}}$.

3. Mathematical modelling of multi-cluster transmission

We consider a multi-cluster UASN model consisting of a source s, destination d and the relay nodes r_1, ..., $r_i, \ldots, r_r$ in each cluster as shown in figure 1. Let us define N_i as the cluster number in the multi-cluster transmission, where $i \in \{1, 2, \ldots, n\}$, and R_r is the relay state variable that provides information on the status of relay nodes. Relay nodes consider the binary value of 1 when decoding a packet successfully, otherwise they consider 0. In R_r, r denotes the decimal number of a vector $[r_r r_{r-1} r_{r-2} \cdots r_2 r_1]$. L_l indicates the number of transmission rounds in a single cluster, where $l \in \{1, 2, \ldots, r+1\}$. The maximum number of transmissions in a single cluster is limited to $r + 1$ rounds due to the possibility that each node will be given a chance of transmission, when other nodes fail to successfully transmit the data packet. The description of the C-HARQ is as follows. Initially the node s broadcasts the hello packets, whenever it has data to send. The nodes that receive the hello packets will provide the depth information along with the acknowledgement (ACK). Upon receiving the ACK, node s will choose the node d and r number of relay nodes to form the (N_i) cluster. Clustering can be done based on depth information, propagation delay and angle of arrival. There are different techniques proposed for clustering in UASNs, but that is beyond the scope of this work. After clustering, the node s transmits the data packets along with the destination and relay nodes addresses in the first

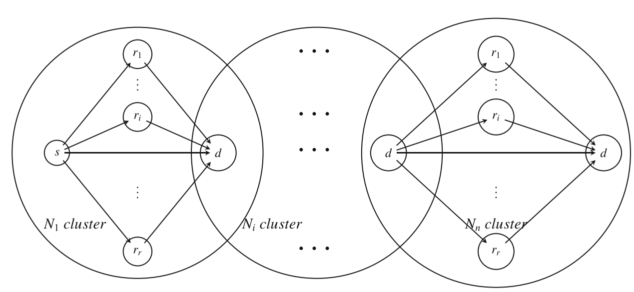

Figure 1. Multi-cluster transmission.

transmission round L_1. In addition to the node d, the relay nodes are also capable of receiving data packets. If the node d decodes the data packet accurately, it sends a positive ACK to the node s. Otherwise, it sends a negative ACK to the node r_1. If the node r_1 decodes the data packet correctly, then it transmits the data packet to the node d in the second transmission round L_2. Otherwise, it sends a negative ACK to the node d. As a result, the node d can approach the remaining relay nodes in the next transmission rounds. If the node d cannot decode the packet, then it will drop the data packet. After the successful reception of data packet in ${N_i}^{th}$ cluster, the node d will act as a source node in ${N_{i+1}}^{th}$ cluster. The data packet is sent to the next successor cluster, forming a chain up to the final destination node.

3.1 *End-to-end PER analysis in multi-cluster transmission*

For the multi-cluster transmission in UASNs, we develop an absorbing Markov chain model to calculate the end-to-end PER. We define the absorbing Markov chain with the sample space given by $\Omega = \{(N_i, R_r, L_l)\} \cup \{success\} \cup \{failure\}$, where $\{success\}$ and $\{failure\}$ are the absorbing states and (N_i, R_r, L_l) are the transient states. The success state represents the successful reception of a data packet at the node d in the end cluster, and the failure state represents the unsuccessful reception of a data packet at the node d of any cluster along the transmission path after the $(r+1)^{th}$ retransmission round. The transition probability matrix for the absorbing Markov chain can be written as $\begin{bmatrix} 0 & I_{2,2} \\ Q & R \end{bmatrix}$, where Q is the transition probability matrix between transient states, R denotes the transition probability matrix from transient to absorbing states, $I_{2,2}$, the identity matrix, denotes the transition probabilities between absorbing states and 0 is all-zero matrix and denotes the transition probabilities from absorbing to transient states. Here, $(N_i, R_r, L_l) \rightarrow (N_{i'}, R_{r'}, L_{l'})$ represents the transition from one transient state (N_i, R_r, L_l) to another transient state $(N_{i'}, R_{r'}, L_{l'})$. $(N_i, R_r, L_l) \rightarrow \{success\}$ and $(N_i, R_r, L_l) \rightarrow \{failure\}$ denote the transition from transient state to absorbing states. The transition probability from $(N_i, R_r, L_l) \rightarrow (N_{i'}, R_{r'}, L_{l'})$ is given by the Q matrix and $(N_i, R_r, L_l) \rightarrow \{absorbingstates\}$ is given by the R matrix. Define PER^i_{sd}, PER^i_{sr}, PER^i_{rd} as the PER of, respectively, the links $s-to-d$, $s-to-r$ and $r-to-d$ in the i^{th} cluster. Now we see probabilities between the transient states: $(N_i, R_r, L_l) \rightarrow (N_i, R_{r'}, L_{l'})$ are the unsuccessful packet reception at the node d of N_i cluster. The transition probability from $(N_i, R_0, L_1) \rightarrow (N_i, R_r, L_2)$ is given by

$$Pr\{(N_i, R_0, L_1) \rightarrow (N_i, R_r, L_2)\} = PER^i_{sd} \prod_{r_k \in A_1} (PER^i_{sr_k}) \prod_{r_g \in A_2} (1 - PER^i_{sr_g}), \tag{4}$$

where $1 \le N_i \le N_n$, $R_0 \le R_r \le R_{2^r-1}$, A_1 is the set of relay nodes that unsuccessfully decoded the data packet and A_2 is the set of relay nodes that successfully decoded the data packet. We assume that the relay nodes transmit directly to the d node at the instance of packet retransmission. Accordingly, the probability of transition from $(N_i, R_r, L_l) \rightarrow (N_i, R_{r'}, L_{l+1})$ is given by

$$Pr\{(N_i, R_r, L_l) \rightarrow (N_i, R_{r'}, L_{l+1})\} = 0, \tag{5}$$

where $1 \le N_i \le N_n$, $R_0 \le R_r \le R_{2^r-1}$, $R_0 \le R_{r'} \le R_{2^r-1}$ and $L_2 \le L_l \le L_r$. $(N_i, R_r, L_l) \rightarrow (N_{i+1}, R_0, L_1)$ represents successful packet reception at the destination node through source or relay nodes in an N_i cluster. The probability of transition from $(N_i, R_0, L_1) \rightarrow (N_{i+1}, R_0, L_1)$ through source node is given by

$$Pr\{(N_i, R_0, L_1) \rightarrow (N_{i+1}, R_0, L_1)\} = 1 - PER^i_{sd},$$

where $1 \le N_i \le N_{n-1}$. The probability of transition from $(N_i, R_r, L_l) \rightarrow (N_{i+1}, R_0, L_1)$ through relay node is given by

$$Pr\{(N_i, R_r, L_l) \rightarrow (N_{i+1}, R_0, L_1)\} = 1 - PER^i_{r_g d},$$

where $r_g \in A_2$, $1 \le N_i \le N_{n-1}$, $R_1 \le R_r \le R_{2^r-1}$ and $L_2 \le L_l \le L_{r+1}$. The probability of transition from $(N_i, R_r, L_l) \rightarrow (N_i, R_r, L_{l+1})$ is given by

$$Pr\{(N_i, R_r, L_l) \rightarrow (N_i, R_r, L_{l+1})\} = PER^i_{r_g d},$$

where $r_g \in A_1$, $1 \le N_i \le N_{n-1}$, $R_1 \le R_r \le R_{2^r-1}$ and $L_2 \le L_l \le L_{r+1}$. The probability of transition from $(N_i, R_r, L_{r+1}) \rightarrow \{failure\}$ is given by

$$Pr\{(N_i, R_{2^r-1}, L_{r+1}) \rightarrow \{failure\}\} = PER^i_{r_g d}, \tag{6}$$

where $1 \le N_i \le N_n$. The probability of transition from $(N_n, R_0, L_1) \rightarrow \{success\}$ through source node in the N_n cluster is given by

$$Pr\{(N_n, R_0, L_1) \rightarrow \{success\}\} = 1 - PER^n_{sd}. \tag{7}$$

The probability of transition from $(N_n, R_0, L_l) \rightarrow \{success\}$ through relay nodes in the N_n cluster is given by

$$Pr\{(N_n, R_r, L_l) \rightarrow \{success\}\} = 1 - PER^n_{r_g d}, \tag{8}$$

where $R_1 \le R_r \le R_{2^r-1}$ and $L_1 \le L_l \le L_{r+1}$.

The probability vector for the states to which the Markov model is absorbed can be obtained by $\chi = \kappa(I - Q)^{-1}R$, where $\kappa = [100 \cdots 0]$ represents the vector with which the Markov model begins. In fact we can write vector χ as

$\chi = [\chi(1)\chi(2)]$, where $\chi(1)$ and $\chi(2)$ are, respectively, the end-to-end probability of success and failure.

3.2 *End-to-end energy consumption modelling*

The aggregate energy consumed in the multi-cluster transmission is the sum of energy consumed in all the individual clusters. It is given by $E_{Total} = \sum_{i=1}^{N} E_{C_i}$, where E_{C_i} is the average energy consumed in the i^{th} cluster. The aggregate energy consumed in an i^{th} cluster is the average sum of energy consumed in all retransmissions. Mathematically $E_{C_i} = \sum_{j=0}^{r} E_{C_i}^{j}$, where $E_{C_i}^{j}$ is the energy consumed in the j^{th} retransmission. The average energy consumed in the first transmission by the source node (*i.e.,* zeroth retransmission) is given by

$$E_{C_i}^{0} = \left((1 - PER_{sd}) + PER_{sd} \prod_{r_k \in A_1} PER_{sr_k} \right) \frac{\Phi(X + C_b)}{R_b}, \tag{9}$$

where $\Phi = [P_t + (r+1)P_r]$, A_1 is the set of r relay nodes that unsuccessfully decoded the data packet and P_r is the receive power consumption. The average energy consumed in the n^{th} retransmission by a relay node from the successful set of relay nodes is given by

$$E_{C_i}^{n} = \left(PER_{sd} \sum_{i=n}^{r} \binom{r}{i} \prod_{r_k \in A_1} PER_{sr_k} \prod_{r_g \in A_2} (1 - PER_{sr_g}) \right) \left(PER_{r_g d}\right)^{n-1} [(n+1)P_t + (r+n+1)P_r] \frac{X + C_b}{R_b}, \tag{10}$$

where A_1 is the set of $r - i$ relay nodes that unsuccessfully decoded the data packet and A_2 is the set of i relay nodes that successfully decoded the data packet. The retransmissions are possible only when one or more relay nodes decoded the packet successfully. Several combinations of relay nodes are possible out of r relay nodes, given by $\binom{r}{i}$. We compare the *EE* performance of different transmission schemes. *EE* is given by $\eta = \frac{X_p(1-PER)}{E_{Total}}$, where X_p is the payload of data packet and *PER* can be obtained using the absorbing Markov model presented in section 3.1. The analytical expressions for calculating *EE* of existing DC and CC schemes are given in [10].

4. Results and discussion

In this section, we present analytical results validated using ns3 simulations. We considered the transmission and energy consumption parameters of an underwater acoustic modem developed by Evologics® as shown in Table 1 [1, 12]. Figure 2 and Table 2 show the variation of *EE* against the distance between S–D links in shallow and deep water scenarios, respectively. Figure 2 and Table 2 also indicate that there is a deciding threshold distance between the source and destination nodes in both shallow and deep water scenarios. This threshold distance determines the scheme that can be used to improve *EE*. It is also noted that the DC and HARQ-DC perform better than CC and C-HARQ when the distance between S–D links is lower

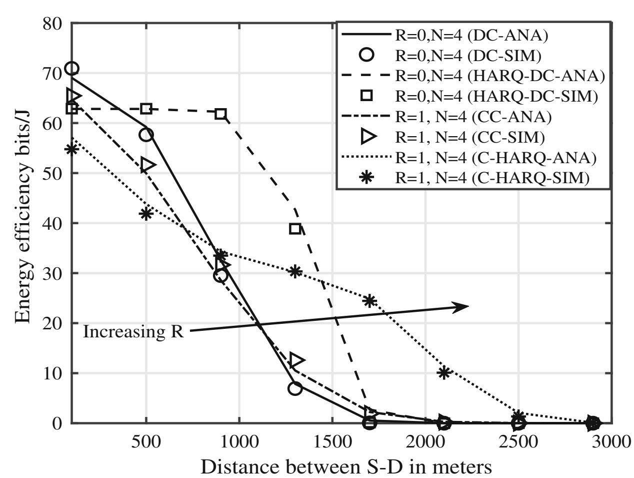

Figure 2. *EE* vs distance between S–D links (shallow water).

Table 1. Parameters considered for numerical analysis.

Parameters	Values	Parameters	Values
Bit rate used in UASNs	13900 bits/s	Depth of shallow water	50 m
Absorption coefficient	Thorp's formula [2]	Depth of deep water	1000 m
X_b	41 bits	Noise model	ns3::UanNoiseModelDefault
X	57 bits	Propagation model	ns3::UanPropModelThorp
Frequency (f)	26 kHz	Transmit mode	ns3::UanTxMode
P_t	35 W	PER model	ns3::UanPhyPerCommonModes
P_r	1.3 W	Energy model	ns3::AcousticModemEnergyModelHelper
(m)	2 bits/symbol	Mobility model	ns3::ConstantPositionMobilityModel

Table 2. EE vs distance between S–D links (deep water).

	Energy efficiency (bits/J)		
Distance (km)	1	1.5	2
DC	39.19	1.5	0.0001
HARQ-DC	62.63	10.2	4.9e–05
CC	40.8	11.4	1.007
HARQ-CC	35.9	30.8	20.7

than the threshold distance. This is due to the extra energy consumed by the relay nodes in CC and C-HARQ. On the other hand, CC and C-HARQ outperform the other DC and HARQ-DC when the distance between S–D links is higher than the threshold distance. It is also observed from the results that an increase in the number of relay nodes improves the performance of C-HARQ in both shallow and deep water scenarios.

5. Conclusion

In this letter, we have developed an accurate stochastic model for multi-cluster transmissions in UASNs by considering different propagation characteristics of underwater channel such as frequency-dependent signal attenuation, acoustic spreading, multi-path fading and aquatic noises. Analytical and simulation results show that there exists a threshold distance between the transceiving nodes in both shallow and deep water scenarios, which plays an important role in deciding the optimal scheme. It is also evident from the results that an increase in the number of relay nodes significantly improves the *EE*.

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