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A cost-causal marginal participation method using min-max fairness for transmission services cost allocation

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Abstract. We consider the problem of fair allocation of the cost of a transmission system among load and generation entities using the marginal participation approach. We show that a cost-causal approach involving capacity-based line cost rate and a min-max fair economic slack bus selection for price-taking entities leads to a rigorously fair and more accurate implementation of marginal participation method. In the existing methods the counter-flows are masked, which is a compromise with fairness and linearity. However, if the counter-flows are incentivized then it can lead to pay-offs to some entities. The proposed approach solves the problem of pay-offs without masking the counter-flows. This is achieved by separation of the total transmission services cost into usage, reliability and residual capacity components. The allocation of the first two components is based on the min-max fairness policy, and the residual capacity costs are allocated on a pro-rata basis. Simulation results on multiple IEEE test systems, Indian utility power systems and extensive comparative evaluations for the contemporary methods demonstrate the claims made.

Keywords. Cost allocation; counter-flows; power flow; marginal pricing; marginal participation; min-max fairness; reliability cost; transmission system.

1. Introduction

The transmission system cost allocation problem involves the sharing of the yearly transmission network cost among the load and/or generator entities. Traditionally, the cost of transmission network usage has been calculated using ad-hoc methods like the postage stamp method or the contract path method [1, 2]. In the postage stamp method, an entity, which could be a load or a generator, pays a network usage cost that is proportional to the MW withdrawn or injected in the grid by it. On the other hand, the contract path approach assigns an arbitrary flow path for a transaction and then assigns a cost to the transaction. Both of these methods are simplistic in nature and do not truly evaluate the 'extent of use' of the network by an entity or a transaction. However, post-deregulation, the pool market scenario has emerged and the principles of the cost allocation have progressed towards the allocation in proportion to the 'extent of use', i.e. the beneficiary pays [3]. This is referred to as *cost causality* [4]. It implies that one should charge the customers the cost they cause. It also helps in fulfilling the regulatory objective of non-discrimination. We consider the problem of allocation of embedded cost of a transmission network among the load and generation entities using the marginal pricing (MP) approach [5–12]. The charges to be recovered are also called as *complementary charges* [7] because these charges represent costs that have to be recovered after adjusting for network revenues accrued from spot pricing of electricity. For example, in India and Europe there are power exchanges that determine area clearing prices using *explicit auctions* while in certain geographies, e.g. PJM market in North America, *implicit auction* is used to determine locational marginal prices. The MP approach is suitable for the pool market where no relationship between buyers and sellers can be established. This approach also meets the requirement that an entity's network usage costs should not depend upon the commercial transactions [13]. The method evaluates the 'extent of use' of the network by an entity by *linearly extrapolating* the marginal flows on the transmission lines. It monetizes the flow share by calculating the fractional usage in a line [5–9, 11] or the network [10, 12]. For monetizing the usage cost, a *line cost rate* has to be defined. The most commonly used definition is that the *line cost rate* is the ratio of the line cost to the line usage. Typically, the line cost is taken for a duration of one year. Usually, the line usage is nothing but the line's power flow in MW obtained from the power flow analysis. It represents the net use of a line by all the

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entities in the system. Once the cost-share of an entity for using the network is known, its *point of connection tariff* (PoC tariff) can be worked out. It is defined as the rate in ₹/MW (or $/MW) that an entity has to pay for the network usage (for a specified duration, e.g. a year) i.e. for its injection or withdrawal, at its point of interconnection (a bus). Typically, the PoC tariff is computed *ex-ante*.

The MP approach and its variants have been used for the cost allocation in different geographies in the world. A variant of this method is implemented by the Central Electricity Regulatory Commission (CERC) in India [14]. Other geographies like Argentina, Chile, Australia and Ireland where this approach has been used are discussed in [13]. The methods reported in the literature differ in many aspects, e.g. the treatment of fairness, computational efficacy, data requirements, simplicity, tariff signal stability, economic efficiency[1], accuracy, political acceptability and appropriate locational signals. Driven by these considerations, research work in the MP approach has focused, primarily, on the following issues.

1. *Fair* selection of *economic slack bus* to evaluate network usage of an entity [8, 9, 12, 15–18].
2. Appropriate definition of the line cost rate [10, 11].
3. Policies to manage incentives from *counter-flows* to the beneficiary entities and impact management on the cost bearers [7, 10, 12].
4. Policies on fair cost allocation by unbundling network usage, reliability and residual capacity costs [11, 19].
5. Modelling of the reactive power variables for improving accuracy of marginal flow component [14, 20, 21].

To the best of our knowledge, no single reported method in the MP framework addresses all these aspects satisfactorily. We have conducted research to resolve these issues. In this paper, we propose a cost-causal marginal participation approach for the cost allocation of the transmission network among the load and generation entities. The focus is on the first four issues mentioned here. The other aspect mentioned will be part of our future work and publications. The rest of the paper is organized as follows. Section 2 reviews various developments and does a thorough gap analysis regarding issues (1–3) mentioned earlier. Section 3 explains the proposed approach regarding our recommendations for issues (1–3) mentioned previously. Case studies for comparison of the proposed cost-causal approach to the other existing methods based on the DC power flow are given in section 4. The design of the proposed cost-causal method with reliability modelling is explained in section 5. The results of the proposed cost-causal method (with and without reliability modelling) for IEEE 14, 30, 57 and 118 bus network test cases and Indian power system are presented in section 6. Section 7 concludes the paper.

[1]the ability to schedule the least cost resource.

2. Gap analysis in MP framework

None of the existing MP approaches are comprehensive enough to address many facets like fairness in *economic slack bus* selection, line-cost-rate selection, managing counter-flow incentives, costing and apportioning of reliability capacity and unused capacity, etc. These aspects are reviewed here.

2.1 *Fair selection of economic slack bus*

Traditionally, a slack bus in the power flow analysis is used to compensate for the losses (and any demand and supply mismatch) that cannot be determined *a priori*. For the marginal participation method, it has been proposed in [8] that a dispersed slack bus can be used. Dispersed slack bus of a generator is a vector of participation factors of the loads; when the generation is incremented by 1 MW, for a lossless network, the loads have to increase their participation factors. Similarly, the dispersed slack bus of a load can be defined by a vector of participation factors of the generators. This participation factor α_{kj} for j_{th} generator for a load k fulfills the condition $\sum_{j=1}^{n_G} \alpha_{kj} = 1$. In principle, the increment in the generation (or the load) could very well be dispersed at multiple loads (or generators). However, in the context of the marginal participation approach, the choice of a slack bus significantly impacts the calculation of the 'extent of use' and hence the usage cost allocated to an entity. Thus, the choice of a slack bus must be acceptable to all the cost bearing entities. Hence, the dispersed slack bus selected for usage cost allocation is called the economic slack bus. In other words, the choice of a slack bus should also be fair. When customized economic slack buses for entities are used, it has to be ascertained that superposition of the marginal contributions of entities leads to line flows that are the same as in the power flow study as symbolized by (1):

$$P^0_{lm} = \sum_{\forall\, L_j \in \mathcal{L}} P(lm, L_j) = \sum_{\forall\, G_i \in \mathcal{G}} P(lm, G_i). \tag{1}$$

The impact of the choice of the economic slack bus on the cost-share (or PoC tariff) of the entities is explained by an illustration given in figure 1. The capacity in MW for the generation and load entities and the line costs in ₹ are mentioned in the diagram. The options considered for the economic slack bus for the load entities for cost allocation of the system are provided in table 1. Table 2 clearly shows the impact of economic slack bus selection for one entity on its cost-share as well as that of the other entity.

In [8], a heuristic based on proportionate tracing or Average Participation (AP) method has been used to define the economic slack bus. In [12], a rigorous application of *min-max fairness policy* has been advocated for choosing

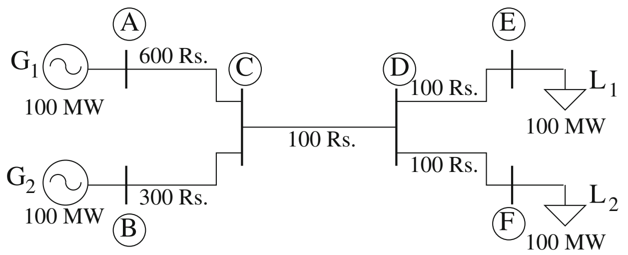

Figure 1. Impact of choice of economic slack bus on cost-sharing.

Table 1. Two options for economic slack bus.

Option	For L1	For L2
Choice-1	$G1 : G2 = 1 : 0$	$G1 : G2 = 0 : 1$
Choice-2	$G1 : G2 = 0 : 1$	$G1 : G2 = 1 : 0$

Table 2. Cost allocation for load entities in ₹.

Option	L1	L2	Total cost of system
Choice-1	750	450	1200
Choice-2	450	750	1200

an economic slack bus for the load and the generation entities. A PoC tariff vector is said to be min-max fair if providing a more favourable economic slack bus to an entity in the network will increase PoC tariff of some other entity(entities) that already pays an equal or higher tariff than the beneficiary entity. Hence, the min-max fair PoC tariff vector represents an equilibrium solution. Reference [22] discusses the application of the min-max fairness policy in the context of determining the long-run marginal cost (LRMC). Application of min-max fairness policy in the context of tracing is discussed in [17, 18]. The Aumann–Shapley (AS) method for transmission system cost allocation [23] is yet another attractive approach to solve the problem of a fair selection of the economic slack bus. A comparison of the proposed method to the AS method is given in section 4.

Beyond the approach to choose the economic slack buses, there are two major differences between the MP approach as developed in [5, 6, 8, 11], and the Min-max fair MP approach [12]. The traditional MP approach does cost accounting on a line-by-line basis whereas the Min-max fair MP approach [12, 24] and a recent paper [10] calculate the aggregate marginal network usage cost. This is achieved by summing the marginal usage cost for each line in the network. The second approach enables the application of the principle of superposition and makes linear programming (LP) viable.

A very desirable outcome of the application of the min-max fairness principle is that it leads to the price clusters. Entities in a cluster have the same PoC tariff. This implies that generators within a cluster can be scheduled using the principle of least marginal cost. This promotes economic efficiency during scheduling. It also provides more choices in the siting of renewable energy sources. This aspect will be illustrated in section 6. Min-max fairness computation can be scaled up for large networks [12, 24] as it requires only an LP tool. Computationally it is simpler than the AS fairness criterion as it requires parameterized LP tool wherein parameterized step influences the accuracy and computational time. Hence, in the proposed work, an entity's economic slack bus selection is based upon the min-max fairness principle.

2.2 *Fair selection of the line cost rate*

The literature so far has defined three ways of defining the *line cost rate*.

2.2a *Approach-1*: Many methods reported in the literature [5, 6, 9, 12] allocate the cost of the transmission line to the current beneficiaries, i.e. those who use the line in a given power flow scenario. Therefore, the line cost rate becomes a function of the loading level. This also distorts the pricing signal as the entire cost of an asset is borne by the current beneficiaries whose requirement may be a small fraction of the capacity built, a consequence of economies of scale and indivisibility of transmission project [25]. In such a scenario, lower the loading level, higher the line cost rate and vice-versa.

2.2b *Approach-2*: Fairness in cost allocation can be improved by separating the 'extent of use' cost and the unused capacity cost. This can be done if the line cost rate is capacity based. The capacity-based line cost rate is defined as the cost of the line divided by the capacity of the line in MW. It also enables cost allocation in the interconnected energy markets [10].

2.2c *Approach-3*: On the other hand, [11] proposes that the maximum flow on a line computed from the $N-1$ credible contingencies should be used to derive the optimal capacity. The 'extent of use' cost and the reliability costs are clubbed into one basket and the residual capacity costs into another. The downside of this approach is also that the line cost rate continues to depend on the power flow scenario.

2.2d *Our recommendation*: We advocate that a static capacity-based line cost rate (see our recent conference proceedings [24]) should be preferred even when accounting for the reliability costs. The capacity-based line cost rate is used in the proposed approach. Our research (see [24]) shows that it leads to stable prices that are easy to grasp for the stakeholders. It also helps in the management of the incentives associated with the counter-flows.

2.3 *Fairness challenge in the presence of counter-flows*

In a power flow scenario, the marginal flow by a specific entity in a line may be in opposition to the net flow through the line. Such a marginal flow component is known as a counter-flow. Since the counter-flow reduces the total flow of the line, it also reduces the congestion of the system. Incentivizing the counter-flow is a fairness requirement. It can potentially reduce the regret associated with the higher PoC tariff takers. However, when the counter-flows are incentivized completely, instances of entities with negative PoC tariffs have been reported [10, 11, 26]. The negative PoC tariffs imply that the positive rate takers or the cost-bearers are over-burdened as the sum of their payments exceeds the cost of the network to be recovered. In such a situation, the policy of incentivizing the counter-flows misfires and harms the positive tariff takers.

The literature on management of the counter-flows is varied and it has advocated all the three possibilities, viz. *partial incentivizing* [5–8, 22]; *complete incentivizing* [10, 12, 24] and *complete suppression of incentives* [9, 21, 23, 27]. Complete suppression schemes would not be acceptable in countries like India, where National Electricity Policy (NEP) [28] stipulates that the tariff mechanism should be both direction- and flow-sensitive. References [11, 25] also discuss a more nuanced partial scheme where the counter-flows are incentivized only if a line is loaded beyond a specific percentage of the installed or the optimal capacity. Since it may not be possible to theoretically rule out the negative PoC tariffs when complete incentivizing of counter-flows is done, reference [10] has proposed a post-facto algorithm to redistribute extra-income arising from the negative PoC tariff on a pro-rata basis among the positive PoC tariff takers of the same type (generator or load).

In the partial counter-flow incentive scheme, only those entities who create positive flow components are allocated a share of the line cost. Thus, entities that create counter-flows do not pay for the asset's usage. At the same time, they are not paid and thus the problem of the negative PoC tariff is resolved. However, while being fairer than the complete suppression scheme, this is not fair enough for those high PoC tariff takers who could have been further benefited by fully incentivizing the counter-flows without creating the negative PoC tariff problem. Also the linearity in the formulation is sacrificed, which hinders the application of rigorous methods that require optimization tools like min-max fairness.

References [10, 12] are examples of a scheme where the counter-flows are fully incentivized. In [12], the line cost rate is defined as the line cost per unit flow. However, now the rate of incentivizing counter-flows becomes extremely high for lightly loaded lines. Hence, the marginal benefits to the lucky few who create counter-flows on them outweigh the marginal costs on lines where their flow component is in the direction of the aggregate flow. Even when a min-max fair economic slack bus is used, this undue advantage persists due to the spatial peculiarity of the loads and the generators. It can be argued that when the negative PoC tariff problem is a consequence of the flow-based line cost rate, using a capacity-based cost rate should solve the problem. However, as shown in [10, 11, 26], even with this rate, the negative PoC tariffs have occurred and the problem now can be traced to inappropriate *economic slack bus* selection, e.g. AP rule. Thus, an integrated approach that takes care of both, the fair line cost rate and the economic slack bus selection, is required. If the line cost rate is based on the line flow and if the counter-flow is found on a lightly loaded line, then a high line cost rate will imply a high reward rate and this component alone may dominate other positive line usage rates. As explained earlier, this adversely affects the other price takers because they not only pay for the entire network cost but also the additional incentives. If an entity gets a negative PoC tariff, it means that the PoC tariffs of other entities will increase. Hence, line cost rate should be based on the line capacity. In the min-max fair selection of economic slack bus the rigorous process of minimizing the maximum PoC tariff in a lexicographic manner prevents the selection of economic slack bus, which leads to negative PoC tariffs. The simultaneous application of a capacity-based line cost rate and the min-max fair economic slack bus selection is the reason that ensures that there will not be any negative PoC tariff. The inference is supported with the help of a comparison of the proposed method to various methods reported in the recent literature using the same and additional real-life case-studies as shown in sections 4.1, 4.2, 4.3 and 6. However, for some pathological cases, negative PoC tariffs may arise that can take recourse to some kind of adjustments.

2.4 *Indian context*

In India, the NEP [28] requires that the interstate and intrastate transmission system cost allocation method should be both direction-sensitive and flow-sensitive. As per the CERC regulations 2010 [14], the MP approach is mandated for allocating the cost of the Central Transmission Utility (CTU) wherein as discussed earlier the economic slack bus selection plays a crucial role in determining the PoC tariff. The economic slack bus is determined by AP rule and from the AC power flow. A critical evaluation of this method is as follows:

1. The AP rule or the proportionate tracing rule for selecting the dispersed slack or the economic slack bus

is a heuristic that is expected to assign close-by economic slack bus generators for the loads and vice-versa. This subjectively implies lower 'extent of use' of the system and hence should decrease the PoC tariff for an entity. However, the logic is inadequate and crude because as the total transmission system cost to be recovered is fixed *a priori*, the PoC tariffs cannot be reduced for all entities. There is no way to figure out who gains, who loses and to what extent. Also, the method is not designed to address the concerns of those who get high PoC tariffs.

2. The implementing agency National Load Dispatch Center (NLDC) faces an intermittent problem where the proportionate tracing method fails when there are circular flows encountered in a network. Such a situation is encountered when there is no pure source or pure sink to begin the tracing. Under such a situation, experimentation is done by making certain lines 'out of service' in the network so that the tracing algorithm converges. As those lines are excluded from the cost allocation, post-facto scaling of PoC tariffs is required to achieve full cost recovery.
3. The economic slack bus decided by the AP rule is a heuristic and it is expected that a better economic slack bus will exist for high PoC tariff taker. Conversely, high PoC takers will claim that the low PoC takers get benefit from the cross-subsidy. This aspect of choosing the economic slack bus fairly can be strengthened.
4. Experience with the MP-AP approach has shown that the high PoC tariff takers find it unfair. Also, many entities have very low PoC tariff and the dispersion or the standard deviation in the spatial PoC is high. The counter-flows are set to zero before calculating the PoC tariff. Hence, the high PoC tariff users who may be responsible for the counter-flow on those lines are not incentivized completely.
5. CERC, vide an amendment order in 2015 [29], has mandated the use of the hybrid approach wherein 90% of the cost is accrued by the MP-AP approach with 10% cost by the postage stamp method. The 10% component is designated as the reliability charges based on the capacity of the entities. However, there is no mechanism to segregate the reliability investments and estimate the share of each entity in it.
6. Moreover, amendment vide [29] is brought in to address the regret of high-price takers. After the computation of PoC tariffs, a post-process of slabbing is executed to create nine price-slabs. The purpose is to reduce the variance in PoC tariffs. However, the procedure is ad-hoc and lacks rigour. It deviates from the 'extent of use' philosophy during post-processing of the PoC tariff. Despite such slabbing, discontent is expressed about the management of regret.

An effort has been made to mitigate the afore-mentioned problems of the Indian power system in the proposed method. The results for case-studies on an Indian power system are also given in section 6, which validate the efficacy of the proposed approach.

3. Proposed cost-causal MP Min-max approach

The Min-max fair MP approach presented in [12] is updated to achieve *cost causality* in the transmission system cost allocation. To avoid repetition, only the concerned portion of the model proposed in [12] is revisited here.

The cost rate c_{lm} based on line power flow used in [12] is tweaked to the line cost rate based on the capacity of the line. The consequence is that the cost recovered by the network's 'extent of use' is less than the total cost to be recovered. ($NetEOUCost < TSU^0$). The remaining cost ($TSU^0 - NetEOUCost$) is then allocated using a postage stamp rate (PSR). The PoC tariff computed by this model is designated as PoC_1. The separation of network cost is outlined by a diagram given in figure 2. It is proposed that 'extent of use' costs be allocated by min-max fair *economic slack bus* selection while the unused capacity costs are allocated pro-rata.

We have shown in [24] that the cost-causal MP Min-max approach leads to the following benefits:

1. the pay-off or negative cost-share is avoided,
2. stability is assured in PoC tariffs under certain scenarios of abnormal usages and
3. PoC tariffs give correct signals for the location of the new loads and generations.

Also, it has been shown through an example in [24] that the incremental network expansion costs are fairly allocated to the entities that needed the expansion. The suggested approach incorporates the optimal selection of the economic slack buses as well as the line cost rate based on capacity. A detailed formulation of the Basic Min-max fair MP method is given in [12]. An overview of the algorithm for the proposed, Min-max fair, cost-causal MP method follows.

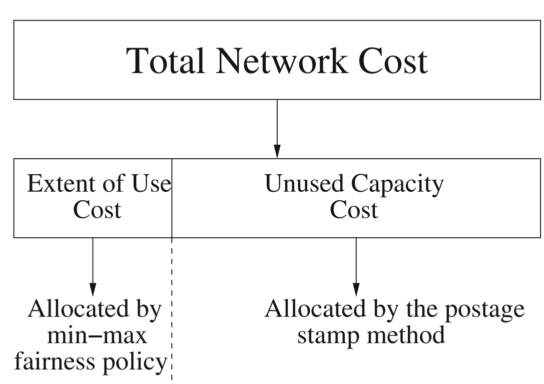

Figure 2. Allocation methodology for proposed PoC_1.

3.1 *Overview of the proposed algorithm*

Let C_{lm} be the cost of line from bus-l to bus-m. Then, $c_{lm} = C_{lm}/S_{lm}$ is the line cost per static capacity for the line lm. The scalar S_{lm} is the line capacity. Let $\mathbf{c}^T$ be the vector built from c_{lm} of dimension equal to number of lines indexed in the same order as that of the line flow vector $\mathbf{F}$. Let the total system cost to be recovered be $TSU^0 = \sum_{\forall lm} C_{lm}$, where C_{lm} is the cost of each line lm, to be recovered in a year. The main steps of the proposed algorithm are as follows.

Step 1 *Read network, injections, and cost data.*
Step 2 *Compute the line cost rates, c_{lm}, on the capacity base.*
Step 3 *Compute the line power flows, P_{lm}, using DC power flow study.*
Step 4 *Compute the 'extent of use' cost NetEOUCost of the network using DC power flow as follows:*

$$NetEOUCost = \sum_{\forall lm} c_{lm} P_{lm}. \quad (2)$$

Step 5 *Using the DC power flow, compute the sensitivities of line flows to injection/withdrawals of the entities at the network buses.*
Step 6 *Allocate the 'extent of use' cost using the min-max fairness principle. Let the PoC tariff component for the NetEOUCost for an entity E_i be $\underline{p}^*_{E_i}$.*
Step 7 *Calculate the residual cost RC as*

$$\mathrm{RC} = TSU^0 - NetEOUCost. \quad (3)$$

Note that $RC \geq 0$. Allocate it pro-rata. Let constant PSR for allocating RC be $\mathcal{K}$.

$$\mathcal{K} = \frac{RC}{TotNetMW} \quad (4)$$

where TotNetMW is the sum of MW of the generator injections and the load withdrawals.
Step 8 *Compute the min-max fair cost-causal PoC tariff, $p^*_{E_i}$, where $E_i \in \{E\}$ is an entity in set of entities $\{E\}$ as*

$$p^*_{E_i} = \underline{p}^*_{E_i} + \mathcal{K}. \quad (5)$$

4. Comparative evaluation for proposed method PoC_1

We show that the simultaneous application of 1) min-max fair *economic slack bus* selection, 2) capacity-based line-cost rate and 3) full incentivizing of the counter-flows leads to a fair MP approach. This approach will be referred to as PoC_1. Using two case-studies, we will bring out the merits of the proposed method, PoC_1, with reference to attributes like isonomy, fairness and the impact of incentivizing the counter-flows.

4.1 *A 3-bus case-study to compare existing MP methods*

The network of 3-bus illustrative example is shown in figure 3. The parameters power flow and power capacity of the lines are presented in table 3. A comparative evaluation of the proposed approach is performed for the following methods:

1) MP Original (Basic) method [5] – with and without counter-flows
2) MP Pro-rata method [8] – with and without counter-flows
3) MP-AP method [8] – with and without counter-flows
4) Equivalent Bilateral Exchanges method (EBE) [9]
5) MP Min-max method [12], which inherently includes counter-flows.

The first four methods implement line by line cost allocation, while the last one implements a network aggregation approach. The dispersed slack bus selection is not optimal or adaptable to avoid negative prices in the first three methods. The fourth method uses an *absolute* approach; hence, there is no chance of getting negative prices. The MP Min-max method performs rigorous optimization to minimize the maximum PoC tariff and economic slack buses can be customized to avoid negative prices. The example is illustrated in figure 3 and table 3.

Results for PoC tariffs obtained with all the MP variant methods along with MP Min-max and the proposed method are shown in table 4. In the prevalent methods with MP approach [5, 8] numbers with negative sign are made zero in the sensitivity matrix [$\mathbf{A}$] and in EBE approach [9], they are made absolute with appropriate modification in formulation. Thus, the negative PoCs are avoided, but, with

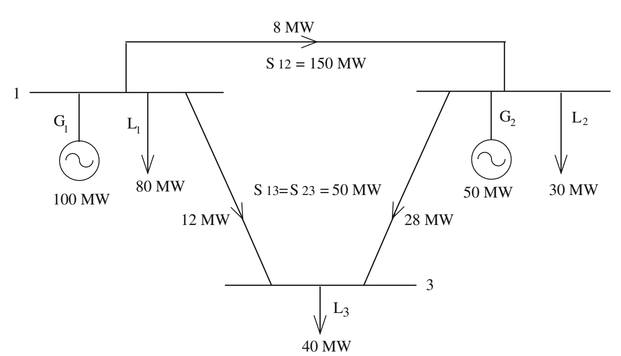

Figure 3. Three-bus illustrative example for MP methods.

Table 3. Data for 3-bus illustrative example.

Line	X in p.u.	Cost in ₹	Flow (MW)	Capacity (MW)
1-2	0.01	1000	8	150
1-3	0.03	333.33	12	50
2-3	0.01	333.33	28	50

loss of linearity and chance of reduction of PoC for high PoC taker. In the proposed approach, the cost per MW rate is reformulated as the cost of line per MW capacity of the line. The negative PoCs and the best statistical parameters are highlighted by italics in table 4. It can be observed that the maximum PoC tariff is the lowest, minimum PoC tariff is the highest and the standard deviation is the lowest in the proposed method, thus proving the superiority.

4.2 *Isonomy and fairness*

The authors in [23] showed that many of the existing methods, including the MP, do not fulfill the fairness attribute of *isonomy*. In the context of transmission system cost allocation, isonomy means that aggregation or splitting of the injections or withdrawals at a bus should not lead to a change in the PoC tariff. Therefore they proposed the AS method to overcome this limitation. We now consider the same motivational case-study (see figure 4) used by the authors in [23] and show that the proposed approach of PoC_1 is consistent about the requirement of isonomy. In the context of this case-study, isonomy means that as the two generators are on the adjacent buses, with a zero cost line connecting them, their tariffs should be the same. The result summary as given in [23] is replicated in table 5 along with the results from the proposed method PoC_1.

It can be seen that the proposed method gives identical PoC tariffs as those of the AS method, which indeed is encouraging. The total cost to be recovered is \$65. The cost allocation criterion between the generators and the loads is fixed as 50% : 50%. Hence, the generator and the demand cost-share should each be \$32.5. It was reported in [23] that the negative tariffs were observed in the LRMC approach [16]; hence, the total generator cost-share exceeds the 50% mark. Even though counter-flows were fully incentivized both in the LRMC approach [16] and the proposed approach, there were no negative PoC tariffs with the proposed method.

4.3 *Impact of counter-flows on negative PoC tariffs*

A recent paper [10] has attempted cost allocation for interconnected market. As mentioned in 2.3, in [10] counter-flows are fully incentivized as a requirement of obtaining precise spare capacity in one market operation to be used in another market operation. It may result in negative cost-shares leading to payoffs. Redistribution algorithm is proposed in [10] to solve the pay-off problem. Our focus is not on the interconnected market operation, but we intend to compare for consistency in the solution of the pay-off problem.

In [10], the IEEE 24-bus reliability test system has been used for testing and the results obtained before and after the redistribution of the payoffs have been discussed. For the same data set, we present the results with the proposed method (PoC_1). It can be observed that the negative PoC

Table 4. Comparison of PoC tariffs (₹/MW) with statistics for MP Basic (with and without counter flows), MP-Prorata (with and without counter flows), MP-AP (with and without counter flows), EBE, MP Minmax and Proposed methods.

Entity Name	MP Basic with CF	MP Basic without CF	MP-Prorata with CF	MP-Prorata without CF	MP-AP with CF	MP-AP without CF	EBE	MP Minmax	Proposed
1-G	0.00	0.00	22.75	7.41	9.53	7.67	4.86	9.33	5.56
2-G	*− 103.17*	1.96	*− 28.84*	1.78	*− 2.40*	3.61	6.94	*− 1.98*	5.56
1-L	− 17.20	0.37	0.00	0.00	3.18	0.00	4.76	3.18	4.49
2-L	103.17	19.70	34.39	10.08	7.12	5.69	6.36	11.90	6.27
3-L	93.25	24.44	29.43	12.61	15.50	13.74	9.7	11.90	7.16
Min.	− 103.17	0.00	− 28.84	0.37	− 2.40	0.00	3.18	− 1.98	*4.49*
Max.	103.17	24.44	34.39	12.61	15.50	13.74	9.70	11.90	*7.16*
std. dev.	84.03	11.88	29.01	5.26	7.25	5.11	2.44	6.71	*0.99*

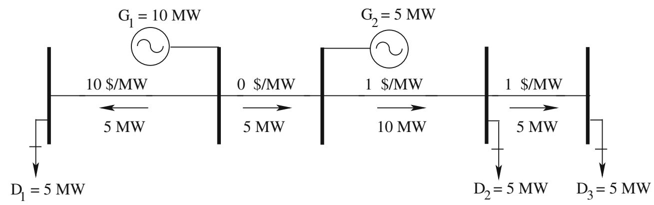

Figure 4. Five-bus sample system.

Table 5. Generator PoC Tariffs in $/MW for Sample System [23].

Gen.	LRMC [16]	APF [15]	Shapley [30]	AS [23]	Proposed PoC_1
G_1 (10 MW)	2.20	2.88	1.88	2.17	2.17
G_2 (5 MW)	2.20	0.75	2.75	2.17	2.17

tariffs are seen in the method of [10]. As demonstrated in this case study, we have been able to solve the payoff problem as compared with the method of [10]. For further comparison of the methods, the minimum, the maximum and the standard deviation of the PoC tariffs are computed and shown in table 6. It can be observed that, with the proposed method PoC_1, the minimum PoC tariff is the highest and the maximum PoC tariff is the lowest, yielding the least standard deviation (all three highlighted in bold in table 6). Thus, the superiority of the proposed method is established as compared with a recent method that retains the counter-flows.

5. Proposed cost-causal method with reliability modelling

So far we have made one very important recommendation, viz. *economic slack bus* selection using Min-max fairness policy, *line cost rate* on capacity base and full incentivizing of counter-flows to be done simultaneously. Yet this does not address significant aspects of the modelling of reliability costs and reactive power injections, which will influence the PoC tariffs of the entities.

Table 6. Result summary for IEEE 24-bus (RTS) system.

Method	PoC tariff in $/MW		
	Min.	Max.	Std. Dev.
Reference [10] (before redistribution)	−0.5963	5.0261	1.68
Reference [10] (after redistribution)	0	5.0261	1.63
Proposed PoC_1	**2.13**	**2.65**	**0.14**

5.1 *Modelling of reliability cost component*

Most of the methods on cost allocation of transmission networks have bundled costs of usage, investments for reliability and unused capacity costs into one basket. However recent evidence [11, 19] suggests that such bundling can cause unacceptable price distortions. Therefore, we notice how research work is directed towards developing pragmatic methods for the valuation of reliability costs for an entity. Consequently, separate cost components for network usage capacity, reliability capacity and residual capacity can be worked out. Recently, two pragmatic approaches for monetizing and allocating the cost of reliability component have been discussed in [11, 19]. Both the approaches consider all $N-1$ contingencies in a network to find the maximum power flow on each line. Reference [11] uses this optimal capacity to redefine the line cost rate. On the other hand, [19] further estimates the capacity of the line by adding future load growth and other factors to this maximum line flow. It evaluates the cost of reliability component by monetizing the difference of maximum line flow and the base case line flow with the capacity-based line cost rate. This cost component is then allocated using the MP approach. However, while allocating the 'extent of use' costs, the benefits of the counter-flows are suppressed by considering only the absolute value of the flow components. Yet, while allocating the reliability cost, counter-flows are partially incentivized by making them zero. This leads to inconsistency, which is difficult to comprehend and explain. Also, as per the policy in [11, 19], the slack bus selection changes during the reliability cost allocation.

There could be many policies on the allocation of reliability cost-shares. We believe that the capacity costs related to maximum excess flow found by $N-1$ contingency simulations correctly captures the reliability capacity requirements of each line. These costs can be shared on the same basis as done for the extent of use costs. This policy is proposed to devise a simple approach that can be easily explained to the policymakers and the stakeholders. It is noted here that the worst-case contingency will also cause changes in the marginal flow associated with the specific entities to different extents depending upon their relative location for the lost line. Efforts to quantify this aspect could lead to more complicity in the method. More intricate methods to allocate the reliability costs can be designed in the future to capture this aspect.

In the proposed integrated approach, the reliability cost component is calculated as follows:

1. First we calculate the baseline power flow on a line.
2. Next we calculate the maximum power flow on a line using the $N-1$ contingencies.
3. The reliability capacity cost is monetized by computing the difference of (2) and (1) on a capacity-based line-cost rate.

Throughout the method, the *economic slack bus* selection is maintained as recommended for the 'extent of use' cost allocation in section 2.1.

5.2 *Overview of proposed cost-causal approach with reliability modelling*

The separation of network cost is outlined by a diagram given in figure 5. The proposed integrated approach as given here can be used for computing both $PoC_{1_{REL}}$.

Step 1 *Read network, injections and cost data.*
Step 2 *Compute line cost rates on the capacity base.*
Step 3 *Compute the 'extent of use' cost NetEOUCost of the network using DC framework as follows:*

$$NetEOUCost = \sum_{\forall lm} c_{lm} P_{lm}. \tag{6}$$

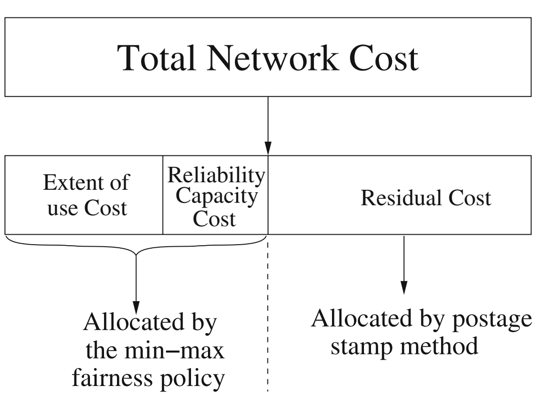

Figure 5. Allocation methodology for the proposed $PoC_{1_{REL}}$.

Step 4 *Using the DC power flow model, compute the sensitivities of line flows to injection/withdrawals of the entities at the network buses.*
Step 5 *The new power flow, $P_{lm,rs}$, on line lm due to outage of line rs can be obtained by adding the additional power flow as explained in [19].*

$$P_{lm,rs} = P_{lm} + \Delta P_{lm,rs}. \tag{7}$$

Now, P_{lm} is line power flow obtained using the DC framework. Likewise, there can be totally n_k such contingencies in $\mathcal{C}$, the set of all the $N-1$ contingencies. The maximum power flow out of all such flows will give the reliability capacity $P_{REL_{lm}}$ of the line lm. The reliability capacity identified for the line lm is given by

$$P_{REL_{lm}} = \max_{k\in\{\mathcal{C}\}}((|P^k_{lm}| - |P_{lm}|), 0). \tag{8}$$

Step 6 *The network reliability capacity cost using the DC framework can be computed as*

$$NetRelCapCost = \sum_{\forall lm} c_{lm} P_{REL_{lm}}. \tag{9}$$

Step 7 *Add the network reliability capacity cost (NetRelCapCost) to the network 'extent of use' cost (NetEOUCost), i.e.*

$$NetCombCost = NetEOUCost + NetRelCapCost. \tag{10}$$

Allocate it using the cost-causal Min-max fair approach presented in [24]. Thus, both the 'extent of use' and the reliability costs are shared by the entities in proportion to the 'extent of use'. Let the PoC tariff component for NetCombCost for entity E_i be $\underline{p}^*_{E_i}$.

Step 8 *The residual cost that is socialized is given by*

$$RC_{REL} = (TSU^0 - NetCombCost). \tag{11}$$

Note that $RC_{REL} \geq 0$. Again, we allocate it pro-rata. Let constant PSR for allocating RC_{REL} be $\mathcal{K}$.

$$\mathcal{K} = \frac{RC_{REL}}{TotNetMW} \tag{12}$$

where TotNetMW is the sum of MW of the generator injections and the load withdrawals.

Step 9 *The proposed integrated PoC tariff can be obtained as*

$$p^*_{E_i} = \underline{p}^*_{E_i} + \mathcal{K}. \tag{13}$$

The PoC tariff thus calculated is designated as $PoC_{1_{REL}}$.

5.3 *Bounds on reliability costs*

The lower bound on the reliability cost component is normally zero. For estimating an upper bound on reliability investment, one redundant line with identical capacity is assumed for each transmission line. This is in tune with common practice of provision of double-circuit lines for transmission network [31, 32]. Suppose that a 'standby' line could be provisioned for the line undergoing outage (lm) with electrical parameters identical to the original line; then the reliability investment corresponding to the lm line is $\frac{C_{lm}}{S_{lm}} P_{lm}^0 = c_{lm} P_{lm}^0$, which is nothing but the 'extent of use' cost of the line. Thus, we can arrive at a conclusion that the upper bound of network reliability investment is nothing but the *NetEOUCost*. Provided that *NetEOUCost* is less than half of TSU^0, the total sum $2 \times NetEOUCost$ can be allocated by Min-max fair composite MP method. The remainder ($TSU^0 - 2 \times NetEOUCost$) is allocated using the PSR. The new PoC tariffs so obtained represent PoC tariff considering upper bound on the reliability investment. In case $2 \times NetEOUCost > TSU^0$, then complete TSU^0 can be allocated by the Min-max fair MP approach but with the line cost rate being capacity based. The PoC tariff obtained by this assumption represents the upper bound of the reliability capacity estimation and is designated as $PoC1_{UB}$. It can be useful in the situations where an arbitrary percentage for reliability costs based on statistical data analysis is fixed by regulatory authorities. Under no circumstances should the reliability cost exceed the 'extent of use' cost.

The PoC tariffs obtained in $PoC1_{REL}$ lie between the lower bound PoC_1 and the upper bound $PoC1_{UB}$. They fulfill the desirable attributes of simplicity, transparency, political implementability and fairness.

6. Results

6.1 *Case-studies on IEEE test systems*

The testing and comparison is performed for IEEE 14, 30, 57 and 118 bus systems. For transformers, we have considered x/r ratio to be 20. The cost of the line is estimated as per [33] with a constant of proportionality set to 10. The line power flow capacity S_{lm} is taken as per St. Clair's curves. The network's extent of use cost, *NetEOUCost*, computed for extent of use for IEEE 14-bus, 30-bus, 57-bus, and 118-bus networks is 26.63%, 43.75%, 21.93% and 33.14% of the total network cost, respectively. Correspondingly, reliability capacity costs, *NetRelCapCost*, are, respectively, 6.89%, 11.46%, 7.85% and 11.36% of the total network cost.

6.2 *Improvement in fairness by proposed approach*

Table 7 captures the effect of the various fairness improvements on the maximum PoC tariff, when applied individually, then in the sub-groups and finally when all of them are simultaneously applied, i.e. proposed tariff formulation PoC_1. With the basic MP approach [5], Table 8 captures corresponding effect on the minimum PoC tariff. With reference to the basic MP method [5], we observe the following.

1. With the basic MP approach [5], complete incentivizing of the counter-flows can lead to the problem of pay-offs (or negative PoC tariffs); compare column A with C in table 8. In turn, the extra payment that the positive rate takers have to make leads to an increase in the maximum PoC tariff. For the four test systems, it is 1.57, 1.82, 3.125 and 1.94 times the case when counter-flows were set to zero. This brings out the criticality of setting the counter-flows to zero in the basic MP method. However, the moment the line cost rate is changed to a capacity base, a drastic reduction in the maximum PoC tariff is seen (column B of table 7) vis-a-vis the basic MP approach. Columns D of tables 7 and 8 consider the case where line cost rate is capacity based and counter-flows are incentivized. We observe that the negative PoC tariff problem is now solved for IEEE 14, 30 and 57 bus systems. As a consequence, the maximum PoC tariff falls down. However, 118-bus system still has the negative PoC tariffs (shown bold in column D of table 8).

Table 7. Maximum PoC Tariffs and Classical Postage Stamp Rates (PSR) for IEEE Networks.

	MP Basic								
IEEE Network	[5] **A**	[5] + cost per capacity **B**	[5] + counter flow retained **C**	[5] + CF retained + cost per capacity **D**	MP Minmax [12] **E**	MP Minmax with cost-rate as per [11] **F**	Proposed PoC_1 **G**	Classical PSR **H**	EBE [9] **I**
14-Bus	62.62	17.3	98.6	19.89	48.77	37.73	13.04	9.56	32.35
30-Bus	58.03	24.77	105.85	23.22	52.46	23.89	13.18	6.81	31.22
57-Bus	284.8	64.41	890.2	83.65	395.89	184.59	43.71	9.94	130.92
118-Bus	118.21	8.2	229.63	11.46	42.77	49.69	4.92	3.98	64.65

Table 8. Minimum PoC Tariffs and Standard Deviation of PoC_1 for IEEE Networks.

	MP Basic								
IEEE Network	[5] **A**	[5] + cost per capacity **B**	[5] + counter flow retained **C**	[5] + CF retained + cost per capacity **D**	MP Minmax [12] **E**	MP Minmax with cost-rate as per [11] **F**	Proposed PoC_1 **G**	Standard Deviation **H**	EBE [9] **I**
14-Bus	0	6.98	− 30.62	4.39	− 15.31	2.1	7.34	1.58	0.99
30-Bus	0	3.81	− 65.09	1.49	− 32.54	− 2.14	4.11	2.34	0.83
57-Bus	0	7.9	− 387.09	2.58	− 193.54	− 1.96	7.76	9.61	3.37
118-Bus	0	2.84	− 144.1	**− 4.08**	− 3.87	1.36	3.16	0.41	2.12

Remark 1: Changing the line cost rate from the flow to the capacity base is an important improvement but it does not fully solve the problem of the negative PoC tariff.

2. Results with the MP Min-max approach of [12] are given in column E of tables 7 and 8. We note that in [12], the economic slack bus selection is min-max fair, but the line cost rate is flow based. We therefore, observe that the negative PoC tariffs arise in all the four networks. However, the payoffs are much lower than those of the Basic MP method with the counter-flows incentivized (compare columns C and E of table 8). This benefit, surely, is a consequence of the min-max fairness principle. Further, even with the negative PoC tariff in IEEE 14-, 30- and 118-bus systems (column E, table 7), maximum PoC tariffs with the MP Min-max method [12] are lower than that in the Basic MP approach (column A, table 7), even though the Basic MP approach sets the counter-flows to zero. This benefit is also a consequence of the min-max fairness policy. However, for the 57-bus system, when the counter-flows are incentivized, the payoffs are too large. Hence, even the maximum PoC tariff seen with the MP Min-max approach is higher than that in the Basic MP approach (column A of table 7).

Remark 2: The min-max fair economic slack bus selection is an important improvement but it does not fully solve the problem of the negative PoC tariff.

3. For a comparative evaluation, the MP Min-max approach is also implemented with the optimal line capacity obtained from $N-1$ contingency studies as proposed in [11] (column F). The maximum PoC tariffs are now lower than that of the MP Min-max method (column E). This is because the optimal line capacity considering the reliability component is higher than the base case line flow. Hence, the line cost rates are lower than with the base flow model. However, the negative PoC tariffs still occur as seen in the column F of table 7.
4. Finally, the columns G of tables 7 and 8 show results with the proposed PoC_1. We now see that the problem of the negative PoC tariff is fully addressed; all the minimum PoC tariffs are above zero, and the maximum PoC tariffs have reduced drastically. For the IEEE 14-, 30-, 57- and 118-bus systems, the maximum PoC tariffs are, respectively, 1.41, 1.94, 4.4 and 1.37 times the classical PSR (column H – table 7). With the basic MP approach, these rates were as high as 6.55, 8.52, 28.65 and 29.7 times PSR, while with MP Min-max approach [12], these rates were 5.1, 7.7, 39.82 and 10.75 times, correspondingly. The column H of table 8 demonstrates low standard deviation in spatial PoC_1 tariff with the proposed approach. It is a consequence of
   (a) a decrease in the maximum PoC tariff,
   (b) an increase in the minimum PoC tariff and
   (c) a clustering of the PoC tariffs that occurs with the application of the min-max fairness principle.
5. Columns I of tables 7 and 8 tabulate the maximum and minimum PoC tariffs obtained using the EBE method [9] that altogether suppresses the directional sensitivity. We note that the results with the proposed approach (column G) are much better than those of the EBE method (columnsI).

Remark 3: The proposed integrated approach outperforms all the other variants of the MP approach and the EBE approach.

6.3 *Cluster formation in proposed PoC tariffs*

Figure 6 shows the PoC tariffs PoC_1 in ascending order for IEEE 118-bus case. A clear clustering of PoC tariffs is confirmed in the proposed tariffs. As explained in section 2, it promotes economic efficiency.

6.4 *Case-study on Indian transmission system*

The results for Indian transmission system are presented here.

1. System: 2016–17 – Quarter 4 – truncated interstate transmission system (ISTS) network
2. No. of buses: 835

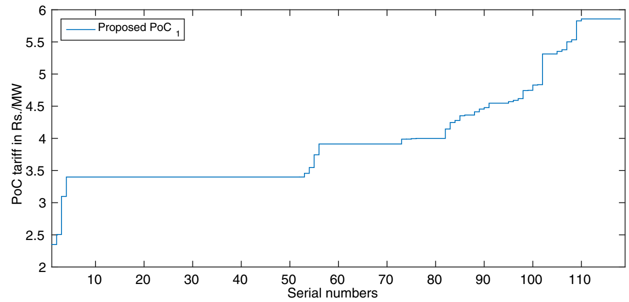

Figure 6. Price cluster formation in proposed PoC_1 tariff for IEEE 118-bus system.

3. Total generation: 113 (GW)
4. Total load: 111 (GW)
5. Total real power loss: 1956 (MW)
6. % Real power loss (on load base): 1.8
7. Cost allocated to both load and generators
8. Total monthly transmission charges in crores: ₹1913.57
9. PSR (paise/kWh):

$$\frac{19135745864 \times 100}{(112808 + 110852) \times 1000 \times 30 \times 24} = 11.88$$

10. PSR in 2011–12: 8.3 paise/kWh
11. Line loading for different voltage level lines:

Voltage Level	No. of Lines	Total ckt kms	Average ckt kms	%Line loading Min	Average	Median	Max
765	105	25834	246.04	0.18	13.51	8.74	48.72
400	1292	153080	118.48	0.00	16.88	12.47	99.77
220	37	2877	77.76	2.33	12.31	12.14	21.77
<132	227	7861	34.63	0.00	17.91	15.63	51.03

Cost allocation methods considered: 1) MP Min-max with cost causality (PoC_1) (see results in figure 7), 2) MP-AP method (see results in figure 8) and 3) MP-AP method with cost causality (see results in figure 9)

Following observations can be made from the results of MP Min-max method with cost causality (PoC_1):

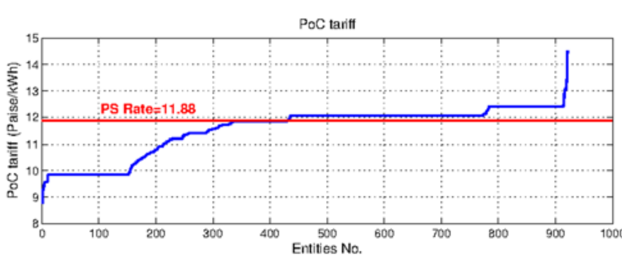

Figure 7. PoC tariff (paise/kWh) of all entities arranged in ascending order for MP Min-max method with cost causality (PoC_1).

Table 9. PoC tariff (paise/kWh) statistics.

	MP Min-max*	MP-AP	MP-AP*
Max	*14.52*	76.00	19.70
Min	8.74	0.00	8.73
Median	12.04	10.15	11.24
Average	11.54	12.01	11.44
Std. dev.	*0.90*	9.94	1.98

*Cost-causal methods

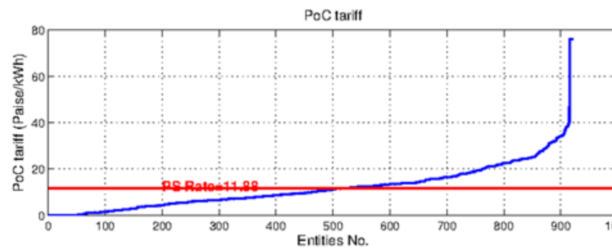

Figure 8. PoC tariffs (paise/kWh) of all entities arranged in ascending order for MP-AP method.

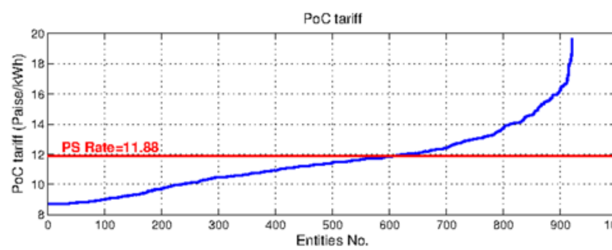

Figure 9. PoC tariffs (paise/kWh) of all entities arranged in ascending order for MP-AP Method with cost causality.

Table 10. Summary of *NetEOUCost* and *NetRelCapCost* percentage of TSU^0 for the proposed method.

IEEE Network	%*NetEOUCost*	Upper bound on %*NetRelCapCost*	N-1 contingency based %*NetRelCapCost*
14-Bus	26.63	26.63	6.89
30-Bus	43.75	43.75	11.46
57-Bus	21.93	21.93	7.85
118-Bus	33.14	33.14	11.56

1. Slabbing is intrinsic (see figure 7).
2. Many entities have PoC tariffs close to PSR (see figure 7).
3. Maximum PoC tariff is the least among the other methods (see highlighted by italics in table 9).
4. Standard deviation is the lowest among the other methods (see highlighted by italics in table 9).

Table 11. Minimum, Maximum and Standard Deviation of $PoC_{1_{REL}}$ Tariff for IEEE Networks.

	$PoC_{1_{REL}}$ tariff		
IEEE Network	Min.	Max.	Std. Dev.
14-bus	6.76	13.94	1.99
30-bus	3.4	14.85	2.96
57-bus	6.98	55.8	13.05
118-bus	2.88	5.25	0.55

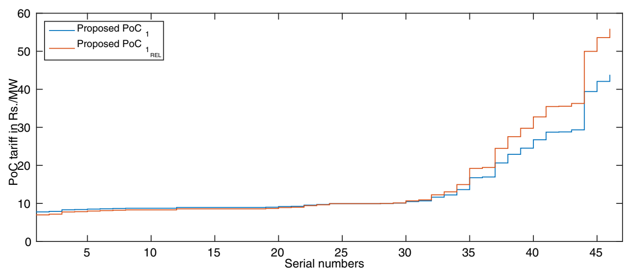

Figure 10. Comparison of the proposed PoC_1 and $PoC_{1_{REL}}$ tariffs for IEEE 57-bus system.

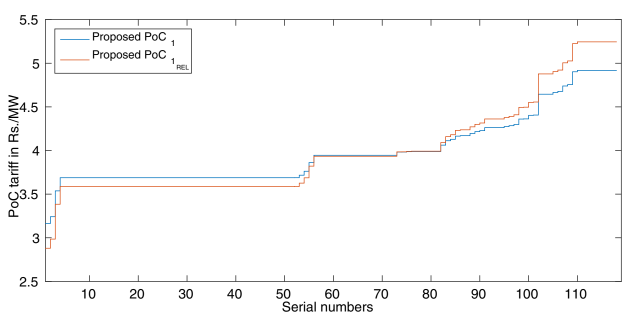

Figure 11. Comparison of the proposed PoC_1 and $PoC_{1_{REL}}$ tariffs for IEEE 118-bus system.

Our power flow study has also confirmed that in a State Transmission Utility and the CTU networks of India, the average line loading is around 37% only. Thus the residual network cost is substantial, as such relative apportioning of line costs can seriously compromise the *cost-causality* requirement. By separating the reliability cost from the residual network cost the unused capacity cost is obtained, which is then allocated on pro-rata or PSR. As the proportion of unused capacity cost component is higher than the extent of use cost, many entities have PoC tariffs that are close to the PSR. Thus, the PoC tariffs are based on a more realistic cost-function and can increase acceptability by the entities as the variance in the PoC tariffs is reduced.

6.5 *Effect of modelling reliability cost component*

The proposed method with the reliability modelling has been programmed in MATLAB [34]. The LP is performed using *Gurobi* Optimizer version 6.0.4 [35]. The testing is performed on the IEEE 14-, 30-, 57- and 118-bus networks as explained in section 6.2. Considering the normal use of double-circuit lines, the number of circuits N_{lm} for *LODF* computation is taken as 2 for all the lines in the networks.

The proportion of the ‘extent of use’ and the reliability capacity costs with reference to the total network cost using the proposed approach and the upper bound analysis are tabulated in table 10. The minimum, maximum and the standard deviation of the proposed $PoC_{1_{REL}}$ tariffs for IEEE networks are tabulated in table 11. The minimum PoC tariffs are now lower than the PoC_1 tariff (column G in table 8). The maximum PoC tariffs in table 11 are higher than the PoC_1 tariff (column G in table 7). This can be explained as follows. When the reliability component is separated from the unused capacity, the postage stamp component falls. Since the net reliability cost is shared in proportion to the extent of use by the beneficiaries, we notice that the min-max fair PoC tariff component increases proportionately. Therefore the lower marginal network users benefit by the predominance of a reduction in the postage stamp component, while the high PoC tariff takers

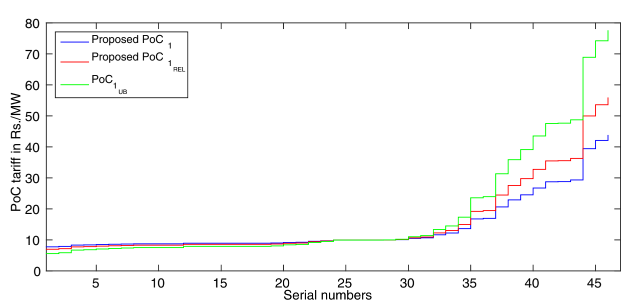

Figure 12. Bound analysis for $PoC_{1_{REL}}$ tariff for IEEE 57-bus system.

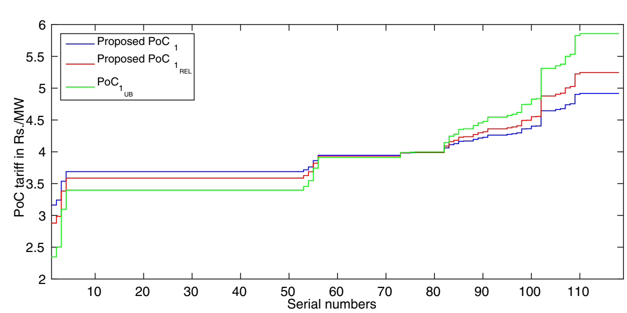

Figure 13. Bound analysis for $PoC_{1_{REL}}$ tariff for IEEE 118-bus system.

for the network 'extent of use' component pay more for a substantial deemed use of the reliability investment.

Figures 10 and 11 show the PoC tariffs PoC_1 and $PoC_{1_{REL}}$ in ascending order for IEEE 57-bus and 118-bus case, respectively. As seen in figures 10 and 11, the minimum PoC tariff decreases and the maximum PoC tariff increases. Hence, the standard deviation shown in table 11 increases for the proposed $PoC_{1_{REL}}$ as compared with the proposed PoC_1 tariff (column H in table 8).

6.6 *Demonstration of the bound analysis for proposed reliability modelling*

The bound analysis explained previously is demonstrated for IEEE 57-bus and 118-bus systems. The PoC tariffs PoC_1, $PoC_{1_{REL}}$ and $PoC_{1_{UB}}$ are plotted together on the same plots for IEEE 57-bus and 118-bus systems as shown in figures 12 and 13, respectively. It can be observed that the proposed $PoC_{1_{REL}}$ is well within the lower and upper bounds.

7. Conclusions

In this paper, we have proposed a rigorously fair approach wherein 1) economic slack bus selection is min-max fair, 2) line cost rate is capacity-based, and 3) benefits of counter-flows are passed fully to entities responsible for it. The residual costs post allocation of the 'extent of use' component are socialized. We show that this solves the problem of pay-offs without any signal distortion. The PoC tariffs are clustered, thus promoting economic efficiency. Further, min-max fair economic slack bus selection and full incentivizing of counter-flows guarantee fairness to high PoC tariff takers as no further benefits can be passed on to them without increasing regret of another entity which leads to equal or high PoC tariff. Proper distribution of the transmission network cost among the network users requires that we separate the 'extent of use' component from the unused capacity. However, the unused capacity is not just a plain overhead that can be distributed across the users. It contains an important spare capacity component that is required for ensuring reliability. The N-1 contingency studies are performed to decide the maximum power flow in a line. The difference between this maximum power and the base case power flow is considered as the reliability capacity of the network. This reliability capacity should be added to the 'extent of use' capacity and then the combined cost is allocated using the composite Min-max fair MP approach. This also ensures the fair allocation of the reliability cost component thereby improves the *cost causality*. Thus, we separate the residual investment component which can be allocated by the postage stamp method. Extensive case studies and comparative evaluations are performed. The lower bound and the upper bound of reliability capacity are also developed. The proposed approach results in PoC tariff $PoC_{1_{REL}}$ which has its minimum and maximum PoC tariffs in between the two bounds. The proposed approach is simple and transparent. Therefore it is expected that it will be acceptable to the stakeholders.

Nomenclature

$\mathcal{C}$	The set of all the N-1 contingencies
$\mathcal{K}$	Constant postage stamp rate to allocate RC
C_{lm}	Cost of line between bus l and m
G	Set of all generator buses
L	Set of all load buses
NetCombCost	Network combined cost for 'extent of use' and reliability capacity using DC framework
NetEOUCost	Network 'extent of use' cost using DC framework
NetRelCapCost	Network cost reliability capacity using DC framework
P_{lm}	Power flow in line lm using DC framework
$P_{REL_{lm}}$	Reliability capacity for line lm using DC framework
PoC_1	PoC tariff for DCPF-based proposed method
RC_{REL}	Residual capacity cost with reliability modelling in DC framework
S_{lm}	Capacity of the line lm in MW
TSU^0	Total network cost to be recovered
UB	Upper bound
DCPF	DC power flow
PoC	Point of connection tariff in ₹/MW
RC	Residual cost of the system

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