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Influences of gear design parameters on dynamic tooth loads and time-varying mesh stiffness of involute spur gears

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Abstract. Gears are one of the most significant machine elements in power transmission due to the many advantages such as high load capacity, long life, and reliability. Due to the increasing power and speed values, the characteristics expected from the transmission elements are also increasing. Significant changes occur in the dynamic behavior of the gears at high speed due to the resonance. For this reason, determining the resonance frequencies is becoming an important issue for designers. This paper presents a method for determining the resonance regions of the gear system under different design parameters. The main purpose of this study is to examine the effects of different gear design parameters on spur gear dynamics. For this aim, the effects of these parameters on the mesh stiffness and contact ratio are examined, and the interaction of mesh stiffness, contact ratio, and dynamic response is presented. Different mesh stiffness calculation methods used to calculate time-varying mesh stiffness and a parametric gear dynamic model are proposed. To solve the equations of motions, a computer program is developed in MATLAB software. Five different design parameters, which are teeth number, pressure angle, reduction ratio, profile shifting factor, addendum factor, and damping ratio, are taken into consideration. The dynamic factor variation is calculated for 1600 rpm a constant pinion speed for each parameter for a single mesh period. Furthermore, the dynamic factor is calculated for the pinion speed between 400–30000 rpm and the frequency response and the resonance regions of the gear system are defined. As a result of the study, the profile shifting and the addendum factor are the most effective two parameters on the gear dynamics. Also, the contact ratio and mesh stiffness have a great effect on the dynamic response of the system. The methods decreasing dynamic load factors are also discussed at the end of the study.

Keywords. Dynamic analysis; spur gear; dynamic tooth loads; mesh stiffness.

1. Introduction

Gears are the most common transmission elements in the industry. They are used from small watches to huge wind turbines. Nowadays, due to the increase in power and speed in the machines, the performances expected from the transmission organs are increasing too. For this reason, the issue of gear dynamics is gaining special importance. Designers must design the gearboxes with minimum dynamic loads because high dynamic loads lead to negative effects on the gearboxes such as vibration, noise, low service life, etc. Consequently, researchers try to minimize these negative effects.

Ozguven *et al* [1] summarized the mathematical models about gear dynamic analysis in their review paper. Dai *et al* [2] investigated dynamic tooth root strains both experimentally and numerically for static and dynamic conditions of the gear pair. The shapes of dynamic strain curves were determined for different pinion speeds. The resonance regions of the gear pairs were defined by using both methods. Khabou *et al* [3] developed a gear dynamic model for transient regime conditions including motor torque, load changes and mesh stiffness variations due to different rotation speeds. Two motor models, which were electric and diesel, were defined for the dynamic analysis. The authors indicated that until the motors reach the nominal rotation speed, the vibration levels were high; therefore, designers should choose suitable couplings to minimize the effect of varying torques and speed on the gear dynamic response. Jeong [4] studied the effects of the one-way clutch on the non-linear behavior of gear dynamics. A mathematical model was developed to get a dynamic response of the one-stage spur gear system. Firstly, without the one-way clutch, the dynamic analysis was done and secondly, the one-way clutch was added to the model. It was seen that a one-way clutch had a great impact on the

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gear dynamics, the dynamic forces were reduced however clutch damping had little effect on the gear system. Kuang *et al* [5] investigated tooth wear effect on the dynamic characteristics of spur gear systems. A gear dynamic model was developed with a wear prediction model. The sliding wear in the initial period on the flanks reduces the dynamic loads as tip relief. However, the gear teeth are damaged due to wear. Guangjian *et al* [6] investigated backlash caused by gear eccentricity effect on the dynamic transmission error by using the finite element method.

Minimizing the dynamic forces of the gear systems is the biggest interest and concern of the designers. Therefore many researchers try to find the most effective solution which is diminishing the dynamic forces of the gear systems. Tooth profile modifications are one of the most powerful ways to reduce dynamic force. Lin *et al* [7] studied the dynamic response of the spur gear systems by using both linear and parabolic tooth profile modification techniques. A computer program was developed to analyze the dynamic performance of the spur gear. The dynamic analyses were done for the various speeds, loading, and the amount of modification. The dynamic forces were decreased with the tooth profile modification but an optimization study was needed. Liu *et al* [8] studied the influence of optimum tooth modification on spur gear dynamic behavior. A 10 degree of freedom parametric gear dynamic model was presented. Various modification types and lengths were investigated. The authors indicated that when the modification amount increased the dynamic load factor behavior changes as a 'V' type. At the end of the study, the optimal points of the dynamic loads were determined. Divandari *et al* [9] developed a six-degree-of-freedom dynamic model for spur gear dynamics. The effects of tooth modification and tooth localized defects were investigated. The root relief method, which reduced the dynamic forces, was defined as the most effective parameter on gear dynamic among the selected parameters. Yoon *et al* [10] offered a new tooth form, which was a cubic spline tooth form, to minimize dynamic loads for the spur gear. A parametric dynamic model was developed and dynamic analyses were conducted. The results indicated that the cubic spline profile reduces the dynamic loads as linear and parabolic tip relief.

The contact ratio also has a critical effect on gear dynamic response. The contact ratio defines the meshing process quality between two teeth. Researchers reduced the dynamic forces with increasing contact ratio in different ways. Liou *et al* [11] investigated the influence of contact ratio on the dynamic response of spur gears with no profile modifications. They varied the contact ratio of spur gear pair from 1.2 to 2.4 also to understand pure contact ratio effect, other gear parameters was taken constantly. The contact ratio was increased with the increase in of tooth addendum factor. When the contact ratio increases the dynamic performance of the gear pair increases, both low and high contact ratio spur gears. The high contact ratio gears had lower dynamic forces than low contact ratio spur gears. Kahraman and Blankenship [12] studied the impact of contact ratio on gear dynamics experimentally. A four-square test setup was used for the experiments. Gear pairs with different contact ratios were tested in the experiments, the dynamic transmission errors were measured. When the contact ratios of gear pairs are increased the dynamic transmission errors of the gear pairs are decreased. Karpat *et al* [13] investigated asymmetric gear dynamic performance with the developed computer program. The different pressure angle effects on dynamic load factor were explained. To decrease the dynamic load factor for asymmetric gears, the authors increased the tooth addendum factor. In this way, considerable reductions were achieved in dynamic loads for asymmetric gears.

Friction and damping also affect the gear dynamic load factors. The following papers Zheng *et al* [14], Xue *et al* [15] are investigated the effect of damping on the dynamic load factor. The damping was shown to suppress the impact of vibration. When the damping increases the gear vibrations are more stable. Li and Mao [16] investigated friction effects on contact analysis of the gear pair statically and dynamically. The authors indicated that reliable friction can decrease noise slightly and the transmission error becomes more stable and regular.

The main purposes of this study are to examine the effects of various gear design parameters on gear dynamic response, time-varying mesh stiffness and contact ratio of the gearbox. To accomplish these objectives, initially finite element model was created to calculate single tooth stiffness, then by using single tooth stiffness values, the time-varying mesh stiffness was calculated for gears under different gear design parameters. A parametric dynamic model was proposed to understand the dynamic behavior of the gear system. The equations of motions of the gear systems which are taken into consideration were created. A computer program was developed to solve these equations of motions and dynamic tooth forces were calculated. Dynamic factors of the gear systems were given as a function of pinion rotation speed and the resonance regions were defined. Thus, the effects of each selected parameter on dynamic loads were determined. The ways of minimizing dynamic loads were also discussed at the end of the study.

2. Material and method

2.1 *Single tooth stiffness*

During the operation of the gears, due to the loads on the tooth, in the direction of the load a certain elastic deformation occurs on the tooth. This deformation is expressed as the summation of tooth bending, shear and Hertzian contact deformation [17]. The total load applied to the tooth will be indicated by ''P'' and the total deformation on the tooth geometry by ''X''. The single tooth stiffness can be calculated as the ratio of the total force to the total deformation acting on the tooth's radius. The single tooth stiffness can be calculated at any radius of the gear using;

$$k_{p\iota} = \frac{P}{X_{p\iota}} \tag{1}$$

$$k_{g\iota} = \frac{P}{X_{g\iota}} \tag{2}$$

$$k_{p\iota\iota} = \frac{P}{X_{p2}} \tag{3}$$

$$k_{g\iota\iota} = \frac{P}{X_{g2}} \tag{4}$$

Tooth stiffness is one of the most important gear parameters in gear dynamics. Thus it has to be calculated very precisely. The literature describes several methods. These methods are divided into three basic areas such as analytical, numerical and experimental methods. Lin [17] calculated single tooth stiffness by using the analytical method. In this method, the single gear tooth thought as a cantilever beam and bending, shear and Hertzian contact deformations were calculated separately. Karpat *et al* [18] developed a new formula for single tooth stiffness of external spur gears with asymmetric gear. In that paper, the authors used the finite element method for calculating single tooth stiffness. Munro *et al* [19] investigated to calculate single tooth stiffness with the experimental method. In that study, a back to back gear dynamics test rig setup was used to measure static transmission errors of the gear pairs. By using this static transmission error values the single tooth stiffness tried to calculate. Kuang *et al* [5–20] developed a formula to calculate single tooth stiffness for standard symmetric spur gears. The formulation of the study can be seen below Eqs. (5)–(9). Also in this study for the calculation of single tooth stiffness of standard symmetric spur gears, these formulas are used. However, in this study, nonstandard gear types are also used for dynamic analysis.

$$\bar{K_i}(r) = (A_O + A_1 x_i) + (A_2 + A_3 x_i)\frac{(r - R_i)}{(1 + x_i) * m} \tag{5}$$

$$A_0 = 3.867 + 1.612 z_i - 0.02916 z_i^2 + 0.0001553 z_i^3 \tag{6}$$

$$A_1 = 17.060 + 0.7289 z_i - 0.0173 z_i^2 + 0.000099 z_i^3 \tag{7}$$

$$A_2 = 2.637 - 1.222 z_i + 0.02217 z_i^2 - 0.0001179 z_i^3 \tag{8}$$

$$A_3 = -6.330 - 1.033 z_i + 0.02068 z_i^2 - 0.000113 z_i^3 \tag{9}$$

The single tooth stiffness of nonstandard gears are calculated by using the finite element method. For this purpose 2D finite element analysis is done. ANSYS Workbench 14.0 software is used for the finite element analysis. The element type is chosen Plane-82 which has 2-DOF and 8 nodes as shown in figure 1. In order that the Hertzian contact deformation can be accurately calculated, the element size is determined using [21];

$$\frac{e}{b_h} = -0,2\left(\frac{c}{e}\right) + 1.2; \quad \text{for} \quad 0.9 \le \frac{c}{e} \le 3. \tag{10}$$

where "c" and "e" are the element length and width as shown in figure 1. In this study, the values of "*c*" and "*e*" are chosen equal to each other and the element length, "*e*", is calculated using Eq. (10). Hertzian contact width $\boldsymbol{b_h}$ is calculated using;

$$b_h = 2,15\sqrt{\frac{P\left(2\rho_p\rho_g/\left(\rho_p + \rho_g\right)\right)}{E}}. \tag{11}$$

where P, is the force exerted on the gear, E, is the modulus of elasticity of the teeth and $\boldsymbol{\rho_P}$ and $\boldsymbol{\rho_g}$ are the radius of curvature at the contact points of the teeth.

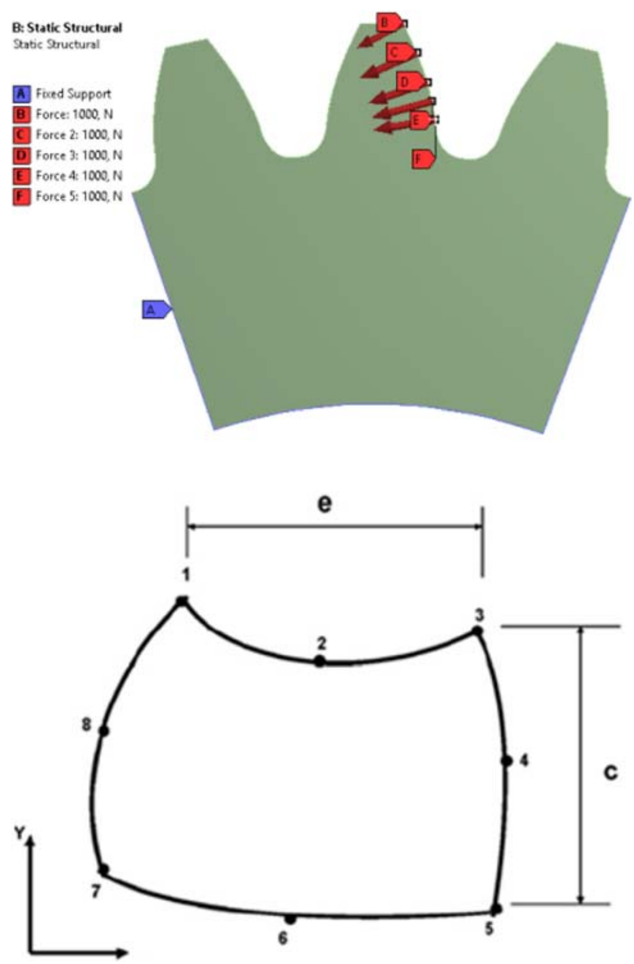

Figure 1. FEA boundary conditions and Plane-82 element model.

The boundary conditions of the analysis are given in figure 1. The force which is defined 1000 N applied on the six different points. Since each selected point has a different radius, the pressure angle that the force will apply on the gear will be different. The pressure angle for each point is calculated according to Eq. (12). As a result of the analysis, the deformation values are given; by using these displacement values and Eqs. (1)–(4), the single tooth stiffness of the gear can be calculated.

$$r_0 * \cos\alpha = r_{(i)} * \cos\alpha_{(i)} \tag{12}$$

where $\boldsymbol{r_0}$: radius of the pitch circle, $\boldsymbol{\alpha}$ = 20° pressure angle on the pitch circle, $\boldsymbol{r_{(i)}}$: any radius of on the gear, $\boldsymbol{\alpha_{(i)}}$: pressure angle on any radius.

2.2 *Gear mesh stiffness*

During the meshing process of the low contact ratio spur gears, the moment transfer is carried out by entering and exiting the one and two gear pairs in turn, respectively. This torque transfer takes place on a straight line in a spur gear with an involute profile. This line can be regarded specifically as the line of action and the whole meshing phenomenon is regarded as taking place on this line as shown in figure 2.

The calculation time-varying mesh stiffness has always been in the interest of researchers. Liang *et al* [22] presented an analytical method to calculate planetary gear set time-varying mesh stiffness with tooth cracks. The crack propagation effect on mesh stiffness was investigated. Liang *et al* [23] also calculated time-varying mesh stiffness of external spur gear with tooth pitting by using the potential energy method. Wan *et al* [24] calculated helical gears, mesh stiffness with different crack levels by using accumulated integral potential energy method and validated the analytic method with the finite element method. Both results matched well after that the dynamic analysis was conducted. Chen *et al* [25] investigated the effects of profile shifting on internal gear time-varying mesh stiffness with the analytical method. The mesh stiffness of internal gears increased with positive profile shifting. However, the negative profile shifting decreased the mesh stiffness of the internal gears.

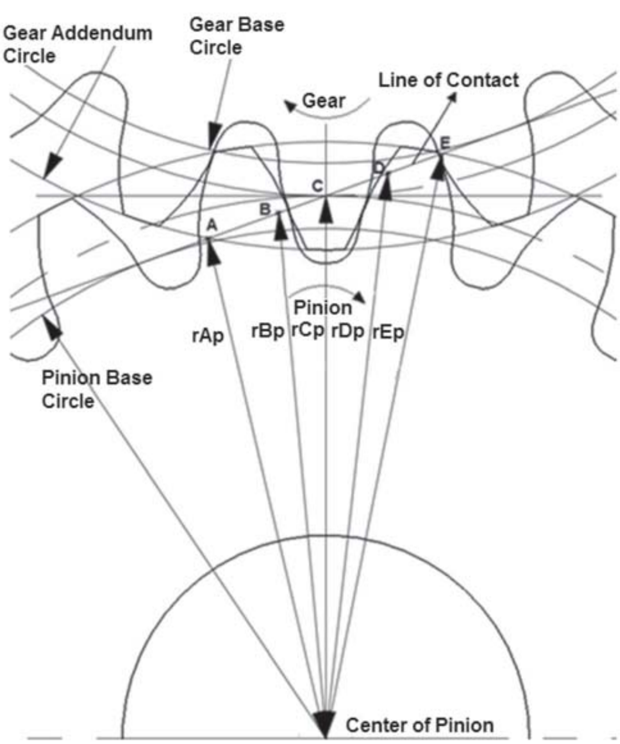

Figure 2. Meshing process in spur gear.

The meshing process starts at the addendum circle of the driven gear. Also for the pinion gear, the meshing process starts at a specific point which is upper from the pinion base circle and illustrated with the point ‘‘A’’ in figure 2. The radius of the point ‘‘A’’ could be calculated with the help of Eq. (13) [26].

$$r_{Ap} = \left[r_{bp}^2 + \left(\left(r_{0p} + r_{0g} \right) \sin\alpha - \left(r_{ag}^2 - r_{bg}^2 \right)^{0.5} \right)^2 \right]^{0.5}. \tag{13}$$

While the contact point moving from point A, a pair of gears comes out of the two pairs of gears in contact, and all of the torque begins to be transmitted over the single gear pair. This special point is called the lowest point of single tooth contact (LPSTC). The LPSTC point for the pinion can be calculated by using Eq. (14).

$$r_{Bp} = \left[r_{bp}^2 + \left(\left(r_{ap}^2 - r_{bp}^2 \right)^{0.5} - \pi m_n \cos\alpha \right)^2 \right]^{0.5}. \tag{14}$$

After LPSTC the torque is transmitted on the on gear pair until the point of D. The ‘‘D’’ is defined as the highest point of single tooth contact (HPSTC). Between B-D the torque is also transmitted to the point of ‘‘C’’ which is the pitch circles of the gears. The radius of pitch circles and HPSTC are calculated the using;

$$r_{Cp} = 0.5 m_n z_p \tag{15}$$

$$r_{Dp} = \left[r_{bp}^2 + \left(\left(r_{bp} + r_{bg} \right) \tan\alpha - \left(r_{ag}^2 - r_{bg}^2 \right)^{0.5} + \pi m_n \cos\alpha \right)^2 \right]^{0.5}. \tag{16}$$

The contact ends at the point of ‘‘E’’ where the addendum circle of the pinion. The radius of point ‘‘E’’ could be calculated according to Eq. (17). Between D-E the second gear pair also enters the contact. Thus, the moment transmitted over the double gear pairs between A-B and D-E.

However, it is transmitted over the single gear pair between B and D. The transmission zones on the tooth profile are illustrated for one tooth in figure 3.

$$r_{Ep} = 0.5\left(m_n z_p + 2m\right). \tag{17}$$

The length of each region on the line of action is given as;

$$|\mathrm{AE}| = \sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - a_d \sin\alpha \tag{18}$$

$$|\mathrm{AD}| = \pi m_n \cos\alpha \tag{19}$$

$$|\mathrm{AC}| = |\mathrm{CE}| = |\mathrm{AE}|/2 \tag{20}$$

$$|\mathrm{AB}| = |\mathrm{AE}| - \pi m_n \cos\alpha. \tag{21}$$

|AE| is called contact length. The contact ratio is also defined as the ratio of contact length to pitch on the base circle. The contact ratio can be calculated with Eq. (22) for the low contact ratio spur gears. The contact ratio indicates how smoothly the gears transmitted the moment. The maximum contact ratio for the standard spur gears can be 1.98.

$$\varepsilon_\alpha = \frac{|\mathrm{AE}|}{\pi m_n \cos\alpha}. \tag{22}$$

When the mesh stiffness is calculated, the teeth that touch each other act as a series of connected springs. The other gear pair from the rear acts as a spring connected parallel to the first gear pair. For this reason, in order to calculate the mesh stiffness, it is necessary to know the region on the line of action.

The equivalent stiffness between the first pair of teeth;

$$\boldsymbol{K_1} = \frac{\boldsymbol{k_{g1} k_{p1}}}{\boldsymbol{k_{g1} + k_{p1}}}. \tag{23}$$

Equivalent stiffness between the second gear pair;

$$\boldsymbol{K_2} = \frac{\boldsymbol{k_{g2} k_{p2}}}{\boldsymbol{k_{g2} + k_{p2}}}. \tag{24}$$

If the contact in the single tooth contact region (Between |BD|);

$$K_1 \neq 0 \text{ and } K_2 = 0.$$

If the contact in the double tooth contact regions (Between |AB| and |DE|);

$$K_1 \neq 0 \text{ and } K_2 \neq 0.$$

The mesh stiffness can be obtained according to the gear contact point. The mesh stiffness expression according to the contact points can also be obtained according to different variables such as gear rotation angle, contact time. In this study, the mesh stiffness is defined according to a contact point on the line of action. Typical mesh stiffness variations with respect to the contact point of the line of action shown in figure 4.

2.3 *Parametric dynamic model of spur gears*

The moment transmission over the gears is not stable. During the moment transmission of the teeth, dynamic loads are acting on the teeth. To determine these dynamic forces the equations of motion between the teeth need to be derived. The equations of motions between two teeth are formulated by using figure 5.

$$J_g \ddot{\theta}_g = r_{bg}(F_1 + F_2) - T_g \pm \rho_{g1}\mu_1 F_1 \pm \rho_{g2}\mu_2 F_2 \tag{25}$$

$$J_P \ddot{\theta}_P = T_p - r_{bp}(F_1 + F_2) \pm \rho_{p1}\mu_1 F_1 \pm \rho_{p2}\mu_2 F_2 \tag{26}$$

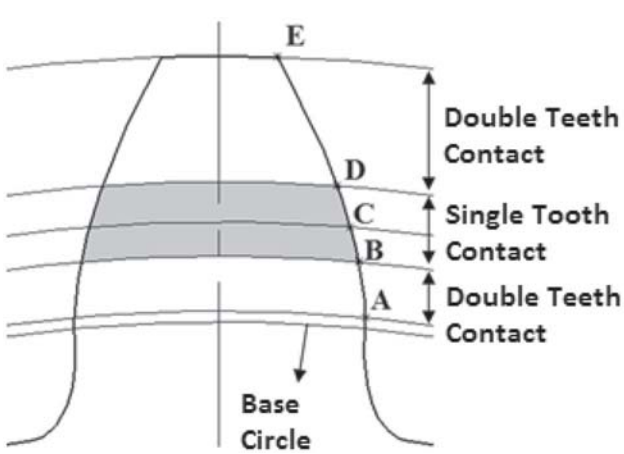

Figure 3. Single and double contact zones on the tooth profile.

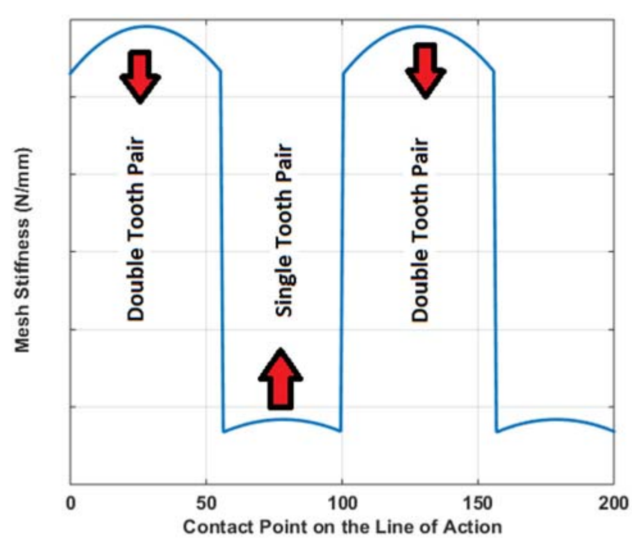

Figure 4. Mesh stiffness variation for one mesh cycle.

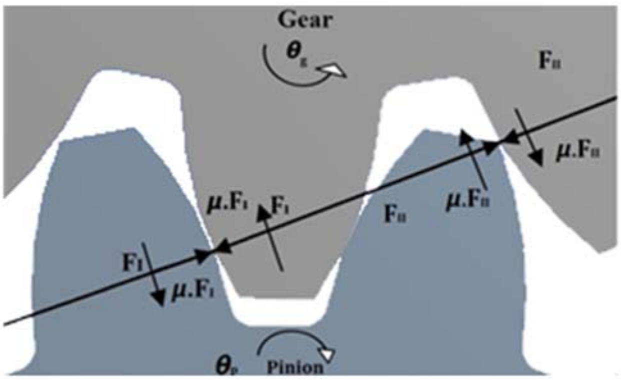

Figure 5. Schematic free body diagram of teeth pairs.

In the above equations, $\boldsymbol{J_g}$ and $\boldsymbol{J_p}$ are defined as the pinion and gear polar mass moment of inertia. $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are the dynamic loads and $\boldsymbol{T_g}$ and $\boldsymbol{T_p}$ are the transmitted moment. $\boldsymbol{\mu_1}$,$\boldsymbol{\mu_2}$ are the coefficient of friction and$\boldsymbol{\rho_p}$, $\boldsymbol{\rho_g}$ are the radius of curvature, which varies according to the contact point. The coefficient of friction between the two teeth is calculated according to Eq. (48) $\boldsymbol{\theta_p}$, $\boldsymbol{\theta_g}$ are the angular displacements, and $\boldsymbol{r_{bp}}$, $\boldsymbol{r_{bg}}$ are the base radii of the pinion and gear respectively.

$$X_g = r_{bg}\theta_g \tag{27}$$

$$X_p = r_{bp}\theta_p. \tag{28}$$

The relative displacement, speed, and acceleration can be expressed as follows;

$$X_r = X_p - X_g \tag{29}$$

$$V_r = \dot{X}_p - \dot{X}_g \tag{30}$$

$$\ddot{X}_r = \ddot{X}_p - \ddot{X}_g. \tag{31}$$

Masses of pinion and gear;

$$m_g = \frac{J_g}{r_{bg}^2}. \tag{32}$$

$$m_p = \frac{J_p}{r_{bp}^2}. \tag{33}$$

The static load that applied on the gears can be expressed as;

$$F_S = \frac{T_g}{r_{bg}} = \frac{T_p}{r_{bp}}. \tag{34}$$

By using upper expressions the new equations of motions can be written as follows.

$$m_g\ddot{X}_g = (F_1 + F_2) - F_S \pm \frac{\rho_{g1}\mu_1 F_1}{r_{bg}} \pm \frac{\rho_{g2}\mu_2 F_2}{r_{bg}} \tag{35}$$

$$m_P\ddot{X}_p = F_S - (F_1 + F_2) \pm \frac{\rho_{p1}\mu_1 F_1}{r_{bp}} \pm \frac{\rho_{p2}\mu_2 F_2}{r_{bp}}. \tag{36}$$

Total profile error can be described;

$$e_1 = e_{g1} + e_{p1} \tag{37}$$

$$e_2 = e_{g2} + e_{p2}. \tag{38}$$

The friction factors can be written in;

$$s_{g1} = 1 \pm \frac{\rho_{g1}\mu_1}{r_{bg}} \tag{39}$$

$$s_{p1} = 1 \pm \frac{\rho_{p1}\mu_1}{r_{bp}} \tag{40}$$

$$s_{g2} = 1 \pm \frac{\rho_{g2}\mu_2}{r_{bg}} \tag{41}$$

$$s_{p2} = 1 \pm \frac{\rho_{p2}\mu_2}{r_{bp}} \tag{42}$$

If the equations (35) and (36) subtract each other and the upper expressions are written in the new equation; the equations of motions can be defined as Eq. (43).

$$\ddot{X} + 2\left[\frac{K_1\left(s_{P1}m_g + s_{g1}m_p\right) + K_2\left(s_{p2}m_g + s_{g2}m_p\right)}{m_g m_P}\right]^{1/2} \xi\dot{X}_r + \frac{K_1\left(s_{p1}m_g + s_{g1}m_p\right) + K_2\left(s_{P2}m_g + s_{g2}m_p\right)}{m_G m_P}X_r = \frac{\left(m_g m_p\right)F_s + K_1 e_1\left(s_{p1}m_g + s_{g1}m_p\right) + K_2 e_2\left(s_{p2}m_g + s_{g2}m_p\right)}{m_g m_p} \tag{43}$$

In short form the equation of motions can be written as;

$$\ddot{X}_r + 2\omega\xi\dot{X}_r + \omega^2 X_r = \omega^2 X_s. \tag{44}$$

The natural frequencies of the system can be written as;

$$\omega^2 = \frac{K_1\left(s_{P1}m_g + s_{g1}m_p\right) + K_2\left(s_{P2}m_g + s_{g2}m_p\right)}{m_g m_p} \tag{45}$$

$$\omega^2 X_s = \frac{\left(m_g m_p\right)F_s + K_1 e_1\left(s_{p1}m_g + s_{g1}m_p\right) + K_2 e_2\left(s_{p2}m_g + s_{g2}m_p\right)}{m_g m_p}. \tag{46}$$

The static transmission error is described in Eq. (47).

$$X_s = \frac{\left(m_g m_p\right)F_s + K_1 e_1\left(s_{p1}m_g + s_{g1}m_p\right) + K_2 e_2\left(s_{p2}m_g + s_{g2}m_p\right)}{K_1\left(s_{P1}m_g + s_{g1}m_p\right) + K_2\left(s_{P2}m_g + s_{g2}m_p\right)}. \tag{47}$$

Finally, the dynamic force between two teeth is defined taking into account profile errors;

$$F_1 = K_1(X_r - e_1) \tag{48}$$

Table 1. Variation of investigated design parameters.

Gear parameters	Case I	Case II	Case III	Case IV	Case V	Case VI
Module (m_n) (mm)	3.18	3.18	3.18	3.18	3.18	3.18
Pinion teeth number (z_p)	28–50–80	28	28	28	28	28
Gear teeth number (z_g)	28–50–80	28	28–42 56–112	28	28	28
Tooth face width, (b) (mm)	25.4	25.4	25.4	25.4	25.4	25.4
Reduction ratio (i)	1	1	1–1.5 2-4	1	1	1
Pressure angele (α) (deg)	20	18–20 25–30	20	20	20	20
Mass of pinion (m_p) (kg)	1.03–3.43 8.88	1.033	1.033	1.033	1.033	1.033
Mass of gear (m_g) (kg)	1.03–3.43 8.88	1.033	1.03–2.40 4.32–17.47	1.033	1.033	1.033
Pinion profile shifting factor (x_p)	0	0	0	−0.3–0.2 0 + 0.5	0	0
Pinion profile shifting factor (x_d)	0	0	0	−0.3–0.2 0 + 0.5	0	0
Addendum factor (h_a)	1	1	1	1	0.9–1 1.1–1.15	1
Dedendum factor (h_f)	1.25	1.25	1.25	1.25	1.25	1.25
Damping ratio, (ξ)	0.1	0.1	0.1	0.1	0.1	0.05 0.1–0.17
Material	Steel	Steel	Steel	Steel	Steel	Steel

$$F_2 = K_2(X_r - e_2). \quad (49)$$

There are various models in the literature for the coefficient of friction calculation between two gears. In this study, the semi-empirical expression of the coefficient of friction between two teeth which is developed by Buckingham (Lin [17]) is used

$$\mu(t) = \frac{0.05}{e^{0.125V_s(t)}} + 0.002\sqrt{V_s(t)}. \quad (50)$$

where $\boldsymbol{V_s(t)}$ expresses the relative sliding velocity tween two teeth and is defined as;

$$V_s(t) = \left[\frac{V\left(r_{0p} + r_{0g}\right)}{r_{0p}r_{0g}}\right]\left(\sqrt{r_{Np1}^2 - r_{bp}^2} - r_{0p}\sin\theta\right) \quad (51)$$

The equations of motions are solved by using the linear iterative method in MATLAB, which is used and detailed in previous studies (Karpat *et al* [13]).

3. Results and discussion

This paper investigates the effects of basic gear parameters on gear instantaneous mesh stiffness and dynamic forces. For this purpose, a computer program is written by using the method, is described in the upper section, in MATLAB. The analysis procedure consisted of six different cases which are defined in table 1. The effect of tooth profile errors is neglected in this study thus, the tooth profile errors are kept zero. In other words, the tooth profiles considered are assumed to be perfect.

The variation of coefficient of friction for one mesh cycle is given in figure 6. The coefficient of friction is calculated according to Eq. (50). According to Buckingham's method [17], the coefficient of friction is varied along the line of action and changes its direction at the point of ''C''. The coefficient of friction decreases continuously in the approach region (between |AC|) and makes a peak at the pitch circle and increases until the end of the contact. The figure is given for the first case I, first analysis 1600 rpm. The coefficient of friction differs from 0.05 to 0.015. However, for the other analysis of the study, the coefficient of friction is differed according to Eq. (50).

Contact ratio is one of the most important factors in the dynamic response of the gear. It gives some clues about the dynamic loads. If the contact ratio is low, it can be inferred that the dynamic loads are higher. If the contact ratio is high, it is expected that dynamic loads will be lower. Furthermore, the low contact ratio gears work with high noise and vibration. Therefore, more contact ratio leads to better performance.

In this study first of all the effect of basic gear parameters on contact ratio is investigated. Table 2, presents all the contact ratios used in this study. When the teeth number, reduction ratio, and addendum factor increase, the contact

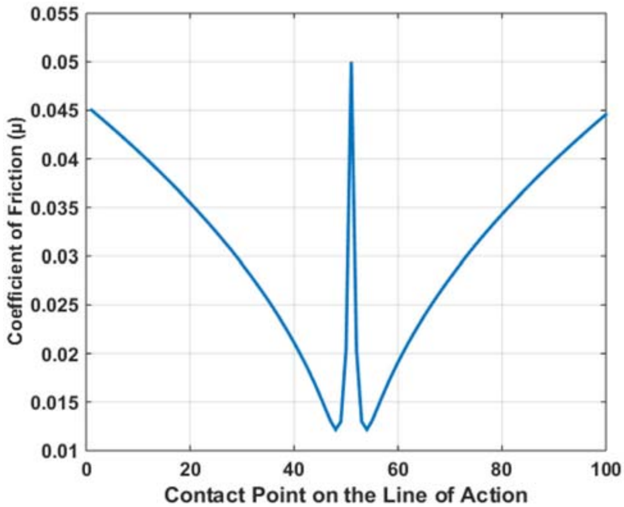

Figure 6. Change of coefficient of friction for one mesh cycle.

Table 2. Contact ratio results for all cases.

	Contact ratio ε_α
Case I (Tooth number) (z_p–z_g)	
28–28	1.6380
50–50	1.7547
80–80	1.8257
Case II (Pressure angle) (α)	
18	1.7280
20	1.6380
25	1.4637
30	1.3465
Case III (Reduction ratio) (i)	
1	1.6380
1.5	1.6805
2	1.7059
4	1.7513
Case IV (Shifting factor) (x)	
−0.3	1.9340
−0.2	1.7991
0	1.6380
0.5	1.5023
Case V (Addendum factor) (h_a)	
0.9	1.4954
1	1.638
1.1	1.7775
1.15	1.8462

ratio is increased too. These parameters have a positive effect on the contact ratio. However, when the pressure angle and profile shifting factor increase the contact ratio is decreased. The negative profile shifting is also increased the contact ratio. The highest contact ratio value is 1.9340 calculated for this study by applying negative profile shifting. This value is too close to 1.98 which is the highest possible contact ratio for low contact ratio spur gears. On the other hand, the minimum contact ratio is calculated when the pressure angle is 30°.

The time-varying mesh stiffness is another important factor for the gear dynamics. Thus the effect of basic gear parameters on time-varying mesh stiffness should be investigated. For the standard gear types, the mesh stiffness of the gear pairs are calculated according to, Kuang and Lin [5] method. The others are calculated by using the finite element method. Both methods provide quite accurate results. In figure 7, for the first five case analyses, the time-varying mesh stiffness variations are shown graphically for one mesh period. There is no effect of damping ratio on gear mesh stiffness thus its graph is not given.

The teeth number effect on the gear mesh stiffness is investigated in the first case. The teeth numbers of the gears selected 28–28, 50–50, and 80–80 in case I. According to mesh stiffness analysis, when the teeth number increased the time-varying mesh stiffness is also increased. In figure 7a, it is seen that the mesh stiffness of the gear increased by 20 % with the number of teeth going up to 28–80. Moreover, the double tooth pair contact region is increased; it means that the contact ratio is increased.

In the second case, the pressure angle effect on the mesh stiffness was investigated. The drive side pressure angle is increased from 18–30. As the pressure angle increases, the tooth thickness also increases, thus the teeth become more rigid. In figure 7b, the effect of pressure angle on gear mesh stiffness is given. According to figure 7b, with the increase in the pressure angle from 18 to 30, the gear mesh stiffness values go up nearly two times. It’s seen that the most effective parameter on the gear mesh stiffness is pressure angle in the selected gear parameters. Furthermore, the single tooth pair region is also increased with the increase of the pressure angle. Thus it can be said that the contact ratio is decreased. The reduction ratio is the third parameter which is investigated. With the increase of the reduction ratio, the gear dimensions are increased but the pinion dimension becomes constant. Figure 7c, shows the effect of the reduction ration on the gear mesh stiffness. When the reduction ratio is going up, the mesh stiffness is increased. The contact ratio of the mechanisms is increased too.

Profile shifting is also an important parameter of the gear design. Due to its many advantages, profile shifting is needed in many applications of power transmission with gears. Thus, the effect of profile shifting on dynamic loads and mesh stiffness should be investigated. The gear geometry is altered by profile shifting. The positive profile shifted gears that have big pressure angles and tooth thickness. However, the negative profile shifted gears have small pressure angles and tooth thickness with the comparison of standard spur gears. Although the positive profile shifting increases the mesh stiffness, the negative profile shifting is decreased. In figure 7d, the single tooth region is increased with a positive profile shifting. The double teeth

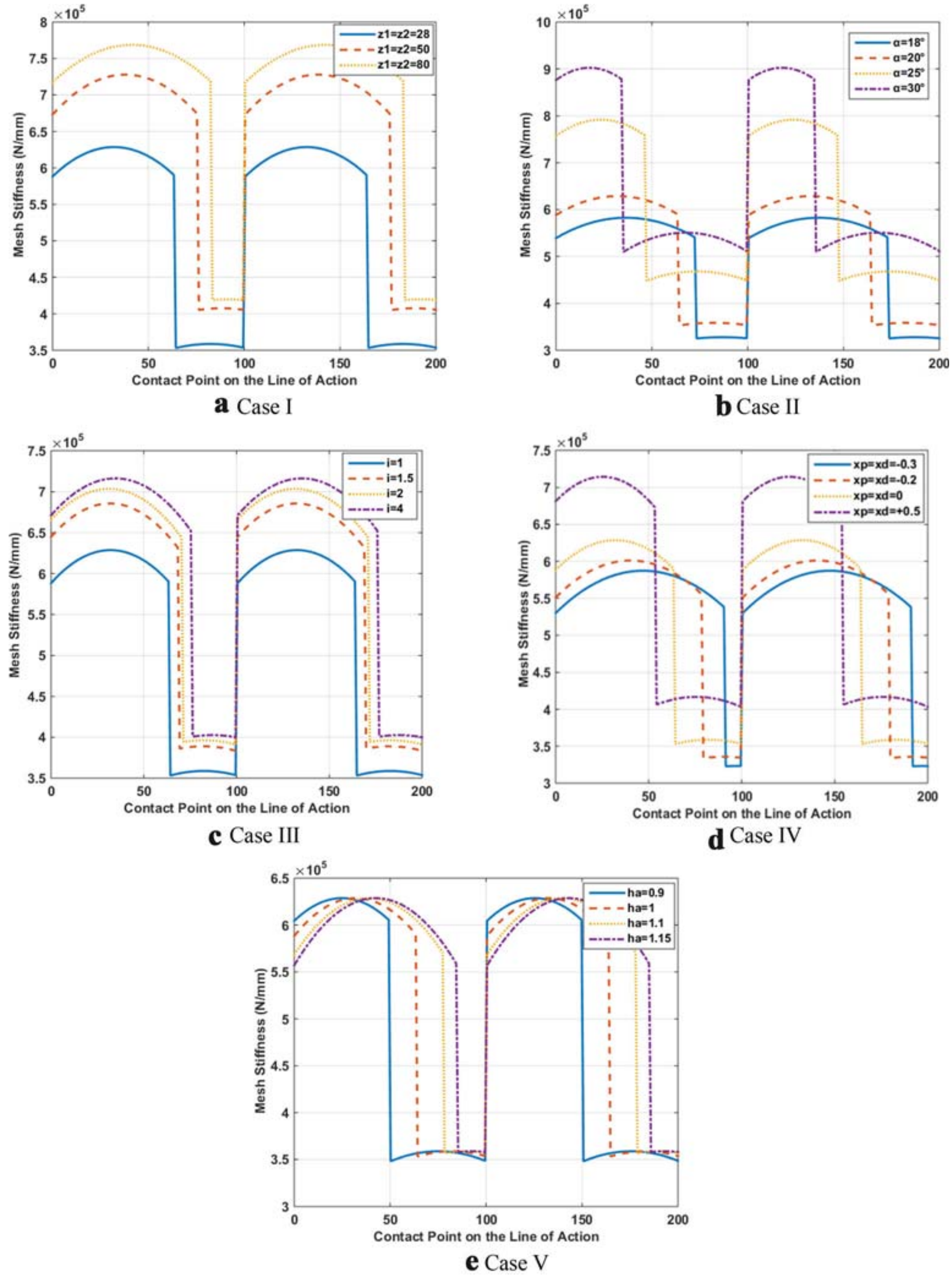

Figure 7. The effects of gear design parameters on mesh stiffness. a) z_1, z_2: Teeth number, b) α: Pressure angle, c) i: Reduction ratio, d) x_p, x_d: Profile shifting factor, e) h_a: Addendum factor.

region and the contact ratio are increased with the negative profile shifting.

The fifth design parameter of this study is the addendum factor. The addendum factor is altered from 0.9 to 1.15. In figure 7e, the effect of addendum factor on the time-varying mesh stiffness is given. According to the figure, there is no huge effect of addendum factor on the maximum and minimum values of the gear mesh stiffness but at the beginning and end of the meshing process the differences between total mesh stiffness can be seen, with the increase of the addendum factor, the gear becomes less stiff and the gear mesh stiffness decreased as shown in figure 7e, also, the contact ratio is increased with the increase of the addendum factor.

Figure 8, shows the effects of basic gear parameters on dynamic tooth load for one mesh cycle at 1600 rpm pinion speed. These results are taken from the Eq. (48) by solving the equations of the motions. To solve these equations, a computer program is written in MATLAB. The results are about 1600 rpm so for the other speed values, the response

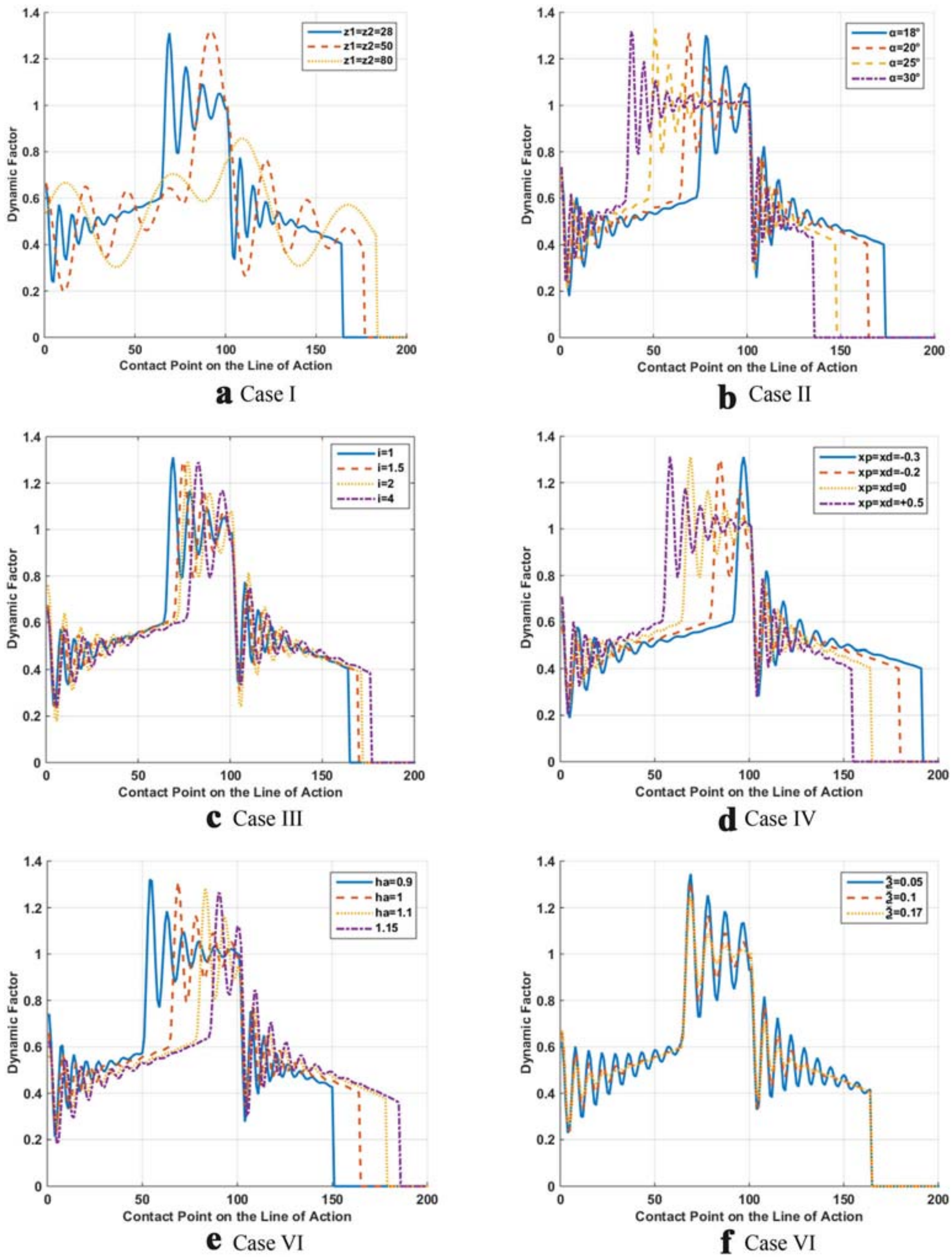

Figure 8. The effects of gear design parameters on dynamic loads for one mesh period. a) z_1, z_2: Teeth number, b) α: Pressure angle, c) *i*: Reduction ratio, d) x_p, x_d: Profile shifting factor, e) h_a: Addendum factor, f) ξ: Damping ratio.

can be different. In the first case when the gear teeth numbers increase, the dynamic loads on the pinion are decreased. The highest difference for the dynamic load for one mesh is seen in this case. With the increase of the tooth number from 28 to 80, the dynamic factor decreased by nearly 40%. But this situation is happening because of the resonance zone of the gears.

The pressure angle effect of the dynamic loads for one mesh cycle can be seen in figure 8b, it is clearly seen that; there are not many differences between the maximum dynamic factors. The maximum dynamic factor for 1600 rpm can be taken at 1.3. Only dynamic loads vary depending on the contact point in this case

As in the second case, there are not many differences between the maximum dynamic factors in the third case. Also, in this case, the maximum dynamic loads can be taken 1.3 in figure 8c.

In positive profile shifted gears, the dynamic force oscillates in the single tooth region because the single tooth region is larger. However, there is no oscillation in the

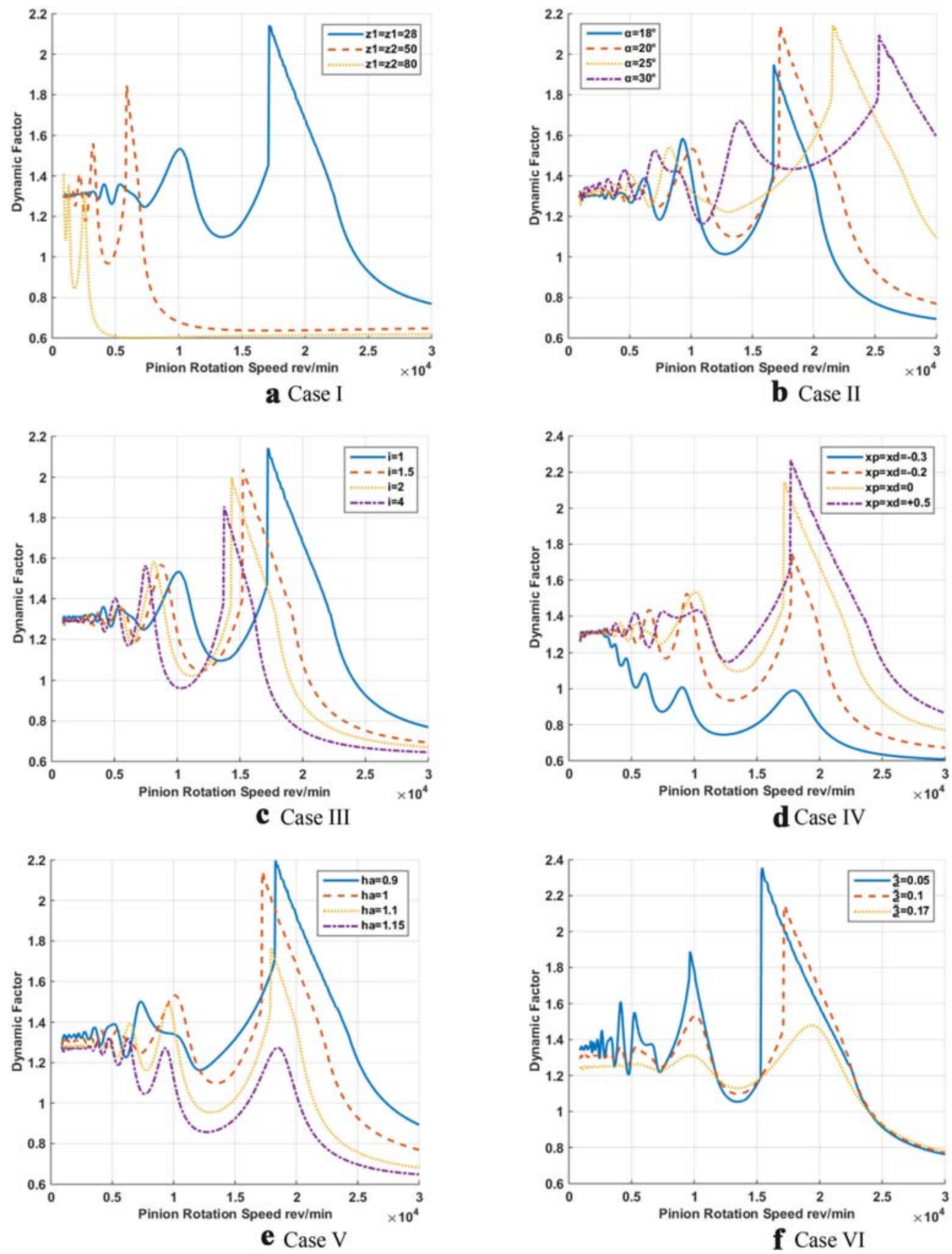

Figure 9. The effects of gear design parameters on dynamic factor between 400–30000 rpm. a) z_p, z_g: Tooth number, b) α: Pressure angle, c) i: Reduction ratio, d) x_p, x_d: Profile shifting factor, e) h_a: Addendum factor, f) ξ: Damping ratio.

single tooth region for the negative profile shifted gears. Similar to other cases here; the maximum dynamic factor is 1.3.

The addendum factor is one of the most important factors for dynamic tooth loads. In the literature to reduce the dynamic forces, the addendum modification is done (Lin *et al* [7], Karpat *et al* [13]). In figure 8e, the dynamic factor variation for the single mesh cycle is given, for different addendum factors.

It's seen that when the addendum factor increased the maximum dynamic factor is decreased for 1600 rpm pinion speed. With the increase of the addendum factor, the dynamic load factor is decreased.

The last parameter, which is investigated in this study, is the damping ratio. The damping ratio is changed from 0.05 to 0.17 in this study. When the damping ratio is going up, the system response is more stable. It can be seen that in figure 8f, the amplitude of the oscillations is greater at low damping ratios. Thus the dynamic factor is increased when the damping ratio is decreased.

The dynamic factor as a function of different pinion speeds is more meaningful than the dynamic factor for single pinion speed. This is because the resonance frequencies of the gearbox can be easily seen. Thus, the designer can select his / her own gear parameters according to resonance regions by using this frequency response curves. For this purpose, the dynamic analysis is done for different pinion speeds between 400 and 30000 rpm with 60 rpm increments. The maximum dynamic factor for each speed is recorded and the frequency response of the gearboxes is created. The effects of selected gear parameters on the dynamic response of the gearboxes are given figure 9.

The effect of teeth number on the dynamic reaction of the gears is given in figure 9a. The maximum dynamic factor is determined when the teeth numbers of the gears are 28–28. The maximum dynamic factor is 2.15 at the speed of 17160 rpm. The dynamic response curve varies when the teeth numbers increase. The maximum dynamic factor is 1.84 at the speed of 5880 for 50–50 teeth number. For the 80–80 teeth number, the dynamic factor is decreased with the increase in pinion speed. This is because of the effective gear mass. When the effective gear mass increases the resonance begins to appear at low pinion speeds. Despite the increase in mesh stiffness, the dynamic factor is reduced because the effective mass and contact ratio is also increased with an increasing number of teeth. The total effects of effective mass and contact ratio have a greater effect than mesh stiffness on the dynamic loads.

The pressure angle is a significant factor for the gear design. Standard spur gears have 20° pressure angle but in many cases, the pressure angle is increased from 20° to 30°, due to strength, high load capacity, and low weight. However the dynamic performances of the gears with high-pressure angles as not well as standard spur gears. The effect of pressure angle on the dynamic factor is given figure 9b. The maximum dynamic factor 2.14 is seen for the 25° pressure angle at 21540 rpm pinion speed. When the average dynamic factors are compared, the minimum dynamic load can be seen 18° and the maximum dynamic factors are seen at 30°. Moreover, when the pressure angle is decreased the contact ratio is decreased thus, the gear dynamic performance gets worse.

The effect of the reduction ratio on the dynamic response of the spur gears is given in figure 9c. The maximum dynamic factor is 2.14 when the reduction ratio is 1. The minimum dynamic factor is 1.85 at 13740 rpm for the reduction ratio 4. As a result of this case when the reduction ratio increases the dynamic response the gear is improved. This is because the contact ratio is increased and the transition between the teeth becomes smoother. As in the first case due to increased effective mass, lower natural frequencies are observed.

Profile shifting is the most effective parameter on the gear dynamic loads in this study. The positive profile shifted gears that have poor dynamic behaviors. However negative profile shifted gears have good dynamic behaviors. The maximum dynamic factor is 2.27 for the +0.5 positive profile shifted gears. The dynamic factor is under 1 for the −0.3 negative profile shifted gears up to 5000 rpm. It is clearly seen in figure 9d, the negative profile shifting is improving dynamic response However, the positive profile shifting is getting worse the dynamic response of the gears. Because the negative profile shifted gears contact ratio is bigger than the positive profile shifted gears. Moreover, the total mesh stiffness of the negative profile shifted gears is lower than the positive profile shifted gears. Profile shifting slightly changes resonance frequencies it can be negligible.

The addendum factor has a great effect on the gear dynamic forces. The dynamic response of the spur gears with different addendums is given in figure 9e. The dynamic loads reduce dramatically when the addendum factor of the gear increase. When the addendum factor decreases the dynamic response of the gear gets worse. While the addendum factor is 0.9, the maximum dynamic factor is 2.2, while the addendum factor is 1.15; the maximum dynamic factor is below 1.2. Nearly 100% reduction is achieved by using high addendum factors.

The damping ratio is the last parameter for this study. According to Kasuba and Evans [27], the damping ratio during one mesh can change between 0.03 and 0.17 and it can be taken as a constant parameter. Thus in this study, the effect of damping ratio on dynamic loads is investigated. When the damping ratio is decreased, the dynamic loads are going up. The damping ratio effect is seen near the resonance frequencies of the system.

3.1 *Design parameters sensitivity analysis*

Design parameters sensitivity analysis is performed to show the effect of each design parameter on the dynamic factor individually by using the frequency response of the dynamic analysis. Maximum dynamic factor values in the first and second resonance regions are compared with the control values. Sensitivity is determined by comparing the percentile changes with the control values of the results obtained for each design parameter. The control values are taken for the standard spur gears. The design parameters are given in table 3 for the standard spur gears.

The control values for the first and second resonance regions (RR) for the standard spur gears are given in table 4.

The effects of design parameters studied in this study on the dynamic response of the spur gears are given in table 5. The design sensitivity analysis results are seen in table 5. Five different design parameters and damping ratio effects on the maximum dynamic factors at first and the second resonance regions are given. Moreover, the comparison of these dynamic factors with the control values is seen. Thus the effects of each parameter on the dynamic response can

Table 3. Design parameters for standard spur gears.

Parameters	Value
Module (m_n) (mm)	3.18
Pinion teeth number (z_p)	28
Gear teeth number (z_g)	28
Tooth face width, (b) (mm)	25.4
Reduction ratio (i)	1.0
Pressure angele (α) (deg)	20
Pinion profile shifting factor (x_p)	0
Pinion profile shifting factor (x_d)	0
Addendum factor (h_a)	1 m
Dedendum factor (h_f)	1.25 m
Damping ratio, (ξ)	0.10
Material	Steel

Table 4. Control values for the resonance regions.

Max. DF first resonance region	Max. DF second resonance region
1.537	2.144

Table 5. Design sensitivity analysis for all design parameters in this study.

	Max. DF. 1.RR.	Max. DF. 2.RR.	% Change for 1.RR	% Change for 2.RR
Control values	1.537	2.144	0	0
z_p–z_g = 50–50	1.561	1.847	−1.56	−13.85
z_p–z_g = 80–80	1.36	1.321	−11.51	−38.36
α = 18	1.586	2.142	3.18	−0.09
α = 25	1.542	2.145	0.32	0.04
α = 30	1.528	2.096	−0.58	−2.23
i = 1.5	1.569	2.040	2.08	−4.85
i = 2.0	1.583	1.988	2.99	−7.27
i = 4.0	1.564	1.855	1.75	−13.47
$x_p = x_d = -0.3$	1.009	0.990	−34.35	−53.82
$x_p = x_d = -0.2$	1.525	1.753	−0.78	−18.23
$x_p = x_d = 0.5$	1.432	2.268	−6.83	5.78
h_a = 0.90	1.503	2.200	−2.21	2.61
h_a = 1.10	1.493	1.766	−2.86	−17.63
h_a = 1.15	1.273	1.272	−17.17	−40.67
ξ = 0.05	1.889	2.356	22.90	9.88
ξ = 0.17	1.313	1.479	−14.57	−31.01

be calculated. The most influential factor is detected as profile shifting factor. The dynamic factor decrease nearly 34% and 54% for first and second resonance regions for negative profile shifting. The second most effective parameter is the addendum factor. The dynamic factor is nearly decreased 17% and 40% for first and second resonance regions with an increase in the addendum factor. The number of teeth is the third effective factor in the dynamic response of the spur gears, which is investigated in this study. The percentile variation of maximum dynamic factor is detected as 12% and 38%. The pressure angle is the least effective parameter for the dynamic response of the spur gear when compared to the other design parameters.

The damping ratio also has great effects on the dynamic response of the spur gears. However, the damping ratio is not a design parameter of the gear. It is a system parameter. Thus, the damping ratio is not classified with the other design parameters. It evaluates by itself. When the damping ratio increases, the dynamic factor of the spur gears is decreases by nearly 31% for this study.

4. Conclusions

In this study, the effects of basic gear design parameters on time-varying mesh stiffness and dynamic factor is investigated. The effects of five different gear design parameters, which are tooth number, reduction ratio, profile shifting, pressure angle, addendum factor, and damping ratio are investigated analytically. The interaction between mesh stiffness, contact ratio, and dynamic loads is also investigated in this study. In the literature, studies are generally investigated just one variable effect on the dynamic loads and time-varying mesh stiffness. Moreover, besides conventional spur gears, asymmetric gear type is used in the dynamic analysis. By changing the pressure angle, the gear type is also varied. Thus this paper offers a broad perspective for the gear designers.

The results are obtained by this study can be summarized as follows.

Profile shifting is the most effective parameter on the gear dynamic loads. The positive profile shifting reduces the dynamic performance; on the other hand, the negative profile shifting has a very positive effect on the dynamic behavior of the gears. Also, positive profile shifting increases, gear mesh stiffness, and decrease contact ratio.

The addendum factor is the second effective parameter on gear dynamic loads. However, it has very little effect on time-varying mesh stiffness. The dynamic loads decreased remarkably when the addendum factor increased. Also, the contact ratio is increased with the incensement of the addendum factor.

The number of teeth has a considerable effect on dynamic loads and mesh stiffness. While the teeth number increases, the mesh stiffness is increased to, but the dynamic loads are decreased. Moreover, the contact ratio of the gears increases. Resonance begins to occur at low pinion speeds due to the increase of the effective mass of the gears.

The reduction ratio has a slight impact on the gear dynamic response and contact ratio when compared with the other parameters. When the reduction ratio is increased, gear dynamic forces are decreased.

The pressure angle is the least effective parameter on the gear dynamics. When the pressure angle goes up, the dynamic conditions of the gear system get worse due to high mesh stiffness and low contact ratio.

The damping ratio is only effective on dynamic loads on resonance regions. When the damping ratio increased, the system behaved more stable. When the damping ratio decreased, the system behaved unsteadily and became worse in terms of dynamics. The effect of the critical damping ratio on the dynamic forces, outside the resonance zones can be neglected.

Taking the above results into consideration, the time-varying mesh stiffness and contact ratio are seen as the two most effective parameters on the gear dynamics. The dynamic response of the gear systems improves by increasing the contact ratio with the change of any gear parameter. However, the increase in the mesh stiffness has a negative effect on gear dynamics. The contact ratio is much more effective than the mesh stiffness in terms of gear dynamics.

5. Future work

In this study, the effects of five different design parameters and damping ratio on time-varying mesh stiffness and dynamic factor are investigated. One design parameter is changed, and the others are taken constant. After this study, the effects of the combinations of these basic gear design parameters on time-varying mesh stiffness and dynamic factor will be studied, and a design sensitivity study will be done.

Acknowledgement

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List of symbols

X	Total deformation on the tooth geometry
P	Applied load on the tooth profile
b_h	Hertzian contact width
c, e	Length and width of finite element mesh size
ρ_p, ρ_g	Radius of curvature pinion and gear
E	Modulus of elasticity
r_0, r	Radius of pitch circle
$r_{(i)}$, R_i	Any radius of on the gear
A_0, A_1, A_2, A_3	Empirical expressions
α	Pressure angle
k_p, k_g	Single tooth stiffness of the pinion and gear
$\bar{K_i}(r)$	Tooth stiffness
r_{bp}, r_{bg}	Base circles of pinion and gear
r_{0p}, r_{0g}	Pitch circles of pinion and gear
r_{ap}, r_{ag}	Addendum circles of pinion and gear
m_n	Normal module of the gears
z_p, z_g	Tooth number for pinion and gear
a_d	Centre distance
$\lvert AE \rvert$	Length of contact
$\lvert AD \rvert$	Pitch on the line of action
$\varepsilon\alpha$	Contact ratio
b	Face width
h_a, h_f	Addendum and dedendum factor
x	Profile shifting factor
J_p, J_g	Polar mass moment of inertias of pinion and gear
F_1, F_2	Dynamic Forces
μ_1, μ_2	Coefficient of friction
T_p, T_g	Transmitted moments
θ_p, θ_g	Angular displacements
m_p, m_g	Masses of pinion and gear
e_1, e_2	Profile errors
s_p, s_g	Friction factors
ω	Natural frequency
X_s	Static transmission error
V_s	Sliding velocity
r_{Np1}	Any radius on the pinion
i	Reduction ratio
ξ	Damping ratio

References

[1] Ozguven H N and Houser D R 1988 Mathematical models used in gear dynamics - a review. *Journal of Sound Vibration* 121: 383–411

[2] Dai X, Cooley C G and Parker R G 2016 Dynamic tooth root strains and experimental correlations in spur gear pairs. *Mechanism and Machine Theory* 101: 60–74

[3] Khabou M T, Bouchala N, Chaari F, Fakhfakh T and Haddar M 2011 Study of a spur gear dynamic behavior in transient regime. *Mechanical Systems and Signal Processing* 25: 3089–3101

[4] Jeong C G 2007 Behavior analysis of spur gear pairs with a one-way clutch. *Journal of Sound Vibration* 301: 760–776

[5] Kuang J H and Lin A D 2001 The effect of tooth wear on the vibration spectrum of a spur gear pair. *Journal of Vibration and Acoustics* 123: 311–317

[6] Guangjian W, Lin C, Li Y and Shuaidong Z 2017 Research on the dynamic transmission error of a spur gear pair with eccentricities by finite element method. *Mechanism and Machine Theory* 109: 1–13

[7] Lin H H, Oswald F B and Townsend D P 1994 Dynamic loading of spur gears with linear or parabolic tooth profile modifications. *Mechanism and Machine Theory* 29: 1115–1129

[8] Liu H, Zhang C, Xiang L and Wang C 2016 Tooth profile modification based on lateral- torsional-rocking coupled nonlinear dynamic model of gear system. *Mechanism and Machine Theory* 105: 606–619
[9] Divandari M, Aghdam B H and Barzamini R 2012 Tooth profile modification and its effect on spur gear pair vibration in presence of localized tooth defect. *Journal of Mechanics* 28: 373–381
[10] Yoon K Y and Rao S S 1996 Dynamic load analysis of spur gears using a new tooth profile. *Journal of Mechanical Design* 188: 1–6
[11] Liou C H, Lin H H, Oswald F B and Townsend D P 1996 Effect of contact ratio on spur gear dynamic load with no tooth profile modifications. *Journal of Mechanical Design* 118: 439–443
[12] Kahraman A and Blankenship G W 1999 Effect of involute contact ratio on spur gear dynamics. *Journal of Mechanical Design* 121: 112–118
[13] Karpat F, Ekwaro-Osire S, Cavdar K and Babalik, F C 2008 Dynamic analysis of involute spur gears with asymmetric teeth. *International Journal of Mechanical Sciences* 50: 1598–1610
[14] Zheng Y, Jian-Zhong S and Zi-Rong L 2013 Effect analysis of friction and damping on anti-backlash gear based on dynamics model with time-varying mesh stiffness. *Journal of Central South University* 20: 3461–3470
[15] Xue J, Li W and Qin C 2014 The scuffing load capacity of involute spur gear systems based on dynamic load sand transient thermal elasto hydrodynamic lubrication. *Tribology International* 79: 74–83
[16] Li Z and Mao K 2013 Frictional effects on gear tooth contact analysis. *Advances in Tribology* 2013: 1–8
[17] Lin H H 1985 Computer aided design and analysis of spur gear dynamics. *Ph.D. Thesis. University of Cincinnati, Ohio*
[18] Karpat F, Dogan O, Ekwaro-Osire S and Yuce C 2014 A novel method for calculation gear tooth stiffness for dynamic analysis of spur gears with asymmetric teeth. *Proceedings of the ASME International Mechanical Engineering Congress and Exposition.* Montreal, Quebec
[19] Munro R G, Palmer D and Morrish L 2001 An experimental method to measure gear tooth stiffness throughout and beyond the path of contact. *Proceedings of the Institution of Mechanical Engineers Part C Journal of Mechanical Engineering Science* 215: 793–803
[20] Kuang J H and Yu J A 1994 Dynamic model for addendum modified gear pair. *Proceedings of ASME 1994 Design Technical Conferences* DE-71: 165–176
[21] Coy J J and Chao C H C 1982 Method of selecting grid size to account for hertz deformation in finite element analysis of spur gears. *Journal of Mechanical Design* 104: 759–766
[22] Liang X, Zuo M J and Pandey M 2014 Analytical evaluating the influence of crack on the mesh stiffness of planetary gear set. *Mechanism and Machine Theory* 76: 20–38
[23] Liang X, Zhang H, Liu L and Zuo M J 2016 The influence of tooth pitting on the mesh stiffness of a pair of external spur gears. *Mechanism and Machine Theory* 106: 1–15
[24] Wan Z, Cao H, Zi Y, He W and Chen Y 2015 Mesh stiffness calculation using an accumulated integral potential energy method and dynamic analysis of helical gears. *Mechanism and Machine Theory* 92: 447–463
[25] Chen Z, Zhai W, Shao Y and Wang K 2016 Mesh stiffness evaluation of an internal spur gear pair with tooth profile shift. *Science China Technological Science* 59: 1328–1339
[26] Colbourne J R 1987 *Geometry of involute gears*, Springer, New Jersey
[27] Kasuba R and Evans J W 1981 An extended model for determining dynamic loads in spur gearing. *ASME Journal of Mechanical Design* 103: 398–409