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A study on critical order of joints with clearances and its effect on kinematic performance of multiloop planar mechanisms

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Abstract. Simulation and study of joint clearances has usually focused on appropriate simulation strategies and their validation against experiments. The effect of joint clearances on the output of a mechanism has been usually evaluated qualitatively. The relative importance of different joints in a mechanism in producing deviation from the output of an ideal mechanism has not been studied. This work identifies the appropriate statistical measure for quantifying this deviation and uses it to rank the joints of one degree of freedom multi-loop planar mechanisms. The inversions of six-bar mechanism have been studied through ADAMS simulations involving different clearance sizes and speed of crank. A trend in clearance location ranking has been identified which can probably be extended to planar mechanisms of higher complexity.

Keywords. Planar mechanism; joint clearance; simulation; kinematic performance.

1. Introduction

A mechanism is designed to perform a specific task and it may produce errors when used in real working conditions. These errors occur due to unavoidable factors such as flexibility of links, joint clearances, friction, lubrication and wear. Links and joints are the main constituents of any mechanism and a designer needs to carefully consider their function and limitations. The links form the basic structure of the system while the joints are used to define the relative interactions between them. In general analysis of the mechanism, it is assumed that the joints are ideal and there is no clearance in them. However, clearances are unavoidable due to machining tolerances, wear, and local deformations. Joint clearances are necessary to allow motion between the links and hence they are essential for correct functioning of the mechanism. The performance of a mechanism is adversely affected by the wear and tear of the joints as a result of impact forces generated due to joint clearances [1].

Due to clearances, there is a deviation from the expected ideal behavior which is considered as error. These errors are small for small clearance size but they cannot be neglected for high precision operations. However, reduction in these joint clearances increases the overall cost of the manufacturing. In order to reach a compromise between the manufacturing cost and the output error, it is necessary to analyze the errors and their patterns to estimate the effects caused by the clearances for the given set of conditions.

Researchers around the world have reported various methods for studying the effects of joint clearances and predicting their effect on a mechanism's performance as accurately as possible. Research in this area is mainly focused on developing mathematical models for joint clearances to predict the mechanism's dynamic and kinematic behavior in a variety of situations [2–5]. Neural networks have also been used for this purpose [6–10].

Models for taking into account both joint clearance and link flexibility/compliance have also been reported [11–35]. Models to capture system dynamics and mechanism optimization for reducing their effect was the natural progression in this field and have been reported by many [36–43].

Low joint clearance is certainly preferable but it increases the cost of manufacturing. Mechanism designers and manufacturers need to take a decision on how much clearance can be allowed at which location in the mechanism. In order to take this decision, one needs to know whether the location of joint clearance makes a difference to the output of a mechanism. Such a study, which does not appear to be reported, has been undertaken in this work. One degree of freedom planar mechanisms with only revolute and prismatic pairs have been simulated in ADAMS and the effect of clearance location, clearance size and crank speed have been studied. The first step necessary for such a study was quantification of the deviation of the kinematic output of a mechanism with clearance from that of an ideal mechanism. The appropriate statistical tool for

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this has been identified and used for analyzing all the simulations.

2. Joint clearance modeling

Modeling of joints with clearance is very crucial to analyze the performance of mechanisms. With the help of these models, the designer can estimate the output and corrective actions can be taken to reduce the output error. The commonly used modeling strategies for mechanisms are shown in figure 1.

a) The clearance is modeled by adding a virtual massless link that has a constant length equal to the radial clearance.
b) Spring-damper can also be used to model the joint clearance.
c) The journal and bearing are considered as two colliding bodies known as momentum exchange approach.

The third approach is more realistic for a dry clearance joint. It allows the contact force models to be applied and it takes into account the dissipation of energy during the impact process. Radial clearances introduce two extra degrees of freedom in the mechanism. Three different types of motion are observed between journal and bearing. They are:

- Free flight: The motion is said to be in free flight mode when the journal moves freely within the bearing's boundary.
- Impact: Impact occurs at the instant when the journal's surface collides with that of the bearing's inner surface. It occurs at the end of free flight.
- Contact: In the contact mode, contact is constantly maintained. The relative penetration between the journal and the bearing may vary during this mode.

Modeling of a mechanism with clearances requires a force model which can accommodate the real conditions as far as possible. The important models of contact force are:

2.1 *Hertz's model [44]*

This non-linear model is the best known contact force law. It is limited to frictionless surfaces and perfectly elastic solids and does not include damping in its original form. The contact process can be defined as two rigid bodies interacting through a nonlinear spring along the line of impact. The deformation is assumed to be concentrated in the vicinity of the contact area, elastic wave motion is neglected, and the total mass of each body moves with the velocity of its center of mass. Hertz model does not account for energy dissipation and the impact force is defined as:

$$F_{\mathrm{N}} = K\delta^{n} \tag{1}$$

where K and n depend on material and geometric properties and are obtained by using elastostatic theory. The exponent n is equal to 1.5 for metallic contacts. K is the stiffness coefficient of the impact body, which is obtained from the impact experiment of two spheres as follows:

$$K = \frac{4}{3\pi\left(\sigma_i + \sigma_j\right)}\left[\frac{R_i R_j}{R_i - R_j}\right]^{1/2} \tag{2}$$

$$\sigma_i = \frac{1 - v_i^2}{\pi E_i} \tag{3}$$

$$\sigma_j = \frac{1 - v_j^2}{\pi E_j} \tag{4}$$

Where, v is the Poisson's ratio and E is the Young modulus, $R_i$and R_j are the radii of the two spheres.

2.2 *Lankarani-Nikravesh model [14]*

Lankarani and Nikravesh [14] developed a contact force model with hysteresis damping for impact in multibody systems. The model uses the general trend of the Hertz contact law, in which a hysteresis damping function is included to represent the energy dissipated during the impact. They suggested separating the normal contact force

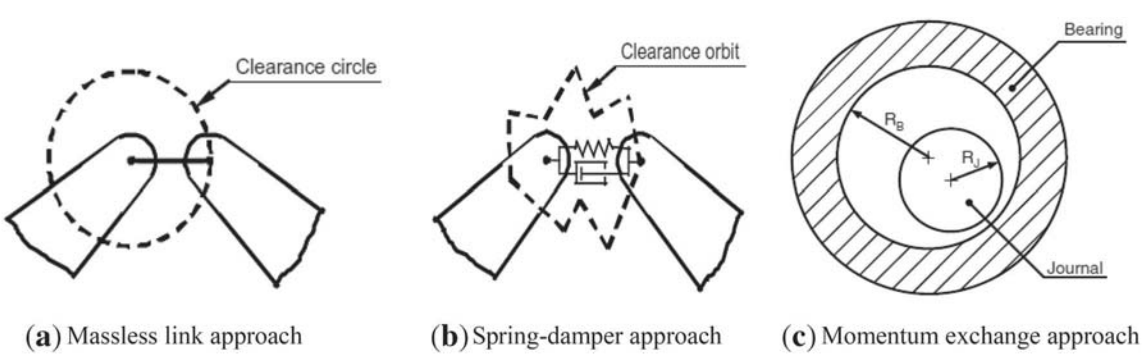

Figure 1. Modeling strategies [2].

into elastic and dissipative components. The impact force is defined as:

$$F_N = K\delta^n + D\dot{\delta} \quad (5)$$

Elastic deformation force is represented by the first term and the energy loss is represented by the second term. δ is the deformation, $\dot{\delta}$ is the relative deformation velocity, K is the contact stiffness coefficient of the impact body.

$$D = \frac{3K\left(1 - c_e^2\right)\delta^n}{4\dot{\delta}^{(-)}} \quad (6)$$

$\dot{\delta}^{(-)}$ is the initial relative velocity of the impact point, c_e is the coefficient of restitution.

ADAMS uses a very similar contact force model, where the dissipative term is given by a step function as discussed in the next section.

3. Modeling in ADAMS

The most common computational tool used for design and analysis of multi-body mechanical systems is ADAMS [45, 46]. Various types of links and joints along with their properties like mass, location of center of mass, moment of inertia and degrees of freedom can be assigned for a mechanism in this tool [18]. The joints defined by this tool are ideal as they do not have any clearance or deformation. Therefore, to compare performance of the mechanisms with joint clearances, clearances at joints were made in ADAMS. The clearance at revolute joint between coupler and slider has been made by making a cylindrical hole in the slider cube and attaching a cylinder to the end of the coupler. So, we can change the size of the hole to set a clearance size. Similarly, the clearance at revolute joint between crank and coupler has been made by attaching a cylinder to the end of the crank and making a hole in the attached cylinder.

3.1 *Input factors*

Various input parameters for modeling of mechanism in ADAMS are as follows:

3.1a *Clearance size*: For a standard journal-bearing of journal diameter 20 mm, the clearance size ranges from 0.02 mm to 0.08 mm. However, due to wear during operation and other environmental factors, the clearance can increase. So, in this research work, the clearance size has been taken in the range of 0.02 mm to 1 mm.

3.1b *Crank speed*: To cover a wide range of cases, the speeds range from 100 rpm to 3000 rpm.

3.1c *Contact conditions*: For the modeling of contacts, ADAMS uses the contact method based on the impact function: IMPACT-Function-Based Contact. In this method, the solver computes the contact force from the IMPACT function available in the ADAMS function library. The normal force of the contact has two components: rigidity and viscous damping. The component of rigidity is a function of the penetration δ. The component of the viscous damping is a function of the speed of penetration. In this model the normal force of contact is given as:

$$\begin{cases} F_N = K\delta^n + STEP(\delta, 0, 0, d_{\max}, C_{\max})\dot{\delta}, & \delta > 0 \\ 0, & \delta \le 0 \end{cases} \quad (7)$$

3.1d *Value of K*: The revolute joint is a case of contact between two cylinders (one inside the other). So, the contact should start with a line contact and then become a 2D rectangular contact. But the value of K in this case will not only depend on the material and geometrical property but also the stress distribution between the cylinders, which cannot be determined accurately unless it is a static case and the force is applied externally.

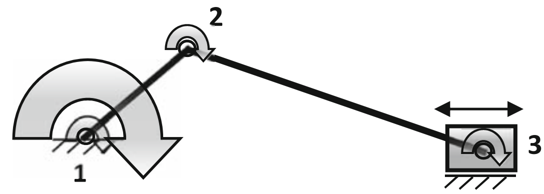

Figure 2. Slider-crank mechanism.

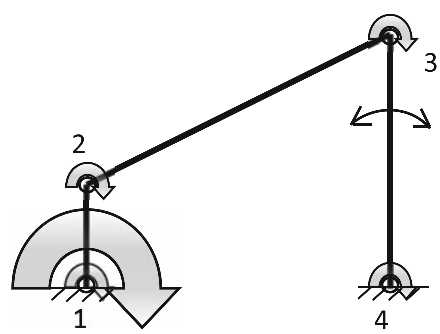

Figure 3. Crank-rocker mechanism.

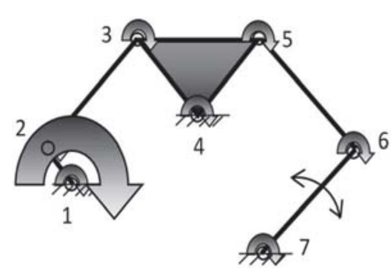

Figure 4. Watt mechanism.

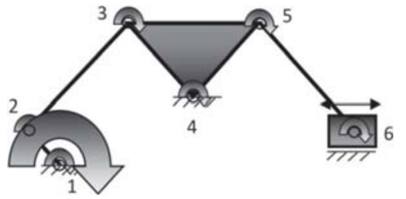

Figure 5. Watt mechanism with slider (1).

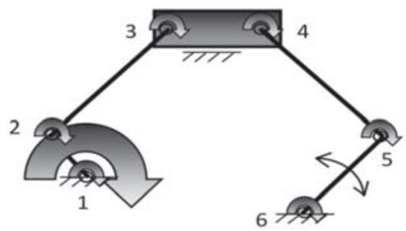

Figure 6. Watt mechanism with slider (2).

So, the researchers solved this problem by stating that the line contact in the revolute joint will only be present for two cylinders aligned with extreme precision. Also, a uniform force distribution over the length of the joint is not possible in real life conditions. Moreover, the force-deformation diagrams for both spherical and cylindrical impact force models were studied in the literature [1–4] and it was found that the spherical and cylindrical force-deformation diagrams are reasonably close. Based on these studies, we used the Hertzian contact force law between two spheres with the different parameters defined in Eq. (8).

$$K = \frac{4}{3\pi\left(h_i + h_j\right)} R^{1/2}, R = \frac{R_i R_j}{R_i + R_j}, h_k = \frac{1 - v_k^2}{\pi E_k}, k = i, j \tag{8}$$

R_i, v_i and E_i represent respectively the radii of the cylinders, the Poisson's ratio and the modulus of elasticity for element i.

For clearance = 0.02 mm, journal radius = 10 mm and bearing radius = 10.02 mm

$E = 2.07*10^5$ N/mm^2; $v = 0.29$

Putting these values in the equation we get $K = 3.37*10^5$ N/m$^{1.5}$

Similarly, for clearance = 0.1 mm, $K = 3.377*10^5$ N/m$^{1.5}$

For clearance = 0.5 mm, $K = 3.4*10^5$ N/m$^{1.5}$

3.1e *Value of n*:The value of n is usually taken to be 1.5 for metallic contacts. So, n = 1.5.

3.1f *Value of damping coefficient*: In ADAMS, the instantaneous damping coefficient is a cubic step function of the penetration given as:

$$STEP(\delta, 0, 0, d_{\max}, C_{\max}) = \begin{cases} 0, & \delta \leq 0 \\ C_{\max}\left(\frac{\delta}{d_{\max}}\right)^2\left(3 - 2\frac{\delta}{d_{\max}}\right), & 0 < \delta < d_{\max} \\ C_{\max} & \delta \geq d_{\max} \end{cases} \tag{9}$$

The value of $C_{\max}$ should be approximately 1 percent of the value of K. $d_{\max}$ = 0.01 mm

3.2 *Output factors*

Two factors were used as parameters for comparing the kinematic performance of different mechanisms, either displacement of the slider or angular rotation of the rocker attached to ground.

4. Quantification of error

ADAMS allows export of data in the form of an Excel Sheet. The data contains many data points of the graph at equal intervals of time. Thus, we have two data sets for the two curves. Now, we need to compare these data sets in order to quantify the difference between the two curves. Some statistical measures which can be used to compare the two data sets are mean deviation (obtained by taking the mean of differences in output of mechanisms with and without clearance), RMS deviation, 90% points tolerance (value of tolerance on both the positive and negative sides of the ideal curve, within which 90% of the data points of real mechanism lie), t-test (to check if the two means are reliably different from each other), F-test and Kolmogorov-Smirnov test.

All these six measures were computed for a set of preliminary simulations with varying levels of clearance. The mean deviation was found to be the most sensitive of these measures. Thus, for the rest of the analysis, we used mean deviation as a measure of deviation of the performance of the mechanism from the ideal mechanism.

5. Planar mechanisms

Different inversions of one degree of freedom of planar four-bar and six-bar mechanisms and one inversions each of eight and ten bar mechanisms were studied in this work. Only one degree of freedom mechanisms were considered. These are described below.

In all these figures, the large single sided arrow shows the location of the actuator and the small double sided arrow shows the location (link) where the displacement (angular or rotary) has been measured as output. The link lengths were proportional to what has been shown in figures 2 to 12. The cross sections of all links were kept same

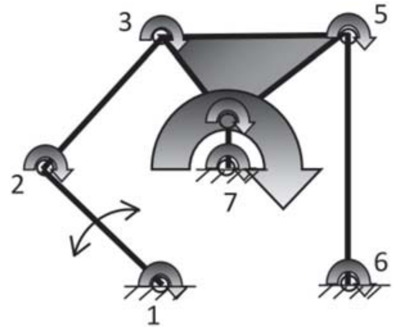

Figure 7. Stephenson1 mechanism.

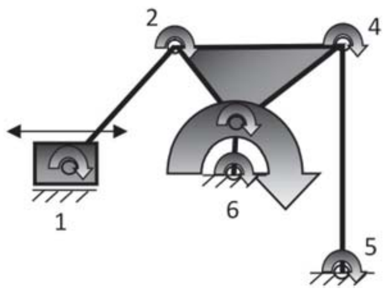

Figure 8. Stephenson1 mechanism with slider.

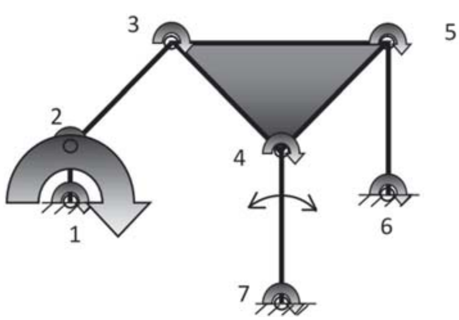

Figure 9. Stephenson2 mechanism (1).

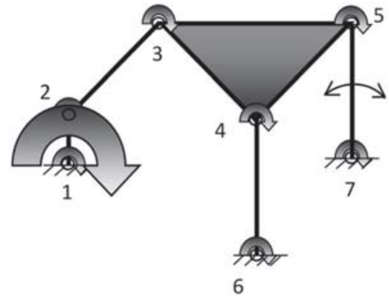

Figure 10. Stephenson2 mechanism (2).

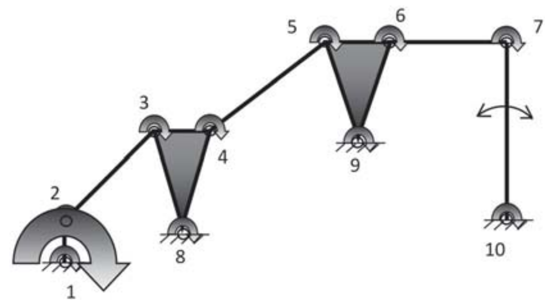

Figure 11. Eight-bar mechanism.

as was the material. Studies with small changes in dimensions of the links showed that the relative ranking of the importance of the joints did not change. For all these simulations, we have not introduced clearance at the joint where the crank is connected to the ground. This is because that joint is the power source which drives the crank. It may be a motor, a set of gears or any other arrangement which are not the part of mechanism we are studying. A motor can be attached to only on an ideal revolute jointin ADAMS.

Clearance has also not been considered at the joint where the output link is attached to the ground because of the problem of output measurement. In order to have a fair comparison between two outputs of the same mechanism in different conditions, the measurement technique of the output factor should be the same. If we introduce a clearance, at the joint where output rocker is attached to the ground, the angle measurement of the rocker will get disturbed. Therefore, in order to keep the input and output conditions same for all the mechanisms and to compare the effects, we have not introduced clearances at the revolute joints between crank and ground and between the output (rocker) and ground.

6. Results and discussions

The primary objective of this study was to identify the joint that producesthe highest error in a mechanism due to clearance. Simulations were carried out for all mechanisms described in the previous section by introducing clearances in different joints, one at a time. The effect of different speeds (of crank) and clearance sizes was also studied. Some important results are discussed here. The results for the four bar mechanisms have been summarized in figures 13, 14 and 15.

These show that Joint 3 is the most critical joint and the values vary significantly for 0.02 mm but they are very similar for 0.5 mm. So, sensitivity to speed is high for low clearance.

Figures 16, 17 and 18 show mean deviation at different joints of different inversions of the Watt chain at 0.1 mm radial clearance. The trend of all the curves is the same.

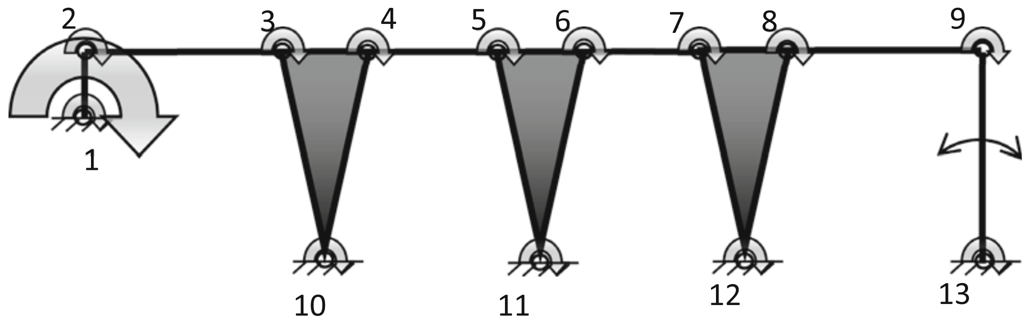

Figure 12. Ten-bar mechanism.

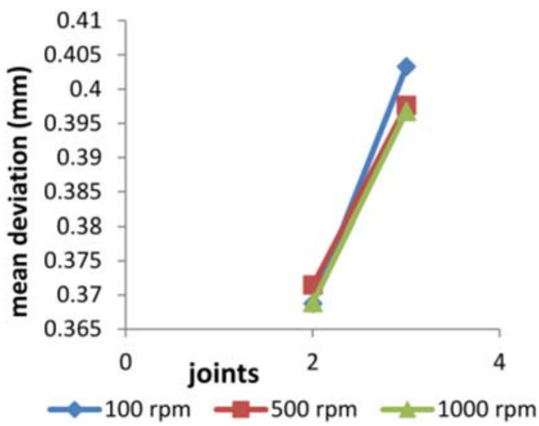

Figure 13. Slider-crank mechanism with 0.5 mm clearance.

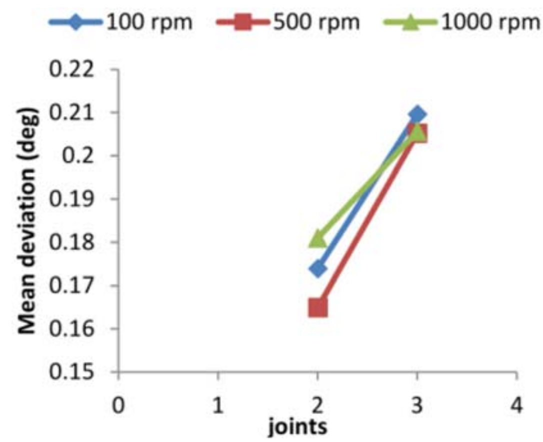

Figure 15. Crank-rocker mechanism with 0.5 mm clearance.

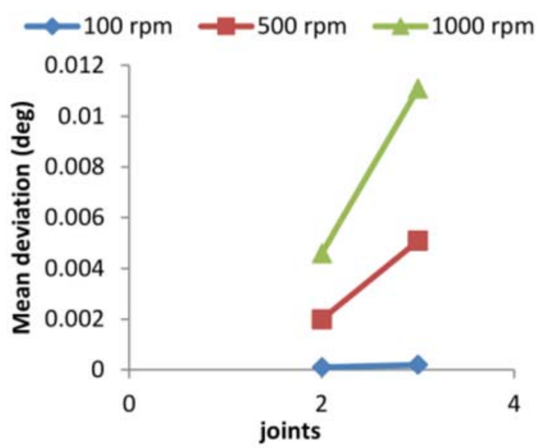

Figure 14. Crank-rocker mechanism with 0.02 mm clearance.

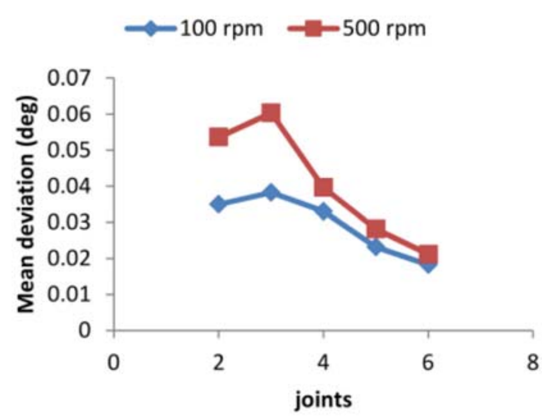

Figure 16. Watt mechanism with 0.1 mm clearance.

Joint 3 is the most critical joint in all the three mechanisms followed by joints 2, 4 and 5, i.e., the criticality order here is: Joint 3 > Joint 2 > Joint 4 > Joint 5. Joint 6 in figures 5 and 6 is the grounded joint which is least critical of them all.

Figures 19 and 20 show the criticality graphs for Stephenson1 mechanism and Stephenson1 mechanism with slider, respectively. Both the graphs show the same trend where the deviation is maximum at the output link and decreases as one proceeds to the other end of the mechanism. A common trend which seems to emerge is that the

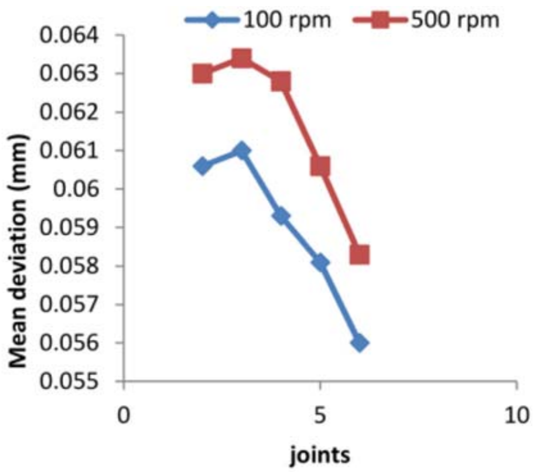

Figure 17. Watt mechanism with a slider (1) and 0.1 mm clearance.

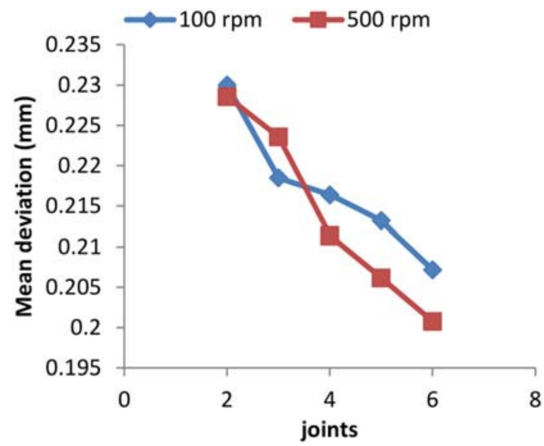

Figure 19. Stephenson1 mechanism with 0.1 mm Clearance.

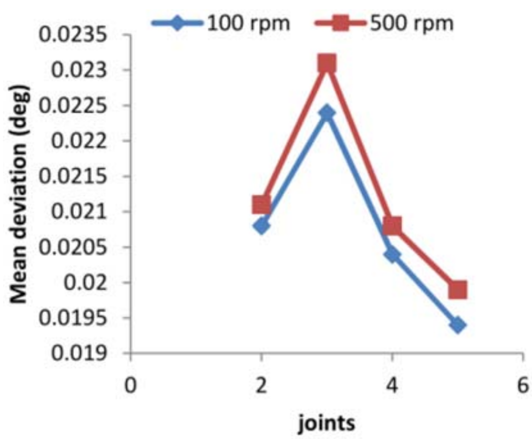

Figure 18. Watt mechanism with a slider (2) and 0.1 mm clearance.

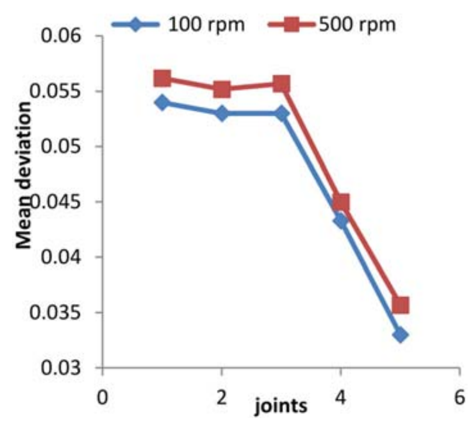

Figure 20. Stephenson1 mechanism with a slider and 0.1 mm clearance.

first joint next to the crank, which has a link connected to ground, is the most critical joint. After that the criticality ranking goes towards the crank and then goes further down to the joints in order of their distances from the crank. We would examine the validity of this hypothesis in the rest of the study.

Figures 21 and 22 show the criticality graphs for Stephenson2 mechanism (1) and Stephenson2 mechanism (2). The order of criticality for both the mechanisms is: Joint4 > Joint3 > Joint2 > Joint5. This is as per the hypothesis stated earlier. The most critical joint is the joint next to the crank, which has a link connected to the ground (i.e., Joint 4). The next most critical joints are the ones towards the crank, i.e., Joint 3 and Joint 2. Then comes the joints which are left, i.e., Joint 5. Joint 6 is the grounded joint. The two graphs look very similar as the structures of mechanisms are the same. The deviations increase significantly with increase in speed, if the clearance is at Joint 4. The values in first graph are slightly higher than the corresponding values in the second graph. The overall conclusion is that deviations are more, if the output link is closer to the crank.

Similar results are seen in eight and ten-bar mechanisms as shown in figures 23 and 24. The joint next to the crank is the most critical joint. It should also be noted that the grounded joints also show a trend. The grounded joint in the first loop has a significant effect but rest of the grounded joints have very little effect on the output.

Error in output generally increases with increase in the crank speed (expect for an aberration shown in figure 19). This happens for all clearance sizes. However, the change in mean deviation is quite small even with a 5 times increase in speed from 100 to 500 rpm. Figures 25 and 26 show the variation of mean deviation with increase in speed when a clearance of 0.1 mm is at Joint 2 and Joint 3, respectively. It can be noticed that the deviations usually increase with speed. This means that a machine working at higher speeds is expected to produce more error than a machine working at lower speeds.

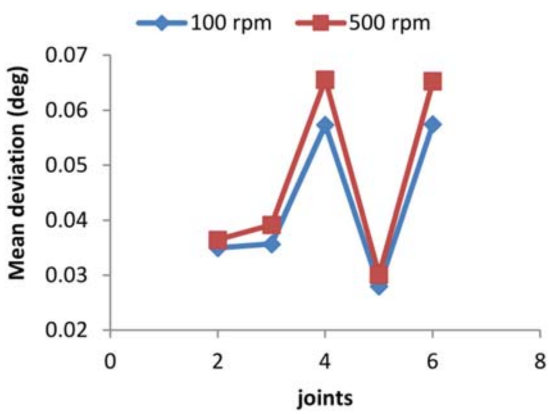

Figure 21. Stephenson2 mechanism (1) with 0.1 mm clearance.

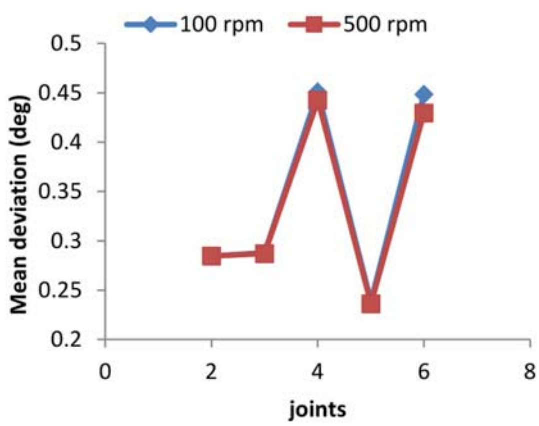

Figure 22. Stephenson2 mechanism (2) with 0.5 mm clearance.

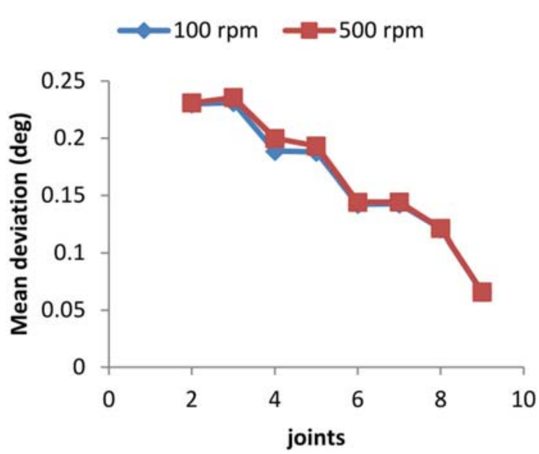

Figure 23. Eight-bar mechanism with 0.1 mm clearance.

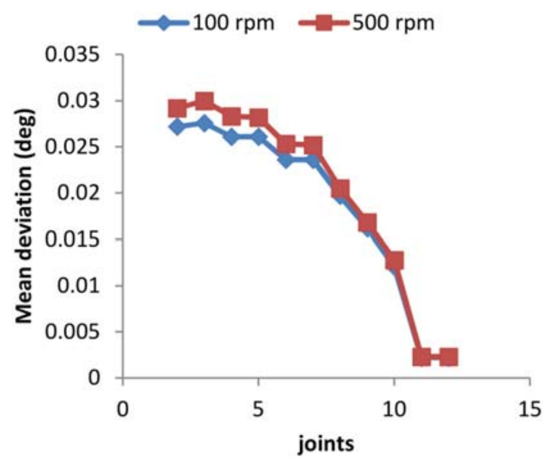

Figure 24. Ten-bar mechanism with 0.1 mm clearance.

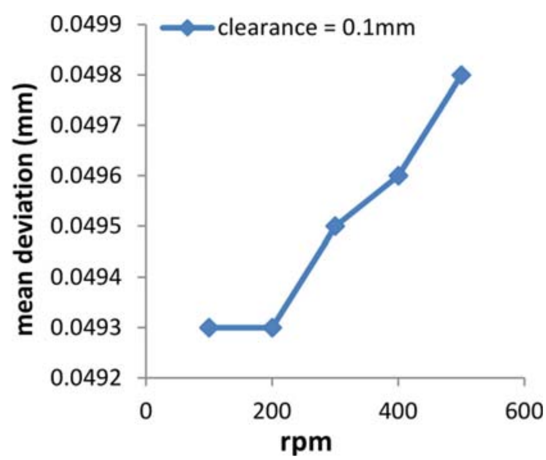

Figure 25. Slider-crank mechanism, Joint 2 deviation v/s speed.

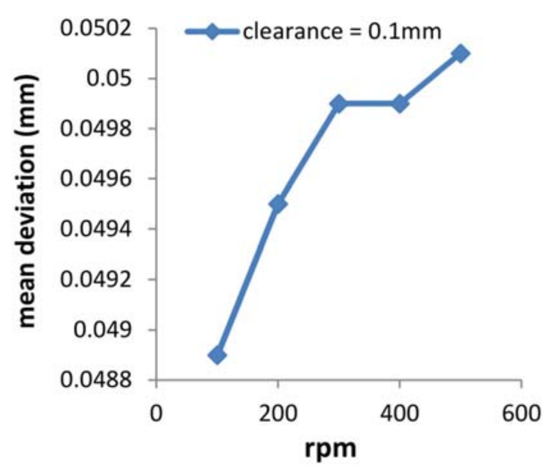

Figure 26. Slider-crank mechanism, Joint 3 deviation v/s speed.

Increase in radial clearance size brings an almost linear increase in the deviation values as seen in figures 27 and 28. It is also worth noting from figure 27 here that the deviation of slider is nearly 50% for 0.1 mm clearance (0.0501 is 50% of 0.1) and it goes on increasing to 80% for 0.5 mm clearance (0.3977 is 80% of 0.5). This happens because at lower values of clearance, the number of

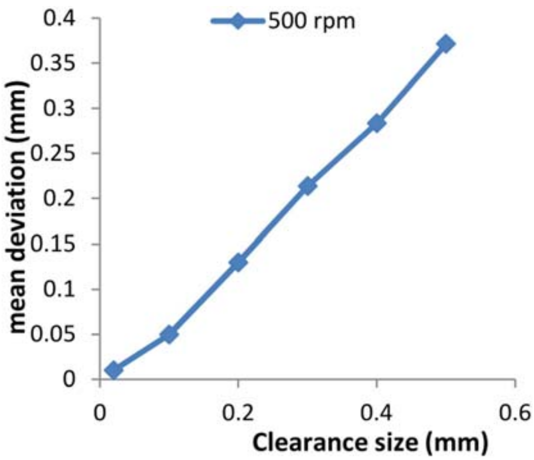

Figure 27. Slider-crank mechanism, Joint 2 deviation v/s clearance size.

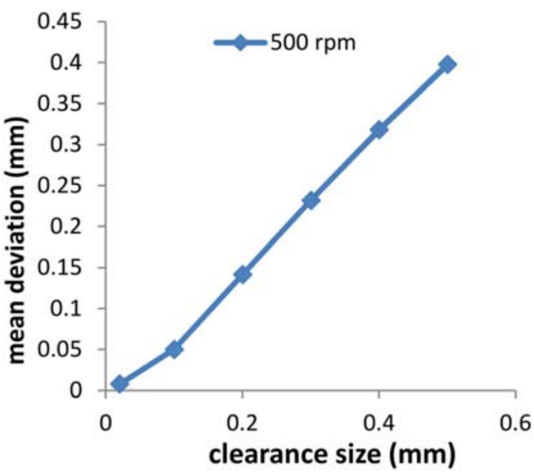

Figure 28. Slider-crank mechanism, Joint 3 deviation v/s clearance size.

collisions between journal and bearing is more. Due to more number of collisions, the journal is sent back to the center again and again, and thus it remains at the center for a larger amount of time as compared to the case of large clearances. Thus, in larger clearances, the journal appears to remain closer to the walls of the bearing for an extended period and spends less time going to the diametrically opposite wall.

7. Conclusions

This work was undertaken to study the effect of joint clearances of multiloop planar mechanisms on their kinematic performance through ADAMS simulations. An attempt was made to identify the joint which leads to the highest deviation of the mechanism's output compared to what is expected in an ideal mechanism (zero joint clearance). An appropriate statistical measure for quantifying this deviation was identified and was used to rank the joints of a mechanism. The effect of clearance location, clearance size and speed (of crank) were studied. The trends which are apparent are as follows:

- The most critical joint in a mechanism is the joint which lies next to the crank in the direction towards the output link, and has a link connected with the ground. It means that if a clearance is present in such a joint, the error in the output of the mechanism will be the highest.
- The next most critical joint is the one towards the crank. After that, the criticality decreases further in rest of the joints in proportion to their distance from the crank.
- The grounded joint in the first loop has a high ranking in the order of criticality. The rest of the grounded joints are not so critical.
- Replacing a revolute joint with a slider as the output does not change the results.
- The deviations increase with an increase in speed of the crank.
- Sensitivity of the deviation to change in speed is high at low clearance.
- Deviation and clearance size have an almost linear relationship in the range of clearances studied in this work.

Previous studies on joint clearance have mostly been qualitative with deviation of output of a mechanism being observed through a plot against time. This work, for the first time, quantifies the deviation and ranks the joints in an order of criticality. A general trend of this ranking has also been identified which can perhaps be extended to planar mechanisms of any level of complexity involving revolute and prismatic pairs. Future work will examine whether this trend can be extended to mechanisms with other lower pairs and spatial mechanisms.

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