Sādhanā (2020)45:284
https://doi.org/10.1007/s12046-020-01522-y

A comparison of novel optimization model and algorithm for solving PMU deployment issues in the grid

ANSHUL AGARWAL[1,*] and KRITHI RAMAMRITHAM[2]

[1]Department of Computer Science and Engineering, VNIT, Nagpur, India
[2]Department of Computer Science and Engineering, IIT, Bombay, India
e-mail: a.anshul215@gmail.com

MS received 22 August 2020; revised 2 October 2020; accepted 9 October 2020

Abstract. Phasor Measurement Unit (PMU) sensors are commonly used nowadays for sensing different line parameters of the grid for making it more efficient and reliable. However, they are costly to procure and maintain. Also, they may fail and produce measurements with errors. Towards these issues, a novel optimization model and a polynomial time algorithm are developed that solve these issues with respect to minimal PMU deployment in the grid. These techniques are compared and tested on the standard IEEE 5, 14, 30, 57 and 118 bus systems. For achieving cross-validation and robustness ability in the grid, the developed optimization model and algorithm deploy about 70% and 141% less number of additional PMUs, respectively, as compared with the baseline approach. The results indicate that the developed techniques are very pragmatic and holistic since they take minimal time for allocating minimum PMU sensors while solving problems of cross-validation of PMU measurements and robustness against PMU outages.

Keywords. Algorithm; minimum PMU allocation; optimization model; PMU deployment issues.

1. Introduction

In a smart grid, line parameters like voltage and current phasors are sensed to estimate the status of the grid for avoiding its breakdown and failure. Phasor Measurement Unit (PMU) sensors [1] are most commonly used for this purpose [2]. However, they require communication facilities, are very expensive and their maintenance is costly [3, 4]. Therefore, it is required that minimum PMUs should be installed in the grid.

While installing PMU sensors in the grid, additional problems related to their deployment may arise like the presence of errors in the measurement [5] and failure of PMU sensors. Thus this work aims to install the minimum number of PMU sensors in the grid while ensuring that the line parameters of all the buses in the grid are sensed (called as full observability of the grid), and issues related to PMU deployment (like errors and failure) in the grid are tackled. Following are the main contributions of this paper:

- A novel optimization model that deploys the minimum number of PMUs for achieving full observability of the grid while satisfying various issues related to PMU deployment.
- A novel (polynomial running time) algorithm to allocate minimal PMU sensors for tackling different PMU deployment problems since minimum PMU allocation is NP-Complete.
- To demonstrate the effectiveness of the newly developed optimization model and algorithm, they are applied on the standard IEEE 5, 14, 30, 57 and 118 bus systems.

The paper also presents some insights on the PMU allocation under different scenarios. We believe no previous work has provided such detailed comparisons and a holistic solution for solving PMU deployment issues in the grid.

2. Methodology

A grid can be represented as a graph (N, E) where N denotes set of nodes representing buses (like substations, power generation centres or load aggregators) and E denotes set of edges representing transmission lines connecting the buses in the grid. For example, the graph model for the standard IEEE 5 bus system [6] consists of 5 nodes and 7 edges (Figure 1).

There are scenarios where the PMU sensors might start drifting from the actual values or produce errors in measurements. Thus, it is necessary to cross-validate readings from PMUs. Also, robustness against a PMU failure is required so that the failure does not lead to loss of observability and cross-validation. Thus, in this paper, five

*For correspondence
Published online: 18 November 2020

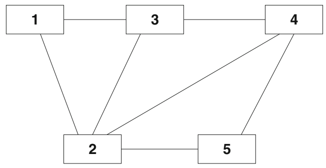

Figure 1. IEEE 5 bus system represented as a graph.

different types of problems are investigated: a) *Pmp* implies minimal PMU allocation, b) *Pcv* denotes *Pmp* with cross-validation and one or zero PMU can be allocated on a node, c) *Pcv** denotes *Pmp* with cross-validation, d) *Pr* denotes *Pmp* with robustness and one or zero PMU can be allocated on a node and e) *Pr** denotes *Pmp* with robustness.

3. Optimization model

The primary objective is to deploy the minimum number of PMU sensors in the grid. This is specified by the objective function (Eq. (1)). Each element i of *used* vector represents the number of PMU sensors deployed on node i.

$$\text{minimize} \quad \sum_{i=1}^{N} used_i \tag{1}$$

$$\text{subject to} \quad \forall i \in \{1...N\} \quad \#sense_i \geq \alpha \tag{2}$$

$$\#sense_i = used_i + \sum_{i \in \mathcal{N}(i)} \#sense_i \tag{3}$$

$$used_i \leq 1 \tag{4}$$

Given a node i, observability status (denoted as $\#sense_i$) is defined as the number of PMU sensors sensing the line parameters corresponding to node i. For solving *Pmp*, the desired observability status (denoted by α) of all the nodes should be at least one since the line parameters corresponding to every node should be sensed by at least one PMU. Similarly, for cross-validation and robustness, the status of all the nodes should be at least two and three, respectively. This constraint is specified by Eq. (2). Eq. (3) specifies that observability status of node i can be determined either by the number of PMUs installed on that node (denoted by the first term), or from the PMUs installed in the neighbouring nodes denoted by $\mathcal{N}(i)$ (denoted by the second term) using Kirchhoff's laws and Ohm's laws [7]. For ensuring that no more than one PMU is installed on a node (*Pcv* and *Pr*), Eq. (4) should be used.

Algorithm 1: Minimal PMU Allocation

```
   Input:
   a) graph model (N, E)
   b) ID: Problem ID
   Output: {loc}: set of nodes where minimal number
           of PMUs are allocated
1  Initialize: {loc} ← Φ, N′ ← N and obs[1 . . . N] ← 0
2  if ID == Pmp then
3      stat ← 1
4  else if ID == (Pcv or Pcv*) then
5      stat ← 2
6  else if ID == (Pr or Pr*) then
7      stat ← 3
8  foreach n ∈ N do
9      𝒩(n) ← Φ
10     foreach node n′ ∈ {N \ n} do
11         if an edge exists between n and n′ then
12             𝒩(n) ← 𝒩(n) ⋃ n′
13 for v ∈ {1, . . . , stat} do
14     while min(obs) < v do
15         alter[1 . . . N] ← 0
16         foreach node n ∈ N′ do
17             if (n ∈ {loc} ) ∧ (ID == (Pcv or Pr) then
18                 continue
19             foreach c ∈ {𝒩(n) ⋃ n} do
20                 if obs[c] < v then
21                     alter[c] ← alter[c] + 1
22             mnode ← argmax_k alter[k]
23         if ((obs[mnode] ≥ v) ∧ (∄n_x ∈ 𝒩(mnode)
           such that obs[n_x] < v)) then
24             N′ ← N′ \ mnode
25             Goto Step 14
26         {loc} ← {loc} ⋃ mnode
27         foreach node n ∈ {𝒩(mnode) ⋃ mnode}
           do
28             obs[n] ← obs[n] + 1
29         N′ ← N′ \ mnode
30     N′ ← N
31 return {loc}
```

4. Algorithm

Minimum PMU placement in the electric grid to achieve full observability is proven to be NP-Complete [8, 9]. This implies that there exists no polynomial time algorithm that can obtain the optimal solution. Thus, this work has developed a novel polynomial running time Algorithm 1 that approximates the minimum number of PMUs required to solve different types of problems listed in Section 2. It outputs the list of nodes {loc} where minimal PMUs are allocated.

The running time complexity of the algorithm (for input size N) is computed as follows: loop starting from Step 19 takes O(N) time since a node can be adjacent to all the other nodes in the graph. For a complete graph this is true for all the nodes, and therefore the outer loop starting from Step 16 takes O(*N*) time. Outer loop denoted by Steps 13–30 takes a fixed *stat* amount of time, and the outer loop starting from Step 14 requires O(*N*) time for the worst case scenario. While calculating running time complexity of an algorithm, the constant times are ignored. Therefore the worst case complexity of the developed Algorithm 1 is $O(N^3)$, which shows that the algorithm has a polynomial running time complexity.

5. Results and evaluation

The IEEE 14 bus system consists of 14 nodes [6]. For solving *Pmp* the optimization model allocates a total of 4 PMUs, one each on nodes 2, 7, 10 and 13, which is explained as follows:

- Since a PMU is allocated on node 2, it becomes observable. Nodes 1, 3, 4 and 5 become observable since they are adjacent to node 2.
- Similarly, nodes 7, 8 and 9 become observable since they are adjacent to node 7 and a PMU is allocated on node 7.
- Allocating a PMU on node 10 leads to nodes 10 and 11 becoming observable.
- Nodes 6, 12, 13 and 14 are observable due to allocation of PMU on node 13. For applying Algorithm 1, initially, node 4 is selected for PMU allocation since it leads to the highest number of change in the observability status of nodes, i.e. it causes six nodes to increment their status value. The algorithm then selects nodes 6, 9, 1 and 7 (in the mentioned order) for allocating PMU sensors. This order is based on the number of changes in the observability status of adjacent nodes. Therefore, the algorithm allocates a total of five PMUs for achieving full observability. For solving other problems, details of PMU allocation are as follows:
- *Pcv:* the model allocates a total of nine PMUs, one each on nodes 2, 4, 5, 6, 7, 8, 9, 11 and 13. Similarly the algorithm also allocates nine PMU sensors, but on the following nodes: 1, 2, 4, 6, 7, 8, 9, 10 and 13.
- *Pcv*:* the model allocates only eight PMUs, two each on nodes 2, 7, 10 and 13, whereas the algorithm allocates one PMU sensor on nodes 1, 2 and 4, and two sensors each on nodes 6, 7 and 9. Therefore the algorithm allocates one extra sensor as compared with the optimal number of eight sensors.
- *Pr:* no feasible solution exists to solve this problem.
- *Pr*:* since now multiple PMU sensors can be allocated on a node, the model allocates three PMU sensors on each of the following nodes: 2, 7, 10 and 13. Algorithm 1 on the other hand allocates a total of thirteen PMU sensors in the following manner: a) one PMU is allocated on nodes 1 and 4, b) two PMU sensors are allocated on node 2 and c) three PMU sensors are allocated on nodes 6, 7 and 9.

A summary of PMU allocation for different IEEE bus systems is presented in Table 1 and average observability status of all the nodes is shown in Figure 2.

5.1 *Evaluation and analysis of the results*

Figure 3 indicates that the difference in the number of PMU sensors allocated by the optimization model and Algorithm 1 for a given problem is almost equal in most of the cases. Even in the worst case scenario for IEEE 118 bus system, the difference in allocation is only about 4% of the total number of nodes. Therefore, the developed algorithm can be used to allocate minimal PMU sensors in the grid while solving different issues related to PMU deployment.

Figure 4 shows the additional number of PMUs allocated by the optimization model and algorithm for satisfying cross-validation and robustness requirements with respect to minimum PMUs allocated for achieving only full

Table 1. Details of PMU allocation in IEEE bus systems.

Problem ID	Algorithm	Optimal number
IEEE 5 bus system		
Pmp	1	1
Pcv	3	3
*Pcv**	2	2
Pr	5	5
*Pr**	3	3
IEEE 14 bus system		
Pmp	5	4
Pcv	9	9
*Pcv**	9	8
Pr	–	–
*Pr**	13	12
IEEE 30 bus system		
Pmp	10	10
Pcv	21	21
*Pcv**	20	20
Pr	–	–
*Pr**	30	30
IEEE 57 bus system		
Pmp	18	17
Pcv	33	32
*pcv**	33	32
Pr	54	54
*Pr**	50	48
IEEE 118 bus system		
Pmp	36	32
Pcv	70	68
*Pcv**	69	64
Pr	–	–
*Pr**	100	96

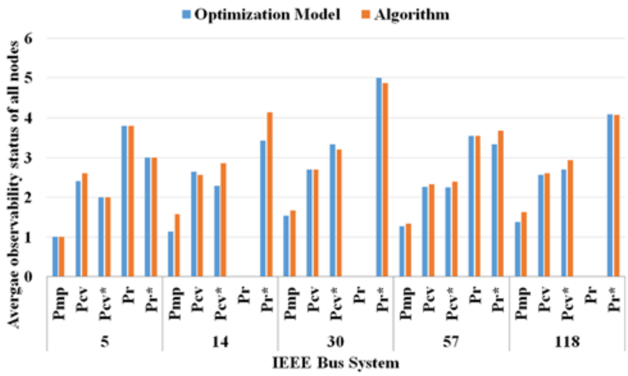

Figure 2. Average observability status of all the nodes in IEEE bus systems.

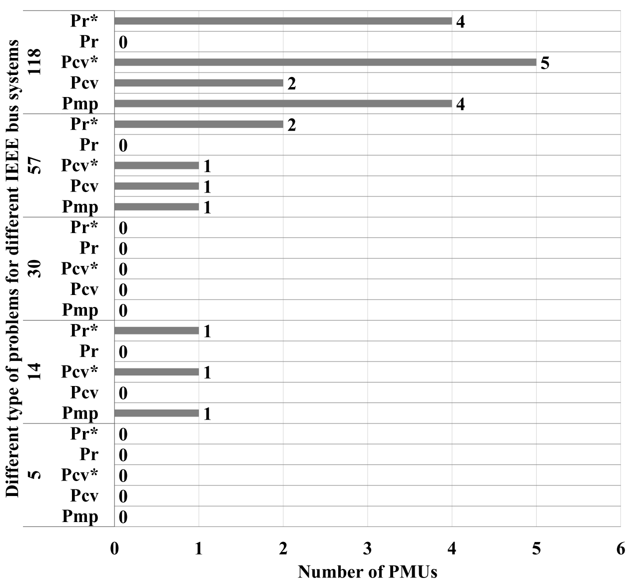

Figure 3. Summary of the difference in the number of PMUs allocated by the optimization model and Algorithm 1.

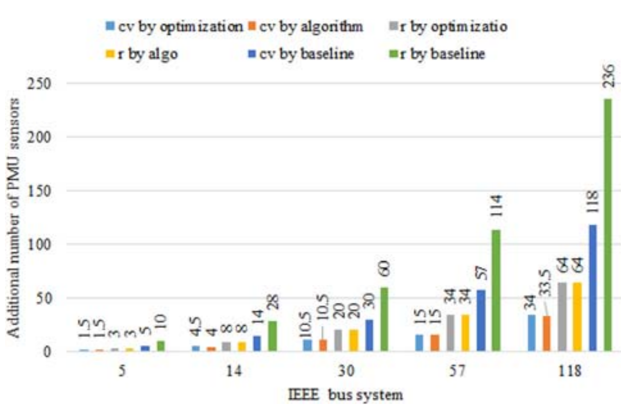

Figure 4. Additional PMUs allocated for solving cross-validation (cv) and robustness (r) problems.

observability. These additional allocations are compared with a baseline approach that allocates one and two additional PMUs to all the buses for ensuring cross-validation and robustness, respectively. Following are the insights drawn from the figure:

- Number of additional PMUs allocated by the optimization model and algorithm for satisfying cross-validation is on an average about 70% less than that by the baseline approach for solving *Pcv* and *Pcv**.
- A huge reduction of about 141% in the number of additional PMU sensors to be deployed for robustness feature by the developed model and algorithm is achieved as compared with the baseline approach.
- For problems like *Pcv** and *Pr**, the total number of PMUs allocated is mostly less than that of the corresponding problems *Pcv* and *Pr*, respectively. This is because, in these problems, multiple PMU sensors can be allocated on a single node.

Therefore, savings of about 70% and 141% in the number of additional PMUs to be deployed by the optimization model and algorithm developed in this paper for achieving cross-validation and robustness ability, respectively, are very impressive and should be adopted in the electric grids while deploying PMU sensors.

Figure 5 shows the percentage of the nodes that are selected for PMU allocation. It can be observed that the highest percentage of nodes are selected for solving *Pr* in IEEE 5 and 57 bus systems (since no feasible solution exists for this problem in other bus systems). On average the highest percentage of nodes selected for PMU allocation is about 61% and is observed in case of cross-validation, with the constraint of zero or one PMU allocation per node. For solving *Pcv** and *Pr**, about 35% of nodes are selected for solving the respective problem. Minimum percentage (about 28%) of nodes are selected in case of problems requiring only full observability.

Figure 6 summarizes the running time taken by the developed techniques. It can be observed that even for a large number of buses, the developed techniques take less than a second for computing the results.

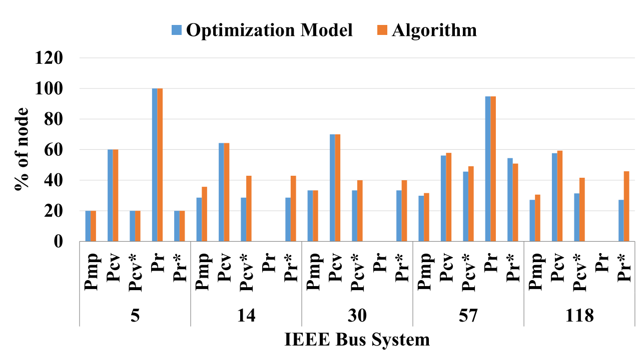

Figure 5. Percentage of nodes selected for PMU allocation.

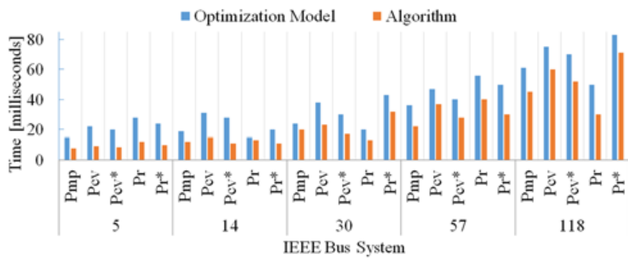

Figure 6. Running time of the developed approaches.

These results demonstrate that the optimization model and algorithm can be used for minimum PMU allocation for achieving full observability, in addition to cross-validation of PMU measurements and robustness against PMU outage.

6. Conclusion

Evaluation of the results showed that the algorithm, which has polynomial running time complexity, allocates PMU sensors very close (equal in many cases) to the optimal number of sensors allocated by the optimization model. Also, these techniques take less than a second to produce the results. Therefore, the algorithm is also a very good alternative for PMU allocation. It is also observed that savings in the extra percentage of PMU sensors required for achieving cross-validation and robustness are 70% and 141%, respectively, as compared with the baseline allocation. Thus, the developed techniques deal with these problems in a holistic manner and therefore should be used to tackle different practical issues related to PMU allocation in the grid.

References

[1] Phadke A G 1993 Synchronized phasor measurements in power systems. *IEEE Comput. Appl. Power* 6(2): 10–15
[2] Ravindra M, Rao R and NagaRaju V 2020 A binary harmony search algorithm approach for security constrained allocation of PMUs to obtain complete observability. *Advances in Intelligent Systems and Computing*, vol. 989
[3] Baldwin T L, Mili L, Boisen M B and Adapa R 1993 Power system observability with minimal phasor measurement placement. *IEEE Trans. Power Syst.* 8(2): 707–715
[4] Ruben C, Dhulipala S C, Bretas A S, Guan Y and Bretas N G 2020 Multi-objective MILP model for PMU allocation considering enhanced gross error detection: a weighted goal programming framework. *Electr. Power Syst. Res.* 182: 106235
[5] Begovic M, Novosel D and Djokic B 2008 Issues related to the implementation of synchrophasor measurements. In: *Proceedings of the 41st Annual HICSS*, p. 164
[6] Al-Roomi A R 2015 *Power flow test systems repository*. URL https://al-roomi.org/power-flow
[7] Dongjie X, Renmu H, Peng W and Tao X 2004 Comparison of several PMU placement algorithms for state estimation. In: *Proceedings of the Eighth IEE International Conference on DPSP,* pp. 32–35
[8] Gyllstrom D, Rosensweig E and Kurose J 2012 On the impact of PMU placement on observability and cross-validation. In: *Proceedings of the 3rd International Conference on e-Energy*, ACM, NY, USA, vol. 20, pp. 1–10
[9] Brueni D J and Heath L S 2005 The PMU placement problem. *SIAM J. Discrete Math.* 19(3): 744–761