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Pricing and market segmentation in an uncertain supply chain

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Abstract. This paper addresses a new location-allocation-pricing problem in designing a three-level uncertain supply chain network with stochastic price-sensitive demands. Using the market segmentation problems, a supply-chain network is developed with two distribution channels that consist of Brick & Mortar and online markets, when some demand leakages occur from the market with a higher price. Due to the lack of physical observation of products in online markets, a return policy is used. So, demand behavior is analyzed in terms of the pricing and return policy. The problem established location, allocation, order quantities, pricing and refund price decisions to optimize the total profit of the chain. Furthermore, it is formulated as a mixed-integer non-linear programming (MINLP) model and solved by a Lagrangian relaxation algorithm. The numerical study and computational results indicate the efficiency and effectiveness of the proposed algorithm.

Keywords. Location-allocation-pricing; stochastic price-sensitive demand; pricing and return policy; market segmentation; Lagrangian relaxation.

1. Introduction

A developed competitive environment in business and the globalized markets have led companies to put significant efforts to supply, procure, construct and distribute their goods to respond to the different needs of their consumers. In this regard, supply chain management (SCM), including the systematic and strategic coordination of business processes, has attracted the attention of researchers as well as managers. Simchi-Levi and Kaminsky [1] described the supply chain network design (SCND) as the first and powerful step to reduce the total supply chain (SC) costs. SCND is the most critical decisions in SCM, especially for new chains, which affects many future tactical and operational decisions.

One of the main significant issues was discussed in the SCND can be referred to the location-allocation (L-A) problems. The L-A aims to locate the facilities and to allocate the needs of the available facilities to the new centers. Usually, strategic decisions are largely influenced by tactical decisions. Therefore, many researchers have integrated the decisions in the SCM.

Motivated by the growth of the number of the Internet users and increasing online sales, in the current research, the intention is to develop the SCND problem with a dual channel consisting of online and offline stores. The mathematical model is formulated as the maximization objective function, and a Lagrangian relaxation algorithm (LRA) is developed to solve the presented model. In the related literature, there are a few SCND problems with the maximization of the profit objective function and price-sensitive demands. Furthermore, most of them have considered a deterministic environment; however, in reality, there are some uncertainties in an SC, especially in demand behavior. The contribution of this paper is three-fold. First, we develop a mathematical model with an integration pricing (including of refund price), inventory, and market segmentation decisions with location-allocation by the focus on the online market (OM) environment. Second, a stochastic price-sensitive demand function is considered when some demand leakages occur from the market at a higher price. Finally, we develop a heuristic algorithm by using the LRA, sub-gradient method, and Karush Kuhn Tucker (KKT) optimality conditions.

The organization of this paper is as follows. In the next section, we present the literature review. The mathematical model and the solution method are presented in sections 3 and 4, respectively. Section 5 reports the computational results. Finally, the conclusions are provided in section 6.

2. Literature review

This section is divided into four sub-sections. First, we review the integrated SCND with inventory decisions. Then, we review the pricing problems and integrated

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SCND with pricing decisions, respectively. Finally, we put forward the research gap that this paper attempts to fill it.

2.1 *Integrated SCND and inventory decisions*

Determining inventory policy is one of the most important SCM tactical decision due to the high volume of related costs. In SCND problems, many researchers have investigated and integrated inventory decisions with other decisions, for example, Miranda and Garrido [2] developed a two-level SCND problem with inventory control decisions. In their model, demand is stochastic and based on the normal distribution, and deterministic lead time is considered. Furthermore, an optimal SCND for a fixed value of the service level is provided. The solution method is done in two steps by heuristic algorithms. In the first stage, customer service levels are optimized, and after that, it is decided to locate the inventories and allocate them to the customers.

Yao *et al* [3] presented a joint facility L-A and inventory (L-A-I) problem to determine locations of warehouses, allocation of stochastic customer's demand (follows the normal distribution with known mean and standard deviation) and inventory levels of warehouses. In their paper, a consumer can be either supplied by a single warehouse or by a manufacturer directly. The inventory system of their model follows a periodic review (r, S) policy. To solve the mathematical model, they proposed a heuristic method. Manatkar *et al* [4] presented the L-A-I for multi-product in a multi-echelon SC. Three types of decisions are considered in their work. The first one is the location of the distribution centre (DC). The second one is allocation, and the third is the maximum amount of inventory at DCs.

Many studies within the field of L-A-I related to the integration of facility capacity decisions with other decisions. Salehi *et al* [5] presented two objectives L-A-I consisting of customers with uncertainty demands, DCs with several levels of capacities and a fixed manufacturer. They considered the proactive lateral transshipment within DCs and analyzed the trade-off between minimization of the costs and maximization of the warehouse utilization.

The SC structure can be divided into two general groups from the flow of a goods point of view. The first one is the forward network that examines the flow of goods from upstream to downstream levels. The second one is the reverse network. Furthermore, in a particular type, many researchers have focused on the SCND for closed-loop networks [6–9].

Ramezani *et al* [10] considered capacity levels decisions in an uncertain closed-loop SC with three objective functions (maximize profit, maximize the service level, and minimize the defective raw material). Niranjan *et al* [11] developed a mathematical model for an omni-channel closed-loop SCND problem. Entity selection and the supplier selection process integrated with the production amounts, inventory levels, stockouts and shipment quantities in their model.

2.2 *Pricing problems*

In pricing problems, many researchers have integrated pricing decisions with other tactical decisions, such as inventory. For example, Shavandi *et al* [12] developed an inventory-pricing model and considered the deterministic exponential demand function. Avinadav *et al* [13] developed a mathematical model to achieve the order quantity, period of replenishment and price of products. Bajwa *et al* [14] presented multi-product, pricing and lot-sizing model for a factory with limited production capacity. Wang *et al* [15] developed a single product inventory-pricing model when the demand is determined from their purchasing utility through a multinomial logit model, and the utility is contingent. Hu *et al* [16] developed a two-echelon SC with the order quantity and pricing (i.e., product retail price, option price, and option exercise price) decisions. A random product retail price-sensitive demand is considered. They analyzed two types of contracts consisting of the option and conventional option contracts.

By considering the concept of willingness to pay (WTP), company segments market demand based on a differentiation price. Some recent studies considered market segmentation in the pricing problem. Zhang *et al* [17] proposed a market segmentation approach in an inventory-pricing problem with monopoly market. Raza [18] proposed an extension to [17] and developed a pricing-inventory problem with market segmentation, in which the uncertain linear demand function depends on a price. This model extended by Raza and Turiac [19] by controlling the process of the manufacturer and considering some reworks in products with the lowly quality level.

Online stores are the principal tools to meet customers' demands. Li *et al* [20] developed several models to examine the impact of pricing, quality of products and return policy in the OM. Zhang *et al* [21] considered an SC with an online retailer, and a capacitated carrier, in which the random demand function depends on the price. They studied the decisions in the centralized and decentralized systems.

Gupta *et al* [22] developed multi-objective pricing, inventory control and fulfillment problem in an omni-channel environment. They considered an omni-channel retailer with Brick & Mortar (B&M) stores and an online channel.

2.3 *Integrated SCND and pricing decisions*

Most research in the SCND focused on the cost minimization. However, paying attention to pricing decisions in the SCND (SCND-P) is another area with maximization of total profit objective function. Ahmadi Javid and Hosseinpour [23] classified SCND-P into two general categories: price-sensitive demands, and demand choice flexibility [24, 25], which prices only act on consumers' decisions to get service or not. Tavakkoli-Moghaddam *et al* [26] presented a multi-objective MINLP model for a location-pricing problem with congestion under a stochastic demand and queue system. They used a multi-objective vibration damping optimization algorithm compared with two well-known evolutionary algorithms to solve their model. Chen *et al* [27] developed a competitive SC problem with a game-theory approach. They studied how pricing strategies affect the decisions of the SC.

Patne *et al* [6] developed a closed-loop SCND with location-allocation and pricing-inventory decisions. They proposed a two-step solution method. Strategic decisions were defined by a particle swarm optimization algorithm in the first step. Operational decisions (such as stock replenishment cycle time, pricing, and refund price) were obtained using the gradient search method.

Zhang *et al* [28] developed a two-level closed-loop SC with pricing, return, refurbishment and resale process decisions. In the model, the retailer sells both new and refurbished products. They proposed a game-theory approach to optimize the profit function of the retailer and the supplier.

Fattahi *et al* [29] developed a dynamic SCND with capacity planning and multi-period pricing, in which consumers have price-sensitive demands. Their model consists of the net income maximization objective function, and they considered a budget limitation in investing. They proposed simulated annealing to solve the deterministic mathematical model. Keyvanshokooh *et al* [30] developed the dynamic pricing problem in a multi-echelon closed-loop SC. The deterministic price-sensitive demand function was considered, and quality and pricing decisions were integrated with the strategic decisions in their paper. Fattahi *et al* [31] developed an SCND and integrated strategic decisions (i.e., installation facilities, determining the capacity level, and redesign decisions) and tactical decisions (i.e., pricing, inventory, and production planning decisions). They considered stochastic potential demands of customer zone (CZ) and generated discrete scenarios.

Ahmadi-Javid and Ghandali [32] developed a capacitated SCND-P to total profit maximization. They proposed an LRA to the deterministic model solving. Ahmadi Javid and Hosseinpour [33] studied a profit-maximization L-A-I problem with price-sensitive demands. They used the markup pricing approach and (r, Q) inventory policy. To the deterministic model solving, they proposed a heuristic algorithm based LRA. Ahmadi Javid and Hosseinpour [23] proposed an extension to the above work by considering limited storage capacity, and different prices offer by each DC. Ghomi-Avili *et al* [34] presented a fuzzy bi-objective bi-level model for the closed-loop SCND with a price-dependent demand under uncertain demand and disruption. They considered pricing decisions to maximize the total profit in a competitive situation while minimizing CO_2 emissions.

2.4 *Literature gap*

A classification of relevant papers summarizes in table 1. From the above literature review, clearly various areas have been investigated by the researchers for designing SC networks. Though we have studied many papers dealing with the SCND, we have observed that a few papers have studied profit maximization SCND models and the literature on the integrated SCND with pricing is still scant, and

Table 1. Summary of relevant studies.

	Decisions						
Papers	L-A	Pricing	Refund price	Market segmentation	Capacity	Inventory	Uncertainty
[6]	✔	✔					
[7]	✔				✔		✔
[9, 33]	✔	✔				✔	
[2–4, 10],[11]	✔					✔	✔
[32]	✔				✔		
[5]	✔				✔	✔	✔
[23, 29, 30]	✔	✔			✔	✔	
[31]	✔	✔			✔	✔	✔
[26]	✔	✔				✔	✔
[12, 13, 15, 16, 22]		✔				✔	
[17–19]		✔		✔		✔	✔
[14]		✔			✔	✔	
[20]		✔	✔				
[21]		✔			✔		✔
This paper	✔	✔	✔	✔		✔	✔

it is a research direction, as mentioned by Ahmadi Javid and Hosseinpour [23] and concluded by Fattahi *et al* [31].

In [17–19], the focus was on tactical SC decisions, in which a revenue management model is developed, without strategic SC decisions integration. In the real world, companies cannot be achieved to sustainable development without considering the downstream and upstream levels of the chain. Many of the actual business variables depend on the decision variables of the other levels. Compared to Raza [18], this paper has two new features: (i) customers are allowed to return their product instead of the refund price on the OM, so online demand depends on the refund price as well and (ii) price and strategic SC decisions are made simultaneously, so the pricing and inventory decisions are integrated with the location and allocation.

In the relevant papers, there are several variants of the SCND-P, such as considering inventory decisions [9, 23, 26, 29–31, 33], capacity decisions or constrictions [23, 29–31], assuming uncertainty in demand [26, 31]. However, the problem of the SCND-P has not been explored sufficiently from the online channel perspective. The distinction between our work and other SCND-P models is that we propose a market segmentation problem and developed the SCND model with a dual channel consisting of online and offline stores. Another gap linked to the SCND-P that has not been considered in the literature so far is the demand leakage from the offline market with a higher price to the OM at a lower price. Furthermore, in a case of uncertainty in demand, we consider a noise term since its probability distribution is undefined.

3. Problem statement and mathematical framework

A three-level SC is considered as a platform of the current study. There is a manufacturer with a fixed location at the upstream level, some DCs at the local level and some CZs at the downstream level of the chain. One of the decisions is the selecting DC location from the determined areas. Another one involves allocating each CZ to a DC. By considering the concept of the Internet-based business, there are two types of distribution channels for the problem. The first channel is the B&M store that sells goods through direct customer contact at the store. The second channel is an OM that customers in each zone can visit the website of the company to buy, and the purchased items will be sent by the DC, which is assigned to that customer. An uncertain price-sensitive demand is assumed for the model. Therefore, another decision of the problem is determining the product's price of both distribution channels in each CZ. By considering the concept of WTP for goods in each distribution channel, a market segmentation takes place based on the differentiation price. Furthermore, demand leakage is usually driven by the demand for more-priced markets towards less-priced markets in the market segmentation problems.

Based on similar research conducted on the OM, these stores typically consider return policies for their customers. The demand for Internet shopping is influenced by these return policies, due to the reduced risk of non-physical visits to the product. Hence, in this paper, the consumer demand function in the OM is also sensitive to the refund price. The schematic structure of the SC is displayed in figure 1.

As described above, L-A and pricing decisions are made to maximize profit in the whole SC. This profit is derived from the difference between revenue (the number of sales multiplied by the price) and the costs of the chain. Expenditure intended for the SC includes fixed investment for installation of the DCs, freight transportation costs between SC levels and inventory costs.

The newsvendor model is considered for the inventory system of the CZ. The essential assumptions of the problem are as follows:

- Two groups of decisions, involving strategic and tactical periods are supposed due to the planning horizon.
- A set of potential locations is considered for installation of the DCs, and the strategic decisions are the selection of these locations.
- Two distribution channels are considered: B&M store, and OM.
- The inventory system at each CZ follows the newsvendor model.
- Each CZ is served exactly by one DC.
- The uncertain demand depends on the price. Furthermore, online demand depends on the refund price too.
- Each DC's inventory has a limited capacity.
- Some leakage occurs from the more-priced markets towards less-priced markets.
- There is no lateral transshipment in the chain.

In reality, the SCND model provided in the present study can be used for companies broadly selling their products through the B&M store, and online channels. In the clothing industry, for instance, authentic global brands and companies have integrated their SC in retail. Regarding the wide extended market area throughout the world, they adopt decisions on opening their brand stores in different cities. Retail clothing industry deals with assigning proper DCs for meeting customers' demands.

In addition to properly appointing DCs and assigning them to CZs, customer market extended scope at this industry may lead to different prime cost. Moreover, different demand behavior, as well as the variety of customers' WTP, has made retail managers deal with products' price valuation at any CZs around the world.

On the other hand, clothing products have experienced remarkable growth in the online sales. Global authentic

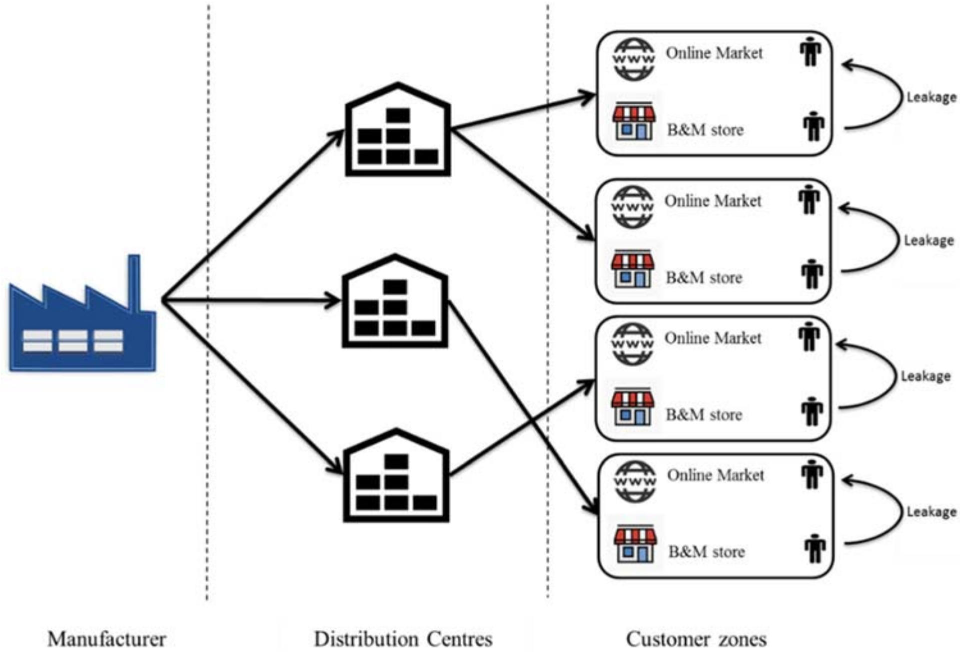

Figure 1. Schematic supply chain structure.

clothing brands present their products at their stores. Additionally, their websites enjoy considerable online purchase terms and return policies. Retail clothing industry managers must meet needs modeled in this study. For instance, a Turkish clothing brand owns over 922 stores at 45 different countries around the world; furthermore, it offers online sale, too. The company sells its products through two traditional and online stores. Online sold products may be returned by rebating if the product was unused. However, return and rebate policies vary depending on different regions and nations.

The proposed mathematical model can be used for adopting decisions on how the aforementioned company operates and to what extent it operates in new regions and nations. Market segmentation and customer zoning to assign stores, determining DCs at different cities to supply stores and online demands, and product valuation at various areas, as well as order volume and return policies regarding transportation costs are of some issues the company is dealing with to develop its business. Using the proposed model, active retail clothing firms can adopt decisions on the presence or absence of different markets to maximize profits and on establishing multiple stores to meet customers' demands.

Table 2 presents a list of notations that are used in the mathematical model.

3.1 *Demand functions*

In the retailer level, the demand of each CZi ($i \in I$) is divided into two segments by different price (v). The first one contains the customers who prefer to visit a product in the store and then buy it. The second contains online customers. The WTP of customers in the OM is lower than the B&M market and therefore, $p_1 \geq \upsilon \geq p_2$ when p_1 and p_2 are the product's prices for the B&M and OM, respectively. The demand functions are defined as linear form and for B&M and OM assumed as Eq. (1) and Eq. (2), respectively in every CZ:

$$u_{1i}(p_{1i}) = [\alpha_i - \beta_i p_{1i}]^+ \tag{1}$$

$$u_{2i}(p_{2i}, r_i) = [\beta_i \upsilon_i - \beta_i p_{2i} + \gamma_i r_i]^+ \tag{2}$$

where u_{hi} represents the PDDD in the hth market segment of CZi and $\alpha_i > 0$ is the total market share of CZi, $\beta_i > 0$ represents the sensitivity of the demand to the market price, and γ_i is the sensitivity of the demand to the refund price.

Some degree of demand leaks from the B&M with a higher price (p_1) to the OM if the price is attractive enough, so the constant demand leakage factor (θ) is considered. According to earlier studies (e.g., [18]), the PDDD with leakage in each market segment is as follows:

$$y_{1i} = (1 - \theta)u_{1i} \tag{3}$$

$$y_{2i} = \theta u_{1i} + u_{2i} \tag{4}$$

Uncertainty in demand occurs due to a variety of factors. Generally, the models of demand are in two forms: additive and multiplicative. In this study, the first one is used. By considering earlier researches [18, 19] the additive form is derived from the sum of the PDDD (y_{hi} as the riskless demand) and a random noise term (ξ_{hi}). So, the PDSD is as follows:

Table 2. Notations.

Indices	
i	Index set of CZs $(i = 1 : I)$
j	Index set of DCs $(j = 1 : J)$
h	Index set of market segments (h = 1; B&M store, h = 2; OM)
Parameters	
TC_{ij}	Transport cost between the jth DC to the ith CZ
$\overline{TC}_j$	Transport cost between the manufacturer to the jth DC
F_j	Fixed cost for installing the jth DC
H_i	Cost of holding a unit of an excess inventory in zone i
S_i	Cost of shortage a unit of an unmet inventory in zone i
p_m	Manufacturer's price
θ	Demand leakage factor
α_i	Maximum perceived cumulative deterministic (riskless) demand (market share)
β	Price sensitivity for cumulative deterministic demand
γ	Return sensitivity for cumulative deterministic demand
ξ_{hi}	Stochastic demand factor for the hth market segment in zone i with μ_{hi} as mean and σ_{hi} as a standard deviation
$f_{hi}(.)$	Probability distribution function for price-dependent stochastic demand (PDSD) in market segment h from zone i
$F_{hi}(.)$	Cumulative probability distribution function for PDSD in market segment h from zone i
$U_{hi}(p)_{hi}$	Price dependent deterministic demand (PDDD) in market segment h from zone i
y_{hi}	PDDD in market segment h from zone i with leakage
d_{hi}	PDSD market segment h from zone i with leakage
R_i	Returned quantity for market segment 2 from zone i
ϕ	Return quantity independent of refund factor
φ	Refund price sensitivity for return quantity
cap_j	Capacity of DC on site j
PH	Planning horizon.
π	Total profit to the total supply chain
Decision variables	
p_{hi}	Price for market segment h in zone i
q_{hi}	Order quantity for market segment h in zone i
υ_i	Differentiation price in zone i
r_i	Refund price for an OM in zone i $(0 \leq r_i \leq p_{2i})$
X_j	1 if a DC is installed on site j; and 0, otherwise
Y_{ij}	1 if the jth DC serves product of the ith customer; and 0, otherwise

$$d_{hi} = y_{hi} + \xi_{hi};\ \forall i \tag{5}$$

where ξ_{hi} represents the stochastic demand function for market segment i (noise term) with μ_{hi} as mean, σ_{hi} as standard deviation and the cumulative probability distribution function $F_{hi}(.)$. Furthermore, the random factor has a density function $f_{hi}(.)$..

3.2 *Return function*

With the growth and extension of information technology and the broader Internet access globally, many companies have started to create online distribution channels in their SC. The creation and development of this distribution channel, has led to growing the demand absorption. Also, it has led to reducing SC costs. On the other hand, the offered price to online customers is lower than the standard one, with the elimination of intermediaries between customer and supplier. Therefore, given the possibility of offering cheaper products (and other online sales benefits such as customer time savings), they have created a great opportunity for companies to attract the market demand, in a competitive business environment.

In online shopping, customers visit the company website and view the images and product specifications to buy it. Due to the lack of physical observation of the product, before it is received, there is a likelihood of customer dissatisfaction with the product received. So online stores use return policy. In this way, customers are allowed to return the good instead of getting all or part of its price, in case of dissatisfaction with the product.

In the return policy, determining the refund price of goods is a tactical decision. By increasing the refund price, more demand is attracted (leading to increased revenue), but the number of returned goods increases too (leading to higher costs). Therefore, a trade-off is required between the demand function and the return function. The refund price is determined in such a way as to maximize profit.

In this paper, a linear return function is used. If the return quantity independent of refund factors is displayed with ∅, then the linear return function in the OM is as follows:

$$R = \phi + \varphi r. \tag{6}$$

3.3 *Profit function*

The problem's profit function is the benefit minus the cost. The benefit is calculated by multiplying the price by the amount of sold products. The cost is divided into two parts, operating cost and fixed investment cost (cost for opening the DCs). Operating costs include the costs essential to the inventory system (holding and shortage costs), shipping, purchasing goods from the manufacturer, and the costs of return policy in OM. By considering the planning horizon, the coefficient PH is used for operating costs and benefit in comparison with fixed investment costs. So the profit function can be modeled in terms of decision variables as follows:

$$\begin{aligned} \text{Max } \pi = {} & PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2} Y_{ij}p_{hi}min\{q_{hi}, d_{hi}\} \\ & - \sum_{j=1}^{J} F_jX_j \\ & - PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2}\left(\overline{TC}_j + TC_{ij}\right)Y_{ij}q_{hi} \\ & - PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2} H_iY_{ij}\mathrm{E}[q_{hi} - d_{hi}]^{+} \\ & - PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2} S_iY_{ij}\mathrm{E}[d_{hi} - q_{hi}]^{+} \\ & - PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2} p_m q_{hi}Y_{ij} \\ & - PH\sum_{j=1}^{J}\sum_{i=1}^{I} R_iY_{ij}(r_i + H_i) \end{aligned} \tag{7}$$

By considering earlier researches [18, 35, 36], we can apply some relationships to clarify Eq. (7) as follows:

$$\min\{q_{hi}, d_{hi}\} = q_{hi} - E[q_{hi} - d_{hi}]^{+};\ \forall h = \{1, 2\} \tag{8}$$

$$\mathrm{E}[d_{hi} - q_{hi}]^{+} = y_{hi} - q_{hi} + \mathrm{E}[q_{hi} - d_{hi}]^{+};\ \forall h = \{1, 2\} \tag{9}$$

$$\begin{aligned} \mathrm{E}[q_{hi} - d_{hi}]^{+} &= \int_{\xi_{hi}}^{q_{hi}-y_{hi}} (q_{hi} - (y_{hi} + \xi_{hi}))f_{hi}(\xi_{hi})d\xi_{hi} \\ &= \int_{\xi_{hi}}^{q_{hi}-y_{hi}} F_{hi}(\xi_{hi})d\xi_{hi} \end{aligned} \tag{10}$$

Substituting Eqs. (8)–(10) into Eq. (7) and after some algebra, the objective function can be rewritten by:

$$\begin{aligned} \text{Max } \pi = {} & PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2} Y_{ij}p_{hi}\left(q_{hi} - \int_{\xi_{hi}}^{q_{hi}-y_{hi}} F_{hi}(\xi_{hi})d\xi_{hi}\right) \\ & - \sum_{j=1}^{J} F_jX_j - PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2}\left(\overline{TC}_j + TC_{ij}\right)Y_{ij}q_{hi} \\ & - PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2} H_iY_{ij}\int_{\xi_{hi}}^{q_{hi}-y_{hi}} F_{hi}(\xi_{hi})d\xi_{hi} \\ & - PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2} S_iY_{ij}\left(y_{hi} - q_{hi} + \int_{\xi_{hi}}^{q_{hi}-y_{hi}} F_{hi}(\xi_{hi})d\xi_{hi}\right) \\ & - PH\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{h=1}^{2} p_m q_{hi}Y_{ij} - PH\sum_{j=1}^{J}\sum_{i=1}^{I} R_iY_{ij}(r_i + H_i) \end{aligned} \tag{11}$$

Subject to:

$$\sum_{j=1}^{J} Y_{ij} \leq 1;\ \forall i \tag{12}$$

$$\sum_{i=1}^{I}\sum_{h=1}^{2} q_{hi}Y_{ij} \leq cap_jX_j;\ \forall j \tag{13}$$

$$p_{1i} \geq \upsilon_i \geq p_{2i} \geq r_i \geq 0;\ \forall i \tag{14}$$

$$X_j, Y_{ij} \in \{0, 1\};\ \forall i, j \tag{15}$$

Eq. (12) shows that each CZ is served at most by one DC. Eqs. (13) and Eq. (14) show the capacity constraint and the relations between prices, respectively. Finally, Eq. (15) states the integer variables.

4. Solution method

4.1 *Lagrangian relaxation*

As the MINLP mathematical model matches a class of NP-hard, a heuristic algorithm based on the LRA is developed for solving it. By relaxing and dualizing Constraints (12) and (14), which are considered as hard constraints, and considering U_i, λ_{1i}, λ_{2i}, λ_{3i} as the Lagrangian multipliers dualized with the ith constraint, the Lagrange Model (LM) can be formulated as follows:

$$L_\pi = \mathrm{Max} \sum_{j=1}^{J}\sum_{i=1}^{I} M_{ij}Y_{ij} - \sum_{j=1}^{J} F_jX_j + \sum_{i=1}^{I} U_i\left(1-\sum_{j=1}^{J} Y_{ij}\right) + \lambda_{1i}(p_{1i}-\upsilon_i) + \lambda_{2i}(\upsilon_i - p_{2i}) + \lambda_{3i}(p_{2i}-r_i) \tag{16}$$

Subject to: Constraints (13) and (15), where:

$$M_{ij} = PH\sum_{h=1}^{2} p_{hi}\min\{q_{hi}, d_{hi}\} - \left(\overline{TC}_j + TC_{ij}\right)q_{hi} - H_i\mathrm{E}[q_{hi}-d_{hi}]^+ - S_i\mathrm{E}[d_{hi}-q_{hi}]^+ - p_m q_{hi} - R_i(r_i+H_i);\ \forall i,j \tag{17}$$

The objective function value of the LM is an upper bound for the original model. To solve the LM, let $X_i^{UB,t}$, $Y_{ij}^{UB,t}$ indicate the optimal results from this model in the tth iteration. The following algorithm can be used as the LR model solving.

Algorithm 1

Step 1. Calculate $\bar{Y}_{ij}^t$ as the solution to the following integer Knapsack problem, which can be solved in polynomial time.

$$\mathrm{Max}\ \pi_1 = \sum_{i=1}^{I}\left(M_{ij}^* - U_i^t\right)\bar{Y}_{ij}^t \tag{18}$$

Subject to:

$$\sum_{i=1}^{I}\sum_{h=1}^{2} q_{hi}\bar{Y}_{ij}^t \le cap_j;\ \forall j \tag{19}$$

$$\bar{Y}_{ij}^t \in \{0,1\};\ \forall i,j \tag{20}$$

Step 2. For $j = 1$to J

$$X_j^{UB,t} = \begin{cases} 1 & \text{if } \sum_{i=1}^{I}\left(M_{ij}^* - U_i^t\right)\bar{Y}_{ij}^t - F_j \ge 0 \\ 0 & \text{if } \sum_{i=1}^{I}\left(M_{ij}^* - U_i^t\right)\bar{Y}_{ij}^t - F_j < 0 \end{cases} \tag{21}$$

Step 3. $Y_{ij}^{UB,t} = \bar{Y}_{ij}^t X_j^{UB,t}$

where M_{ij}^* and the optimal decisions for pricing, market segmentation, return policy and order quantities are the solutions to the following sub-problems:

$$\mathrm{Max}\ \pi_2 = \sum_{i=1}^{I}\sum_{h=1}^{2}(p_{hi}+s_i-p_m)q_{hi} - s_i y_{hi} - (p_{hi}+s_i+H_i)\int_{\xi_{hi}}^{q_{hi}-y_{hi}} F_{hi}(\xi_{hi})d\xi_{hi} - \sum_{i=1}^{I} R_i(r_i+H_i) \tag{22}$$

Subject to: Constraint (14)

The KKT optimality condition is used to calculate the optimal solution of π_2 in each CZ. By considering $\rho_{hi} = (p_{hi}+s_i-p_m)/(p_{hi}+s_i+h_i), \forall i,h$ and $y_1 = (1-\theta)(\alpha_i - \beta p_{1i})$, $y_2 = \theta(\alpha_i-\beta p_{1i}) + (\beta v_i - \beta p_{2i} + \gamma r_i)$ optimization of the $\pi_2$in each of the CZ is suggested by Algorithm 2.

Algorithm 2

Step 1. The optimal order quantities are as follows:

$$q_{hi}^* = y_{hi}^* + F^{-1}\rho_{hi}^*;\ \forall i \tag{23}$$

Step 2. The optimal differentiation price is equal to the optimal price for market segment 1 in each CZ $\left(\upsilon_i^* = p_{1i}^*; \forall i\right)$.

Step 3. The optimal prices for each market segment and the optimal refund price for the OM are determined as follows:

Step 3.1. Calculate the first response by solving the system of Eq. (24).

$$\begin{cases} y_{1i} + F_{1i}^{-1}(\rho_{1i}) - \beta(1-\theta)(p_{1i}-p_m) + \beta(1-\theta)(p_{2i}-p_m) \\ -\int_{\xi_{1i}}^{F_{1i}^{-1}(\rho_{1i})} F_{1i}(\xi_{1i})d\xi_{1i} = 0 \\ y_{2i} + F_{2i}^{-1}(\rho_{2i}) - \beta(p_{2i}-p_m) - \int_{\xi_{2i}}^{F_{2i}^{-1}(\rho_{2i})} F_{2i}(\xi_{2i})d\xi_{2i} = 0 \\ r_i - \dfrac{\gamma(p_{2i}-p_m)-\phi}{\varphi} = 0 \end{cases} \tag{24}$$

Step 3.2. Calculate the second response by solving the system of Eq. (25).

$$\begin{cases} y_{1i} + F_{1i}^{-1}(\rho_{1i}) - \beta(1-\theta)(p_{1i}-p_m) + \beta(1-\theta)(p_{2i}-p_m) \\ -\int_{\xi_{1i}}^{F_{1i}^{-1}(\rho_{1i})} F_{1i}(\xi_{1i})d\xi_{1i} = 0 \\ y_{2i} + F_{2i}^{-1}(\rho_{2i}) - (\gamma-\beta)(p_{2i}-p_m) - (\phi+\varphi r_i) \\ -\int_{\xi_{2i}}^{F_{2i}^{-1}(\rho_{2i})} F_{2i}(\xi_{2i})d\xi_{2i} = 0 \\ r_i - p_{2i} = 0 \end{cases} \tag{25}$$

Step 3.3. In the solutions obtained from the system of Eq. (25), if $p_{2i} < \frac{\gamma p_m + \phi}{\gamma - \varphi}$, then the solutions obtained from the system of Eq. (24) are the optimal values; otherwise, the solutions obtained from the system of Eq. (24) and the system of Eq. (25), each having a larger objective function value, must be selected.

Proof See "Appendix A". □

4.2 *Heuristic algorithm for lower bounds*

Given that the solution obtained from the LM (from Algorithm 1 presented in section 4.1), may be infeasible, the upper bound obtained must be converted into a feasible solution. Accordingly, a lower bound of the problem is obtained through the following heuristic algorithm:

Algorithm 3

Step 1. Initialize the sets: $I_1^{LB} = \left\{ i | \sum_{j=1}^{J} Y_{ij}^{UB} \geq 1 \right\}$, $I_2^{LB} = \left\{ i | \sum_{j=1}^{J} Y_{ij}^{UB} = 0 \right\}$, and define $K_{ij}(i) = M_{ij}^* - U_i^t$, $RC(j) = cap_j$.

Step 2. Set $Y_{ij}^{LB} = 0$ and $X_j^{LB} = X_j^{UB}$ for $i \in I, j \in J$.

Step 3. While $I_1^{LB} = \emptyset$, for $i \in I_1^{LB}$ remove i from I_1^{LB} and find $\hat{j}$ that:

$$K_{i\hat{j}}(i) = \text{Max}\left\{ K_{ij}(i) | Y_{ij}^{UB} = 1, X_j^{LB} = 1, RC(j) \geq (q_{1i} + q_{2i}) \right\}$$

If there is such a $\hat{j}$, then $Y_{i\hat{j}}^{LB} = 1$ and modify $RC(\hat{j}) = RC(\hat{j}) - (q_{1i} + q_{2i})$.

Step 4. While $I_2^{LB} = \emptyset$, for $i \in I_2^{LB}$ remove i from I_2^{LB} and find $\hat{j}$ that:

$$K_{i\hat{j}}(i) = Max\left\{ K_{ij}(i) | X_j^{LB} = 1, RC(j) \geq (q_{1i} + q_{2i}) \right\}$$

If there is such a $\hat{j}$, then $Y_{i\hat{j}}^{LB} = 1$ and modify $RC(\hat{j}) = RC(\hat{j}) - (q_{1i} + q_{2i})$.

Step 5. For each j, if $X_j^{LB} = 1$ and $\sum_{i=1}^{I} Y_{ij} = 0$, set $X_j^{LB} = 0$.

4.3 *Update the Lagrangian multipliers*

A sub-gradient method is used to improve the obtained results. According to this, Lagrangian multipliers have to be updated in each iteration using by:

$$U_i^t = U_i^{t-1} - k^t \left(1 - \sum_{j=1}^{J} Y_{ij}^{UB} \right) \tag{26}$$

By considering Ω as a control parameter, k^t is the step size and calculates as follows:

$$k^t = \Omega \frac{UB^* - LB^*}{\sum_{j=1}^{J} \sum_{i=1}^{I} \left(1 - Y_{ij}^{UB} \right)^2} \tag{27}$$

To evaluate the efficiency of the results in each iteration, a quasi-optimality gap (QOG) is used and calculated by:

$$\text{QOG} = 100 \times \frac{UB^* - LB^*}{UB^*} \tag{28}$$

Furthermore, the condition QOG $< \varepsilon$ is the stopping condition, where ε is a user-driven tolerance percentage. The whole of the solution method (LRA) is shown in figure 2 and Algorithm 4.

Algorithm 4

Step 1. Set $t = 0$ as the iteration number.

Step 2. Set $LB^* = -\infty$, $UB^* = +\infty$, $U_i^t = 0; \forall i$.

Step 3. Solve the LM and obtain Y_{ij}^{UB} and the upper bound (UB) of objective function value as described in section 4.1.

Step 4. Create a feasible solution as the lower bound (LB) of the problem as described in section 4.2.

Step 5. If $UB < UB^*$ then $UB^* = UB$ and if $LB > LB^*$ then $LB^* = LB$.

Step 6. Finish the algorithm, if $QOG < \varepsilon$; otherwise, continue.

Step 7. Set $t = t + 1$.

Step 8. If there is no improvement after m iteration, then $\Omega = \frac{\Omega}{2}$.

Step 9. Update the Lagrangian multipliers as follows:

$$U_i^t = U_i^{t-1} - k^t \left(1 - \sum_{j=1}^{J} Y_{ij}^{UB} \right)$$

where $k^t = \Omega \frac{UB^* - LB^*}{\sum_{j=1}^{J} \sum_{i=1}^{I} \left(1 - Y_{ij}^{UB} \right)^2}$

Step 10. Go to Step 3.

5. Computational results

5.1 *Test problem generation*

To explain the application of the proposed LRA, a computational study is presented throughout this section. 10 test problems with different sizes are developed by benchmarking of similar research in the literature review. Table 3 shows the framework of the test problems. We considered $p_m = 6$, $\beta = 800$, $\theta = 0.5$, $\gamma = 20$, $\phi = 10$, $\varphi = 8$, and PH = 1000 as the fixed value of parameters for all test problems. Other parameters were drawn independently from the uniform distribution reported in table 4. The normally distributed price-dependent demand is considered, and $\xi_i = \xi \in \left[-\sqrt{3}\sigma_i, \sqrt{3}\sigma_i \right]$ is the stochastic demand factor with μ_i=0 as mean and σ_i=10 as standard deviation.

5.2 *Algorithm performance*

The solution algorithms were coded with MATLAB 2016 programming language and all test problems run on a PC with core i5 (480 M) 2.66 GHz processor and 4.00 GB of RAM. As described in section 4 to evaluate the efficiency of the results, a quasi-optimality gap (QOG) is used and the condition QOG $< \varepsilon$ is considered as the stopping criterion. By considering earlier researches [23, 33], we can apply QOG and elapsed time (as a reasonable time for large-sized problems) to determine the performance of the algorithm. The elapsed times of each test problem are stated for three error tolerances ε = 5%, 2% and 1% in table 5. As shown in the table, the proposed LRA is noticeably successful at performing high-quality results within acceptable times, for large-sized test problems, which marks the efficiency and

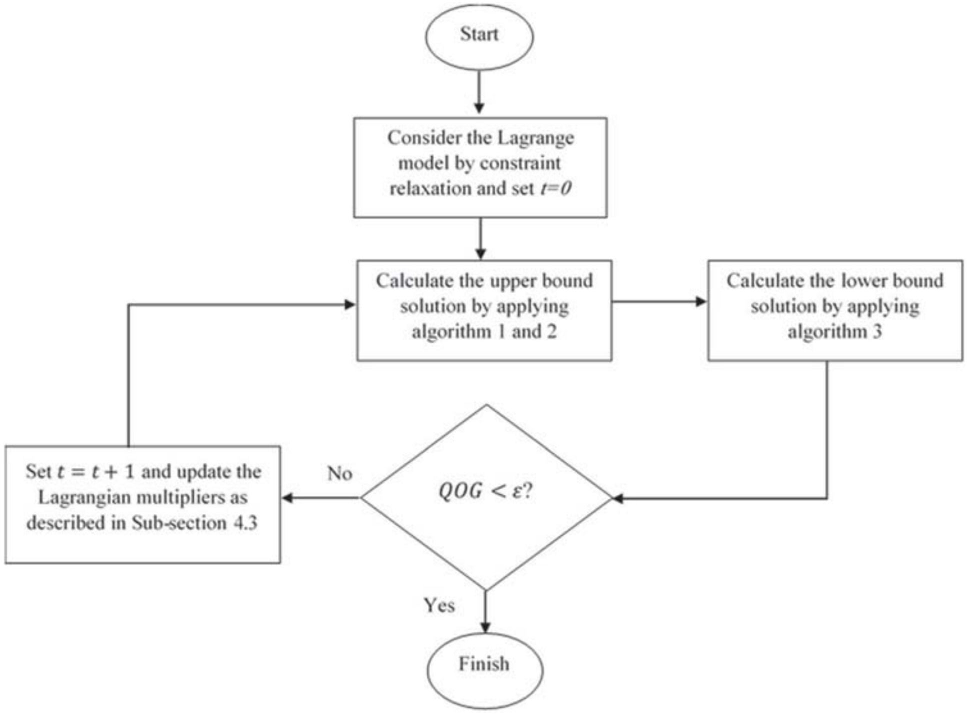

Figure 2. Framework of the proposed LRA.

Table 3. Framework of test problems.

Problem set	No. of DC $\|J\|$	No. of CZ $\|I\|$	Instance size $\|I\| \times \|J\| \times \|h\|$
1	5	10	100
2	7	15	210
3	10	20	400
4	10	25	500
5	12	25	600
6	15	30	900
7	18	35	1260
8	20	40	1600
9	20	50	2000
10	25	50	2500

Table 4. Distribution function of parameters.

Parameter	Distribution function
TC_{ij}	Uniform [1, 5]
$\overline{TC}_j$	Uniform [1, 5]
F_j	Uniform [8E+06, 12E+06]
H_i	Uniform [1, 5]
S_i	Uniform [1, 5]
α_i	Uniform [1E+04, 2E+04]
cap_j	Uniform [8E+03, 12E+03]

Table 5. Computational results.

Problem set	No. of constraints	No. of integer variables	Elapsed time (s)		
			$\varepsilon = 5\%$	$\varepsilon = 2\%$	$\varepsilon = 1\%$
1	70	55	98	115	137
2	104	112	115	115	115
3	140	210	249	260	260
4	170	260	514	514	514
5	174	312	385	478	478
6	210	465	749	796	796
7	246	648	967	967	1087
8	280	820	2475	3106	3106
9	340	1020	755	825	851
10	350	1275	940	954	1040

effectiveness of the presented LRA. Upper bound and lower bound values of the objective function for different iteration numbers for some of the problem sets (1, 5, 6, and 10) are shown in figure 3.

5.3 *Sensitivity analysis*

A sensitivity analysis is performed to understand the impact of changes in parameters on the results. For this purpose, one of the problem sets is selected (#5). Table 6 shows the results of the algorithm for the test problem and the

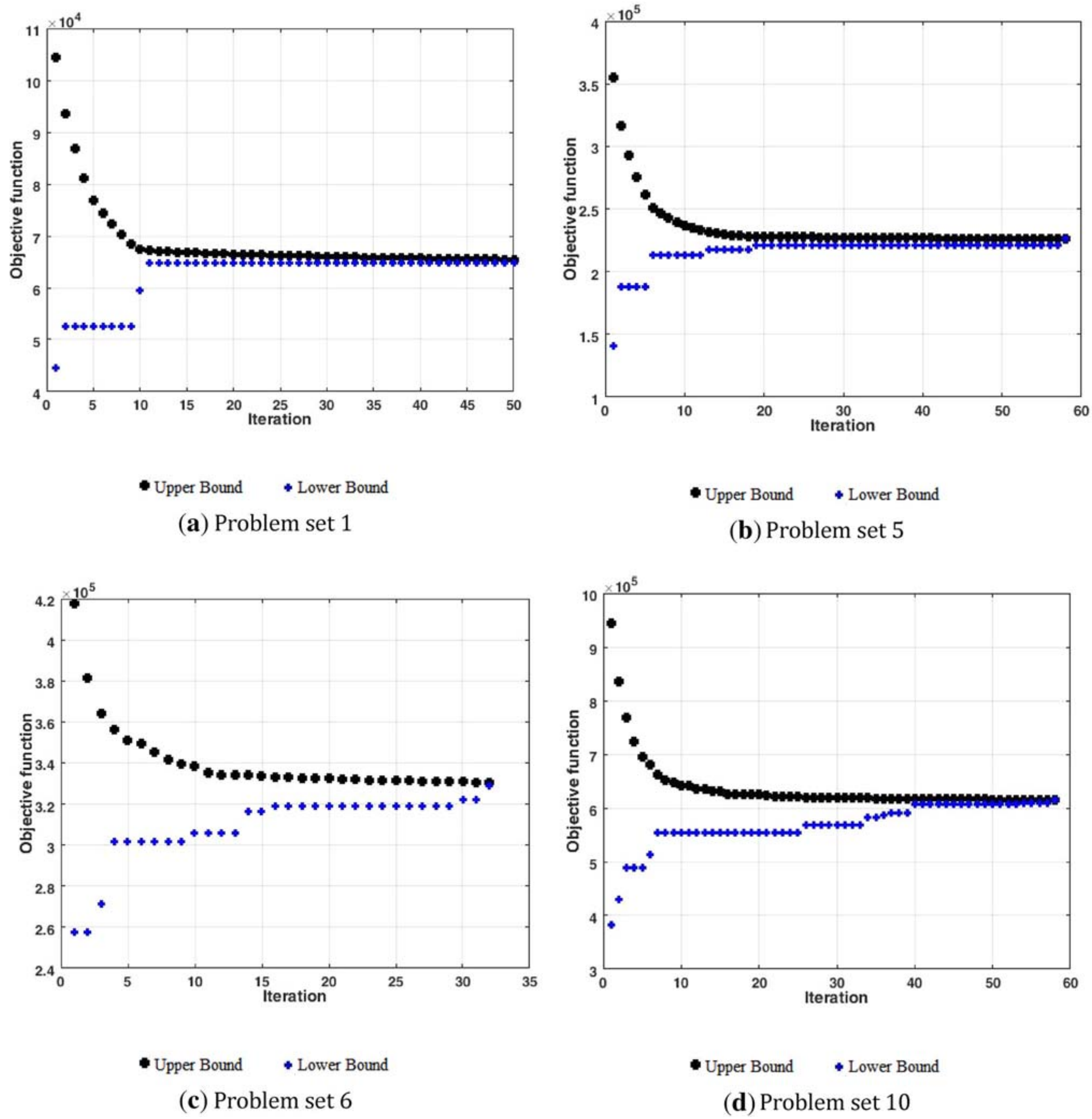

Figure 3. Lower and upper bounds versus the iteration number.

obtained results of changing in $\beta, \gamma, \phi, \theta$ and σ are presented in table 7 and figure 4–6. According to table 7, changing $\beta, \gamma, \theta,$ and large varying σ may affect the strategic decisions. Changes in location and allocation variables are presented in table 7.

By 20% varying in β, for instance, location and allocation variables would change as shown by 'Yes' in table 7. Furthermore, 10% changing of β shows no changes in location and allocation variables (as shown by 'No' in the table).

As shown in figure 4, the order quantities and prices decrease by increasing β but good profit will be obtained by decreasing this parameter. As shown in figure 5 by reducing γ, pricing decisions decrease, while the order quantity increases in both markets when this parameter changed. From figure 6, whenϕ increases, both pricing decisions and profit function decrease, while the order quantity increases

Table 6. Results for problem set 5.

		Tactical variables					
Location variables	Allocation variables	p_{1i}	p_{2i}	r_i	q_{1i}	q_{2i}	Total profit
X_1 = 1	$Y_{23,1}$ = 1	17.73	13.22	13.22	1795	5669	2.26E+08
X_2 = 1	$Y_{19,2}$ = 1	17.59	13.13	13.13	1774	5602	
X_3 = 1	$Y_{13,3}$ = 1	13.68	10.71	10.53	1191	3774	
	$Y_{25,3}$ = 1	15.51	11.85	11.85	1465	4635	
X_9 = 1	$Y_{6,9}$ = 1	18.82	13.90	13.90	1958	6178	
X_{10} = 1	$Y_{12,10}$ = 1	19.72	14.45	14.45	2091	6595	
X_{11} = 1	$Y_{24,11}$ = 1	18.90	13.94	13.94	1968	6212	
X_{12} = 1	$Y_{16,12}$ = 1	13.37	10.52	10.06	1144	3624	
	$Y_{20,12}$ = 1	16.40	12.40	12.40	1597	5048	

Table 7. Sensitivity analysis for problem set 5.

		Changes in (%)							
Parameters	Changes	Location variables	Allocation variables	$\overline{p_{1i}^*}$	$\overline{p_{2i}^*}$	$\overline{r_i^*}$	$\sum q_{1i}^*$	$\sum q_{2i}^*$	π^*
β	20%	Yes	Yes	−16.76	−13.92	−19.76	−10.01	−10.75	−45.80
	10%	No	No	−8.31	−6.93	−8.76	−3.48	−3.89	0.32
	−10%	Yes	Yes	−0.86	−0.55	−1.15	66.93	67.92	40.87
	−20%	Yes	Yes	12.49	10.75	11.38	82.73	85.29	123.24
γ	50%	Yes	Yes	−1.33	0.81	1.38	10.12	15.38	6.22
	20%	Yes	Yes	−2.25	−1.19	−0.63	14.67	16.73	2.34
	−20%	Yes	Yes	−3.12	−3.07	−11.65	18.99	16.94	−1.09
	−50%	Yes	Yes	−3.38	−3.74	−48.39	20.26	15.49	0.02
ϕ	50%	No	No	−0.06	−0.11	−1.24	0.23	0.08	−0.34
	20%	No	No	−0.02	−0.05	−0.50	0.09	0.03	−0.14
	−20%	No	No	0.02	0.05	0.43	−0.09	−0.04	0.14
	−50%	No	No	0.06	0.12	0.69	−0.25	−0.12	0.38
σ	=5	No	No	0.02	0.02	0.02	−0.09	−0.01	0.32
	=50	No	No	−0.17	−0.14	−0.19	0.69	0.04	−0.34
	=100	No	No	−0.38	−0.32	−0.42	1.56	0.10	−5.82
	=150	No	No	−0.60	−0.49	−0.66	2.43	0.15	−9.05
	=500	Yes	Yes	1.74	1.43	1.12	−11.12	−17.35	−29.17
θ	80%	Yes	Yes	2.27	6.06	10.32	−81.94	12.90	−2.14
	40%	No	No	1.24	3.20	5.73	−43.31	6.85	−4.11
	−40%	Yes	Yes	−1.41	−3.60	−6.65	48.86	−7.75	4.00
	−80%	Yes	Yes	−3.00	−7.67	−14.71	104.37	−16.61	10.58

in both markets, and vice versa. Considering the σ changes from table 7, by increasing this parameter; we should increase the order quantities and decrease pricing decisions, and the worse profit will be obtained.

Figure 7 shows the fixed cost of opening DCs and the total profit of the fifth test problem for various values of β, γ, and σ. If the demand behavior changes and price sensitivity for cumulative deterministic demand decreases to 20% (β = 640), it would probably increase prices and meet larger demands (table 7). In this case, as seen in figure 7, the firm must adopt an aggressive strategy to meet higher demands and to enhance market share. Therefore, it is required that more DCs are provided. The normal problem requires 7 DCs; whereas, in this context, to meet larger existing demands, the enterprise can double supply chain total profit through increasing DCs up to 12 (at potential nodes). Adopting this aggressive strategy may necessitate higher fixed capital and cost for constructing DCs. Figure 7 clearly shows the increased fixed cost for DCs construction. On the contrary, if price sensitivity for cumulative deterministic demand increases up to 20% (β = 960); then, given the defensive strategy, the enterprise should reduce prices, order size, as well as the number of DCs.

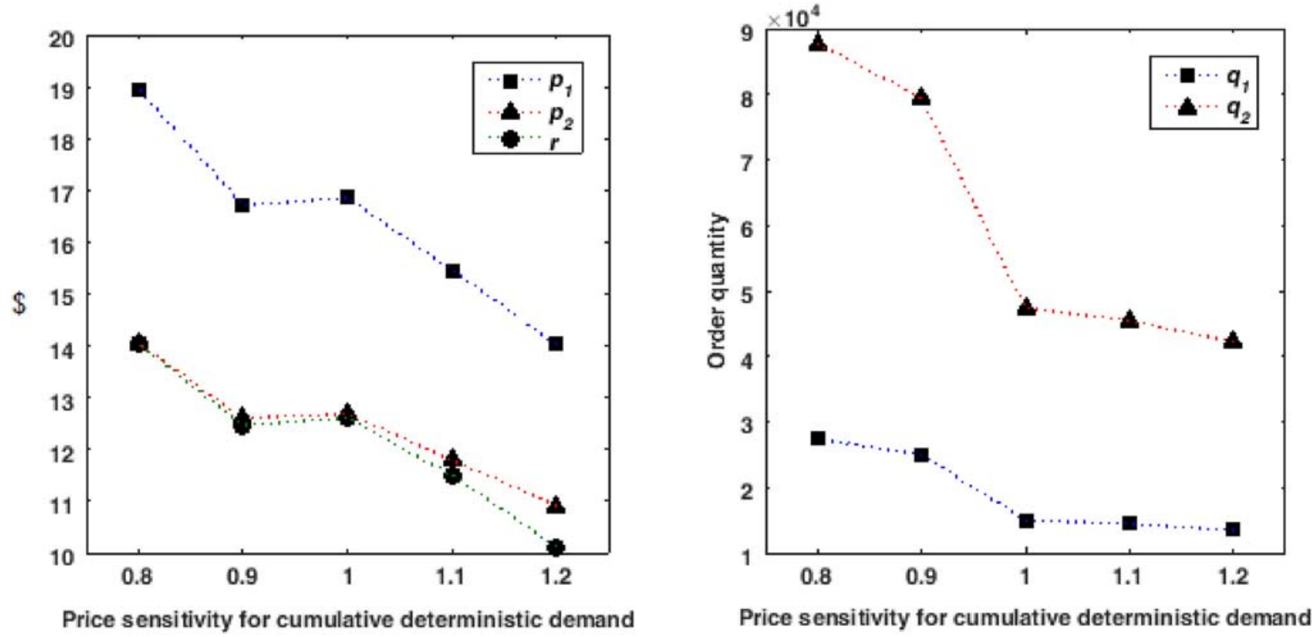

Figure 4. Optimal decisions under different β.

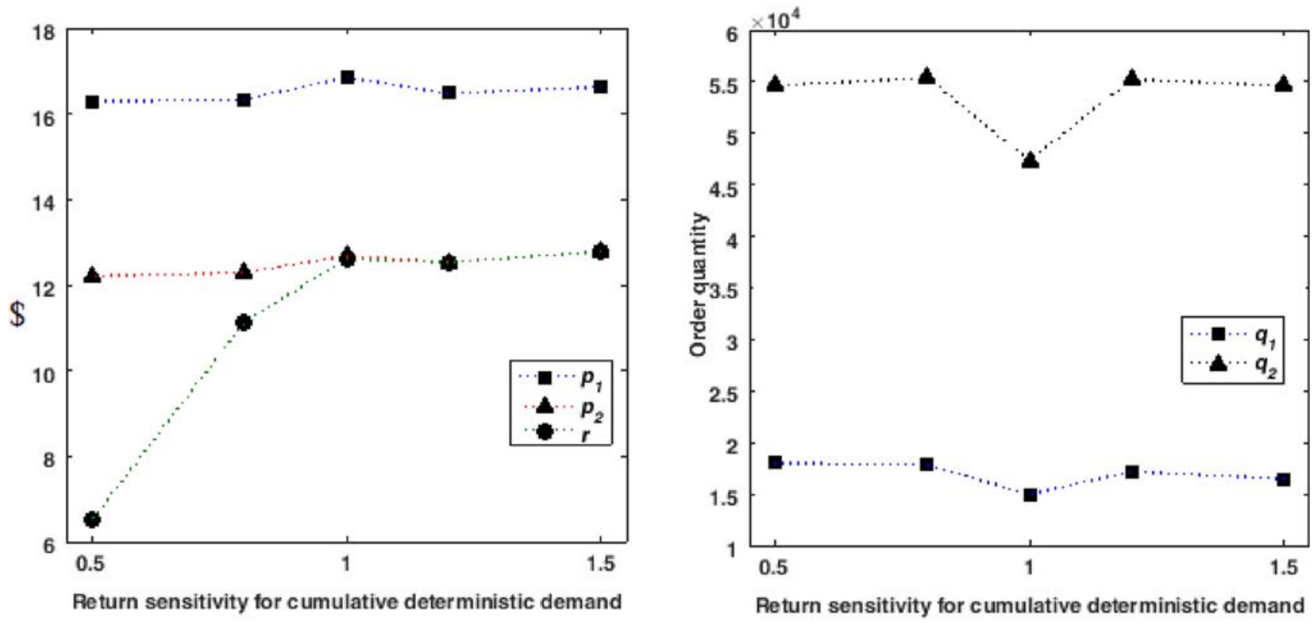

Figure 5. Optimal decisions under different γ.

In the case of study return sensitivity for cumulative deterministic demand, if the parameter decreases 50% (γ= 10), the firm can increase market share and meet higher demands by lower prices. Although adopting this aggressive strategy may cause higher fixed investment costs for DCs, it results in increased profit for the whole SC. In the case of study the effects of demand uncertainty on the SC, increasing σ_{hi} as the standard deviation of the demand function's random noise term, the firm may require defensive strategy, smaller-sized order quantities, and consequently making less DCs. Thus, this may lead the firm to lower fixed investment costs, limited corporate activities, and ultimately, decreased supply chain profit.

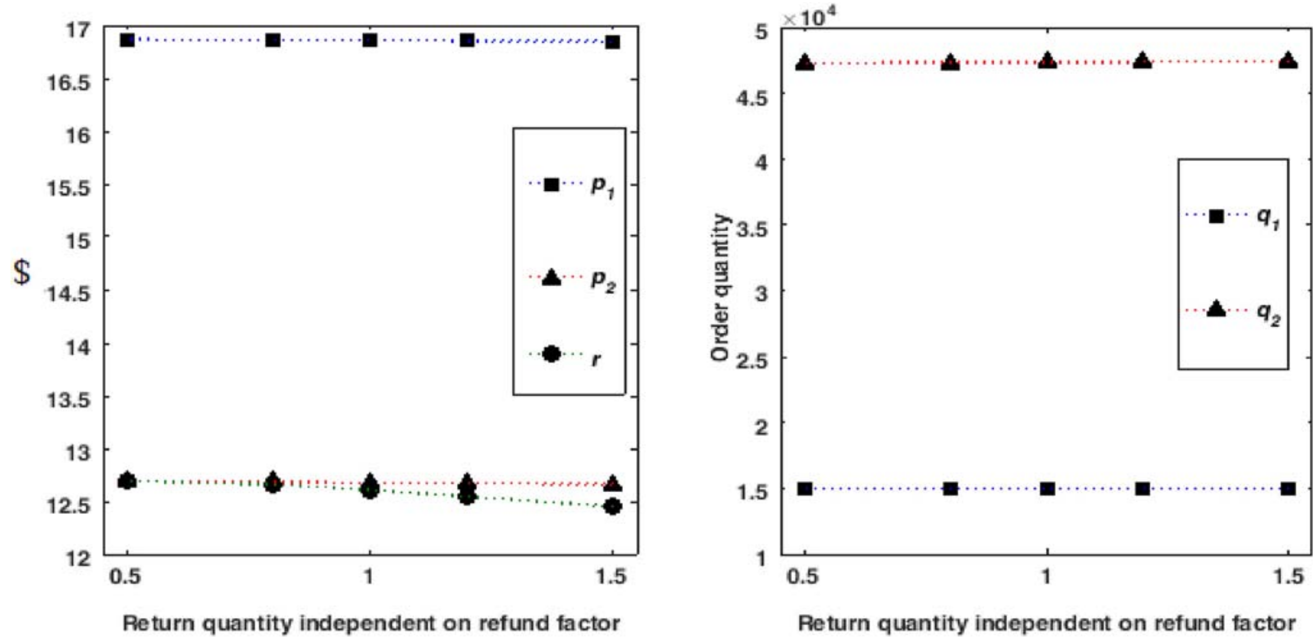

Figure 6. Optimal decisions under different ϕ.

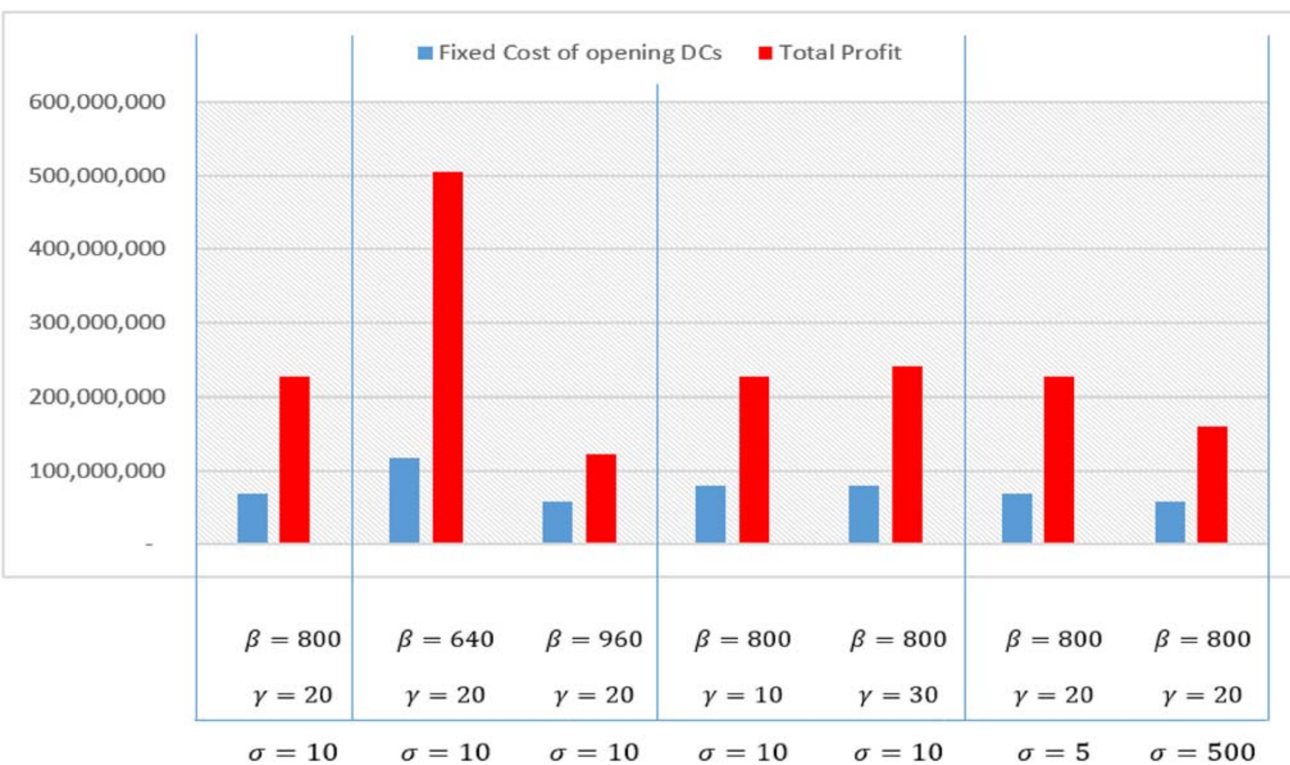

Figure 7. Fixed cost and total profit of Problem set 5 in different parameters.

5.4 *Managerial insights*

The best locations for installation of the DCs can be determined using this model. To achieve the expected rate of return and avoid the loss of investment, an investor can apply this model for the economic analysis of different locations to adopt decisions on the presence or absence of various markets (i.e., defining the market target). The model can be used for the SC with offline and online stores. Furthermore, different demand behavior has made retail managers deal with products' price valuation at any CZs around the world. So, the results of the paper will be beneficial for taking the other decisions consisting of revenue management and inventory management.

Managerial insights are supposed from the sensitivity analysis, which introduced in the previous sub-section. The first insight relates to the impact of the demand uncertainty on the SC formation. Table 7 shows how the number of installed DCs and the order quantities are affected when the parameters are varied. We can see that the number of DCs is more significantly impacted by the return sensitivity for cumulative deterministic demand (γ) and price sensitivity for cumulative deterministic demand (β). So, as shown in figure 7, if the firm reduces price sensitivity for cumulative deterministic demand (e.g., by considering the impact of quality on the demand), the total profit increases sharply.

The second insight can be inferred by observing the varying in return quantity independent of the refund factor (ϕ). Table 7 shows when this parameter changes, we can improve the total profit of the chain by changing in the tactical decisions, and we do not need to change the strategic decisions like that location and allocation. The third insight relates to the customer's behavior. As shown in table 7, when the population of the B&M market is high (i.e., decreasing in demand leakage factor), the refund price reduces when the market size decreases as compared when the population of the OM is high.

Another managerial insight refers to the previous subsection. As described above, by changing parameters, the strategy of the firm needs to define in the case of aggressive or defensive strategies.

6. Conclusions

The main goal of the current study was to develop a new SCND problem. This research proposed a profit maximization SCND-P problem with the stochastic price-sensitive customer demand. A market segmentation model is used to enable DCs to optimally segment their markets and present their products with different distribution channels. So, demand is satisfied in B&M stores and OM. Furthermore, leakage demand from the B&M with a higher price to the OM with a lower price was considered. In the OM, due to the lack of physical observation of the product before it is received, a return policy is considered so that customers are allowed to return the good instead of getting all or part of its price, in case of dissatisfaction with the product. The problem is formulated as an MINLP and a heuristic method based on the LRA and KKT optimality conditions developed to solve it. In this algorithm, the upper bound is obtained by minimizing the dual Lagrangian through a sub-gradient method. Additionally, a heuristic algorithm was used to create a feasible solution as the lower bound of the problem.

The computational result shows the efficiency and effectiveness of the presented solution method. It is found that strategic decisions depend on tactical decisions, and both types of decisions should be considered simultaneous in SCND problems. For the future study, the model can be extended by considering a multi-product system and quality dependent demand and return functions. Furthermore, it is possible to consider the planning of reverse flows in the SCND.

Appendix A. Proof of Algorithm 2

The Lagrange function for the sub-problem in each of the CZ and the KKT optimality conditions are as follows:

$$L_{\pi_2} = \left(\sum_{h=1}^{2} (p_{hi} + s_i - p_m) q_{hi} - s_i y_{hi} - (p_{hi} + s_i + H_i) \int_{\xi_{hi}}^{q_{hi} - y_{hi}} F_{hi}(\xi_{hi}) d\xi_{hi} \right) - (r_i + H_i)(\emptyset + \varphi r) + \lambda_{1i}(p_{1i} - \upsilon_i) + \lambda_{2i}(\upsilon_i - p_{2i}) + \lambda_{3i}(p_{2i} - r_i) \tag{A-1}$$

$$\frac{\partial L_{\pi_2}}{\partial q_{hi}} = (p_{hi} + s_i - p_m) - (p_{hi} + s_i + H_i) F_{hi}(q_{hi} - y_{hi}) = 0 \tag{A-2}$$

$$\frac{\partial L_{\pi_2}}{\partial p_{1i}} = q_{1i} - s_i \frac{\partial y_{1i}}{\partial p_{1i}} - s_i \frac{\partial y_{2i}}{\partial p_{1i}} + (p_{1i} + s_i + H_i) F_{1i}(q_{1i} - y_{1i}) \frac{\partial y_{1i}}{\partial p_{1i}} - \int_{\xi_{1i}}^{q_{1i} - y_{1i}} F_{1i}(\xi_{1i}) d\xi_{1i} + (p_{2i} + s_i + H_i) F_{2i}(q_{2i} - y_{2i}) \frac{\partial y_{2i}}{\partial p_{1i}} + \lambda_{1i} = 0 \tag{A-3}$$

$$\frac{\partial L_{\pi_2}}{\partial p_{2i}} = q_{2i} - s_i \frac{\partial y_{2i}}{\partial p_{2i}} + (p_{2i} + s_i + H_i) F_{2i}(q_{2i} - y_{2i}) \frac{\partial y_{2i}}{\partial p_{2i}} - \int_{\xi_{2i}}^{q_{2i} - y_{2i}} F_{2i}(\xi_{2i}) d\xi_{2i} - \lambda_{2i} + \lambda_{3i} = 0 \tag{A-4}$$

$$\frac{\partial L_{\pi_2}}{\partial \upsilon_i} = ((p_{2i} + s_i + H_i)F_{2i}(q_{2i} - y_{2i}) - s_i)\frac{\partial y_{2i}}{\partial \upsilon_i} - \lambda_1 + \lambda_2 = 0 \quad (A-5)$$

$$\frac{\partial L_{\pi_2}}{\partial r_i} = ((p_{2i} + s_i + H_i)F_{2i}(q_{2i} - y_{2i}) - s_i)\frac{\partial y_{2i}}{\partial r_i} - (\phi + \varphi r_i) - \lambda_3 = 0 \quad (A-6)$$

$$\lambda_{1i}(p_{1i} - \upsilon_i) = 0 \quad (A-7)$$

$$\lambda_{2i}(\upsilon_i - p_{2i}) = 0 \quad (A-8)$$

$$\lambda_{3i}(p_{2i} - r_i) = 0 \quad (A-9)$$

where $\frac{\partial y_{1i}}{\partial p_{1i}} = -\beta(1-\theta)$, $\frac{\partial y_{2i}}{\partial p_{2i}} = -\beta$, $\frac{\partial y_{2i}}{\partial p_{1i}} = -\beta\theta$, $\frac{\partial y_{2i}}{\partial \upsilon_i} = \beta$, $\frac{\partial y_{2i}}{\partial r_i} = \gamma$, $F_{hi}(q_{hi} - y_{hi}) = (p_{hi} + s_i - p_m)/(p_{hi} + s_i + H_i) = \rho_{hi}$ and $\lambda_{hi} \geq 0$.

Taking the partial derivative of the Lagrange function for q_{hi}, setting it equal to zero (Eq. A-2) and after some algebra, this results in:

$$q_{hi} = y_{hi} + F_{hi}^{-1}(\rho_{hi}), \forall h = \{1, 2\} \quad (A-10)$$

where π_2 is jointly concave in q_{hi}. To prove that, the Hessian matrix (H) equation can be used.

$$H = \begin{bmatrix} \frac{\partial^2 \pi_2}{\partial q_{1i}^2} & \frac{\partial^2 \pi_2}{\partial q_{1i}\partial q_{2i}} \\ \frac{\partial^2 \pi_2}{\partial q_{2i}\partial q_{1i}} & \frac{\partial^2 \pi_2}{\partial q_{2i}^2} \end{bmatrix} = \begin{bmatrix} -(p_{1i} + s_i + H_i)f_{1i}(q_{1i} - y_{1i}) & 0 \\ 0 & -(p_{2i} + s_i + H_i)f_{2i}(q_{2i} - y_{2i}) \end{bmatrix} \quad (A-11)$$

Therefore, $|H| \geq 0$. This proves the joint concavity of π_2.

To consider KKT conditions, λ_{hi} is analyzed with eight cases as follows:

Case (1): $\lambda_{1i} = \lambda_{2i} = \lambda_{3i} = 0$; this case will yield unrestricted pricing, and therefore this solution is infeasible.

Case (2): $\lambda_{1i}, \lambda_{2i}, \lambda_{3i} > 0$; this case would yield to obtain $p_{1i} = p_{2i} = r_i = \upsilon_i$ as a feasible solution but not preferred.

Case (3): $\lambda_{1i}, \lambda_{2i} > 0, \lambda_{3i} = 0$; this case would yield to obtain $p_{1i} = p_{2i} = \upsilon_i$ as a feasible solution but not preferred.

Case (4): $\lambda_{1i}, \lambda_{3i} > 0, \lambda_{2i} = 0$; this case would yield to $p_{1i} = \upsilon_i$, $p_{2i} = r_i$, $\lambda_{1i} = \beta(p_{2i} - p_m)$ (using Eq. A-5), and $\lambda_{3i} = \gamma(p_{2i} - p_m) - (\phi + \varphi r_i)$ (using Eq. A-6), this case has a feasible solution if $p_{2i} > \frac{\gamma p_m + \phi}{\gamma - \varphi}$ and by considering other KKT conditions as described in Eqs. A-2 to A-9, we calculate Eq. (25).

Case (5): $\lambda_{1i} = \lambda_{2i} = 0, \lambda_{3i} > 0$; this case would yield unrestricted pricing, and therefore this solution is infeasible.

Case (6): $\lambda_{1i} = \lambda_{3i} = 0, \lambda_{2i} > 0$; by using Eq. A-5 and after some algebra we have $\lambda_{2i} = -\beta(p_{2i} - p_m)$ and by considering $\lambda_{hi} > 0$ according to KKT conditions, this solution is infeasible.

Case (7): $\lambda_{1i} = 0, \lambda_{2i}, \lambda_{3i} > 0$; by using Eq. A-5 and after some algebra we have $\lambda_{2i} = -\beta(p_{2i} - p_m)$ and by considering $\lambda_{hi} > 0$ according to KKT conditions, this solution is infeasible.

Case (8): $\lambda_{1i} > 0, \lambda_{2i} = \lambda_{3i} = 0$; yield to $p_{1i} = \upsilon_i$, $\lambda_{1i} = \beta(p_{2i} - p_m)$ (using Eq. A-5). This case has a feasible solution and by considering other KKT conditions as described in Eqs. A-2 to A-9, we calculate Eq. (24).

References

[1] Simchi-Levi D, Kaminsky P and Simchi-Levi E 2004 *Managing the supply chain: definitive guide*. Tata McGraw-Hill Education, New York, USA, 308 pages

[2] Miranda P A and Garrido R A 2009 Inventory service-level optimization within distribution network design problem. *International Journal of Production Economics* 122: 276–285

[3] Yao Z, Lee L H, Jaruphongsa W, Tan V and Hui C F 2010 Multi-source facility location–allocation and inventory problem. *European Journal of Operational Research* 207: 750–762

[4] Manatkar R P, Karthik K, Kumar S K and Tiwari M K 2016 An integrated inventory optimization model for facility location-allocation problem. *International Journal of Production Research* 54: 3640–3658

[5] Salehi H, Tavakkoli-Moghaddam R and Nasiri G R 2015 A multi-objective location-allocation problem with lateral transshipment between distribution centres. *International Journal of Logistics Systems and Management* 22: 464–482

[6] Patne K, Shukla N, Kiridena S and Tiwari M K 2018 Solving closed-loop supply chain problems using game theoretic particle swarm optimisation. *International Journal of Production Research* 56: 5836–5853

[7] Amin S H and Baki F 2017 A facility location model for global closed-loop supply chain network design. *Applied Mathematical Modelling* 41: 316–330

[8] Chen Y T, Chan F T S and Chun S H 2015 An integrated closed-loop supply chain model with location allocation problem and product recycling decisions. *International Journal of Production Research* 53: 3120–3140

[9] Kaya O and Urek B 2016 A mixed integer nonlinear programming model and heuristic solutions for location, inventory and pricing decisions in a closed loop supply chain. *Computers & Operations Research* 65: 93–103

[10] Ramezani M, Bashiri M and Tavakkoli-Moghaddam R 2013 A new multi-objective stochastic model for a forward/reverse logistic network design with responsiveness and quality level. *Applied Mathematical Modelling* 37: 328–344

[11] Niranjan T, Parthiban P, Sundaram K and Jeyaganesan P N 2019 Designing a omnichannel closed loop green supply chain network adapting preferences of rational customers. *Sādhanā* 44: 60

[12] Shavandi H, Mahlooji H and Nosratian N E 2012 A constrained multi-product pricing and inventory control problem. *Applied Soft Computing* 12: 2454–2461

[13] Avinadav T, Herbon A and Spiegel U 2014 Optimal ordering and pricing policy for demand functions that are separable into price and inventory age. *International Journal of Production Economics* 155: 406–417

[14] Bajwa N, Fontem B and Sox C R 2016 Optimal product pricing and lot sizing decisions for multiple products with nonlinear demands. *Journal of Management Analytics* 3: 43–58

[15] Wang Q, Zhao N, Wu J and Zhu Q 2019 Optimal pricing and inventory policies with reference price effect and loss-averse customers. *Omega*: 102174

[16] Hu B, Qu J and Meng C 2018 Supply chain coordination under option contracts with joint pricing under price-dependent demand. *International Journal of Production Economics* 205: 74–86

[17] Zhang M, Bell P C, Cai G G and Chen X 2010 Optimal fences and joint price and inventory decisions in distinct markets with demand leakage. *European Journal of Operational Research* 204: 589–596

[18] Raza S A 2015 An integrated approach to price differentiation and inventory decisions with demand leakage. *International Journal of Production Economics* 164: 105–117

[19] Raza S A and Turiac M 2016 Joint optimal determination of process mean, production quantity, pricing, and market segmentation with demand leakage. *European Journal of Operational Research* 249: 312–326

[20] Li Y, Xu L and Li D 2013 Examining relationships between the return policy, product quality, and pricing strategy in online direct selling. *International Journal of Production Economics* 144: 451–460

[21] Zhang Y, He Y, Yue J and Gou Q 2019 Pricing decisions for a supply chain with refurbished products. *International Journal of Production Research* 57: 2867–2900

[22] Gupta V K, Ting Q U and Tiwari M K 2019 Multi-period price optimization problem for omnichannel retailers accounting for customer heterogeneity. *International Journal of Production Economics* 212: 155–167

[23] Ahmadi-Javid A and Hoseinpour P 2015 A location-inventory-pricing model in a supply chain distribution network with price-sensitive demands and inventory-capacity constraints. *Transportation Research Part E: Logistics and Transportation Review* 82: 238–255

[24] Shu J, Li Z, Shen H, Wu T and Zhong W 2012 A logistics network design model with vendor managed inventory. *International Journal of Production Economics* 135: 754–761

[25] Shen Z J M 2006 A profit-maximizing supply chain network design model with demand choice flexibility. *Operations Research Letters* 34: 673–682

[26] Tavakkoli-Moghaddam R, Noshafagh S V, Taleizadeh A A, Hajipour V and Mahmoudi A 2017 Pricing and location decisions in multi-objective facility location problem with M/M/m/k queuing systems. *Engineering Optimization* 49: 136–160

[27] Chen X, Wu S, Wang X and Li D 2019 Optimal pricing strategy for the perishable food supply chain. *International Journal of Production Research* 57: 2755–2768

[28] Zhang J, Zhao S, Cheng T E and Hua G 2019 Optimisation of online retailer pricing and carrier capacity expansion during low-price promotions with coordination of a decentralised supply chain. *International Journal of Production Research* 57: 2809–2827

[29] Fattahi M, Mahootchi M, Govindan K and Husseini S M M 2015 Dynamic supply chain network design with capacity planning and multi-period pricing. *Transportation Research Part E: Logistics and Transportation Review* 81: 169–202

[30] Keyvanshokooh E, Fattahi M, Seyed-Hosseini S M and Tavakkoli-Moghaddam R 2013 A dynamic pricing approach for returned products in integrated forward/reverse logistics network design. *Applied Mathematical Modelling* 37: 10182–10202

[31] Fattahi M, Govindan K and Keyvanshokooh E 2018 A multi-stage stochastic program for supply chain network redesign problem with price-dependent uncertain demands. *Computers & Operations Research* 100: 314–332

[32] Ahmadi-Javid A and Ghandali R 2014 An efficient optimization procedure for designing a capacitated distribution network with price-sensitive demand. *Optimization and Engineering* 15: 801–817

[33] Ahmadi-Javid A and Hoseinpour P 2015 Incorporating location, inventory and price decisions into a supply chain distribution network design problem. *Computers & Operations Research* 56: 110–119

[34] Ghomi-Avili M, Jalali Naeini S G, Tavakkoli-Moghaddam R and Jabbarzadeh A 2018 A joint fuzzy pricing network design model for a competitive green closed-loop supply chain design in the presence of disruptions. *Journal of Cleaner Production* 188: 425–442

[35] Alfares H K and Elmorra H H 2005 The distribution-free newsboy problem: extensions to the shortage penalty case. *International Journal of Production Economics* 93: 465–477

[36] Chen F Y, Yan H and Yao L 2004 A newsvendor pricing game. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans* 34: 450–456