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Performance criteria and tuning of fractional-order cascade control system

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Abstract. Industrial process control systems suffer from the overshoot problem. Designing the controller plant models by conventional Proportional Integral Derivative (PID) may increase the rise time, settling time and overshoot. Cascade control is a remedial measure undertaken to overcome these problems. In this paper, we present a new cascade model using fractional-order PIDs. The fractional-order cascade controller can be expressed by fractional-order differential equations. Different laws proposed in the field of fractional calculus form the theoretical part in evaluating the equations and designing the controllers. The new structure gives improved responses for the first-order and second-order systems with time delay. Better simulation results are obtained by introducing Smith predictor in primary and secondary loops. Detailed analyses have been done on the stability, performance criteria and disturbance rejection. The usefulness of this proposed cascade structure and its superiority over normal cascade are illustrated with examples.

Keywords. Cascade control; fractional-order PID controllers; model mismatch; robust; Smith predictor; stability analysis.

1. Introduction

Proportional-Integral-Derivative (PID) controllers, being easier to implement and understand, are widely utilized in industries. However, advances in the field of fractional calculus enhance designing of fractional-order PID (FOPID) controllers. The advantage is that in FOPID we have five tuning factors, i.e. proportional gain (Kp), integral gain (Ki), derivative gain (Kd), lambda (λ) and mu (μ), to get better performance than in classical PID controllers [1]. To tune the FOPID, one of the tuning methods is by the PSO algorithm as established in [2] and [3]. To tune fractional-order PI (FOPI) controllers, relay auto-tuning method is established in [4]. This method is appropriate when the process model is unable to be decided precisely. Stability boundary locus method can be applied to design $FOPI^{\lambda}$ by satisfying the required Gain Margin (GM) and Phase Margin (PM) of the system [5]. A new tuning rule is illustrated in [6], where a new set of Optimal Fractional-Order Proportional Integral (OFOPI) tuning rules are proposed. Composition of DE and Smith predictor methods gives efficient control of time delay processes [7]. A new method is represented to reduce the overshoot and increase robustness by cascading a sliding mode controller in outer loop [8].

The conventional PID is not efficient enough to adjust with load disturbance. Therefore, cascade controllers are utilized to produce a stable output. Among the primitive tuning methods of cascade control, relay auto-tuning is one of them. It allows tuning of secondary loop without placing the primary controller in manual mode [9]. Another tuning approach is to apply the Z–N tuning rules to the inner PI controller and outer PID controller [10]. Alfaro in [11] showed that the use of (2-DOF) PID controllers is capable of producing good performance for regulation and set point tracking and allowing smooth control. Azar A and Serrano F [12] reported the use of Internal Model control plus Proportional-Integral-Derivative (IMC-PID) tuning procedure for cascade control systems. Ibrahim Kaya used the method of controller synthesis to tune the primary and

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secondary loops of cascade controller [13]. Hemavathy showed that the optimal values of FOPI controller in primary can be obtained by minimizing Integral Absolute Error (IAE) using Genetic Algorithm [14]. Padhan and Reddy [15] reported that tuning the secondary loop by IMC and incorporating a filter dead time compensator in primary loop improves servo and regulatory response of series cascade controller [15]. Garai *et al* [16] depicted the superiority of IMC technique for cascade controller compared with Z–N and modified Z–N methods for tuning of heating furnace.

In the present work, cascade control has been combined with Smith predictor tuning for time delay process using fractional-order PIDs. First-, second- and third-order systems were tested with the proposed control technique and produced good responses; improved time response parameters were obtained like settling time, rise time and overshoot using objective function. Integral Square Error (ISE), Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE) robustness and model-mismatch analyses were also performed to get a clear idea about the plant's responses for different conditions.

2. Problem formulation

2.1 *Proposed FO-PID cascade control structure*

A new FO-PID cascade control structure is incorporated for controlling first-, second- and third-order process with time delay.

The modified structure is formed by adding a Smith predictor in the primary and secondary loops of the cascade control system. The primary and secondary processes are thus represented by Gp_1 and Gp_2, respectively. $G_{mp} = G_m e^{-\theta_m} s$ and $G_{ms} = G_{m1} e^{-\theta m1} s$ are the process models. G_m and G_{m1} are the models of the primary and secondary process, respectively, without time delay and θ_{mp} and θ_{ms} are the overall time delay of the primary and secondary process, respectively. GFC1 and GFC2 are, respectively, the primary and secondary loop FO-PID controllers. The controllers are useful in eliminating load disturbances from entering the primary and secondary loops.

The closed loop transfer function for servo response is

$$\frac{y}{u1} = \frac{G_{FC1} G_{FC2} G_{P1} G_{P2} G_{mp} G_{ms} \theta_{dp} \theta_{ds}}{\left[(G_{FC1} + G_{mp})\theta_{dp} + G_{FC1}\right]\left[(G_{FC2} + G_{ms})\theta_{ds} + G_{FC2}\right]\left[1 + G_{P2} G_{ms} \theta_{ds}\right]\left[1 + G_{P1} G_{mp} \theta_{dp}\right]}$$

The closed loop transfer function for regulatory response is

$$\frac{y}{d} = \frac{G_{FC1} G_{FC2} G_{P1} G_{P2} G_{ms} \theta_{ds}}{\left[G_{FC1} G_{FC2} G_{P2} G_{ms} G_{mp} \theta_{ms} (G_{mp} + \theta_{mp})\right]\left[(G_{FC2} + G_{ms})\theta_{ds} + G_{FC2}\right]\left[1 + G_{P2} G_{ms} \theta_{ds}\right]\left[1 + G_{P1} G_{mp} \theta_{dp}\right]}$$

2.2 *PID controller*

The controllers GFC1 and GFC2 have the following transfer functions:

$$G_{FC1} = K_{C1}\left[1 + \frac{1}{\tau_{i1^{\lambda}s}} + \tau_{d1^{\mu}s}\right],$$

$$G_{FC2} = K_{C2}\left[1 + \frac{1}{\tau_{i2^{\lambda}s}} + \tau_{d2^{\mu}s}\right].$$

2.3 *Process models*

For controller design purpose the transfer functions taken for both the primary and secondary loops are the following: first order plus dead time (FOPDT) model

$$G_{P1} = G_{P2} = \frac{K_1^* e^{-\theta_{1}s}}{\tau_{1s+1}}$$

second order plus dead time (SOPDT) model

$$G_{P1} = G_{P2} = \frac{K_1^* e^{-\theta_{1}s}}{(\tau_{1s+1})(\tau_{2s+1})}$$

where K_1 is known as steady-state gain; τ_1 and τ_2 are time constants and $\theta_1$is time delay of primary and secondary processes and primary and secondary models, respectively.

2.4 *Tuning procedure*

While tuning the controllers at first the secondary controller is tuned by adjusting the proportional gain (Kp), then the integral gain (Ki) and finally the derivative gain (Kd). After this the same procedure is followed to tune the primary controller. The fractional part (λ and μ) is tuned by the objective function method starting with the secondary part first and later the primary part. The Smith predictor has been incorporated in both the loops to give a much better response.

3. Heating furnace

The transfer functions of primary and secondary processes of heating furnace are obtained from [17]. They are

$$G_1(s) = \frac{1/90e^{-s}}{\left(s + \frac{1}{30}\right)(s + 1/3)}$$

$$G_2(s) = \frac{1/10e^{-s}}{\left(s + \frac{1}{10}\right)(s + 1)^2}$$

where $G_1(s)$ is primary loop's transfer function and $G_2(s)$ is secondary loop's transfer function.

Table 1. FO-PID parameters.

Controller	Kp	Ki	Kd	λ	μ
Primary	3	0.08	2	2	0.9
Secondary	5	0.7	4	1.2	1.3

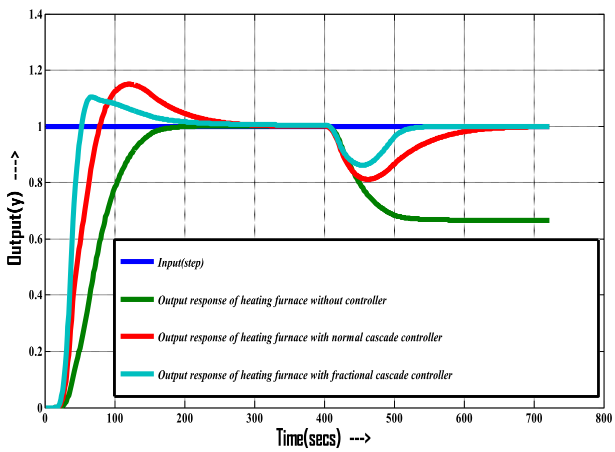

Figure 1. Output response of heating furnace by FO-PID cascade controller.

Tuning values of different parameters of the processes are presented in Table 1.

3.1 *Output response*

The output response is presented in Figure 1.

3.2 *Performance criteria*

Performances are presented in Table 2.

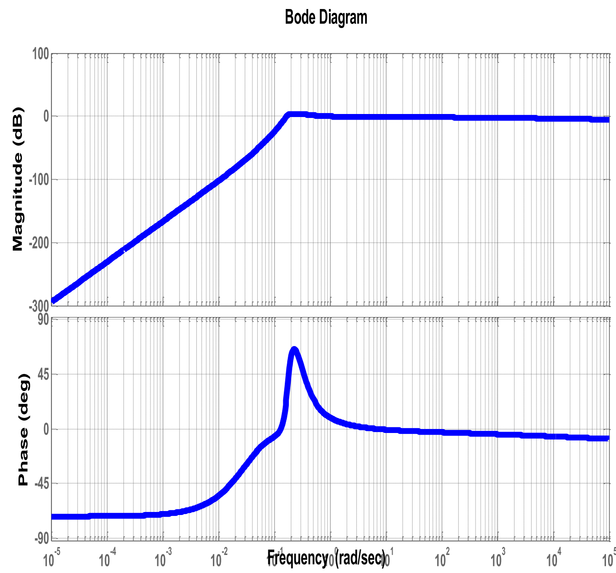

Figure 2. Bode plot of sensitivity of heating furnace using primary controller.

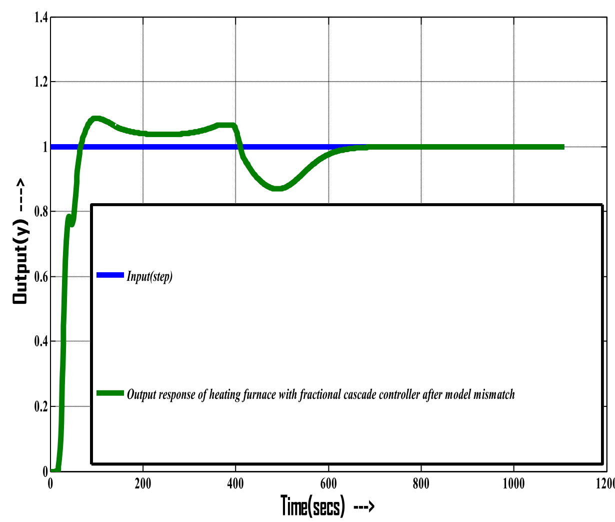

Figure 3. Output response of heating furnace after model mismatching.

Table 2. Performances.

Time response parameters	Without controller	Normal cascade controller	Fractional cascade controller
Rise time Tr (s)	123	70	46
Peak time Tp (s)	210	118	66
Settling time Ts (s)	157	239	186
Overshoot (%)	0	15	10
IE	97.23	3.85e + 0.53	11.57
ISE	21.79	1.05e + 106	95.56
IAE	97.59	3.85e + 0.53	166.1
ITAE	58758.08	3.81e + 0.56	64440.10

Table 3. Performance before and after model mismatching.

	Rise time (Tr)	Peak time (Tp)	Settling time (Ts)	Overshoot
Before model mismatching	46 s	66 s	186 s	10%
After model mismatching	58 s	99 s	407 s	9%

3.3 *Robustness analysis by sensitivity checking method*

$$\text{Sensitivity } (S_G) = \frac{1}{1 + [G_{FC1}(s)G_{FC2}(s)]G(s)}$$

where $G_{FC1}(s)$ is transfer function of primary controller, G_{FC2}(s) is transfer function of secondary controller and $G(s)$ is transfer function of plant.

$$G_{FC1}(\mathrm{s}) = \frac{2s^{2.9} + 3s^2 + 0.08}{s^2}$$

$$G_{FC1}(\mathrm{s}) = \frac{4s^{2.5} + 5s^{1.2} + 0.7}{s^{1.2}}$$

$$S_G = \frac{1}{1 + [G_{FC1}(s)G_{FC2}(s)]G(s)} = \frac{s^{5.2} + 0.367s^{4.2} + 0.011s^{3.2}}{0.088s^{5.4} + s^{5.2} + 0.132s^{4.5} + 0.367s^{4.2} + 0.11s^{4.1} + 0.176s^{3.2} + 0.0154s^{2.9} + 0.00352s^{2.5} + 0.0231s^2 + 0.0044s^{1.2} + 0.000616}$$

Figure 2 shows that the system is sensitive at low frequencies. At high frequencies, sensitivity is zero.

3.4 *Model mismatch analysis*

Figure 3 shows that the system provides stable response after model mismatching.

A comparison is made between performance criteria before and after model mismatching of fractional cascade controller, shown in Table 3.

4. Conclusion

A detailed analysis has been performed on fractional-order cascade controller and its tuning using first-, second- and third-order systems. One practical process has also been designed using fractional-order cascade controllers and by Smith predictor tuning. The output responses are found to be improved compared with those of normal cascade controllers. As evident from the comparisons given in tables the simulations of FO-PID controllers had lesser rise time and settling time, than normal cascade controllers. It is observed that FOPID applied to cascade controller design with first-, second- and third-order processes with dead time yields improved performance with disturbance rejection, stability and robustness.

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