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On the static deformation of FG sandwich beams curved in elevation using a new higher order beam theory

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Abstract. This article presents the static analysis of FG sandwich beams curved in elevation. Navier-type semi-analytical solutions are obtained based on polynomial type fifth order shear and normal deformation theory. The beam has FG skins and isotropic core. Material properties of FG skins are graded in z-direction according to the power-law distribution. The present theory accounts for a fifth-order distribution of axial displacement and fourth-order distribution of transverse displacement. The present theory considers the effect of thickness stretching and gives a realistic variation of transverse shear stress through the thickness of the beam. The governing equations are obtained within the framework of the principle of virtual work. Semi-analytical static solutions for the simply supported FG sandwich beams curved in elevation are obtained using Navier's technique. The beam is subjected to uniformly distributed load. The non-dimensional numerical values for displacements and stresses are obtained for various power-law index and thickness of the core. The present results are found in good agreement with previously published results.

Keywords. FG sandwich; curved beams; shear and normal deformation theory; fifth-order; static analysis.

1. Introduction

Functionally graded material (FGM) is a combination of two materials (ceramic and metal) in which elastic properties are graded on either along the length, along the thickness or along both the directions using different gradation laws. A beam (straight or curved) is an important structural element of any engineering structure. In curved beam, the neutral axis in the unloaded condition is curved instead of straight. These are widely applied in aerospace, civil, automotive, mechanical and shipbuilding industries. Analyses of thick and thin composite beams are carried out using elasticity theory; classical beam theory (CBT) [1], first-order shear deformation theory (FSDT) [2] and higher-order shear deformation theories (HSDTs). However, CBT and FSDT are not suitable for the analysis of thick composite beams due to their many limitations. Therefore, one should use higher-order shear deformation theory for the accurate analysis of composite beams either straight or curved in nature. One can refer the review articles presented by Sayyad and Ghugal [3–5] to understand various higher order shear deformation theories available in the literature for the analysis of laminated composite, sandwich and functionally graded beams. Khdeir and Reddy [6] have presented bending analysis of cross-ply laminated composite beams of various boundary conditions using third-order shear deformation theory. Aithraju and Averill [7] have used a zig-zag distribution of the in-plane displacement field through the thickness to satisfy the interlaminar shear stress continuity across the layer interfaces. Shimpi and Ghugal [8] have presented a new layer-wise trigonometric shear deformation theory for the analysis of two-layered cross-ply laminated beams using Navier's solution technique. Sankar [9] and Venkataraman and Sankar [10] have presented elasticity solutions for bending of functionally graded sandwich beams. Rastgo *et al* [11] have presented the thermal buckling analysis of a FG curved beam with doubly symmetric cross section. Piovan and Cortinez [12] developed the theoretical model for the linear analysis of composite thin-walled curved beams with open and closed cross-sections. Kadoli *et al* [13] have presented a static thermal analysis of functionally graded beams using higher order shear deformation theory. Tessler *et al* [14] have presented a zig-zag shear deformation theory for the analysis of laminated composites and sandwich beams. Yaghoobi and Fereidoon [15] investigated the position of the neutral surface for the bending analysis of functionally graded simply supported beam subjected to uniformly distributed load. Yousefi and Rastago [16] investigated free vibration analysis of functionally graded spatial curved beams by taking into account the effects of

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thickness-curvature based on first-order shear deformation theory and Ritz method.

Vo and Thai [17] have presented a static analysis of laminated composite beams using various refined shear deformation theories. Guinta *et al* [18] presented a unified method for the analysis of functionally graded sandwich beams. Wang and Liu [19] have presented a elasticity solution for functionally graded curve beams using the Airy stress function. Qu *et al* [20] have presented a free vibration analysis of laminated composite beams using a generalized higher-order shear deformation theory. Carrera *et al* [21] have developed a number of polynomial and non-polynomial type refined beam theories by expanding the unknown displacement variables over the beam section axes. Pradhan and Chkraverty [22] have presented a free vibration analysis of functionally graded beams of different boundary conditions using classical and first order shear deformation beam theories based on Rayleigh–Ritz method. Li *et al* [23] have presented a free vibration analysis of laminated composite beams of various boundary conditions using various higher-order shear deformation beam theories and spectral finite element method. Hadji *et al* [24] have developed a refined exponential shear deformation theory for the static and free vibration analysis of functionally graded beams. Vo *et al* [25, 26] have developed a new quasi-3D theory for the static, free vibration and buckling analysis of functionally graded sandwich beams using a finite element method. Osofero *et al* [27] developed higher order shear deformation theories considering the effects of thickness stretching on bending behavior of simply supported functionally graded sandwich beams. Fereidoon *et al* [28] have studied bending analysis of curved sandwich beams with FG core. Authors have modeled the face sheets using CBT and the core using HSDT. Nanda and Kapuria [29] have presented a wave propagation analysis of laminated composite curved beams using the CBT and FSDT based on the spectral finite element method. Luu *et al* [30] investigated the bending and buckling analyses of shear deformable laminated composite curved beams using the NURBS-based isogeometric method. Chen *et al* [31] used Timoshenko beam theory for the buckling and static analysis of shear deformable functionally graded porous beams. Kurtaran [32] has presented a geometrically nonlinear analysis of functionally graded curved beams using the FSDT. Nguyen *et al* [33] have obtained a Ritz-type analytical solution for the buckling and free vibration analysis of functionally graded sandwich beams using a quasi-3D beam theory. Ye *et al* [34] investigated the vibration characteristics of thick composite laminated and sandwich curved beams with variable curvatures as well as general restraints. Khdeir and Aldraihem [35] have investigated free vibration behaviour of soft core sandwich beam by using zig-zag beam theory. Ugurcan [36] has studied large deflection analysis of planar functionally graded curved beams. Liu *et al* [37] have developed a layer-wise shear deformation theory for the free vibration analysis of functionally graded sandwich and laminated composite shells using a differential quadrature finite element method. Huynh *et al* [38] presented the bending, buckling and a free vibration analysis of functionally graded curved beams with variable curvatures using the NURBS-based isogeometric method. Gua *et al* [39] have presented the static, free, forced and transient vibrations of isotropic and composite laminated curved beams of various boundary conditions using the FSDT. Mohamad *et al* [40] have presented a nonlinear free and forced vibration analysis of clamped-clamped curved beams. Li *et al* [41] developed a new higher-order shear deformable mixed beam element model for static and vibration analysis of functionally graded sandwich beams. Chen *et al* [42] presented a thermal buckling analysis of sandwich beam. Sayyad and Avhad [43] have presented bending, buckling and free vibration analysis of sandwich beams by using hyperbolic shear deformation theory. Sayyad and Ghugal [44] have presented a sinusoidal beam theory considering the effects of transverse normal strain for the bending analysis of FG sandwich beams curved in elevation.

Kiani *et al* [45] presented linear thermal buckling analysis of piezoelectric functionally graded beams using Timoshenko beam theory. Kiani *et al* [46] studied effects of low velocity impact on functionally graded beams under thermal environment by using third order shear deformation theory. Komijani *et al* [47] and Kargani *et al* [48] presented nonlinear buckling analysis of piezoelectric functionally graded beams by using Timoshenko beam theory under the effects of thermal and electrical loads. Ghiasian *et al* [49] presented non-linear vibration analysis of functionally graded beams subjected to thermal loads.

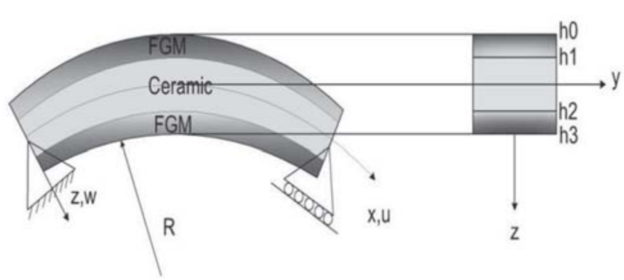

Figure 1. FG sandwich curved beam.

Table 1. Lamination schemes and thickness of layer.

Lamination schemes	Thickness of layers			
	h_0	h_1	h_2	h_3
1-1-1	−0.5h	−0.167h	0.167h	0.5h
1-2-1	−0.5h	−0.25h	0.25h	0.5h
2-1-2	−0.5h	−0.1h	0.1h	0.5h

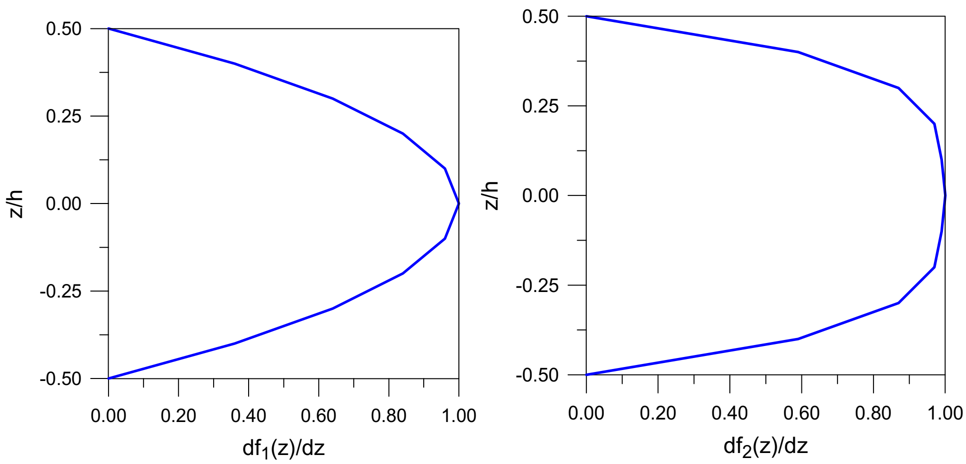

Figure 2. Variation of first derivatives of shape function through-the-thickness of curved beam.

1.1 *Novelty of the present work*

In the present study, a higher order shear and normal deformation theory is presented for the static analysis of functionally graded sandwich beams curved in elevation. Plenty of research articles are available in the literature on static analysis of straight beams using higher-order beam theories. However, limited research articles are found in the literature on static analysis of curved beams. To the best of the authors' knowledge, only one paper by Sayyad and Ghugal [44] is available in the literature on FG sandwich beams curved in elevation. Therefore, static analysis of FG sandwich curved beam under uniform load presented in this paper will be the important novelty of the present work.

In this paper, a fifth-order shear and normal deformation theory is developed for the curved beam. The theory accounts for a fifth-order distribution of axial displacement and fourth-order distribution of transverse displacement. In a whole variety of the literature, only Sayyad and Ghugal [44] have considered the effect of transverse normal strain for developing refined beam theory for the analysis of curved sandwich beams. Therefore, the present theory considers the effect of thickness stretching for the analysis of curved beam.

2. Functionally graded sandwich beam curved in elevation

Consider a sandwich beam curved in elevation and made up of functionally graded skins and isotropic core as shown in figure 1.

The beam occupies the region $0\leq x\leq L$; $-b/2\leq y\leq b/2$; $-h/2\leq z\leq h/2$ in Cartesian coordinate systems. The beam has a rectangular cross-section ($b\times h$). The total thickness of the beam is divided into three layers. Top and bottom layers (skins) are made up of functionally graded materials and central layer (core) is made up of homogeneous isotropic material. Eq. (1) stated the power-law used for the gradation of Young's modulus of FG skins.

$$E(z) = E_m + (E_c - E_m)V_c(z) \tag{1}$$

where,

V_c represents the volume fraction function. Values of V_c different layers of sandwich beam are given below.

$$\begin{aligned} V_c(z) &= \left(\frac{z-h_0}{h_1-h_0}\right)^p \quad \text{for} \quad z\in[h_0,\ h_1] \\ V_c(z) &= 1 \quad \text{for} \quad z\in[h_1,\ h_2] \\ V_c(z) &= \left(\frac{z-h_3}{h_2-h_3}\right)^p \quad \text{for} \quad z\in[h_2,\ h_3] \end{aligned} \tag{2}$$

where,

E (m=metallic, c=ceramic) represents the modulus of elasticity and p is the power law index. Three types of lamination schemes are solved in the present study (see table 1).

2.1 *Mathematical formulation*

The mathematical formulation of the present theory is based on the following kinematic assumptions.

1) The axial displacement (x-direction) is presented in-terms of the extension, bending and shear terms.
2) The transverse displacement (z-direction) is a function of both x and z coordinates, i.e., it considered the effects of transverse normal deformations ($\varepsilon_z \neq 0$)
3) Shape functions used to account the effects of transverse shear and normal deformations are polynomial type and expanded up to fifth-order.

Based on the aforementioned assumptions, the displacement field of the present theory considering the effects of both transverse shear and normal deformations is written as follows:

$$u(x,z) = \left(1+\frac{z}{R}\right)u_0(x) - z\frac{\partial w_0}{\partial x} + \left(z - \frac{4z^3}{3h^2}\right)\phi_x(x) + \left(z - \frac{16z^5}{5h^4}\right)\psi_x(x)$$
$$w(x,z) = w_0(x) + \left(1 - \frac{4z^2}{h^2}\right)\phi_z(x) + \left(1 - \frac{16z^4}{h^4}\right)\psi_z(x) \tag{3}$$

where u and w are the displacements of any point of the beam domain in x- and z- directions, respectively; u_0 is the x- directional displacement of a point on the middle surface of the beam and w_0 is the z- directional displacement of a point on the top and bottom surfaces of the beam i.e., $w = w_0$ at $z=\pm h/2$. The non-zero normal strains and transverse shear strain at any point of the beam are obtained using the theory of elasticity.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} + \frac{w}{R} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - \frac{u_0}{R} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} + f_1(z)\frac{\partial \phi_x}{\partial x} + f_2(z)\frac{\partial \psi_x}{\partial x} + \frac{w_0}{R} + f_1'(z)\frac{\phi_z}{R} + f_2'(z)\frac{\psi_z}{R} \\ f_1''(z)\phi_z + f_2''(z)\psi_z \\ f_1'(z)\phi_x + f_2'(z)\psi_x + f_1'(z)\frac{\partial \phi_z}{\partial x} + f_2'(z)\frac{\partial \psi_z}{\partial x} \end{Bmatrix} \tag{4}$$

where,

$$f_1(z) = \left(z - \frac{4z^3}{3h^2}\right), \quad f_2(z) = \left(z - \frac{16z^5}{5h^4}\right), \quad f_1'(z) = \left(1 - \frac{4z^2}{h^2}\right), \quad f_2'(z) = \left(1 - \frac{16z^4}{h^4}\right) \tag{5}$$

Eq. (4) shows that the present theory satisfies the zero shear strain conditions at top and bottom surfaces of the beam (see figure 2).

The two-dimensional constitutive relation used to obtain stresses.

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{13} & 0 \\ Q_{13} & Q_{33} & 0 \\ 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} \tag{6}$$

where,

$$Q_{11} = Q_{33} = \frac{E(z)}{1-\mu^2}, \quad Q_{13} = \frac{\mu E(z)}{1-\mu^2}, \quad Q_{55} = \frac{E(z)}{2(1+\mu)} \tag{7}$$

The principle of virtual work (PVW) is used to derive variationally consistent governing differential equations and associated boundary conditions. An analytical form of the PVW is as follows:

$$\int_0^L \int_{-h/2}^{h/2} \left[\sigma_x \delta\varepsilon_x + \sigma_z \delta\varepsilon_z + \tau_{xz}\delta\gamma_{xz}\right] dz\,dx = \int_0^L q\delta w\,dx \tag{8}$$

where δ denotes the variational operator. From Eq. (2) using the values of strain components and introducing stress resultants in Eq. (6), one can obtain following equation.

$$\int_0^L \left(\begin{aligned} & N_x \frac{\partial \delta u_0}{\partial x} + N_x \frac{\delta w_0}{R} - M_x^b \frac{\partial^2 \delta w_0}{\partial x^2} + M_x^{s1}\frac{\partial \delta \phi_x}{\partial x} \\ & + M_x^{s2}\frac{\partial \delta \psi_x}{\partial x} + V_x^1 \frac{\delta \phi_z}{R} \\ & + V_x^2 \frac{\delta \psi_z}{R} + Q_z^1 \delta\phi_z + Q_z^2 \delta\psi_z + Q_{xz}^1 \delta\phi_x + Q_{xz}^2 \delta\psi_x \\ & + Q_{xz}^1 \frac{\partial \delta \phi_z}{\partial x} + Q_{xz}^2 \frac{\partial \delta \psi_z}{\partial x} \end{aligned} \right) dx - \int_0^L q\left(\delta w_0 + f_1'(z)\phi_z + f_2'(z)\psi_z\right) dx = 0 \tag{9}$$

where $N_x, M_x^b, M_x^{s1}, M_x^{s2}, V_x^1, V_x^2, Q_z^1, Q_z^2, Q_{xz}^1$ and Q_{xz}^2 are the stress resultants defined as follows:

$$N_x = \int_{-h/2}^{h/2} \sigma_x\,dz, \quad M_x^b = \int_{-h/2}^{h/2} \sigma_x z\,dz, \quad M_x^{s1} = \int_{-h/2}^{h/2} \sigma_x f_1(z)\,dz,$$
$$M_x^{s2} = \int_{-h/2}^{h/2} \sigma_x f_2(z)\,dz, \quad V_x^1 = \int_{-h/2}^{h/2} \sigma_x f_1'(z)\,dz, \quad V_x^2 = \int_{-h/2}^{h/2} \sigma_x f_2'(z)\,dz,$$
$$Q_z^1 = \int_{-h/2}^{h/2} \sigma_z f_1''(z)\,dz, \quad Q_z^2 = \int_{-h/2}^{h/2} \sigma_z f_2''(z)\,dz, \quad Q_{xz}^1 = \int_{-h/2}^{h/2} \tau_{xz} f_1'(z)\,dz,$$
$$Q_{xz}^2 = \int_{-h/2}^{h/2} \tau_{xz} f_2'(z)\,dz. \tag{10}$$

By substituting stresses from Eq. (4) into Eq. (8), the stress resultants are expressed as follows

$$N_x = A_{11}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R}\right) - B_{11}\frac{\partial^2 w_0}{\partial x^2} + C_{11}\frac{\partial \phi_x}{\partial x} + D_{11}\frac{\partial \psi_x}{\partial x} + E_{11}\frac{\phi_z}{R} + F_{11}\frac{\psi_z}{R} + G_{13}\phi_z + H_{13}\psi_z$$

$$M_x^b = B_{11}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R}\right) - I_{11}\frac{\partial^2 w_0}{\partial x^2} + J_{11}\frac{\partial \phi_x}{\partial x} + K_{11}\frac{\partial \psi_x}{\partial x} + L_{11}\frac{\phi_z}{R} + M_{11}\frac{\psi_z}{R} + N_{13}\phi_z + O_{13}\psi_z$$

$$M_x^{s1} = C_{11}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R}\right) - J_{11}\frac{\partial^2 w_0}{\partial x^2} + P_{11}\frac{\partial \phi_x}{\partial x} + QQ_{11}\frac{\partial \psi_x}{\partial x} + R_{11}\frac{\phi_z}{R} + S_{11}\frac{\psi_z}{R} + T_{13}\phi_z + U_{13}\psi_z$$

$$M_x^{s2} = D_{11}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R}\right) - K_{11}\frac{\partial^2 w_0}{\partial x^2} + QQ_{11}\frac{\partial \phi_x}{\partial x} + V_{11}\frac{\partial \psi_x}{\partial x} + W_{11}\frac{\phi_z}{R} + X_{11}\frac{\psi_z}{R} + Y_{13}\phi_z + Z_{13}\psi_z$$

$$V_x^1 = E_{11}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R}\right) - L_{11}\frac{\partial^2 w_0}{\partial x^2} + R_{11}\frac{\partial \phi_x}{\partial x} + W_{11}\frac{\partial \psi_x}{\partial x} + BA_{11}\frac{\phi_z}{R} + BB_{11}\frac{\psi_z}{R} + BC_{13}\phi_z + BD_{13}\psi_z$$

$$V_x^2 = F_{11}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R}\right) - M_{11}\frac{\partial^2 w_0}{\partial x^2} + S_{11}\frac{\partial \phi_x}{\partial x} + X_{11}\frac{\partial \psi_x}{\partial x} + BB_{11}\frac{\phi_z}{R} + BE_{11}\frac{\psi_z}{R} + BF_{13}\phi_z + BG_{13}\psi_z$$

$$Q_z^1 = G_{13}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R}\right) - N_{13}\frac{\partial^2 w_0}{\partial x^2} + T_{13}\frac{\partial \phi_x}{\partial x} + Y_{13}\frac{\partial \psi_x}{\partial x} + BC_{13}\frac{\phi_z}{R} + BF_{13}\frac{\psi_z}{R} + BH_{33}\phi_z + BI_{33}\psi_z$$

$$Q_z^2 = H_{13}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R}\right) - O_{13}\frac{\partial^2 w_0}{\partial x^2} + U_{13}\frac{\partial \phi_x}{\partial x} + Z_{13}\frac{\partial \psi_x}{\partial x} + BD_{13}\frac{\phi_z}{R} + BG_{13}\frac{\psi_z}{R} + BI_{33}\phi_z + BJ_{33}\psi_z$$

$$Q_{xz}^1 = BK_{55}\left(\phi_x + \frac{\partial \phi_z}{\partial x}\right) + BL_{55}\left(\psi_x + \frac{\partial \psi_z}{\partial x}\right)$$

$$\text{and}\quad Q_{xz}^2 = BL_{55}\left(\phi_x + \frac{\partial \phi_z}{\partial x}\right) + BM_{55}\left(\psi_x + \frac{\partial \psi_z}{\partial x}\right) \tag{11}$$

$$(A_{11}, B_{11}, C_{11}, D_{11}, E_{11}, F_{11}) = \int_{-h/2}^{h/2}\left[1, z, f_1(z), f_2(z), f_1'(z), f_2'(z)\right] Q_{11}\, dz$$

$$(I_{11}, J_{11}, K_{11}, L_{11}, M_{11}) = \int_{-h/2}^{h/2}\left[z, f_1(z), f_2(z), f_1'(z), f_2'(z)\right] z Q_{11}\, dz$$

$$(G_{13}, H_{13}) = \int_{-h/2}^{h/2}\left[f_1''(z), f_2''(z)\right] Q_{13}\, dz \quad (N_{13}, O_{13}) = \int_{-h/2}^{h/2}\left[f_1''(z), f_2''(z)\right] z\, Q_{13}\, dz$$

$$(P_{11}, QQ_{11}, R_{11}, S_{11}) = \int_{-h/2}^{h/2}\left[f_1(z), f_2(z), f_1'(z), f_2'(z)\right] f_1(z) Q_{11}\, dz$$

$$(T_{13}, U_{13}) = \int_{-h/2}^{h/2}\left[f_1''(z), f_2''(z)\right] f_1(z) Q_{13}\, dz,$$

$$(V_{11}, W_{11}, X_{11}) = \int_{-h/2}^{h/2}\left[f_2(z), f_1'(z), f_2'(z)\right] f_2(z) Q_{11}\, dz,$$

$$(BA_{11}, BB_{11}) = \int_{-h/2}^{h/2}\left[f_1'(z), f_2'(z)\right] f_1'(z) Q_{11}\, dz$$

$$(Y_{13}, Z_{13}) = \int_{-h/2}^{h/2}\left[f_1''(z), f_2''(z)\right] f_1(z) Q_{13}\, dz \quad (BC_{13}, BD_{13}) = \int_{-h/2}^{h/2}\left[f_1''(z), f_2''(z)\right] f_1'(z) Q_{13}\, dz$$

$$(BF_{13}, BG_{13}) = \int_{-h/2}^{h/2}\left[f_1''(z), f_2''(z)\right] f_2'(z) Q_{13}\, dz \quad (BH_{33}, BI_{33}) = \int_{-h/2}^{h/2}\left[f_1''(z), f_2''(z)\right] f_1''(z) Q_{33}\, dz$$

$$(BK_{55}, BL_{55}) = \int_{-h/2}^{h/2}\left[f_1'(z), f_2'(z)\right] f_1'(z) Q_{55}\, dz,$$

$$BJ_{33} = \int_{-h/2}^{h/2}\left[f_2''(z) f_2''(z)\right] Q_{33}\, dz,$$

$$BE_{11} = \int_{-h/2}^{h/2}\left[f_1'(z) f_1'(z)\right] Q_{11}\, dz,$$

$$BM_{55} = \int_{-h/2}^{h/2}\left[f_2'(z) f_2'(z)\right] Q_{55}\, dz \tag{12}$$

Governing differential equations are obtained by integrating Eq. (9) by parts and collecting the coefficients of six unknown variables (δu_0, δw_0, $\delta\phi_x$, $\delta\psi_x$, $\delta\phi_z$, $\delta\psi_z$).

$$\begin{aligned}
\delta u_0 : \quad & \frac{\partial N_x}{\partial x} = 0 \\
\delta w_0 : \quad & \frac{\partial^2 M_x^b}{\partial x^2} - \frac{N_x}{R} + q = 0 \\
\delta \phi_x : \quad & \frac{\partial M_x^{s1}}{\partial x} - Q_{xz}^1 = 0 \\
\delta \psi_x : \quad & \frac{\partial M_x^{s2}}{\partial x} - Q_{xz}^2 = 0 \\
\delta \phi_z : \quad & \frac{\partial Q_{xz}^1}{\partial x} - \frac{V_x^1}{R} - Q_z^1 + q f_1'(z) = 0 \\
\delta \psi_z : \quad & \frac{\partial Q_{xz}^2}{\partial x} - \frac{V_x^2}{R} - Q_z^2 + q f_2'(z) = 0
\end{aligned} \tag{13}$$

The boundary conditions along the edges at x=0 and x=L are of the following form

$$N_x = 0 \quad \text{or} \quad u_0 = 0 \tag{14}$$

$$M_x^b = 0 \quad \text{or} \quad \frac{\partial w_0}{\partial x} = 0 \tag{15}$$

$$\frac{\partial M_x^b}{\partial x} = 0 \quad \text{or} \quad w_0 = 0 \tag{16}$$

$$M_x^{s1} = 0 \quad \text{or} \quad \phi_x = 0 \tag{17}$$

$$M_x^{s2} = 0 \quad \text{or} \quad \psi_x = 0 \tag{18}$$

$$Q_{xz}^1 = 0 \quad \text{or} \quad \phi_z = 0 \tag{19}$$

$$Q_{xz}^2 = 0 \quad \text{or} \quad \psi_z = 0 \tag{20}$$

By substituting stress resultants from Eq. (11) into Eq. (13), the governing equations are expressed as follows in terms of unknown variables:

$$\begin{aligned}
\delta u_0 : -A_{11}\left(\frac{\partial^2 u_0}{\partial x^2} + \frac{1}{R}\frac{\partial w_0}{\partial x}\right) + B_{11}\frac{\partial^3 w_0}{\partial x^3} - C_{11}\frac{\partial^2 \phi_x}{\partial x^2} \\
- D_{11}\frac{\partial^2 \psi_x}{\partial x^2} - \frac{E_{11}}{R}\frac{\partial \phi_z}{\partial x} - \frac{F_{11}}{R}\frac{\partial \psi_z}{\partial x} \\
- G_{13}\frac{\partial \phi_z}{\partial x} - H_{13}\frac{\partial \psi_z}{\partial x} = 0
\end{aligned} \tag{21}$$

$$\begin{aligned}
&\delta w_0 : A_{11}\left(\frac{1}{R}\frac{\partial u_0}{\partial x} + \frac{w_0}{R^2}\right) - B_{11}\left(\frac{\partial^3 u_0}{\partial x^3} + \frac{2}{R}\frac{\partial^2 w_0}{\partial x^2}\right) \\
&+ I_{11}\frac{\partial^4 w_0}{\partial x^4} - J_{11}\frac{\partial^3 \phi_x}{\partial x^3} + \frac{D_{11}}{R}\frac{\partial \psi_x}{\partial x} - K_{11}\frac{\partial^3 \psi_x}{\partial x^3} \\
&\quad + \frac{E_{11}}{R}\phi_z + \frac{L_{11}}{R}\frac{\partial^2 \phi_z}{\partial x^2} - N_{13}\frac{\partial^2 \phi_z}{\partial x^2} + \frac{F_{11}}{R^2}\psi_z + \frac{G_{13}}{R}\phi_z \\
&+ \frac{H_{13}}{R}\psi_z - \frac{M_{11}}{R}\frac{\partial^2 \psi_z}{\partial x^2} - O_{13}\frac{\partial^2 \psi_z}{\partial x^2} = q
\end{aligned} \tag{22}$$

$$\begin{aligned}
&\delta \phi_x : -C_{11}\left(\frac{\partial^2 u_0}{\partial^2 x} + \frac{1}{R}\frac{\partial w_0}{\partial x}\right) + J_{11}\frac{\partial^3 w_0}{\partial x^3} - P_{11}\frac{\partial^2 \phi_x}{\partial x^2} \\
&+ BK_{55}\phi_x - QQ_{11}\frac{\partial^2 \psi_z}{\partial x^2} + BL_{55}\psi_x - \frac{R_{11}}{R}\frac{\partial \phi_Z}{\partial x} \\
&\quad - T_{13}\frac{\partial \phi_Z}{\partial x} + BK_{55}\frac{\partial \phi_Z}{\partial x} - \frac{S_{11}}{R}\frac{\partial \psi_Z}{\partial x} \\
&- U_{13}\frac{\partial \psi_Z}{\partial x} + BL_{55}\frac{\partial \psi_Z}{\partial x} = 0
\end{aligned} \tag{23}$$

$$\begin{aligned}
&\delta \psi_x : -D_{11}\left(\frac{\partial^2 u_0}{\partial^2 x} + \frac{1}{R}\frac{\partial w_0}{\partial x}\right) + K_{11}\frac{\partial^3 w_0}{\partial x^3} - Q_{11}\frac{\partial^2 \phi_x}{\partial x^2} \\
&+ BL_{55}\phi_x - V_{11}\frac{\partial^2 \psi_z}{\partial x^2} + BM_{55}\psi_x - \frac{W_{11}}{R}\frac{\partial \phi_Z}{\partial x} \\
&\quad - Y_{13}\frac{\partial \phi_Z}{\partial x} + BL_{55}\frac{\partial \phi_Z}{\partial x} - \frac{X_{11}}{R}\frac{\partial \psi_Z}{\partial x} - Z_{13}\frac{\partial \psi_Z}{\partial x} \\
&+ BM_{55}\frac{\partial \psi_Z}{\partial x} = 0
\end{aligned} \tag{24}$$

$$\begin{aligned}
&\delta \phi_z : E_{11}\left(\frac{1}{R}\frac{\partial u_0}{\partial x} + \frac{w_0}{R^2}\right) + G_{13}\left(\frac{\partial u_0}{\partial x} + \frac{w_0}{R}\right) - \frac{L_{11}}{R}\frac{\partial^2 w_0}{\partial x^2} \\
&- N_{13}\frac{\partial^2 w_0}{\partial x^2} + \frac{R_{11}}{R}\frac{\partial^2 \phi_x}{\partial x^2} + T_{13}\frac{\partial \phi_x}{\partial x} \\
&\quad - BK_{55}\frac{\partial \phi_x}{\partial x} + \frac{W_{11}}{R}\frac{\partial \psi_x}{\partial x} + Y_{13}\frac{\partial \psi_x}{\partial x} - BL_{55}\frac{\partial \psi_x}{\partial x} + \frac{BA_{11}}{R}\phi_Z \\
&+ 2.\frac{BC_{13}}{R}\phi_Z + BH_{33}\phi_Z + \frac{BB_{11}}{R}\psi_Z \\
&\quad - BK_{55}\frac{\partial^2 \phi_Z}{\partial x^2} + \frac{BD_{13}}{R}\psi_Z + \frac{BF_{13}}{R}\psi_Z \\
&+ BI_{33}\psi_Z - BL_{55}\frac{\partial^2 \psi_Z}{\partial x^2} = q f_1'(z)
\end{aligned} \tag{25}$$

$$\begin{aligned}
&\delta \psi_z : F_{11}\left(\frac{1}{R}\frac{\partial u_0}{\partial x} + \frac{w_0}{R^2}\right) + H_{13}\left(\frac{\partial u_0}{\partial x} + \frac{w_0}{R}\right) - \frac{M_{11}}{R}\frac{\partial^2 w_0}{\partial x^2} \\
&- O_{13}\frac{\partial^2 w_0}{\partial x^2} + \frac{S_{11}}{R}\frac{\partial^2 \phi_x}{\partial x^2} + U_{13}\frac{\partial \phi_x}{\partial x} \\
&\quad - BL_{55}\frac{\partial \phi_x}{\partial x} + \frac{X_{11}}{R}\frac{\partial \psi_x}{\partial x} + Z_{13}\frac{\partial \psi_x}{\partial x} - BM_{55}\frac{\partial \psi_x}{\partial x} \\
&\quad + \frac{BB_{11}}{R}\phi_Z + \frac{BF_{13}}{R}\phi_Z + \frac{BD_{13}}{R}\phi_Z \\
&\quad + BI_{33}\phi_Z - BL_{55}\frac{\partial^2 \phi_Z}{\partial x^2} + \frac{BE_{11}}{R^2}\psi_Z \\
&+ 2\frac{BG_{13}}{R}\psi_Z + BJ_{33}\psi_Z - BM_{55}\frac{\partial^2 \psi_Z}{\partial x^2} = q f_2'(z)
\end{aligned} \tag{26}$$

3. The Navier's solution

In the present study, the Navier's solution technique is used to investigate the static behaviour of a simply-supported functionally graded sandwich beams curved in elevation. The boundary conditions of the simply-supported beams curved in elevation (at x=0 and x=L) are as follows,

$$u_0 = 0,\ w_0 = 0,\ M_x^b = 0,\ M_x^{s1} = 0,\ M_x^{s2} = 0,\ \phi_z = 0,\ \psi_z = 0 \tag{27}$$

For simply-supported boundary conditions stated in Eq. (27), the unknown variables are assumed to be of the following form:

$$\begin{aligned} u_0 &= u_m \sum_{m=1}^{\infty} \cos\left(\frac{m\pi x}{L}\right), & w_0 &= w_m \sum_{m=1}^{\infty} \sin\left(\frac{m\pi x}{L}\right), \\ \phi_x &= \phi_{xm} \sum_{m=1}^{\infty} \cos\left(\frac{m\pi x}{L}\right), & \phi_z &= \phi_{zm} \sum_{m=1}^{\infty} \sin\left(\frac{m\pi x}{L}\right), \\ \psi_x &= \psi_{xm} \sum_{m=1}^{\infty} \cos\left(\frac{m\pi x}{L}\right), & \psi_z &= \psi_{zm} \sum_{m=1}^{\infty} \sin\left(\frac{m\pi x}{L}\right), \end{aligned} \tag{28}$$

The transverse uniform load q is expanded in the following form:

$$q_0 = \sum_{m=1}^{\infty} \frac{4q_0}{m\pi} \sin\left(\frac{m\pi x}{L}\right) \tag{29}$$

where, u_m, w_m, ϕ_{xm}, ψ_{xm}, ϕ_{zm} and ψ_{zm} are the unknown coefficients to be determined. By substituting Eqs. (28) and (29) into Eqs. (21)-(26), the static solution can be obtained from the following equations:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \times \begin{Bmatrix} u_0 \\ w_0 \\ \phi_x \\ \psi_x \\ \phi_z \\ \psi_z \end{Bmatrix} = \frac{4}{m\pi} \begin{Bmatrix} 0 \\ q_0 \\ 0 \\ 0 \\ q_0 f_1'(z) \\ q_0 f_2'(z) \end{Bmatrix} \tag{30}$$

where

$$\begin{aligned} & K_{11} = A_{11}\alpha^2, \quad K_{12} = \left(-\frac{A_{11}}{R}\alpha - B_{11}\alpha^3\right), \quad K_{13} = C_{11}\alpha^2, \\ & K_{14} = \left(D_{11}\alpha^2\right), \\ & K_{15} = \left(-\frac{E_{11}}{R}\alpha - G_{13}\alpha\right), \quad K_{16} = -\frac{F_{11}}{R}\alpha - H_{13}\alpha, \\ & K_{22} = \left(\frac{A_{11}}{R^2} + I_{11}\alpha^4 + 2\frac{B_{11}}{R}\alpha^2\right), \\ & K_{23} = \left(-\frac{C_{11}}{R}\alpha - J_{11}\alpha^3\right), \quad K_{24} = \left(-\frac{D_{11}}{R}\alpha - K_{11}\alpha^3\right), \\ & K_{25} = \left(\frac{E_{11}}{R^2} + \frac{G_{13}}{R} + \frac{L_{11}}{R}\alpha^2 + N_{13}\alpha^2\right), \\ & K_{26} = \left(\frac{F_{11}}{R^2} + \frac{H_{13}}{R} + \frac{M_{11}}{R}\alpha^2 + O_{13}\alpha^2\right), \\ & K_{33} = \left(P_{11}\alpha^2 + BK_{55}\right), \quad K_{34} = \left(QQ_{11}\alpha^2 + BL_{55}\right), \\ & K_{35} = \left(-\frac{R_{11}}{R}\alpha - T_{13}\alpha + BK_{55}\alpha\right), \\ & K_{36} = \left(-\frac{S_{11}}{R}\alpha - U_{13}\alpha + BL_{55}\alpha\right), \quad K_{44} = \left(V_{11}\alpha^2 + BM_{55}\right), \\ & K_{45} = \left(-\frac{W_{11}}{R}\alpha - Y_{13}\alpha + BL_{55}\alpha\right), \\ & K_{46} = \left(-\frac{X_{11}}{R}\alpha - Z_{13}\alpha + BM_{55}\alpha\right), \\ & K_{55} = \left(\frac{BA_{11}}{R^2} + 2\frac{BC_{13}}{R} + BH_{33} + BK_{55}\alpha^2\right), \\ & K_{56} = \left(\frac{BB_{11}}{R^2} + \frac{BD_{13}}{R} + \frac{BF_{13}}{R} + BI_{33} + BL_{55}\alpha^2\right), \\ & K_{66} = \left(\frac{BE_{11}}{R^2} + 2\frac{BG_{13}}{R} + BJ_{33} + BM_{55}\alpha^2\right). \end{aligned} \tag{31}$$

Stiffness matrix is symmetric matrix i.e., K_{ij}=K_{ji}

4. Illustrative examples and numerical results

In this section, an accuracy of the present theory is proved by applying it to the bending problems of simply supported functionally graded beam curved in elevation. The beam is made up of ceramic (E_c= 380 GPa, μ = 0.3) and metal (E_m= 70 GPa, μ = 0.3). These properties are varied through-the-thickness according to the power-law. For the convenience, numerical results are presented in the following forms.

Table 2. Comparison of non-dimensional transverse deflection of simply-supported FG sandwich straight beams.

P	Theory	L/h=5			L/h=20		
		2-1-2	1-1-1	1-2-1	2-1-2	1-1-1	1-2-1
0	CBT [1]	-	2.8783	2.8783	-	2.8783	2.8783
	FSDT [2]	-	3.1657	3.1657	-	2.8963	2.8963
	HSDT1 [26]	-	3.1397	3.1397	-	2.8947	2.8947
	HSDT2 [43]	3.1241	3.1241	3.1241	2.8585	2.8585	2.8585
	SSDT [44]	-	3.1296	-	-	2.8848	-
	Present	3.1825	3.1825	3.1825	2.8973	2.8973	2.8973
1	CBT [1]	-	5.9181	5.0798	-	5.9181	5.0798
	FSDT [2]	-	6.3128	5.4408	-	5.9428	5.1024
	HSDT1 [26]	-	6.2098	5.3612	-	5.9364	5.0975
	HSDT2 [43]	6.8424	6.3011	5.0341	6.4978	5.9561	5.3415
	SSDT [44]	-	6.1917	-	-	5.9155	-
	Present	6.9666	6.2941	5.4446	6.5857	5.9384	5.1055
2	CBT [1]	-	8.0074	6.4056	-	8.0074	6.4056
	FSDT [2]	-	8.4582	6.8003	-	8.0356	6.4302
	HSDT1 [26]	-	8.3893	6.6913	-	8.0262	6.4235
	HSDT2 [43]	9.5104	8.2734	6.3359	9.1626	7.9201	6.6697
	SSDT [44]	-	8.2828	-	-	7.9984	-
	Present	9.7377	8.4250	6.7858	9.2900	8.0292	6.4293
5	CBT [1]	-	10.811	8.1409	-	10.811	8.1409
	FSDT [2]	-	11.337	8.5762	-	10.844	8.1681
	HSDT1 [26]	-	11.117	8.4276	-	10.830	8.1589
	HSDT2 [43]	13.032	11.070	8.0576	12.589	10.676	8.4045
	SSDT [44]	-	11.078	-	-	10.789	-
	Present	13.367	11.297	8.6012	12.766	10.832	8.2220
10	CBT [1]	-	12.132	9.0232	-	12.132	9.0232
	FSDT [2]	-	12.132	9.4800	-	12.167	9.0518
	HSDT1 [26]	-	12.445	9.3099	-	12.151	9.0413
	HSDT2 [43]	14.434	12.391	8.9290	13.953	11.979	9.2824
	SSDT [44]	-	12.397	-	-	12.103	-
	Present	14.811	12.684	9.4751	14.094	12.155	9.0774

$$\bar{w} = 100\frac{E_m h^3}{q_0 L^4} w\left(\frac{L}{2}\right), \quad \bar{u} = 100\frac{E_m h^3}{q_0 L^4} u\left(0, -\frac{h}{2}\right), \quad \bar{\sigma}_x = \frac{h}{q_0 L}\sigma_x\left(\frac{L}{2}, \frac{h}{2}\right), \quad \bar{\tau}_{xz} = \frac{h}{q_0 L}\tau_{xz}(0,0) \tag{32}$$

4.1 *FG sandwich straight beams*

For the validation of the present formulation, initially the present theory is applied to the bending analysis of FG straight beams. Numerical results for the straight beams are recovered by setting $R = \infty$ in the present formulation. The present results are compared with previously published results and found in excellent agreement. The numerical results are presented for three lamination schemes (1-1-1, 1-2-1, 2-1-2), various values of power law index (p = 0,1,2,5,10) and aspect ratios (L/h = 5, 20). The present results are compared with Bernoulli-Euler [1], Timoshenko [2], Vo *et al* [26], Sayyad and Avhad [43] and; Sayyad and Ghugal [44].

Table 2 shows a comparison of non-dimensional transverse deflection of simply-supported FG sandwich straight beam subjected to uniform load. The examination of table 2 reveals that the present results are in excellent agreement with those presented by Vo *et al* [26], Sayyad and Avhad [43] and Sayyad and Ghugal [44] using higher-order theories. The CBT of Bernoulli-Euler [1] underestimates the deflections due to neglect of shear and normal deformations. It is observed that the non-dimensional transverse deflection decreases with increase in thickness of the core, whereas increases with an increase in the power law index. Comparison of axial stresses of simply-supported FG sandwich straight beam subjected to uniform load is shown in table 3. It is to be noted that the axial stress at the bottom surface of the beam, i.e., $z=h/2$ are presented in this table which may not be the maximum axial stress in the beam. The present results are in excellent agreement with previously published results. It is also observed that the axial stress decreased with an increase in thickness of the core. Table 4 shows comparison of transverse shear stress

Table 3. Comparison of non-dimensional axial stress of simply-supported FG sandwich straight beams.

P	Theory	L/h=5			L/h=20		
		2-1-2	1-1-1	1-2-1	2-1-2	1-1-1	1-2-1
0	FSDT [2]	-	3.7500	3.7500	-	15.000	15.000
	HSDT1 [26]	-	3.8005	3.8005	-	15.012	15.012
	HSDT2 [43]	3.8025	3.8025	3.8025	15.0136	15.013	15.013
	SSDT [44]	-	3.8582	-	-	15.253	-
	Present	3.7927	3.7928	3.7927	14.8451	15.009	15.009
1	FSDT [2]	-	1.4203	1.2192	-	5.6814	4.8766
	HSDT1 [26]	-	1.4330	1.2315	-	5.6845	4.8797
	HSDT2 [43]	1.5900	1.4614	1.2331	6.3020	5.7370	4.8802
	SSDT [44]	-	1.4602	-	-	5.7994	-
	Present	1.5953	1.4409	1.2413	6.2374	5.6834	4.8883
2	FSDT [2]	-	1.9218	1.5373	-	7.6871	6.1493
	HSDT1 [26]	-	1.9352	1.5505	-	7.6904	6.1526
	HSDT2 [43]	2.2384	1.9369	1.5530	8.8940	7.6154	6.1534
	SSDT [44]	-	1.9732	-	-	7.8527	-
	Present	2.2423	1.9572	1.5678	8.7975	7.7310	6.1562
5	FSDT [2]	-	2.5948	1.9538	-	10.379	7.8152
	HSDT1 [26]	-	2.6079	1.9672	-	10.382	7.8185
	HSDT2 [43]	3.0733	2.6101	1.9707	12.222	10.271	7.8196
	SSDT [44]	-	2.6575	-	-	10.596	-
	Present	3.0575	2.6078	1.9116	12.063	10.372	7.5215
10	FSDT [2]	-	2.9117	2.1656	-	11.646	8.6623
	HSDT1 [26]	-	2.9245	2.1788	-	11.650	8.6655
	HSDT2 [43]	3.4047	2.9268	2.1829	13.545	11.523	8.6667
	SSDT [44]	-	2.9776	-	-	11.880	-
	Present	3.3800	2.9146	2.2156	13.213	11.636	8.7457

Table 4. Comparison of non-dimensional transverse shear stress of simply-supported FG sandwich straight beams.

P	Theory	*L/h*=5			*L/h*=20		
		2-1-2	1-1-1	1-2-1	2-1-2	1-1-1	1-2-1
0	FSDT [2]	-	0.5976	0.5976	-	0.5976	0.5976
	HSDT1 [26]	-	0.7233	0.7233	-	0.7432	0.7432
	HSDT2 [43]	0.7285	0.7285	0.7285	0.7355	0.7355	0.7355
	SSDT [44]	-	0.7431	-	-	0.7596	-
	Present	0.7318	0.7318	0.7318	0.7362	0.7362	0.7362
1	FSDT [2]	-	0.8208	0.7507	-	0.8208	0.7507
	HSDT1 [26]	-	0.8444	0.7993	-	0.8657	0.8193
	HSDT2 [43]	0.9050	0.8767	0.8056	0.9107	0.8784	0.8106
	SSDT [44]	-	0.8623	-	-	5.7994	-
	Present	0.8529	0.8331	0.7920	0.8393	0.8340	0.7919
2	FSDT [2]	-	0.9375	0.8208	-	0.9375	0.8208
	HSDT1 [26]	-	0.9084	0.8349	-	0.9316	0.8556
	HSDT2 [43]	0.9103	0.9170	0.8424	0.9149	0.9222	0.8486
	SSDT [44]	-	0.9233	-	-	0.9405	-
	Present	0.8994	0.8423	0.8666	0.9898	0.8408	0.8700
5	FSDT [2]	-	1.0929	0.9053	-	1.0929	0.9053
	HSDT1 [26]	-	0.9931	0.8763	-	1.0194	0.8986
	HSDT2 [43]	1.1766	1.0048	0.8851	1.1835	1.0101	0.8897
	SSDT [44]	-	1.0010	-	-	1.0208	-
	Present	0.9142	0.9679	0.8840	0.9898	0.9686	0.8855
10	FSDT [2]	-	1.1819	0.9497	-	1.1819	0.9497
	HSDT1 [26]	-	1.0458	0.8980	-	1.0736	0.9214
	HSDT2 [43]	1.2982	1.0586	0.9083	1.3055	1.0642	0.9128
	SSDT [44]	-	1.0487	-	-	1.0698	-
	Present	1.0115	1.0886	0.8947	1.0956	1.0915	0.8888

Table 5. Non-dimensional axial displacement of simply-supported FG sandwich curved beams.

R/h	Theory	*p*	*L/h*=5			*L/h*=20		
			2-1-2	1-1-1	1-2-1	2-1-2	1-1-1	1-2-1
5	Present	0	1.9111	1.9111	1.9111	3.5597	3.5593	3.5597
	SSDT [44]		-	1.8244	-	-	-	-
	Present	1	4.1660	3.7665	3.2614	8.0805	7.9353	6.2672
	SSDT [44]		-	3.6707	-	-	-	-
	Present	2	5.8149	5.0378	4.0631	11.3903	9.8481	7.8859
	SSDT [44]		-	4.9327	-	-	-	-
	Present	5	7.9700	6.7387	5.1209	15.6414	13.2728	10.0641
	SSDT [44]		-	6.6188	-	-	-	-
	Present	10	8.8669	7.5607	5.6581	17.3437	14.8888	11.1229
	SSDT [44]		-	7.4123	-	-	-	-
10	Present	0	1.4856	1.4856	1.4856	1.9888	1.9888	1.9888
	SSDT [44]		-	1.3995	-	-	-	-
	Present	1	3.2378	2.9276	2.5352	4.5168	4.0733	3.5025
	SSDT [44]		-	2.8300	-	-	-	-
	Present	2	4.5195	3.9165	3.1593	6.3690	5.5055	4.4090
	SSDT [44]		-	3.8081	-	-	-	-
	Present	5	6.1938	5.2376	4.9257	8.7490	7.4235	5.6309
	SSDT [44]		-	5.1148	-	-	-	-
	Present	10	6.8900	5.8762	4.3993	9.6998	8.3286	6.2209
	SSDT [44]		-	5.7294	-	-	-	-

Table 5 continued

R/h	Theory	p	L/h=5			L/h=20		
			2-1-2	1-1-1	1-2-1	2-1-2	1-1-1	1-2-1
20	Present	0	1.2539	1.2539	1.2539	1.1333	1.1333	1.1333
	SSDT [44]		-	1.1682	-	-	-	-
	Present	1	2.7320	2.4706	2.1397	2.5746	2.3217	1.9962
	SSDT [44]		-	2.3726	-	-	-	-
	Present	2	3.8136	3.3055	2.6668	3.6309	3.1384	2.5133
	SSDT [44]		-	3.1962	-	-	-	-
	Present	5	5.2255	4.4193	3.3597	4.9886	4.2327	3.2112
	SSDT [44]		-	4.2963	-	-	-	-
	Present	10	5.8122	4.9577	3.7132	5.5305	4.7495	3.5471
	SSDT [44]		-	4.8135	-	-	-	-

Table 6. Non-dimensional vertical displacement of simply-supported FG sandwich curved beams.

R/h	Theory	p	L/h=5			L/h=20		
			2-1-2	1-1-1	1-2-1	2-1-2	1-1-1	1-2-1
5	Present	0	3.1775	3.1775	3.1775	2.8551	2.8551	2.8551
	SSDT [44]		-	3.1294	-	-	-	-
	Present	1	6.9461	6.2763	5.4307	6.5667	5.6738	5.0931
	SSDT [44]		-	6.1913	-	-	-	-
	Present	2	9.7014	8.3955	6.7647	9.2565	7.2235	7.2235
	SSDT [44]		-	8.2823	-	-	-	-
	Present	5	13.309	11.248	8.5579	12.711	8.5781	8.5781
	SSDT [44]		-	11.077	-	-	-	-
	Present	10	14.811	12.625	9.4354	14.094	9.5025	9.5025
	SSDT [44]		-	12.396	-	-	-	-
10	Present	0	3.1775	3.1775	3.1775	2.8570	2.8570	2.8570
	SSDT [44]		-	3.1295	-	-	-	-
	Present	1	6.9461	6.2763	5.4307	6.5667	5.6612	5.0931
	SSDT [44]		-	6.1916	-	-	-	-
	Present	2	9.7014	8.3955	6.7647	9.2565	7.2069	7.2069
	SSDT [44]		-	8.2827	-	-	-	-
	Present	5	13.309	11.248	8.5579	12.711	8.5601	8.5601
	SSDT [44]		-	11.078	-	-	-	-
	Present	10	14.811	12.625	9.4354	14.094	9.4802	9.4802
	SSDT [44]		-	12.397	-	-	-	-
20	Present	0	3.1775	3.1775	3.1775	2.8928	2.8928	2.8928
	SSDT [44]		-	3.1295	-	-	-	-
	Present	1	6.9461	6.2763	5.4307	6.5667	5.6520	5.0931
	SSDT [44]		-	6.1916	-	-	-	-
	Present	2	9.7014	8.3955	6.7647	9.2565	7.1941	6.4085
	SSDT [44]		-	8.2828	-	-	-	-
	Present	5	13.309	11.248	8.5579	12.711	8.5457	8.1799
	SSDT [44]		-	11.078	-	-	-	-
	Present	10	14.811	12.625	9.4354	14.094	9.4632	9.0394
	SSDT [44]		-	12.397	-	-	-	-

Table 7. Non-dimensional axial stress of simply-supported FG sandwich curved beams.

R/h	Theory	p	L/h=5 2-1-2	L/h=5 1-1-1	L/h=5 1-2-1	L/h=20 2-1-2	L/h=20 1-1-1	L/h=20 1-2-1
5	Present	0	3.7557	3.7557	3.7556	14.845	14.845	14.845
	SSDT [44]		-	3.8221	-	-	-	-
	Present	1	1.5801	1.5068	1.2301	6.2374	6.1237	4.8387
	SSDT [44]		-	1.4449	-	-	-	-
	Present	2	2.2208	1.9403	1.5552	8.7975	7.6559	6.0980
	SSDT [44]		-	1.9518	-	-	-	-
	Present	5	3.0234	2.5838	1.8933	12.063	10.2648	7.4370
	SSDT [44]		-	2.6274	-	-	-	-
	Present	10	3.3396	2.8837	2.1970	13.213	11.4982	8.6609
	SSDT [44]		-	2.9432	-	-	-	-
10	Present	0	3.7557	3.7557	3.7556	14.845	14.845	15.167
	SSDT [44]		-	3.8402	-	-	-	-
	Present	1	1.5801	1.5068	1.2301	6.2374	6.1237	4.9527
	SSDT [44]		-	1.4526	-	-	-	-
	Present	2	2.2208	1.9403	1.5552	8.7975	7.6559	6.2515
	SSDT [44]		-	1.9625	-	-	-	-
	Present	5	3.0234	2.5838	1.8933	12.063	10.264	7.6693
	SSDT [44]		-	2.6424	-	-	-	-
	Present	10	3.3396	2.8837	2.1970	13.213	11.498	8.8873
	SSDT [44]		-	2.9604	-	-	-	-
20	Present	0	3.7557	3.7557	3.7556	14.845	14.845	15.167
	SSDT [44]		-	3.8492	-	-	-	-
	Present	1	1.5801	1.5068	1.2301	6.2374	6.1237	4.9527
	SSDT [44]		-	1.4564	-	-	-	-
	Present	2	2.2208	1.9403	1.5552	8.7975	7.6559	6.2515
	SSDT [44]		-	1.9679	-	-	-	-
	Present	5	3.0234	2.5838	1.8933	12.063	10.264	7.6693
	SSDT [44]		-	2.6499	-	-	-	-
	Present	10	3.3396	2.8837	2.1970	13.213	11.498	9.0394
	SSDT [44]		-	2.9690	-	-	-	-

Table 8. Non-dimensional shear stress of simply-supported FG sandwich curved beams.

R/h	Theory	P	L/h=5 2-1-2	L/h=5 1-1-1	L/h=5 1-2-1	L/h=20 2-1-2	L/h=20 1-1-1	L/h=20 1-2-1
5	Present	0	0.7318	0.7318	0.7318	0.7267	0.7363	0.7363
	SSDT [44]		-	0.7431	-	-	-	-
	Present	1	0.8529	0.9090	0.7919	0.8508	0.9105	0.7920
	SSDT [44]		-	0.8623	-	-	-	-
	Present	2	0.8989	0.8406	0.8660	0.8954	0.8406	0.8694
	SSDT [44]		-	0.9233	-	-	-	-
	Present	5	0.9998	0.9676	0.8828	0.9894	0.9684	0.8846
	SSDT [44]		-	1.0010	-	-	-	-
	Present	10	1.1089	1.0879	0.8928	1.0953	1.0909	0.8873
	SSDT [44]		-	1.0487	-	-	-	-
10	Present	0	0.7318	0.7318	0.7318	0.7267	0.7363	0.7363
	SSDT [44]		-	0.7431	-	-	-	-
	Present	1	0.8529	0.9090	0.7919	0.8508	0.9105	0.7920
	SSDT [44]		-	0.8623	-	-	-	-
	Present	2	0.8989	0.8406	0.8660	0.8954	0.8406	0.8694
	SSDT [44]		-	0.9233	-	-	-	-
	Present	5	0.9998	0.9676	0.8828	0.9894	0.9684	0.8846
	SSDT [44]		-	1.0010	-	-	-	-

Table 8 continued

R/h	Theory	P	L/h=5 2-1-2	L/h=5 1-1-1	L/h=5 1-2-1	L/h=20 2-1-2	L/h=20 1-1-1	L/h=20 1-2-1
	Present	10	1.1089	1.0879	0.8928	1.0953	1.0909	0.8873
	SSDT [44]		-	1.0487	-	-	-	-
20	Present	0	0.7318	0.7318	0.7318	0.7267	0.7363	0.7363
	SSDT [44]		-	0.7431	-	-	-	-
	Present	1	0.8529	0.9090	0.7919	0.8508	0.9105	0.7920
	SSDT [44]		-	0.8623	-	-	-	-
	Present	2	0.8989	0.8406	0.8660	0.8954	0.8406	0.8694
	SSDT [44]		-	0.9233	-	-	-	-
	Present	5	0.9998	0.9676	0.8828	0.9894	0.9684	0.8846
	SSDT [44]		-	1.0010	-	-	-	-
	Present	10	1.1089	1.0879	0.8928	1.0953	1.0909	0.8873
	SSDT [44]		-	1.0487	-	-	-	-

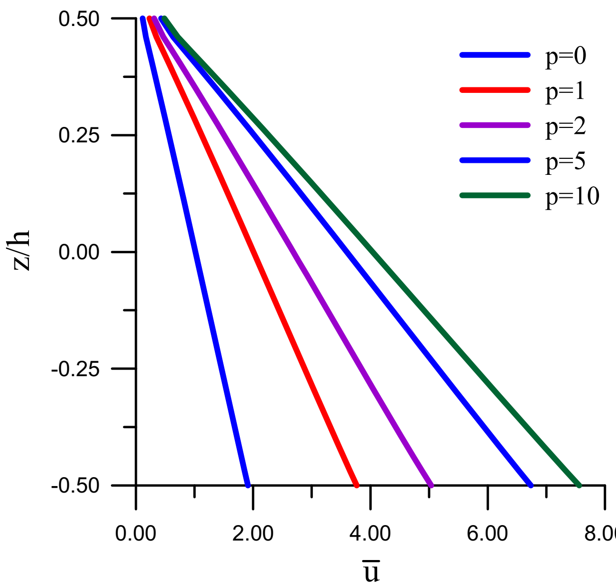

Figure 3. Through thickness variation of axial displacement for functionally graded sandwich beams curved in elevation (1-1-1, *R/h=5* and *L/h*=5).

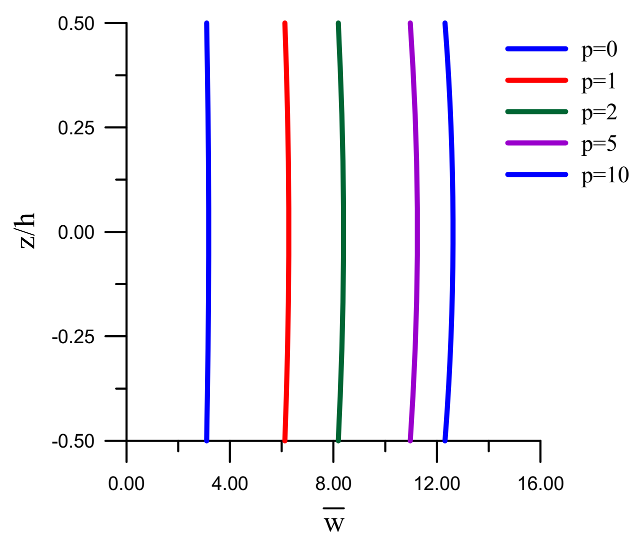

Figure 4. Through thickness variation of transverse displacement for functionally graded sandwich beams curved in elevation (1-1-1, *R/h=5* and *L/h*=5).

of simply-supported FG sandwich straight beam subjected to uniform load. The present results are found in excellent agreement with other theories. The examination of table 4 reveals that the non-dimensional transverse shear stress increases with respect to increase in the power law index.

The HSDT1 [26] and HSDT2 [43] are used for the comparison of the present results. The HSDT1 considered the effect of transverse normal deformation; however, thickness coordinate in this theory is expanded upto third order only. The HSDT2 neglect the effects of transverse normal deformations i.e., thickness stretching which plays an important role to predict the accurate bending behaviour of thick FG sandwich beams. The present theory considers the effects of both transverse shear and normal deformations; and fifth order expansion of thickness coordinate which improve the accuracy of the present theory compared to these HSDTs.

4.2 *FG sandwich curved beams*

In this section, static analysis of FG sandwich beam curved in elevation are considered (1-1-1, 1-2-1, 2-1-2). Non-dimensional displacements, axial stresses and shear stresses of functionally graded sandwich beam curved in elevation for various values of the power law index (p=0,1,2,5,10) and radii of curvature (R=5,10,20,50,100) are presented in

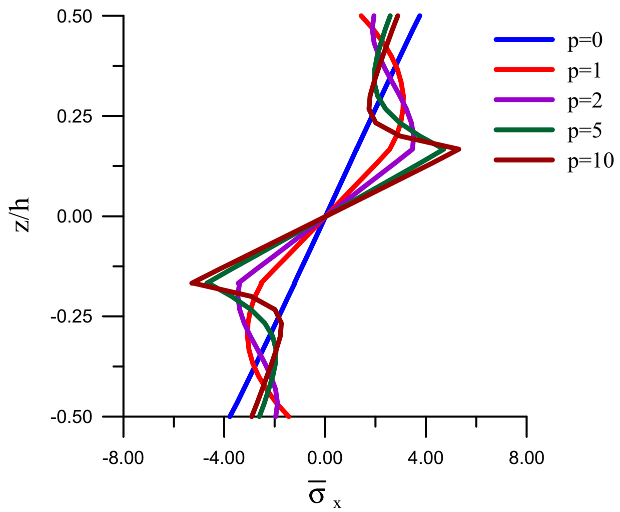

Figure 5. Through thickness variation of axial stress for functionally graded sandwich beams curved in elevation (1-1-1, *R/h=5* and *L/h*=5).

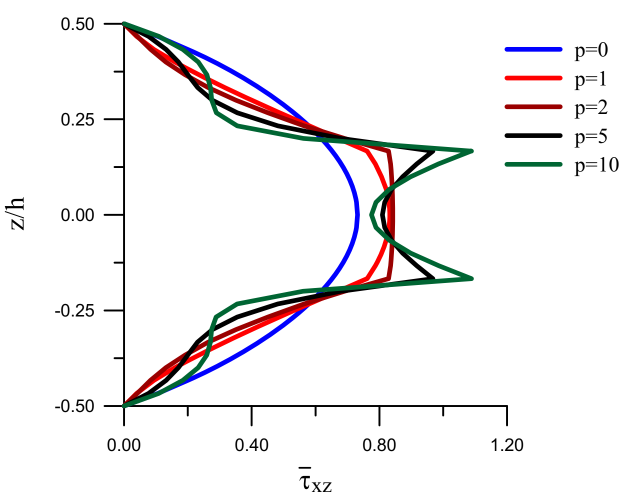

Figure 6. Through thickness variation of transverse shear stress for functionally graded sandwich beams curved in elevation (1-1-1, *R/h=5* and *L/h*=5).

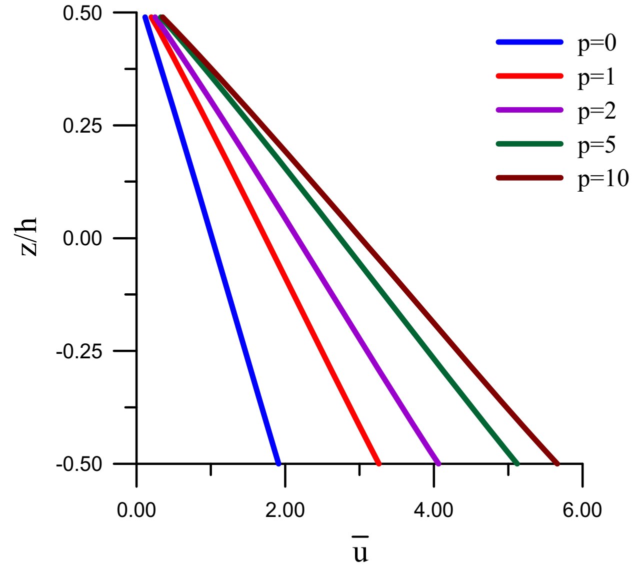

Figure 7. Through thickness variation of axial displacement for functionally graded sandwich beams curved in elevation (1-2-1, *R/h=5* and *L/h*=5).

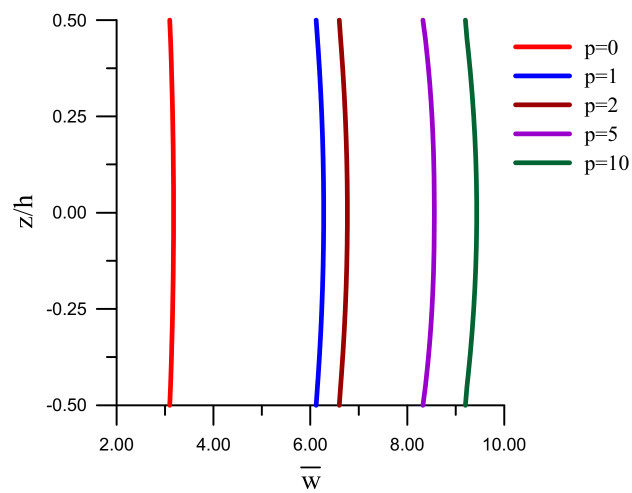

Figure 8. Through thickness variation of transverse displacement functionally graded sandwich beams curved in elevation (1-2-1, *R/h=5* and *L/h*=5).

tables 5-8 and plotted in figures 3-14. In the variety of literature, authors found only one paper by Sayyad and Ghugal [44] in which displacements and stresses for FG sandwich curved beams are presented. Therefore, the present results are compared with those presented by Sayyad and Ghugal [44].

Table 5 shows a comparison of non-dimensional axial displacement of simply-supported FG sandwich curved beam under uniform load. It is observed that the value of axial displacement in the curved beam is higher for the lower value of the radius of curvature, whereas the value of axial displacement is lower for the higher value of the radius of curvature. From tables 6-8 it is observed that the transverse displacement, axial stress and transverse shear stress in curved beams are similar to straight beam. Overall, the present results are in good agreement with those presented by Sayyad and Ghugal [44].

From figures 3-14, it is pointed out that through-the-thickness variation of axial stresses is nonlinear along the thickness of face sheets and linear along the thickness of the core. Similarly nonlinear variation is observed in case of transverse shear stress also. From these figures, it is also

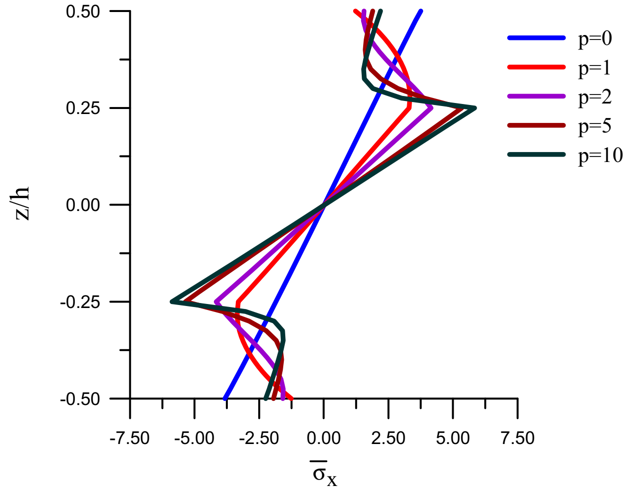

Figure 9. Through thickness variation of axial stress for functionally graded sandwich beams curved in elevation (1-2-1, *R/h=5* and *L/h*=5).

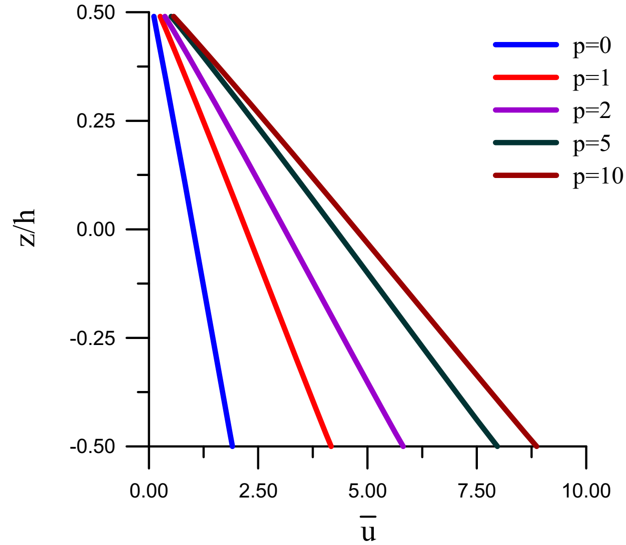

Figure 11. Through thickness variation of axial displacement, for functionally graded sandwich beams curved in elevation (2-1-2, *R/h=5* and *L/h*=5).

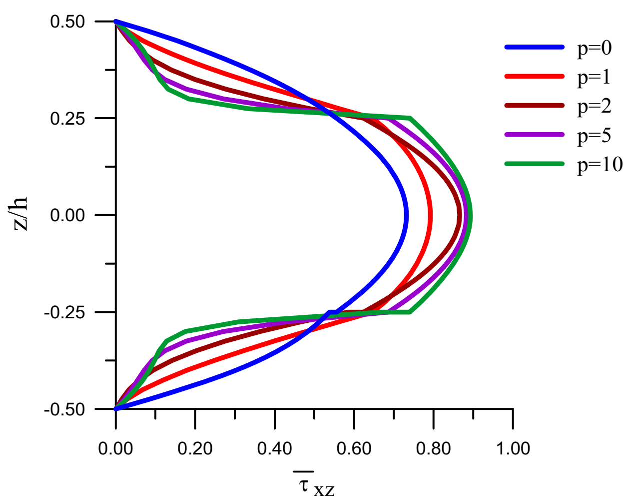

Figure 10. Through thickness variation of transverse shear stress for functionally graded sandwich beams curved in elevation (1-2-1, *R/h=5* and *L/h*=5).

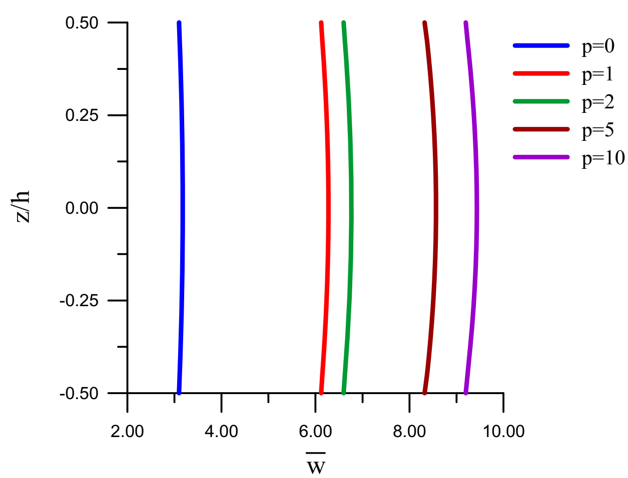

Figure 12. Through thickness variation of transverse displacement for functionally graded sandwich beams curved in elevation (2-1-2, *R/h=5* and *L/h*=5).

pointed out that the zero transverse shear stress condition at top and bottom surfaces of the beam are achieved using constitutive relations.

The kinematics of the sinusoidal shear deformation theory (SSDT) used for the comparison is based on non polynomial trigonometric functions in terms of thickness coordinate which can be expanded up to infinite power series. Therefore, the SSDT involves four unknowns to consider effects of transverse shear and normal deformations. The kinematics of the present theory is based upon polynomial type functions which is expanded up to fifth-order in terms of thickness coordinate. Also, every power of thickness coordinate (*z/h*) in the displacement field introduces one unknown variables in the present theory. Therefore, the kinematics of the present theory is much richer than the SSDT. Also, the present theory involves six unknowns and also accounts for transverse shear and normal deformations. Therefore the numerical results of the present theory are higher than SSDT.

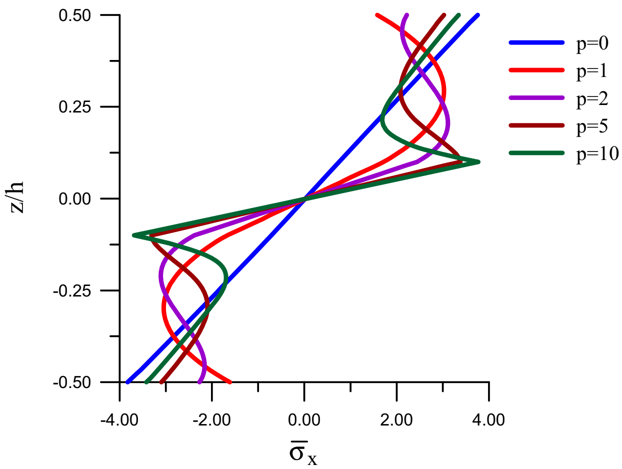

Figure 13. Through thickness variation of axial stress for functionally graded sandwich beams curved in elevation (2-1-2, *R/h=5* and *L/h*=5).

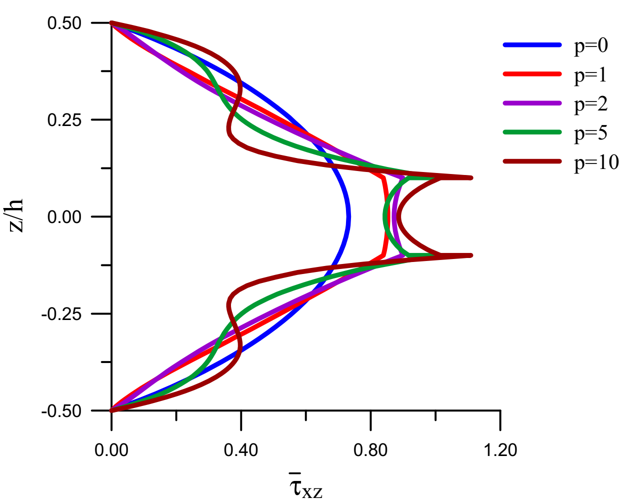

Figure 14. Through thickness variation of transverse shear stress for functionally graded sandwich beams curved in elevation (2-1-2, *R/h=5* and *L/h*=5).

5. Conclusions

In the present study, a higher order shear and normal deformation theory is presented for the static analysis of functionally graded sandwich beams curved in elevation. Simply-supported curved beams are solved using the Navier's solution technique. Nondimensional displacements and stresses are presented for FG sandwich straight and curved beams. This theory includes both shear and normal deformation effects, i.e., thickness stretching effect. Effects of power-law index, radius of curvature and skin-core-skin thickness ratios on the displacements and stresses are discussed and found in excellent agreement with previously published results.

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