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Experimental investigation on the filtration characteristics of a commercial diesel filter operated with raw and processed karanja-diesel blends

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Abstract. With soaring fuel prices, engine emissions, and demand for energy security, the development of alternative fuels from renewable resources has become significant. The current work explores the filtration performance of a commercial diesel filter operated with diesel blends of raw and processed karanja oils. Three processes viz. esterification, transesterification, and degumming were performed with karanja oil, and their diesel blends (20% v/v) were tested in a continuous run filtration system. The fuel filter plays a significant role in removing the contaminants/impurities from the fuel. Hence, the pressure difference across the fuel filter was monitored at the flow rates (0.03 m^3/h and 0.075 m^3/h) and fuel temperatures (30°C and 70°C). Besides, the effects of weight gain and surface morphology on the filter characteristics were observed during the study. Further, the head loss factors of fuel filter under varying operating conditions were deduced from the experimental measurements. The results of different processed oil blends were compared with fossil diesel to find the suitable blend. By considering the fossil diesel as the reference case, the average pressure difference for the degummed blends was significantly lower compared to other tested blends, which highlights the importance of the degumming process. Furthermore, the tested images of filter revealed the presence of minor components such as sludge or gums on the filter surface for the fuel without the degumming process.

Keywords. Degumming; fuel filter; diesel engines; contaminants; pressure drop; biodiesel.

1. Introduction

Diesel engines have numerous applications viz. transportation, agricultural farm equipment, power generators, locomotives, and marine, etc. To reduce the dependency on crude oil and harmful emissions, the search for sustainable alternative fuel has been the major focus of research. In this regard, raw and processed oils have been the close counterpart to fossil diesel. Raw vegetable oils mainly contain triglycerides that can be used directly or blended with fossil diesel to operate compression ignition engines. In fact, Rudolf Diesel demonstrated his patented engine using peanut oil. However, several problems like carbon deposits, corrosion, and pour point have impaired their widespread usage as fuel. Hence, to overcome these issues, the raw oil was processed by methods like esterification and transesterification. Processed oils like biodiesel comprise of long-chain fatty acid alkyl esters produced from various edible and non-edible oils.

Biodiesel preparation is done by different processes viz. preheating, transesterification, micro emulsification, degumming, and thermal cracking [1, 2]. These processes are applied to remove the triglycerides from the oil, thereby decreasing the density and viscosity of the parent oil. The idea of using biodiesel became popular around the nineteenth century, and since then, numerous studies have been carried out. In particular, several experimental works have been conducted in a single cylinder, naturally aspirated, constant speed diesel engine of Kirloskar make [3–8]. Although earlier studies claim the potential of biofuels and demand no major modifications on the engine side, the fuel supply system viz. fuel tank, pump, strainer, fuel lines, and fuel filter needs to be analyzed as they are primarily designed for fossil diesel. A comprehensive review of the compatibility of automotive systems in biodiesel could be noticed in the literature [9–11]. Sorate and Bhale [9] have opined that biodiesel could damage the fuel supply system by filter plugging, injector cocking, corrosion, fusion of moving components, and hardening of elastomeric components. Further, the components of the fuel supply system are directly influenced by the changes in fuel property or the presence of impurities in biofuels. Hence, the current work explores the filter performance of a commercial diesel filter employed for agricultural applications. For this

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purpose, non-edible oil of Indian origin viz. karanja oil was chosen, and three different processing methods viz. degumming, esterification, and transesterification were attempted. The focus of the study is to observe the filtration characteristics of different oil blends and concomitantly recommend a suitable fuel processing strategy, which could result in reduced filter blocking tendencies. The fuel filter is an integral element of the fuel supply system. The separation of solid particles, which are suspended in a liquid utilizing a porous medium to retain the solids and permit the liquid, is termed filtration. The rate of filtration depends on the pressure drop across the filter medium, the viscosity of the filtrate, and the area of the filtering surface [12]. Typical materials used for the filter cartridge is the paper element and aluminum or plastic for the filter housing [11]. Kenneth *et al* [13] compared the fuel system related maintenance issues in automotive transit buses operated with B20 and fossil diesel. Among the five buses tested, two buses were reported for engine stalling caused by plugged fuel filters. After careful examination, the authors observed a brown "grease-like" material in the filter paper element and opined it to be the cause of the filter plugging. The authors concluded that the fuel filter plugging was the major problem and demands filter replacements or additional maintenance costs for the buses operated with B20. Field trials performed by Petiteaux *et al* [14] with B30 (blends of rapeseed methyl ester) indicated no signs of accelerated filter clogging. The same filter media (8 μm) also indicated no accelerated clogging in laboratory testing with B20 (blends of soy methyl ester). However, the filter media having smaller pore sizes shows the tendency to clog faster with biodiesel blends (B10 & B20), indicating their limitations [14, 15]. Accelerated filter clogging with biodiesel blend depends on the feedstock and amount of saturated monoglycerides, sterol glycosides, carboxylate salts formed in biodiesel. Usage of fuel additives can delay the filter clogging experienced with biodiesel operation [15]. To avoid filter clogging, Bari *et al* [16] suggested heating the fuel (waste cooking oil) temperature above 55°C. To reduce the head loss, the authors also recommended to place the fuel tank at a higher elevation and to utilize two filters. Winston-Galant *et al* [17] studied the effect of fuel temperature on the filtration performance for neat biodiesel and biodiesel blends (B10, B20). The following parameters viz. precipitate mass on the filter element; fuel flow rate and the pressure drop across the filter were measured. The authors concluded that the amount of precipitate on the filter element increased with the increase in biodiesel blend concentration. Fersner *et al* [15] studied the effect of filter blocking tendency for various biodiesel feedstocks viz. used cooking methyl ester, soy methyl ester, rapeseed methyl ester, and coconut methyl ester. Using the bench-top filtration testing equipment, the authors have shown that the specific feedstock of biodiesel can alter the filterability characteristics. Recently, Anant *et al* [18] studied the effect of biodiesel and biodiesel-ethanol blend on the fuel filter characteristics by measuring the pressure difference and contaminants in a continuous run filtering system. The authors recommended purging the fuel wetted parts with fossil diesel after the completion of the neat biofuel operation. For the smooth operation of biodiesel fuels, there are enough studies in the literature recommending modifications on the filter end (e.g., heated filters, double filters). However, investigations about the effect of various fuel processing strategies on the performance of fuel filters are scarce. As observed in the open literature, most studies are generally encouraging blending and additive characterization. Therefore, in this study, the impact of fuel processing techniques to enhance the filter performance of a commercial diesel filter was investigated. Hence, this study is essential for seamless and long term operation of biodiesel or their blends in an unmodified diesel engine.

2. Experimental set-up and methodology

The experimental methods carried out to test the diesel filter and processed karanja oil is discussed in this section. The experimental filter set-up simulates the real engine conditions of a diesel supply system, and two major tests viz. pressure difference and contaminant retention test are carried out following the Japanese standard D1617:1998. As shown in figure 1, a continuous fuel flow set-up was established to monitor the pressure difference across the fuel filter. The test set-up includes a fuel tank, pump, electric resistance heater, battery, pressure transducers, thermocouples, flow meter, and the test filter. The biodiesel blend was stored in a fuel tank, closed with a lid having inlet and outlet pipes for the blend to flow. A 12 V DC battery powered the fuel pump (Bosch turbine pump) and the electric heater. The maximum flow capacity of the turbine pump is 255 liters/h. The filter used in the experiment was basically employed for single cylinder agricultural engines (Kirloskar make). A thermocouple placed along the fuel line monitors the inlet fuel temperature, while the inlet and outlet pressures are measured via pressure transducers. The fuel flow was regulated with the help of a control valve and a flow meter. Initially, the experiment was conducted with the test fuels to measure the pressure difference across the fuel filter. During the test, the fuel was continuously circulated, and the static pressure across the filter was monitored at ambient temperature conditions. After the test completion, the amount of contaminants retained by the filtering element was evaluated by measuring the weight gained in the filter. The filtering element was removed from its casing and was dried in a desiccator to prevent the interaction of the filtering element with the atmospheric dust particles. After the drying process, the filter's weight was measured with a precision balance. Further, a qualitative inspection of the filtered membrane was carried out with an optical microscope

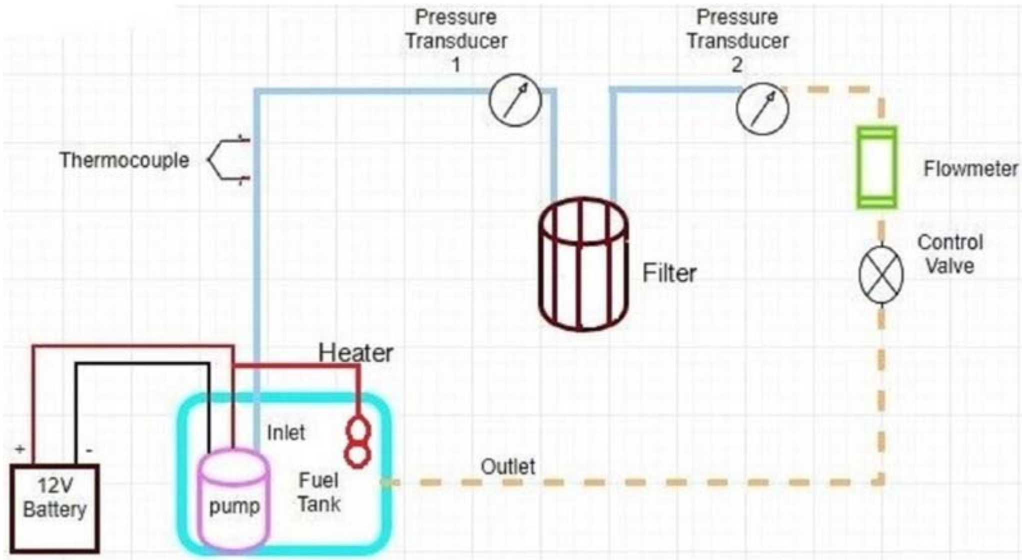

Figure 1. Schematic diagram of the experimental filter set-up.

(ZEISS Stemi 508) to analyze the change in surface morphology with fossil diesel and degummed karanja diesel blends.

Also, the volume flow rates were measured to analyze the pump performance for the tested fuels. A fresh fuel filter was employed for different fuel blends, and after each test run, the fuel lines were purged by fossil diesel. The measurement uncertainty is evaluated following Holman [19] for five observations, according to the normal distribution based on 95% confidence interval. The next section describes the three fuel processing procedure attempted in this work to overcome the issues, especially higher density and viscosity with that of raw oil.

2.1 *Esterification*

Esterification is a chemical process, wherein an acid reacts with oil blended alcohol in the presence of a catalyst. For this process, 150 ml of methanol and 1 ml of concentrated sulphuric acid was used in the molar ratio (methanol to oil) of 3:1 at ambient conditions. Simultaneous heating and mixing process was performed with the help of a magnetic stirrer and hot plate up to 500°C. After the completion of the reaction, the mixture was allowed to settle in a separating funnel for an overnight period. Esterified oil was taken in a separate container, and the conversion efficiency was found to be 97%.

2.2 *Transesterification*

Transesterification is a chemical process of exchanging the organic alkyl groups of raw oil with the alcohol group, namely, methanol. The significant factors affecting the transesterification process are the content of alcohol, catalyst, reaction temperature, time, and the content of free fatty acid (FFA). In this process, alkaline catalyst (NaOH) was dissolved with methanol by 0.68 w/w %. This mixture was then allowed to mix with the raw karanja oil and heated up to 60 minutes on a magnetic stirrer. Later, it was allowed to settle in a separating funnel for 8–10 h. Due to the density difference, karanja methyl ester gets separated and rises on the upper layer, and glycerol gets collected from the bottom layer. The isolated karanja methyl ester was washed with distilled water of 5% v/v ratio to remove the excess catalyst.

2.3 *Degumming*

Degumming is a chemical process assayed to get rid of the phospholipids and gums. 250 ml of karanja oil was taken and heated on a hot water bath up to a temperature of 70°C and stirred at a speed of 500 rpm. 5% v/v of distilled water was added drop-by-drop, and the mixture is again agitated for 30 minutes. The hydrated gum was then treated with 3 to 5% v/v of orthophosphoric acid to remove the phospholipids and other gums (see figure 2).

It was interesting to observe the color of the gum material obtained after the degumming of karanja oil to be brown "grease-like" substance, as mentioned by Kenneth *et al* [13]. Further, Tziourtzioumis and Stamatelosthe [20] reported the presence of similar dense slurry rich in the fuel filter of a CRDI engine operated with B70 biodiesel blend. It was then stirred up in a centrifuge machine at a speed of 4000 rpm for an hour. Subsequently, the top layer of oil was filtered out and dried. In this case, for every one liter of karanja oil, around 660 ml of degummed karanja oil was obtained. Figure 3 depicts the summary of the three

Figure 2. Brown "grease-like" substance obtained after degumming of karanja oil.

processing methods carried out with the Karanja oil. The processed oils were blended with diesel and represented as RK20 (20% raw karanja oil blended with 80% diesel by vol.), EK20 (20% esterified karanja blended with 80% diesel by vol.) and TK20 (20% transesterified karanja blended with 80% diesel by vol.).

3. Results and discussion

The results concerning the effect of fuel processing techniques such as degumming, esterification, and transesterification of karanja oil on the filter performance are

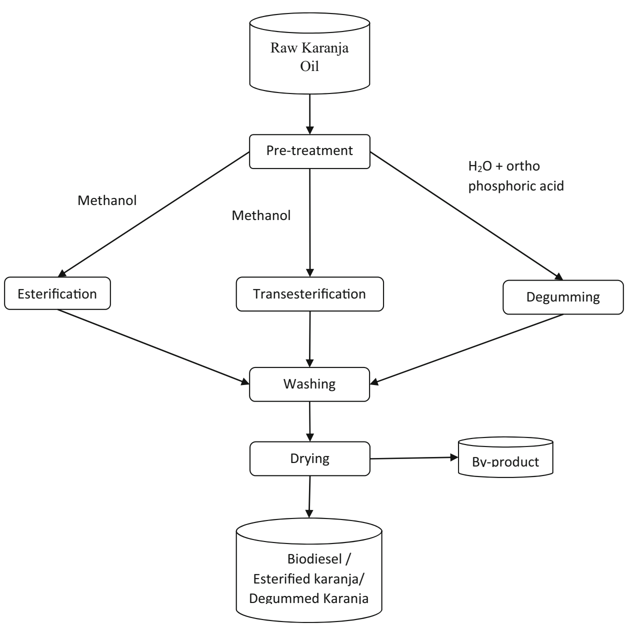

Figure 3. Various processing methods carried out with Karanja oil.

Table 1. Measured fatty acid composition of karanja

Type of fatty acid	Carbon chain	% (by mass)
Palmitic	C 16:0	9.22
Stearic	C 18:0	6.00
Oleic	C 18:1	54.10
Linoleic	C 18:2	18.00
Linolenic	C 18:3	4.32
Arachidic	C 20:0	1.22
Behenic	C 22:0	4.88
Lignoceric	C 24:0	1.41

discussed in this section. EN 14103 test method is employed for measuring the composition of fuel samples. A gas chromatograph fitted with flame ionization detector (GC-FID) (model-CS5800), which contained a fused silica capillary column (Porapak Q, 30 m × 0.250 mm × 0.25 µm) coated with polyethylene glycol is used for determining the ester contents. The fatty acid composition of the karanja oil is provided in table 1. It is explicit that karanja has unsaturated compounds as their major constituents (76.43%), with monounsaturated oleic acid (54.1%) as the dominant ester.

The measured results of density, viscosity, flash point, and calorific value are provided in table 2. The measurements in this study were obtained after repeating the procedures for three times and calculated arithmetic mean to compute the measured property values. It can be noted that the diesel possesses maximum calorific value compared to raw karanja oil and its blends. However, the density and viscosity of raw karanja oil are significantly higher among the other fuel. Interestingly the viscosity of the degummed fuel blend (TK20) is closer to the value of fossil diesel. The calculated acid value (ASTM D974 method) of karanja oil was found to be 5 mg KOH/mg.

The filter performance tests for the fuel samples were conducted at two fuel flow rates (0.03 m^3/h and 0.075 m^3/h) and fuel temperatures (30°C and 70°C). Ejim *et al* [21] had mentioned that the typical temperature of fuel during the time of injection is 80°C. Hence, the experiments are carried out close to this condition, because the excess fuel from the injector returns to the fuel tank. An attempt has been made to compare the pressure values of the blended biofuels with diesel as the reference case. Hence the pressure values of diesel are considered to be the baseline reading (marked with a dotted line in the graphical figures). Other measurement values that surpass the baseline are not preferred since a higher amount of work needs to be supplied to the main injection pump to overcome the pressure loss. Hence, it is preferred to operate any alternative fuel closer to the diesel value. As shown in figure 4, the pressure difference gradually decreases with the various refining process for the test condition at 30°C and 0.03 m^3/h.

Figure 4 highlights the importance of the degumming process on the filtration characteristics of biofuel blends, as

Table 2. Engine related properties of the test fuels

Test fuel	Density (Kg/m^3)	Viscosity at 40°C (cSt)	Flash point (°C)	Calorific value (MJ/kg)
Test methods	ASTM D4052	ASTM D445	ASTM D93	ASTM D240
Fossil diesel	830 ± 1	2.9 ± 0.3	78 ± 2	43.72 ± 1
Raw karanja oil	950 ± 1	39.2 ± 0.3	225 ± 2	40.05 ± 1
EK20	870 ± 1	3.8 ± 0.3	144 ± 2	43.07 ± 1
TK20	850 ± 1	3.5 ± 0.3	135 ± 2	42.40 ± 1
TK20 (with degumming)	840 ± 1	3.1 ± 0.3	132 ± 2	43.63 ± 1

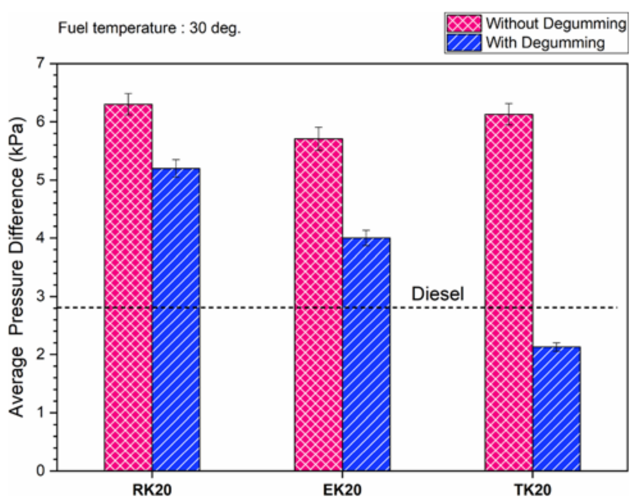

Figure 4. Pressure difference comparison at 30°C and 0.03 m^3/h.

low-pressure difference value is observed for all the three samples which have undergone degumming compared to the fuel blends without the degumming process.

Figure 5 provides the pressure difference comparison at 30°C and a higher flow rate (0.075 m^3/h) for the test samples. It is clearly observed that the pressure difference for all the test samples has got significantly increased than the values obtained at a low flow rate condition. However, the degummed blends have resulted in lower pressure difference, due to the possible reduction in contaminants/gums. The drop in average pressure difference is ~ 22.54% in the case of TK20 compared to RK20. With degumming, the average pressure difference is further reduced to 25.78% for TK20 compared to RK20. This highlights the combined effectiveness of the degumming and transesterification process of karanja biodiesel on the filter performance characteristics.

The result of the average pressure difference at a low flow rate and higher fuel temperature (70°C) is shown in figure 6. A similar trend is observed that the pressure difference for the degummed process is comparatively lower than in other cases. However, a substantial decrease is found at a high temperature, which coheres with the result of Bari *et al* [15] that the effect of filter clogging will be lessened at high temperatures. At high temperature, the drop in viscosity with degummed fuels could be significant and hence resulted in a drastic reduction in their pressure values.

Figure 7 represents the test results for high temperature and flow rate conditions. It is observed that at this test condition, the magnitude of pressure difference is more compared to the test conducted at ambient temperature conditions. This could be attributed to the effect of viscosity at high temperature; fuel becomes less viscous, so the pumping work becomes less, and thereby, with more

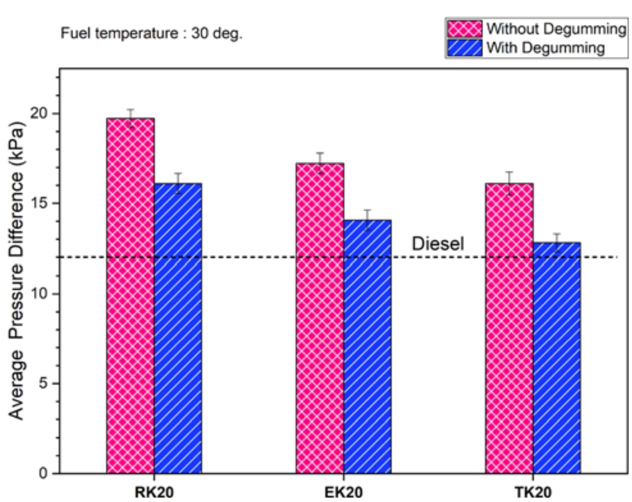

Figure 5. Pressure difference comparison at 30°C and 0.075 m^3/h.

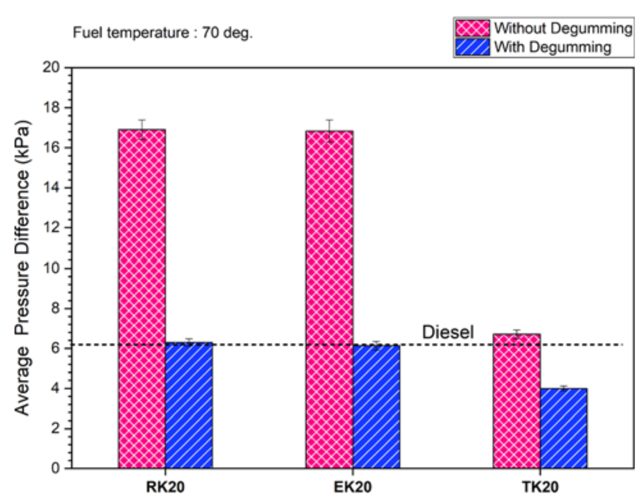

Figure 6. Pressure difference comparison at 70°C and 0.03 m^3/h.

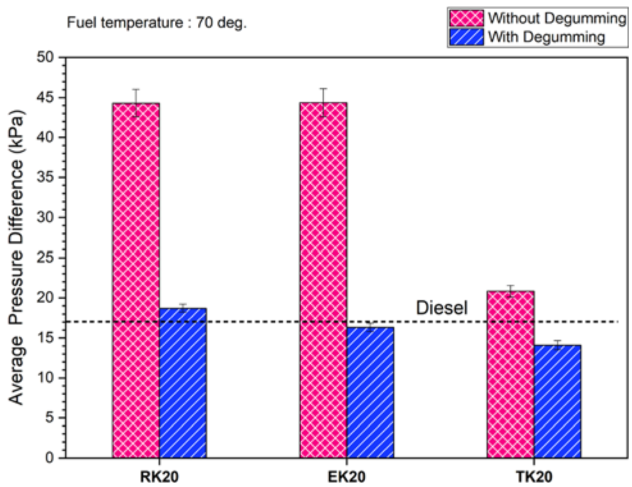

Figure 7. Pressure difference comparison at 70°C and 0.075 m^3/h.

pressure, it passes into the fuel filter. In the case of degummed blends, the average pressure difference has reduced significantly, and the transesterifed degummed karanja has the least pressure difference (47.73%) compared to all other cases. To analyze the change in pressure difference over a period of time, a short time duration study was conducted for the case of transesterifed degummed karanja and diesel fuel. The system was allowed to operate continuously for a certain time duration (3 hours), and the change in pressure difference was monitored intermittently, as shown in figure 8. The transesterifed degummed karanja exhibits lower pressure difference during the entire test than fossil diesel.

The significant losses across the filtering medium comprise friction and total head loss coefficient (HL/Q^2), where HL is the head loss, and Q is the flow rate [22]. Table 3 indicates the calculated head loss for the tested blends; the coefficient increased for all the samples than the diesel value. Further, a maximum of 23% increase in the head loss was observed for TK20 without degumming in comparison to the degummed TK20.

Table 3. Head loss of the karanja blends times the diesel value

Fuel blends	RK20	EK20	TK20
Degummed	1.515	1.323	1.230
Non degummed	1.698	1.610	1.515

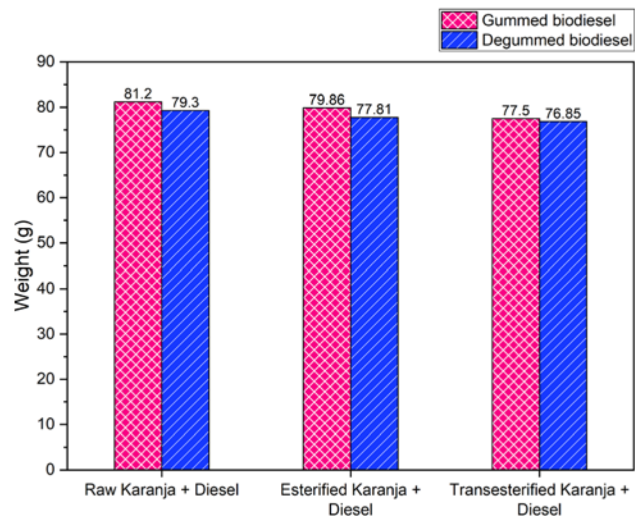

Figure 9. Effect of contaminants on the mass of fuel filter.

Gravimetric analysis was attempted to analyze the weight of contaminant retained on the filter during experiments. The weight of the fresh fuel filter is found to be 55.2 g and has been taken as the reference limit in figure 9.

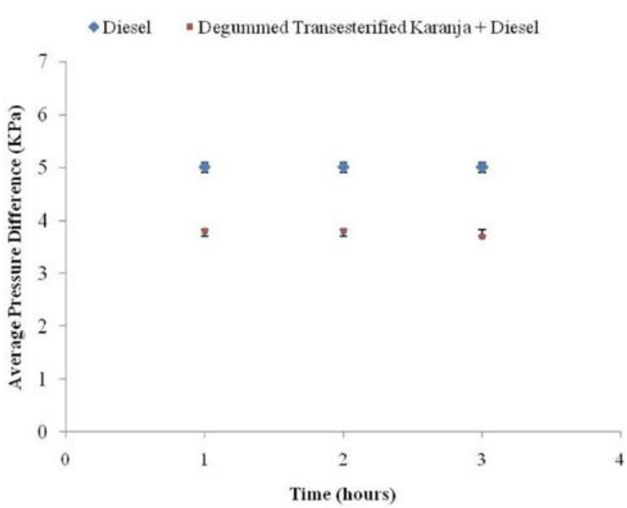

Figure 8. Time duration study at 0.03 m^3/h and 30°C.

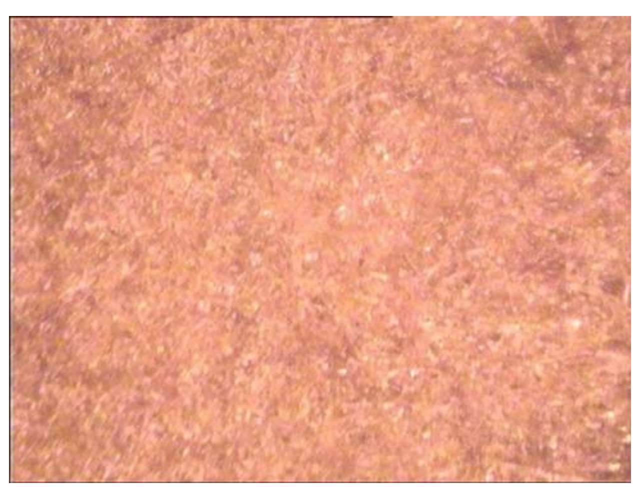

Figure 10. Surface morphology of the filter elements with the degumming process.

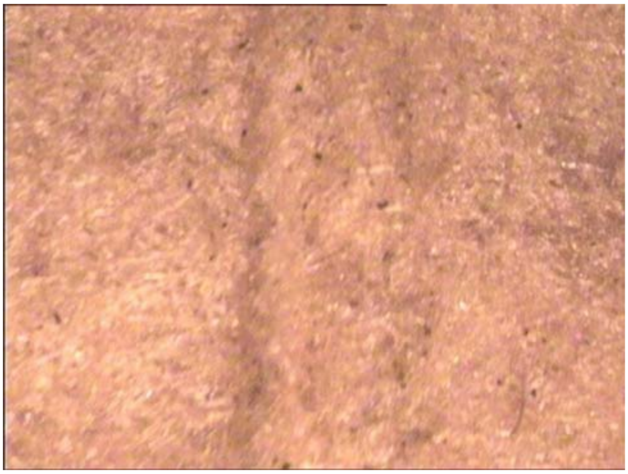

Figure 11. Surface morphology of the filter elements without the degumming process.

All the test samples are above the reference limit, and transesterifed degummed karanja has the minimum weight gain compared to other samples. However, the effect of degumming is not significant (0.8%) as compared to the difference in pressure difference across the filter, considering the short time duration study.

The surface morphology of the filter paper element was analyzed with the help of an optical microscope (magnification 400x). The corresponding images are shown in figure 10 for the case of transesterifed karanja with degumming and without degumming (figure 11) process. It is clearly observed that the minor components as sludge/gums are deposited on the filter surface for the fuel without the degumming process (figure 11). Hence a change in filter design/material or inclusion of degumming with the biodiesel production process is recommended for the seamless operation of biofuels.

4. Conclusions

The quality of the biodiesel determines the longevity of a fuel filter and also the durability of other parts of the fuel supply system. The experimental analysis of the filtration characteristics of a commercial diesel filter operated with raw and processed karanja-diesel blends revealed the following:

- Raw karanja oil had unsaturated compounds as their major constituents (76.43%) with mono-unsaturated oleic acid (54.1%) as the dominant ester and degumming process lowers both the density and viscosity of raw karanja oil.
- Among the tested conditions, degummed transesterifed karanja (TK20) blend yielded a drop in average pressure difference (22.54%) in comparison to raw karanja (RK20) blend.
- Degumming reduced the pressure drop about three times and the head loss across the fuel filter compared to transesterified karanja without the degumming process.
- The short time duration study highlights that the degummed transesterifed karanja exhibited lower pressure difference than that of fossil diesel.

Hence from the observation, it is concluded that the degumming process helped in enhancing the filtration performance and can lower the maintenance cost for the automotive systems operated with biodiesel blends. Future study is intended towards the filter simulation studies for a better understanding of the property characteristics of biofuels, the impact of degumming on the internal deposits of a fuel injector, and in evaluating the composition of gum deposited on the filter.

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