Sādhanā (2020) 45:267
https://doi.org/10.1007/s12046-020-01480-5

Cost-efficiency measurement for two-stage DEA network using game approach: an application to electrical network in Iran

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MS received 9 May 2020; accepted 5 July 2020

Abstract. This study proposes a two-stage game–data envelopment analysis (DEA) approach for cost-efficiency (CE) measurement using centralized and Stackelberg game models. Each decision-making unit (DMU) is proposed to make up two-stage network structures (or processes), where all the first-stage outputs are the only second-stage inputs. The main contribution of this study is the development of cooperative and non-cooperative models for CE measurement and implies a unique CE decomposition. We apply centralized and Stackelberg approaches for cooperative and non-cooperative assumption between two stages. We proposed a simplified version of the two-stage DEA network CE model. Applying the proposed model for CE calculation reduces both the number of constraints and variables, resulting in a sharp reduction in computational requirements. The applicability of the presented model is demonstrated in the context of studying the output of the electrical network in Iran. The results of the case study show that in a two-stage network, the proposed DEA model can provide accurate estimates of CE. This paper aids the two-stage network structures to control the costs by weak DMUs management. To the authors' knowledge, this paper is the first research on the CE in a network by the DEA approach so that the relationships between the internal stages of the network are considered.

Keywords. Data envelopment analysis; two-stage network; cost efficiency; electrical network; centralized; Stackelberg.

1. Introduction

Position assessment of any decision-making units (DMUs) based on costs has a significant importance. In order to excel competitors, companies have to assess their position and efficiency compared to other competitors. In recent years, different measures have been used to assess the performance of organizations in which efficiency benchmark is one of the most important and useful ones. In the past studies, many approaches have been used to calculate efficiency score. For data organizing and analysing in comparison to others, data envelopment analysis (DEA) is known to be a better method, since it has no need for any background of efficiency frontier. Also, it allows considering multiple inputs and outputs to any DMU [1]. DEA is one of the most important techniques in efficiency measurement that measures units' relative efficiency using mathematical modelling. This paper concentrates on cost efficiency assessment.

Cost-efficiency (CE) concept, first introduced by Farrell [2], means the rate of output generation in relation to potential assessment with minimum input cost. Since CE also considers input cost, it is more trustworthy than technical efficiency measurement. A DMU computing in CE based on the Farrell method may be efficient enough at technical efficiency (TE) point, but not efficient from CE view. One of the most important issues for the business plan and production analysis of firms is the measurement of CE. Since many of the DMUs have network structures, the performance measurements of the total cost of the network and each subunit are one of the most important issues the company faces. Hence, we perform assessment of CE in implicit two-stage networks in this study and answer these questions:

- How efficient would a two-stage network and each network stage be based on CE?
- What are the relations and differences between Stackelberg and centralized models in a two-stage network?
- How can the complexity of the proposed model be reduced as much as possible?

Different approaches have been proposed to network structural modelling and each one has its own advantages and disadvantages. Game theory is one of them that

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Published online: 24 October 2020

attracted DEA researchers in recent years because of its significant advantages. This paper is the first research about CE assessment in network processes using game theory algorithm. Cooperation or non-cooperation of network stages affects total network efficiency in problems with network structures. We have considered a pure two-stage (PTS) series network and assessed CE based on centralized and non-cooperative conditions across stages using game theory algorithm and finally calculated CE of each network stage.

The main research objectives of this study are summarized as follows:

- Proposing the centralized and Stackelberg approaches to calculate CE in PTS networks.
- Proposing a simplified version of the DEA CE models in a two-stage network, in terms of the number of constraints and decision variables.
- Implementing the proposed model in the electrical network in Iran consisting of power plants and regional electricity companies.

This paper is summarized as follows. In the next section, literature of past works is reviewed. In Section 3, CE–DEA concepts are discussed. In the fourth section, proposed models are debated and will be tested for a numerical example in the fifth section. Finally, we conclude and sum up the discussion in the sixth section.

2. Literature review

In this section, literature of past works is reviewed on network DEA, CE and research gap sections.

2.1 *Network DEA*

Network DEA employs DEA methods for DMUs, which are considered as their internal processes. Cook *et al* [3] grouped all types of two-stage DEA models to four general divisions. (1) Standard DEA approach: This method conducted modelling by assuming that the second-stage input was the first-stage output and got the total efficiency through multiplying each stage efficiency measurements. Some of the articles that have used this method are [4–6]. (2) Efficiency decomposition approach: Total network efficiency is calculated through element efficiencies (for example [7–9]). (3) Network DEA approach: Network may have more than two levels. It is also possible that the input of each stage will not be only from the past stage or the outputs in this method may not necessarily be the input for the next stage. In this approach, modelling is conducted based on network element relations ([10–15] are some examples of this model). (4) Game theoretic approach: Network element relations are considered in this model. This relation may be cooperative or non-cooperative ([16–21] have applied game theoretic approach). Of course, a number of articles have been used to calculate the efficiency in the three-stage networks by the game theory approach and [22, 23] are its examples.

Koronakos *et al* [24] reformulate some of the basic DEA methodologies of the network within a specific simulation setting. They prove that the leader–follower approach, the multiplicative and additive decomposition approaches can all be modelled within a multi-objective programming context.

After calculating the efficiency of the network, in order to calculate the performance of individual stages, we deal with multiple optimal weights in decomposition of efficiency scores which may lead to confusion. Cook and Zhu [25] submit that this issue is due to the fact that the intermediate relationship between the two stages of the network is assumed to be unchanged in the DEA models. That is the outputs of the first component are unchanged and are considered as the input of the second component, while the input of the second component may be less than the output of the first component. To overcome this problem, researchers have suggested different proposals. For example, Kao and Hwang [26] assume that the efficiency of leader is more important than efficiency of follower and then the efficiency of follower is considered to hold leader's efficiency. Chen *et al* [27] calculated a weighted additive model to solve the problem. For the first time, Liang *et al* [16] applied the concept of game theory to solve the problem. They used Stackelberg's game or the leader–follower game to calculate the efficiency of each component of the network. Maleki and Matin [28] developed an improved Russell graph model for efficiency assessment of two-stage development network structures with convex hulls for intermediates.

2.2 *CE*

CE is the minimum total cost ratio. In addition to this concept, there is another point defining CE. TE curve considers the amount of output relevant to the used input; however, the cost is missing in this method. CE not only considers the related input and output but also measuring the cost is a highlighted feature in it. This concept was proposed first by Farrell [2]. He calculated CE when input costs were constant. Certain knowledge about costs is difficult and prices may change in short time. This was first proposed by Cooper *et al* [29]. His study was advanced by Camanho and Dyson [30]. They considered costs in a range and calculated CE in pessimistic and optimistic conditions. Jahanshahloo *et al* [31] simplified the model proposed by Camanho for calculating CE in constant cost. Bagherzadeh Valami [32] evaluated CE under fuzzy input costs conditions. Lei and Li [33] calculated CE by the same Camanho method, but with a two-objective mathematical programming under pessimistic condition. Conceição *et al* [34] used the Farrell CE method and considered inputs and input costs variable; this uncertainty is implemented by a coefficient, which has been defined by the authors. Toloo [35] used the vendor selection problem, but minimized the subject function between DMU and CE point. Garfamy [36] suggests and illustrates the use of the DEA approach in assessing overall performance of suppliers on various parameters based on the total cost of ownership. Sarkar [37] prepared the CE model for finding true cost leaders in private primary education. He questioned whether or not any of them had the same goal.

Unfortunately most studies in this field have not considered network structure and multi-stage processes, whereas, actually, many DMUs consist of network structures or have process-oriented nature.

Past researchers considered DMUs as a single entire unit, using DEA algorithm for calculating efficiency, and did not pay attention to these units' internal structures. In other words the system has been considered as a black box, which is entered by several inputs and exited by several outputs. Many researches show that this is not always true. Hence, one must consider units' internal structures to make DMUs evaluation and also calculate their efficiency better [7]. In general, all systems that have more than one process that are correlated to each other are named as networks. The network DEA model is needed to measure network systems efficiencies. Supply chains with their network nature are related to each other through front and rear relations and each one has different processes and activities, and their outputs are value production through products and services that reach final costumers.

Though the DEA model is common, the network DEA model has no standard form and depends on considered network structure [38]. Series and parallel structures in network systems are basic. In both of them, system efficiency (or inefficiency) can be decomposed to element processes efficiency (or inefficiency). Each DMU in the network structure has individual subunits, and each subunit has an internal competition with other subunits of the same DMU. Therefore, to calculate these system types there is a need for models that consider subunits internal competition in addition to the network internal structures. PTS networks are the simplest networks, in which the first stage outputs are the only inputs of the second stage.

Fukuyama and Matousek [39] considered a standard network for Turkish banking system and calculated banking network CE with a concentrated model. Lozano [40] determined CE by separating assigned efficiency and TE through a concentrated model for Taiwanese BCP factories. Wanke and Barros [41] have considered CE as the first stage and manufacturing efficiency as the second stage and determined CE through a concentrated TE model. The DEA model for measuring profit efficiency was developed by Petridis *et al* [42] for each Turkish electrical distribution company. Electrical distribution firms aim to maximize revenue while increasing expenses. Using this reality, Petridis *et al* [42] calculated the profit efficiency of electric distribution companies.

It can be seen from literature that CE in network structures has seldom been studied, whereas there are many DMUs actually that follow these structures. Table 1 shows a review of studies evaluating CE in DEA and the last row defines our study. This study is the first that evaluates CE in network through combining DEA and game theory algorithms. As shown in Table 1, we conclude that CE at network structure has not been evaluated and the relationships between stages have not been examined. The present study evaluates this and simplifies the final model.

In this study, a centralized and a Stackelberg two-stage DEA models are applied to calculate the CE score. The Liang *et al* [16] method is used in this study to avoid achieving several solutions in determining network stages of CE. Later the offered model is simplified in the CE of the total network, so is each stage efficiency. This simplification in the models decreases constraints and decision variables. In the end, in order to evaluate the CE and capability of the proposed model, we analyse electricity network of Iran as a case study.

3. CE measurement

CE assesses the given output amount at the minimum value of the input cost. Therefore, we expect the amount of inputs prices to be imported into the CE calculations. Farrell [2] proposed the CE measure for each DMU in input-oriented projection when the input prices are known as follows:

$$\begin{aligned} & C_o^* = \min \frac{\sum_{i=1}^m p_{io} x_{io}^*}{\sum_{i=1}^m p_{io} x_{io}}, \quad o \in J = \{1, \ldots, n\} \\ & \text{s.t.} \\ & \sum_{j=1}^n \lambda_j x_{ij} \le x_{io}^*, \quad (i = 1, \ldots, m) \\ & \sum_{j=1}^n \lambda_j y_{rj} \ge y_{ro}, \quad (r = 1, \ldots, s) \\ & \lambda_j \ge 0, \quad (j = 1, \ldots, n) \\ & x_{io}^* \text{ is free} \end{aligned} \tag{1}$$

For each DMU_j $(j = 1, \ldots, n)$, we denote, respectively, the input and output vectors as (X_j, Y_j), $j = 1, \ldots, n$ where$X_j = x_{1j}, \ldots, x_{mj}$ and $Y_j = y_{1j}, \ldots, y_{sj}$. We also employ X to denote the $m \times n$ matrix of inputs and Y to denote the $s \times n$ matrix of outputs. We assume that $X > 0$ and $Y > 0$. In this model, $p_{ij}$is the price of input i for DMU_j and x_{io}^* is the minimum amount of i input for DMU_o under review.

For example, in Figure 1, we assume the existence of two inputs (x_1, x_2) and one output (Y) under constant return to scale (CRS) situation and y is fixed. The input prices are

Table 1. Features of this study versus existing studies.

References	Feature				
	Total efficiency computation	Components efficiency computation	A two-stage network	Communication between the stages	Reduction in complexity of the model
[38]	√	√	√		
[39]	√	√	√		
[40]		√	√		
[41]	√	√	√		
[21]	√	√		√	
[30]	√	√			√
[32]	√				
[36]	√	√			
This study	√	√	√	√	√

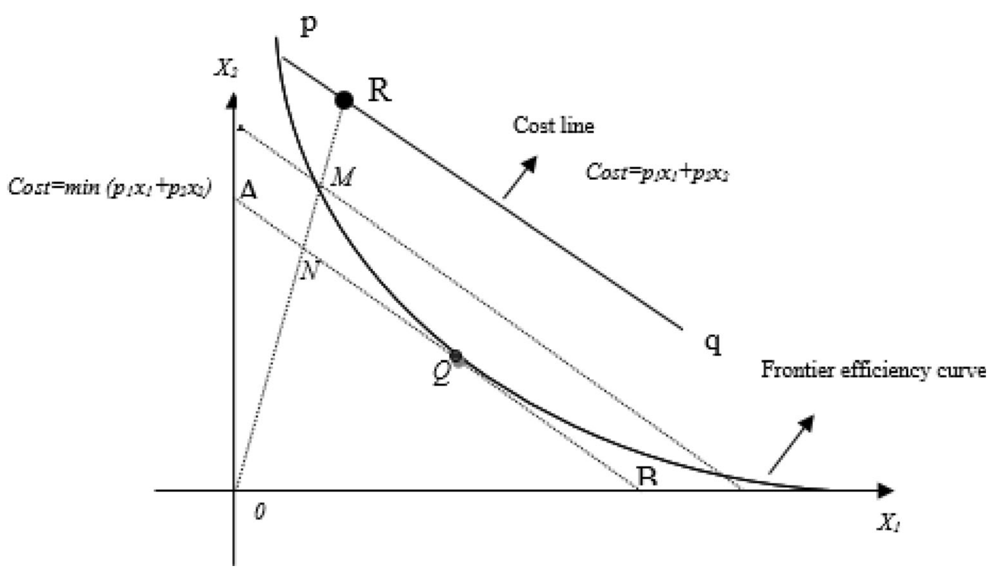

Figure 1. Cost line and frontier efficiency curve.

represented by cost line (see Figure 1). DMU_Q is efficient as it produces along the output *Y*, using the minimum value of inputs. Let us suppose that there is a DMU that operates in *R*, producing the same level of output as is produced along *Y*. DMU_R uses more inputs than DMU_Q to produce output *Y*. This is why *R* is qualified as inefficient, with a TE score of $\frac{OM}{OR}$ [2].

If DMU_R was at point M, it would be maximum as TE but not for CE because the Q point can be efficient at a lower cost. Moving M towards Q actually locates the DMU in an efficient point of CE. If the relevant amount of inputs is shown by *pq* cost line, the CE is calculated as $\frac{ON}{OR}$. These are the concepts stated by Farrell [2] first.

Schaffnit *et al* [43] have converted the Farrell model to the standard DEA formulation. They propose the resulting CE model, which is as follows:

$$
\begin{aligned}
& C_o^* = Max \sum_{r=1}^{s} u_r y_{ro}, \quad (o \in J = 1, \ldots, n) \\
& \text{s.t.} \\
& \sum_{i=1}^{m} \nu_i x_{io} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} \le 0 \quad (j = 1, \ldots, n) \\
& \nu_{i^a} - \frac{p_{i^a o}}{p_{i^b o}} \nu_{i^b} = 0 \quad (i^a < i^b (i^b = 1, \ldots, m)) \\
& \nu_i, u_r \ge 0
\end{aligned} \tag{2}
$$

The objective function and the first two constraints show the input-oriented CCR primal model (multiplier). The only difference is the third constraint, which changes the model to a CE model. In this model, v_{i^a} and v_{i^b} are the weights used for inputs; $p_{i^a o}$ and $p_{i^b o}$ are the inputs prices; a and b are used to prevent repetition of i. Schaffnit *et al* [43] prove that if weight constraints are included in Model (1), the amount of CE converts to Model (2). In Model (1), the input weights are related to input prices. The relative value between input prices and input weights is as follows:

$$v_{i^a} - \frac{p_{i^a o}}{p_{i^b o}} v_{i^b} = 0 \quad i^a < i^b (i^b = 1, \ldots, m))$$

Jahanshahloo *et al* [31] simplified Model (2), as Model (3). This model has m variables fewer than Model (2), and has less computational complexity.

$$C_o^* = Max \sum_{r=1}^{s} u_r y_{ro}, \quad o \in J = \{1, \ldots, n\}$$
$$\text{s.t.}$$
$$\sum_{i=1}^{m} u_r y_{rj} \le \frac{\sum_{i=1}^{m} p_{io} x_{ij}}{\sum_{t=1}^{m} p_{to} x_{to}} \quad (j = 1, \ldots, n) \tag{3}$$
$$u_r \ge \varepsilon \quad (r = 1, \ldots, s)$$

In this paper, we want to propose CE measurement in a two-stage network. Figure 2 shows a PTS process. As shown in Figure 2, each DMU$_j$ consists of two stages, which are connected as series. Furthermore, only total outputs of each stage are consumed as the next stage inputs. In other words, stage 1 and stage 2 have no external outputs and external inputs, respectively.

Let us consider a two-stage DEA system and assume that each DMU$_j$ $(j = 1, \ldots, n)$ consumes m inputs x_{ij} $(i = 1, \ldots, m)$ to produce D outputs z_{dj}, $(d = 1, \ldots, D)$ in the first stage. The D of the first stage is used as input in the second stage to produce y_{rj} $(r = 1, \ldots, s)$ outputs where p_{ij} is input price, p'_{dj} is intermediate price and p''_{rj} is output price.

In order to state the problem better, the assumptions are as follows:

- The system studied in this paper has two stages, so the total inputs of the second stage are equal outputs of the first stage (pure series network).
- The model is CRS.
- The proposed model is general and it can be used for any problem that has two-stage network structure, generally.
- The costs have been considered to be certain.

In the following, the centralized and Stackelberg models are proposed to compute the CE of the two-stage network in Figure 2.

4. Proposed models

In this section, using the approach introduced by Liang *et al* [8, 16], centralized and Stackelberg models are proposed for evaluating the CE of DMUs that have certainty prices. Then, by simplifying the models, we decrease the computational complexity under the CRS condition.

4.1 *Centralized model*

If each stage is considered as a player, we can propose CE in a two-stage network in input-oriented form and CRS with centralized cooperation, as follows:

$$e_o^{centralized} = Min \frac{\sum_{i=1}^{m} p_{io} x_{io}^*}{\sum_{i=1}^{m} p_{io} x_{io}}, \quad o \in J = \{1, \ldots, n\}$$
$$\text{s.t.}$$
$$\sum_{j=1}^{n} \lambda_j^1 x_{ij} \le x_{io}^*, \quad (i = 1, \ldots, m) \tag{4a}$$
$$\sum_{j=1}^{n} \lambda_j^1 z_{dj} \ge z_{do}^*, \quad (d = 1, \ldots, D) \tag{4b}$$
$$\sum_{j=1}^{n} \lambda_j^2 z_{dj} \le z_{do}^*, \quad (d = 1, \ldots, D) \tag{4c}$$
$$\sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{ro}, \quad (r = 1, \ldots, s)$$
$$\lambda_j^1, \lambda_j^2 \ge 0, z_{do}^*, x_{io}^* \text{ is free} \quad (j = 1, \ldots, n) \tag{4d}$$

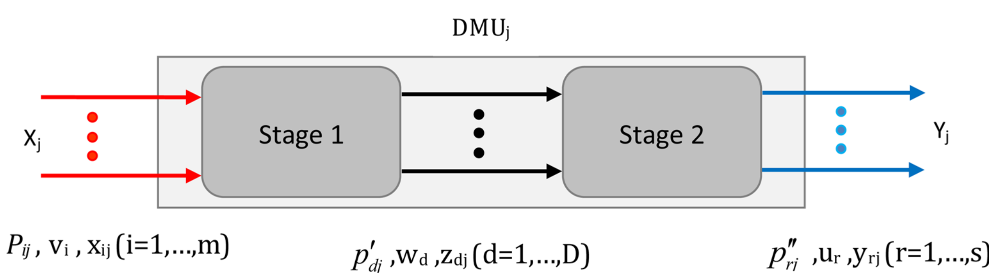

Figure 2. Two-stage process of DMU$_j$.

According to Figure 2 DMU_j are considered as a total, in which two internal processes run and viewed from a centralized perspective and input-oriented condition. Therefore, the objective function measures the minimum cost of inputs relative to the cost for the production of the network outputs. The constraints of (4a) and (4c) indicate that the most optimal inputs of the first and second stage are on the cost line, and this line is at the intersection with the frontier efficiency curve. The constraint of (4b) indicates that the outputs of first stage are the same inputs of second stage and since we assume that the optimal input value produces an optimal output, z_{do} is equal to z^*_{do}. The constraint of (4d) restricts the output of the second stage. In Model (4), λ^1_j and λ^2_j are, respectively, the weights used for first stage and second stage, and x^*_{io} and z^*_{do} are the optimal inputs of first and second stages. Dual form of Model (4) is the same as that of Model (5).

$$ce_o^{centralized} = Max \sum_{r=1}^{s} u_r y_{ro}, \quad o \in J = \{1, \ldots, n\}$$

s.t.

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d^2 z_{dj} \le 0, \quad (j = 1, \ldots, n) \tag{5a}$$

$$\sum_{d=1}^{D} w_d^1 z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad (j = 1, \ldots, n) \tag{5b}$$

$$v_i = \frac{p_{io}}{\sum_{i=1}^{m} p_{io} x_{io}}, (i = 1, \ldots, m) \tag{5c}$$

$$w_d^1 = \frac{p'_{do}}{\sum_{d=1}^{D} p'_{do} z_{do}}, \quad (d = 1, \ldots, D) \tag{5d}$$

$$w_d^2 = \frac{p'_{do}}{\sum_{d=1}^{D} p'_{do} z_{do}}, \quad (d = 1, \ldots, D) \tag{5e}$$

$$u_r, v_i, w_d^1, w_d^2 \ge 0$$

In this model, v_i are the weights used for first-stage inputs and w_d are the weights of z_{dj}; p_{io} are the first-stage inputs prices and p'_{do} are the second-stage inputs prices. If we consider $w_d^1 = w_d^2 = w_d$ then (5d) or (5e) is deleted. Substituting (5c) and (5d) in Model (5), this model is converted to the following:

$$ce_o^{centralized} = Max \sum_{r=1}^{s} u_r y_{ro}, \quad o \in J = \{1, \ldots, n\}$$

s.t.

$$\sum_{d=1}^{D} w_d z_{dj} \le \frac{\sum_{i=1}^{m} p_{io} x_{ij}}{\sum_{i=1}^{m} p_{io} x_{io}}, \quad (j = 1, \ldots, n), \tag{6a}$$

$$\sum_{r=1}^{s} u_r y_{rj} \le \frac{\sum_{d=1}^{D} p'_{do} z_{dj}}{\sum_{d=1}^{D} p'_{do} z_{do}}, \quad (j = 1, \ldots, n), \tag{6b}$$

$$u_r, w_d \ge 0, \tag{6c}$$

Model (6) gives a total of $2j$ linear inequality constraints and $d + r$ variables.

In Model (5), we let $t_1 = \left(\sum_{i=1}^{m} p_{io} x_{io}\right)^{-1}$ and $t_2 = \left(\sum_{d=1}^{D} p'_{do} z_{do}\right)^{-1}$ so that $v_i = p_{io} t_1$, $w_d = p'_{do} t_2$ and $\sum_{i=1}^{m} p_{io} t_1 x_{io} = 1$ or $\sum_{i=1}^{m} v_i x_{io} = 1$, $\sum_{d=1}^{D} p'_{do} t_2 z_{do} = 1$ or $\sum_{d=1}^{D} w_d z_{do} = 1$. We suppose that all the outputs of stage 1 are the inputs of stage 2, so $w^1 = w^2 = w$. By applying Lemma 1 of Podinovski [44], CE of the two-stage network in the centralized mode is also proposed as Model (7). Using Propositions 3 and 4, we demonstrate that Model (7) can be converted into Model (6). Both of them compute the CE of the two-stage network in the centralized mode. Considering DMU_j as a total, two internal processes run in it; viewing them from a centralized perspective, the CE can be written in input-oriented condition as follows:

$$ce_o^{centralized} = Max \sum_{r=1}^{s} u_r y_{ro}, \quad o \in J = \{1, \ldots, n\}$$

s.t.

$$\sum_{i=1}^{m} v_i x_{io} = 1 \tag{7a}$$

$$\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad (j = 1, \ldots, n) \tag{7b}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0, \quad (j = 1, \ldots, n) \tag{7c}$$

$$v_{i^a} - \frac{p_{i^a o}}{p_{i^b o}} v_{i^b} = 0, \quad i^a < i^b (i^b = 1, \ldots, m) \tag{7d}$$

$$w_{d^p} - \frac{p'_{d^p o}}{p'_{d^q o}} w_{d^q} = 0, \quad d^p < d^q (d^q = 1, \ldots, D) \tag{7e}$$

$$u_r, v_i, w_d \ge 0$$

In this model v_{i^a} and v_{i^b} are the weights used for first-stage inputs; w_{d^p} and w_{d^q} are the weights of z_{dj}; $p_{i^a o}$ and $p_{i^b o}$ are the first-stage inputs prices; $p'_{d^p o}$ and $p'_{d^q o}$ are the second-stage inputs prices; a and b are used to prevent repetition of i, p and q are inputs and d is index desire selection. Model (6) has $C_2^m + C_2^D + 1$ constraints and m variables fewer than Model (7). In this model, the frontier is the lowest cost line of any DMU that cuts the PPS frontier. For example, in Figure 1, line $Cost = \min(p_1 x_1 + p_2 x_2)$ defines the value frontier of the CE of DMU_R. The objective function represents the minimum cost line that is written in Model (7). The constraints (7a)–(7c) ensure that the value frontier is against intersects PPS frontier. The constraints (7d) and (7e) show the difference between CE and TE models.

Models (6) and (7) are proposed to measure a two-stage network's CE. If the spaces of the Models (6) and (7) solutions are assumed, respectively, to be the sets S and S', it can be proven that these sets are closed and feasible as well as convex, and the spaces of the two solutions are the same.

Proposition 1 *The areas (sets) S and S' are possible and compact.*

Proof It is clear that (0,0,0) is a feasible solution for Models (6) and (7). Since all the constraints of Models (6) and (7) are linear, the sets S and S' are closed. Therefore, in Euclidean space the closed sets S and S' are compact.

Proposition 2 *The areas (sets) S and S' are convex.*

Proof Assume $\left(w'_1,\ldots,w'_d,u'_1,\ldots,u'_s\right)\in S$ and $\left(w''_1,\ldots,w''_d,u''_1,\ldots,u''\right)\in S$. From the constraint (6c) for every $\lambda\in[0,1]$ the following relationship is established:

$$\begin{aligned}&\lambda w'_d+(1-\lambda)w''_d\geq 0,\quad d=1,\ldots,D\\&\lambda u'_r+(1-\lambda)u''_r\geq 0,\quad r=1,\ldots,s.\end{aligned}$$

Thus, we have

$$\begin{aligned}\sum_{d=1}^{D}\left[\lambda w'_d+(1-\lambda)w''_d\right]z_{dj}&=\lambda\sum_{d=1}^{D}w'_dz_{dj}+(1-\lambda)\sum_{d=1}^{D}w''_dz_{dj}\\&\leq\lambda\frac{\sum_{i=1}^{m}p_{io}x_{ij}}{\sum_{i=1}^{m}p_{io}x_{io}}+(1-\lambda)\frac{\sum_{i=1}^{m}p_{io}x_{ij}}{\sum_{i=1}^{m}p_{io}x_{io}}\\&\leq\frac{\sum_{i=1}^{m}p_{io}x_{ij}}{\sum_{i=1}^{m}p_{io}x_{io}}\end{aligned}$$

$$\begin{aligned}\sum_{r=1}^{s}\left[\lambda u'_r+(1-\lambda)u''_r\right]y_{rj}&=\lambda\sum_{r=1}^{s}u'_ry_{rj}+(1-\lambda)\sum_{r=1}^{s}u''_ry_{rj}\\&\leq\lambda\frac{\sum_{d=1}^{D}p'_{do}z_{dj}}{\sum_{d=1}^{D}p'_{do}z_{do}}+(1-\lambda)\frac{\sum_{d=1}^{D}p'_{do}z_{dj}}{\sum\limits_{d=1}^{D}p'_{do}z_{do}}\\&\leq\frac{\sum_{d=1}^{D}p'_{do}z_{dj}}{\sum\limits_{d=1}^{D}p'_{do}z_{do}}.\end{aligned}$$

Based on this

$$\begin{aligned}&\lambda w'_d+(1-\lambda)w''_d\in S,\quad d=1,\ldots,D\\&\lambda u'_r+(1-\lambda)u''_r\in S,\quad r=1,\ldots,s.\end{aligned}$$

Hence

$$\lambda\left(w'_1,\ldots,w'_d,u'_1,\ldots,u'_s\right)+(1-\lambda)\left(w''_1,\ldots,w''_d,u''_1,\ldots,u''_s\right)\in S.$$

Therefore, it can be concluded that the set S is convex. Similarly, the area set of S' is convex. Appendix A proves this.

Proposition 3 *The v_i of Model (7) is equal to $v_i=\frac{p_{io}}{\sum_{i=1}^{m}p_{io}x_{io}}$.*

Proof Considering $i^a=1$ and $i^b=i,\ i=2,3,\ldots,m$ in Eq. (7d) we have

$$v_1-\frac{p_{1o}}{p_{io}}v_i=0,\ i=2,\ldots,m.$$

Thus

$$v_i=\frac{p_{io}}{p_{1o}}v_1,\ i=2,\ldots,m\tag{8}$$

Substituting the values of v_i in Eq. (7a), we get

$$\begin{aligned}1=\sum_{i=1}^{m}v_ix_{io}&=v_1x_{1o}+\sum_{i=2}^{m}\left(\frac{p_{io}}{p_{1o}}v_1\right)x_{io}\\&=v_1\left(x_{1o}+\sum\nolimits_{i=2}^{m}\frac{p_{io}}{p_{1o}}x_{io}\right)\end{aligned}$$

which implies that

$$\begin{aligned}v_1=\frac{1}{x_{1o}+\sum_{i=2}^{m}\frac{p_{io}}{p_{1o}}x_{io}}&=\frac{p_{1o}}{p_{1o}x_{1o}+\sum_{i=2}^{m}p_{io}x_{io}}\\&=\frac{p_{1o}}{\sum_{i=1}^{m}p_{io}x_{io}}\end{aligned}\tag{9}$$

Using (8) and (9), the values of other v_i are obtained as

$$v_i=\frac{p_{io}}{\sum_{i=1}^{m}p_{io}x_{io}}.$$

Proposition 4 *The w_d of Model (7) satisfies $w_d\leq\frac{p'_{do}}{\sum_{d=1}^{D}p'_{do}z_{do}}$.*

Proof Considering $d^p=1$ *and* $d^q=d,\ d=2,3,\ldots,D$ in Eq. (7e) we have

$$w_1-\frac{p'_{1o}}{p'_{do}}w_d=0,\ d=2,\ldots,D.$$

Thus

$$w_d=\frac{p'_{do}}{p'_{1o}}w_1,\ d=2,\ldots,D\tag{10}$$

Since inequality (7b) is established for any j, we have this for DMU_o as follows:

$$\sum_{d=1}^{D}w_dz_{do}-\sum_{i=1}^{m}v_ix_{io}\leq 0\tag{11}$$

Substituting the values of v_i and w_d in Eq. (11), we have
$w_1 z_{1o} + \sum_{d=2}^{D} \left(\frac{p'_{do}}{p'_{1o}} w_1\right) z_{do} - \sum_{i=1}^{m} \left(\frac{p_{io}}{\sum_{i=1}^{m} p_{io} x_{io}}\right) x_{io} \le 0$

Thus

$$w_1 \le \frac{\sum_{i=1}^{m} (\frac{p_{io}}{\sum_{i=1}^{m} p_{io} x_{io}}) x_{io}}{z_{1o} + \sum_{d=2}^{D} (\frac{p'_{do}}{p'_{1o}}) z_{do}} = \frac{1}{\sum_{d=1}^{D} (\frac{p'_{do}}{p'_{1o}}) z_{do}}$$

so

$$w_1 \le \frac{p'_{1o}}{\sum_{d=1}^{D} p'_{do} z_{do}} \tag{12}$$

Substituting the values of w_1 in Eq. (12), we have

$$w_d \le \frac{p'_{do}}{\sum_{d=1}^{D} p'_{do} z_{do}} \tag{13}$$

The values of other v_i are obtained via Eq. (13).

Proposition 5 *The areas (sets) S and S′ are equal.*

Substituting Propositions 3 and 4 in Model (7), this model is converted to Model (6).

Model (6) presents complete $2j$ constraints and $d+r$ variables. This equivalent model has $C_2^m + C_2^D + 1$ constraints and m variables less than Model (7) and in view of this the model is computationally economical, resulting in a sharp reduction in computational requirements. In summary, the efficiency measurement obtained from Model (6) is equal to the CE measurement obtained from Model (7). Thus for computational purposes we recommend using Model (6) instead of Model (7).

If we denote the optimal value to Model (6) as $ce_o^{centralized}$ centralized, then we have

$$ce_o^{centralized} = ce_o^{1,centralized} \times ce_o^{2,centralized}. \tag{14}$$

Note that optimal multipliers from Model (6) may not be unique, meaning that $ce_o^{1,centralized}$ and $ce_o^{2,centralized}$ may not be unique. To test for uniqueness, we can first determine the maximum achievable value of $ce_o^{1,centralized}$ via

$$\begin{aligned}
& ce_o^{1+} = Max \sum_{d=1}^{D} w_d z_{do}, \quad o \in J = \{1,\ldots,n\} \\
& \text{s.t.} \\
& \sum_{r=1}^{s} u_r y_{ro} = ce_o^{centralized} \\
& \sum_{d=1}^{D} w_d z_{dj} \le \frac{\sum_{i=1}^{m} p_{io} x_{ij}}{\sum_{i=1}^{m} p_{io} x_{io}}, \quad (j=1,\ldots,n), \\
& \sum_{r=1}^{s} u_r y_{rj} \le \frac{\sum_{d=1}^{D} p'_{do} z_{dj}}{\sum_{d=1}^{D} p'_{do} z_{do}}, \quad (j=1,\ldots,n), \\
& u_r, w_d \ge 0,
\end{aligned} \tag{15}$$

From Eq. (14), $ce_o^{2-} = \frac{ce_o^{centralized}}{ce_o^{1+}}$. The maximum of $ce_o^{2,centralized}$, denoted by ce_o^{2+}, can be determined in a manner similar to that discussed earlier, and the minimum of $ce_o^{1,centralized}$ is then calculated as $ce_o^{1-} = \frac{ce_o^{centralized}}{ce_o^{2+}}$. Note that if $ce_o^{1-} = ce_o^{1+}$ and only if $ce_o^{2-} = ce_o^{2+}$, also if $ce_o^{1-} = ce_o^{1+}$ or $ce_o^{2-} = ce_o^{2+}$, then $ce_o^{1,centralized}$ and $ce_o^{2,centralized}$ are calculated by Model (6) uniquely. If $ce_o^{1-} \ne ce_o^{1+}$ or $ce_o^{2-} \ne ce_o^{2+}$, then there will certainly be some flexibility in determining values for $ce_o^{1,centralized}$ and $ce_o^{2,centralized}$ [16].

4.2 *Non-cooperative model*

Leader–follower is one type of a non-cooperative game according to the concept of Stackelberg. Each stage is assumed to be a player; the first and second stages are leader and follower, respectively, so that the first-stage efficiency is more important and the second-stage efficiency (follower) is measured, according to the DEA efficiency of the leader stage. Liang *et al* [16] solved the first-stage efficiency, using the CCR model as follows:

$$\begin{aligned}
& e_o^{1*} = Max \sum_{d=1}^{D} w_d z_{do}, \quad o \in J = \{1,\ldots,n\} \\
& \text{s.t.} \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad (j=1,\ldots,n) \\
& w_d \ge 0; \quad v_i \ge 0;
\end{aligned} \tag{16}$$

Liang *et al* [16] showed that the leader-stage maintains its DEA efficiency by (17d). They solved the second-stage (follower) efficiency, using the CCR model as follows:

$$e_o^{2*} = Max(\sum_{r=1}^{s} u_r y_{ro})/e_o^{1*}, \quad o \in J = \{1,\ldots,n\}$$

s.t.

$$\sum_{i=1}^{m} v_i x_{io} = 1 \tag{17a}$$

$$\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad (j=1,\ldots,n) \tag{17b}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad (j=1,\ldots,n) \tag{17c}$$

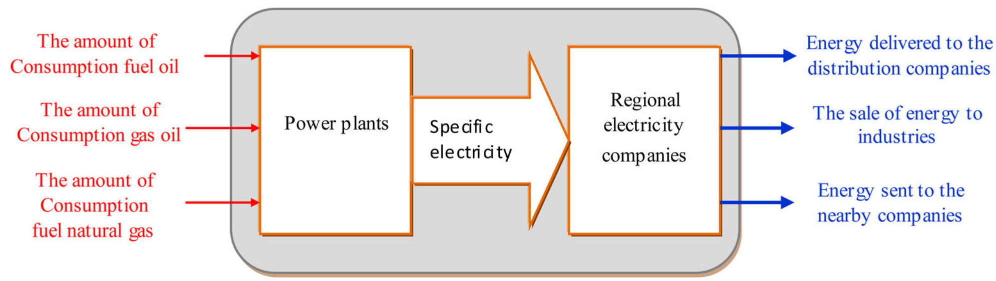

Figure 3. Conceptual model of production and distribution.

$$\sum_{d=1}^{D} w_d z_{do} = e_o^{1*} \tag{17d}$$
$$u_r, v_i, w_d \geq 0$$

When we consider the second stage to be the leader and the first stage to be the follower, then the same thing is done. First, we calculate the usual DEA efficiency for the second stage; then we solve the first stage (follower) efficiency with the constraint that the second stage value has already been calculated.

Now, we want to calculate CE in a two-stage network using Stackelberg game. From Figure 2, considering DMU$_j$ as a total in which two internal processes run and assuming that the first stage is the leader as in the Stackelberg model or leader–follower network point of view, CE for first stage (leader) can be written through Model (3) in input-oriented condition as Model (18). In this model, we consider the second stage output instead of the total network output.

$$ce_o^{1*} = Max \sum_{d=1}^{D} w_d z_{do}, \quad o \in J = \{1, \ldots, n\}$$
s.t.
$$\sum_{d=1}^{D} w_d z_{dj} \leq \frac{\sum_{i=1}^{m} p_{io} x_{ij}}{\sum_{t=1}^{m} p_{to} x_{to}} \quad (j = 1, \ldots, n) \tag{18}$$
$$w_d \geq \varepsilon \quad (d = 1, \ldots, D)$$

Based on Models (1) and (17), the CE of the second stage (follower) is expressed as follows:

$$ce_o^{2*} = \min \frac{\sum_{d=1}^{D} p'_{do} z^*_{do}}{\sum_{d=1}^{D} p'_{do} z_{do}}, \quad o \in J = \{1, \ldots, n\}$$
s.t. (19a)
$$\sum_{j=1}^{n} \lambda_j z_{dj} \leq z^*_{do}, \quad (d = 1, \ldots, D)$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad (r = 1, \ldots, s) \tag{19b}$$

$$\frac{\sum_{i=1}^{m} p_{io} x^*_{io}}{\sum_{i=1}^{m} p_{io} x_{io}} = ce_1^*, \quad o \in J = \{1, \ldots, n\} \tag{19c}$$

The constraints of (19a) and (19b) indicate that the optimal inputs of the second stage are on the cost line, and this line is at the intersection with the frontier efficiency curve. Since the second stage is the follower the CE model of the second stage should consider the CE of first stage, so the constraint (19c) is also added to Model (19). Dual form of Model (19) is as follows:

$$ce_o^{2*} = Max(\sum_{r=1}^{s} u_r y_{ro} + ce_o^{1*}\delta_1 - ce_o^{1*}\delta_2), \quad o \in J = \{1, \ldots, n\}$$
s.t.
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} + v_i\delta_1 - v_i\delta_2 \leq 0 \quad (j = 1, \ldots, n) \tag{20a}$$

$$w_d = \frac{p'_{do}}{\sum_{d=1}^{D} p'_{do} z_{do}}, \tag{20b}$$
$$u_r, w_d \geq 0, \delta_1 \ \ and \ \ \delta_2 \ are \ free$$

In this model w_d and u_r are the weights used for the first and second-stage outputs. Variables δ_1 and δ_2 are applied to the model due to restriction (19c); v_i is the same as $\frac{p_{io}}{\sum_{i=1}^{m} p_{io} x_{io}}$, it is certain and p'_{do} are the second-stage input prices. Substituting w_d in Model (20), this model is converted to the following:

$$ce_o^{2*} = Max(\sum_{r=1}^{s} u_r y_{ro} + ce_o^{1*}\delta_1 - ce_o^{1*}\delta_2),$$
$$o \in J = \{1, \ldots, n\}$$
s.t.
$$\sum_{r=1}^{s} u_r y_{rj} + v_i\delta_1 - v_i\delta_2 \leq \frac{\sum_{d=1}^{D} p'_{do} z^*_{dj}}{\sum_{d=1}^{D} p'_{do} z_{do}} (j = 1, \ldots, n)$$
$$u_r \geq 0, \delta_1, \delta_2 \text{ are } free \tag{21}$$

Table 2. Price of inputs.

DMU	Price of the first-stage inputs during time period 2006–2016						Specific electricity (million kWh)
	Fuel oil (million m^2)		Gas oil (million m^2)		Natural gas (million m^2)		
	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound	
Azerbaijan	23424.2	3007970	41374.5	1469264	8089.49	2795194	14140
Isfahan	34901	487666	85783	1513488	49.812	2985.5	17151
Bakhtar	26793	375927	58017.9	1036339	213.34	5943	13180
Tehran	30401	349736	206408.7	5033504	51288.8	647025	40964
Khorasan	1891.8	691730	101429.4	3060432	8526	156187	16530
Khuzestan	3435	176554	77221.6	2153528	1594.8	496118	13782
Sistan	11668	718276	0	0	21783.4	346964	4087
West	8588.1	145536	22978.9	1015723	1910	161227	8784
Fars	0	0	84441.5	3024748	8694.18	208388	18976
Kerman	3082	243580	28279.3	1138716	15222.8	255754	10047
Gilan	0	0	50751.8	1330760	8976.73	164690	9525
Mazandaran	15519	230377	74634.7	1702175	0	22.538	13661
Hormozgan	20714	171815	60410.1	1875824	1812.65	623301	12169
Yazd	0	0	13837.4	622741	4318.56	939459	4336

Table 3. Inputs and outputs values.

DMU	Inputs values (million m^2)			Intermediates values (million kWh)	Outputs values (million kWh)		
	Fuel oil	Gas oil	Natural gas	Specific electricity	Energy delivered to the distribution companies	The sale of energy to industries	Energy sent nearby companies
Azerbaijan	272092	3011	838814	3875137	10155	423	1593
Isfahan	0	0	764080	2539940	13098	5663	7468
Bakhtar	0	327	784911	11040722	9268	3394	8265
Tehran	486674	986	2648459	14239863	33990	2048	8425
Khorasan	332304	262682	3579700	53454	13750	1342	1415
Khuzestan	0	213720	1546932	10453851	21641	4165	5070
Sistan	0	57000	285119	3954744	3487	262	42
West	326252	335019	707753	4268176	4948	557	1764
Fars	702832	368	452567	1020660	13467	1121	5088
Kerman	0	19588	426760	7431531	7825	906	3000
Gilan	117071	98768	2342397	1442343	3786	119	5797
Mazandaran	0	62826	380052	10644237	8057	376	6161
Hormozgan	965690	0	1891574	13903268	5392	3161	3946
Yazd	842929	76664	3378252	4627749	3417	1070	1371

The second stage's CE (follower's cost efficiency) can be expressed as Model (21). This equivalent model has j linear constraints and $r+2$ variables. In the similar TE model (Model (17)), there are $2j+2$ constraints and $i+d+r$ variables. Model (21) is mathematically economical, leading to a strong reduction in computational requirements.

We calculated the CE of the first stage from Model (16) and the CE of the second stage from Model (21). The CE of the total network can be calculated from different methods. For example, we can multiply the CE of stages 1 and 2 or obtain their average. Eq. (22) shows the average of stages 1 and 2:

$$\frac{\sum_{d=1}^{D} w_d z_{do} + \sum_{r=1}^{s} u_r y_{ro} + ce_o^{1*}\delta_1 - ce_o^{1*}\delta_2}{2} \tag{22}$$

Furthermore, using an application to electrical network in Iran, we calculate the CE in both modes of cooperation and non-cooperation and we will investigate the results.

Table 4. Cost-efficiency measures by centralized and Stackelberg models.

	Centralized			Stage 1 as the leader in Stackelberg model			Stage 2 as the leader in Stackelberg model		
DMU	Stage 1	Stage 2	Overall	Stage 1	Stage 2	Overall	Stage 1	Stage 2	Overall
1 Azerbaijan	0.4942	0.3101	0.1532	0.4942	0.3101	0.1532	0.4045	0.3117	0.1261
2 Isfahan	0.7201	0.6900	0.4971	0.7201	0.6899	0.4968	0.7001	0.6781	0.4747
3 Bakhtar	0.6451	0.4319	0.2791	0.6451	0.4319	0.2786	0.6331	0.4101	0.2596
4 Tehran	0.4905	0.4411	0.22	0.4905	0.4411	0.2163	0.4895	0.4201	0.2056
5 Khorasan	1	1	1	1	1	1	1	1	1
6 Khuzestan	0.4601	0.3925	0.1812	0.4601	0.3931	0.1809	0.4578	0.3721	0.1703
7 Sistan	0.526	0.0639	0.0343	0.526	0.0639	0.0336	0.426	0.0558	0.0238
8 West	0.4253	0.3493	0.1541	0.4253	0.3493	0.1486	0.4153	0.3093	0.1284
9 Fars	1	0.9601	0.9601	1	0.9601	0.9601	0.9524	0.9801	0.9334
10Kerman	0.7341	0.2001	0.1501	1	0.1501	0.1469	0.7341	0.1951	0.1432
11Gilan	0.9972	0.8515	0.8561	0.9972	0.8515	0.8491	0.9972	0.8515	0.8491
12Mazandaran	0.9371	0.2297	0.2162	1	0.2162	0.2153	0.9371	0.2127	0.19932
13Hormozgan	0.4129	0.2308	0.1064	0.4129	0.2308	0.0953	0.4129	0.2308	0.0953
14Yazd	0.3673	0.2963	0.1103	0.3673	0.2959	0.1087	0.3573	0.2659	0.095

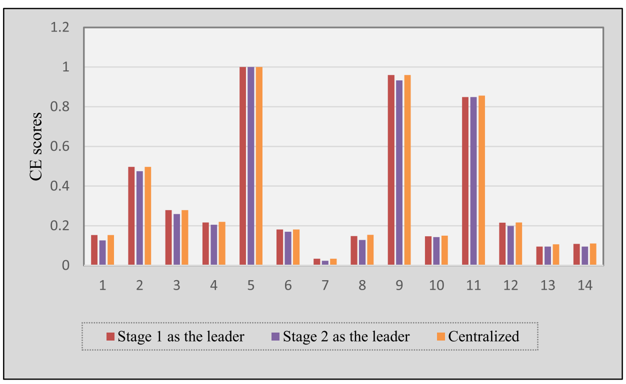

Figure 4. Comparison of centralized and Stackelberg cost-efficiency scores.

5. An application to electrical network in Iran

In this section a case study stated in Khalili-Damghani and Shahmir [45]' is used, which is related to a supply chain consisting of supplier and distributor of the electrical industry in Iran. Figure 3 shows the supply graphical structure with its inputs, outputs and intermediate variables.

The inputs of electricity power plants include the consumed fuel oil, gas oil and natural gas, and the output is the specific electricity. The input of the electricity distribution sector (regional electricity companies) is the same as the output of the production sector, and the output of the distribution sector consists of delivery of electricity to distribution companies, the sale of electricity to industries and the transmission of electricity to nearby regional electricity distribution companies.

In this study, we investigate 14 homogenous production and distribution networks and the input prices of stages 1 and 2 are considered. For this purpose, the input prices are monitored during the 2006–2016 period and their average values are considered, which are shown in Table 2.

Values of the input and output are collected from detailed reports of Iran's electricity power generation and energy balances in 2016. The definitions of inputs and outputs of Figure 3 are presented in Table 3.

CE measures, by centralized and Stackelberg models, yielded unique efficiency scores for all DMUs. Table 4 reports the efficiency results obtained from both non-co-operative (leader–follower) model and centralized model. The fifth column of Table 4 shows the CE of the overall in the centralized model, which is the result of Model (6). The columns 6 and 7 are results from Models 10 and 21, respectively. The eighth column shows multiplied scores of the columns 6 and 7. The results shown in columns 9–11 are related to the case where the second stage is leader and first stage is follower. This information is useful for identifying the sub-processes and the DMUs with weak efficiencies, and can be helpful for improving overall performance of electrical network in Iran. Table 4 indicates that the DMUs have significant scope for improvement of CE, since the average CE measures in centralized and Stackelberg models are only around 35%. Also, their input mix combination is not favourable to minimize costs in light of the current prices. In order to become fully efficient, these DMUs must reduce the input levels used and consider that the input prices will maintain the current values; they should also adjust the relative proportions between the inputs.

Figure 4 compares centralized and Stackelberg CE. As shown in Figure 4, centralized CE can be seen to be more than Stackelberg CE.

As shown in Figure 4 the CE and centralized model scores are almost similar when the first stage is the leader in the Stackelberg model, while the CE scores are reduced by about 2% in the second-stage leader. This research and case studies investigated bring the following managerial insights:

- Electricity costs can have a significant impact on the energy market; cost management and CE measurement can help to organize the energy market. Managers can easily identify and manage poor DMUs.
- The efficiency analysis reported in this paper uncovered the existence of considerable inefficiencies in electrical network in Iran. Managers have to increase the CE of poor DMUs by allocating optimal input factors.
- These large inefficiencies may be the fact that CE is not the sole driver in the management of the commercial activity of electrical networks. The policy of expanding cooperation in order to improve the CE of the network can lead to energy savings and increased efficiency.

6. Conclusion remarks and future research directions

This study proposes the centralized and Stackelberg models in order to make a fair and unique calculation of cost effectiveness in a situation where the prices of input data, intermediate data and output data are constant. This study examines the CE of DMUs that have two-stage network structures (or processes), where all the outputs from the first stage are the only inputs to the second stage. We proposed a simplified version of the two-stage DEA network CE model. The proposed model is applied in the electrical network of Iran. Applying the proposed model for CE measurement decreases the number of constraints as well as variables, which leads to a strong reduction in the computational requirements. The simplified CE DEA method can provide robust estimates of CE with prices constant. The results reported uncovered the existence of considerable inefficiencies in electrical network in Iran. An explanation for the presence of such large inefficiencies may be the fact that CE is not the sole driver in the management of the commercial activity of electrical networks. The policy of expanding cooperation in order to improve the CE of the network may lead to energy savings and increased efficiency.

For the future research, considering fuzzy or probable price in the proposed model is suggested. Also, input data, intermediate data or output data can be considered in an uncertain environment. Many of the two-stage networks are not pure networks and this model can be applied for other types of networks. Besides, the proposed model can be extended for three-stage networks and it can be the basis for pricing with the DEA approach.

Appendix A

In this part, we prove that the area set of S' is convex.

Proof Assume $\left(v'_1,\ldots,v'_m,w'_1,\ldots,w'_d,u'_1,\ldots,u'_s\right)\in S$ and $\left(v''_1,\ldots,v''_m,w''_1,\ldots,w''_d,u''_1,\ldots,u''\right)\in S$. Then for every $\lambda\in[0,1]$ the following relationship is established:

$$\begin{aligned}&\lambda v'_i+(1-\lambda)v''_i\geq 0,\quad i=1,\ldots,m\\&\lambda w'_d+(1-\lambda)w''_d\geq 0,\quad d=1,\ldots,D\\&\lambda u'_r+(1-\lambda)u''_r\geq 0,\quad r=1,\ldots,s.\end{aligned}$$

Thus, we have

$$\begin{aligned}\sum_{i=1}^{m}[\lambda v'_i+(1-\lambda)v''_i]x_{ij}=1\\&=\lambda\sum_{i=1}^{m}v'_ix_{ij}+(1-\lambda)\sum_{i=1}^{m}v''_ix_{ij}\\&=\lambda+(1-\lambda)=1\end{aligned}$$

$$\begin{aligned}\sum_{d=1}^{D}\left[\lambda w'_d+(1-\lambda)w''_d\right]z_{dj}&=\lambda\sum_{d=1}^{D}w'_dz_{dj}+(1-\lambda)\sum_{d=1}^{D}w''_dz_{dj}\\&\leq\lambda\sum_{i=1}^{m}v_ix_{ij}+(1-\lambda)\sum_{i=1}^{m}v_ix_{ij}\\&\leq\sum_{i=1}^{m}[\lambda v_i+(1-\lambda)v_i]x_{ij}\end{aligned}$$

$$\begin{aligned}\sum_{r=1}^{s}\left[\lambda u'_r+(1-\lambda)u''_r\right]y_{rj}&=\lambda\sum_{r=1}^{s}u'_ry_{rj}+(1-\lambda)\sum_{r=1}^{s}u''_ry_{rj}\\&\leq\lambda\sum_{d=1}^{D}w'_dz_{dj}+(1-\lambda)\sum_{d=1}^{D}w''_dz_{dj}\\&\leq\sum_{d=1}^{D}\left[\lambda w'_d+(1-\lambda)w''_d\right]z_{dj}\end{aligned}$$

$$\begin{aligned}&\lambda v'_{i^a}+(1-\lambda)v''_{i^a}-\frac{p_{i^ao}}{p_{i^bo}}(\lambda v'_{i^b}+(1-\lambda)v''_{i^b})\\&=\lambda(v'_{i^a}-\frac{p_{i^ao}}{p_{i^bo}}v'_{i^b})+(1-\lambda)(v''_{i^a}-\frac{p_{i^ao}}{p_{i^bo}}v''_{i^b})\\&=0\end{aligned}$$

The last constraint is similarly convex.
Based on this:

$$\begin{aligned}&\lambda w'_d+(1-\lambda)w''_d\in S,\quad d=1,\ldots,D\\&\lambda u'_r+(1-\lambda)u''_r\in S,\quad r=1,\ldots,s\\&\lambda v'_i+(1-\lambda)v''_i\in S,\quad i=1,\ldots,m.\end{aligned}$$

Hence

$$\lambda\left(v_1', \ldots, v_m', w_1', \ldots, w_d', u_1', \ldots, u_s'\right) + (1 - \lambda)\left(v_1'', \ldots, v_m'', w_1'', \ldots, w_d'', u_1'', \ldots, u_s''\right) \in S.$$

Therefore, it can be concluded that the set S is convex.

References

[1] Kao C 2014 Network data envelopment analysis: a review. *Eur. J. Oper. Res.* 239(1): 1–16

[2] Farrell M J 1957 The measurement of productive efficiency. *J. R. Stat. Soc. Ser. A* 120: 253–281

[3] Cook W D, Liang L and Zhu J 2010 Measuring performance of two-stage network structures by DEA: a review and future perspective. *Omega* 38 (6): 423–430

[4] Sexton T R and Lewis H F 2003 Two-stage DEA: an application to major league baseball. *J. Prod. Anal.* 19: 227–249

[5] Seiford L M and Zhu J 1999 Infeasibility of super-efficiency data envelopment analysis models. *Inf. Syst. Oper. Res.* 37(2): 174–186

[6] Zhu J 2000 Multi-factor performance measure model with an application to fortune 500 companies. *Eur. J. Oper. Res.* 123(1): 105–124

[7] Khalili-Damghani K, Taghavi-Fard M and Abtahi A R 2012 A fuzzy two-stage DEA approach for performance measurement: real case of agility performance in dairy supply chains. *Int. J. Appl. Decis. Sci.* 5: 293–318

[8] Liang L, Yang F, Cook W D and Zhu J 2006 DEA models for supply chain efficiency evaluation. *Ann. Oper. Res.* 145: 35–49

[9] Guo J, Liu J and Qiu L 2006 Research on Supply Chain Performance Evaluation Based on DEA/AHP model. In: *Proceedings of the Asia–Pacific Conference on Services Computing*, pp. 609–612

[10] Simsek B and Tüysüz F 2018 An application of network data envelopment analysis with fuzzy data for the performance evaluation in cargo sector. *J. Enterp. Inf. Manag.* 31(1)

[11] Chaluvadi S, Raut R and Gardas B B 2018 Measuring the performance efficiency of banks in a developing economy. *Benchmark. Int. J.* 25(2): 575–606

[12] Azadeh A, Zarrin M and Salehi N 2016 Supplier selection in closed loop supply chain by an integrated simulation-Taguchi-DEA approach. *J. Enterp. Inf. Manag.* 29(3): 302–326

[13] Kahraman C and Çebi F 2018 Fuzzy approaches in production research and information management. *J. Enterp. Inf. Manag.* 31(4): 490–491

[14] Ajirlo S F, Amirteimoori A and Kordrostami S 2019 Two-stage additive integer-valued data envelopment analysis models. *J. Modell. Manag.* 14(1): 199–213

[15] [15] Jalali-Naini S G, Paydar M and Fathian M 2013 Fuzzy stochastic linear programming-based approach for multiple departures single destination multiple travelling salesman problem. *Int. J. Oper. Res.* 17(4): 417–435

[16] Liang L, Cook W D and Zhu J 2008 DEA models for two-stage processes: game approach and efficiency decomposition. *Naval Res. Logist.* 55: 643–653

[17] [17] Shafiee M, HosseinzadehLotfi F, Saleh H and Ghaderi M 2016 A mixed integer bi-level DEA model for bank branch performance evaluation by Stackelberg approach. *Ind. Eng. Int.* 12: 81–91

[18] Du J, Liang L, Chen Y, Cook W D and Zhu J 2011 A bargaining game model for measuring performance of two-stage network structures. *Eur. J. Oper. Res.* 210: 390–397

[19] Chen Li, Wang W et al 2014 Measurement of China's regional economic efficiency and environmental efficiency: a DEA-NBG approach. In: *Proceedings of the International Conference on Management Science & Engineering*, pp. 687–694

[20] Li Y, Xie J, Wang M and Liang L 2016 Super efficiency evaluation using a common platform on a cooperative game. *Eur. J. Oper. Res.* 255: 884–892

[21] Jalali-Naini S G, Moini A and Jahangoshai Rezaee M 2013 Nash bargaining game model for two parallel stages process evaluation with shared inputs. *Int. J. Adv. Manuf. Technol.* 67: 475–484

[22] An Q, Wen Y, Ding T and Li Y 2019 Resource sharing and payoff allocation in a three-stage system: integrating network DEA with the Shapley value method. *Omega* 85: 16–25

[23] Amirkhan M, Didehkhani H, Khalili-Damghani K and Hafezalkotob A 2018 Measuring performance of a three-stage network structure using data envelopment analysis and Nash bargaining game: a supply chain application. *Int. J. Inf. Technol. Decis. Mak.* 17(05): 1429–1467

[24] Koronakos G, Sotiros D and Despotis D K 2019 Reformulation of network data envelopment analysis models using a common modelling framework. *Eur. J. Oper. Res.* 278(2): 472–480

[25] Cook W D and Zhu J 2012 Data envelopment analysis. a handbook on the modeling of internal structures and networks. International Series in Operations Research & Management Science 208

[26] Kao C and Hwang S N 2008 Efficiency decomposition in two-stage data envelopment analysis: an application to non-life insurance companies in Taiwan. *Eur. J. Oper. Res.* 185(1): 418–429

[27] Chen Y, Cook W D, Li N and Zhu J 2009 Additive efficiency decomposition in two stage DEA. *Eur. J. Oper. Res.* 196: 1170–1176

[28] Maleki S and Matin R K 2019 A Russell-based model for estimating overall and divisional efficiency in two-stage production systems with sets of convex hulls in intermediate products. *Sādhanā* 44(3): 69

[29] Cooper W W, Thompson R G and Thrall R M 1996 Extensions and new developments in DEA. *Ann. Oper. Res.* 66: 3–45

[30] Camanho A S and Dyson R G 2005 Cost efficiency measurement with price uncertainty: a DEA application to bank branch assessments. *Eur. J. Oper. Res.* 161: 432–446

[31] Jahanshahloo G R, Soleimani-damaneh M and Mostafaee A 2008 A simplified version of the DEA cost efficiency model. *Eur. J. Oper. Res.* 184: 814–815

[32] Bagherzadeh Valami H 2009 Cost efficiency with triangular fuzzy number input prices: an application of DEA. *Chaos Solitons Fractals* 42(3): 1631–1637

[33] Lei F and Li H 2013 Duality and efficiency computations in the cost efficiency model with price uncertainty. *Comput. Oper. Res.* 40: 594–602

[34] Conceição M, Portela A S and Thanassoulis E 2014 Economic efficiency when prices are not fixed: disentangling quantity and price efficiency. *Omega* 47: 36–44
[35] Toloo M 2016 A cost efficiency approach for strategic vendor selection problem under certain input prices assumption. *Measurement* 85: 175–183
[36] Garfamy R M 2006 A data envelopment analysis approach based on total cost of ownership for supplier selection. *J. Enterp. Inf. Manag.* 19(6): 662–678
[37] Sarkar S 2019 Detection of cost leaders using non-central principal component analysis and data envelopment analysis. *Benchmark. Int. J*. 26(2): 548–566
[38] Cooper W W and Seiford L M 2000 Data envelopment analysis. Kluwer Academic Publishers
[39] Fukuyama H and Matousek R 2011 Efficiency of Turkish banking: two-stage network system. Variable returns to scale model. *J. Int. Financ. Mark. Inst. Money* 21: 75–91
[40] Lozano S 2011 Scale and cost efficiency analysis of networks of processes. *Expert Syst. Appl.* 38: 6612–6617
[41] Wanke P and Barros C 2014 Two-stage DEA: an application to major Brazilian banks. *Expert Syst. Appl.* 41: 2337–2344
[42] Petridis K, Ünsal M G, Dey P K and Örkcü H H 2019 A novel network data envelopment analysis model for performance measurement of Turkish electric distribution companies. *Energy* 174: 985–998
[43] Schaffnit C, Rosen D and Paradi J C 1997 Best practice analysis of bank branches: an application of DEA in a large Canadian bank. *Eur. J. Oper. Res.* 98: 269–289
[44] Podinovski V V 2001 Validating absolute weight bounds in data envelopment analysis (DEA) models. *J. Oper. Res. Soc.* 52(2): 221–225
[45] Khalili-Damghani K and Shahmir Z 2015 Uncertain network data envelopment analysis with undesirable outputs to evaluate the efficiency of electricity power production and distribution processes. *Comput. Ind. Eng.* 88: 131–150