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On pulse-width pulse-frequency modulator control strategy: An adaptation-based describing function representation

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Abstract. This research describes an adaptation-based control strategy to deal with the pulse-width pulse-frequency modulator. The subject behind the research is to control the above-referenced modulators the hard limiter block via closed-loop describing function representation. In a word, this modulator has a set of parameters, which are all in need of adjusting, accurately, in order to guarantee its high performance, in the entire areas of overall systems under control. The investigated results indicate that parameter's adjustment of the aforementioned pulse-width pulse-frequency modulator can be recognized as the state-of-the-art outcomes via adaptation rules.

Keywords. State-of-the-art adaptation rules; pulse-width pulse-frequency modulator; describing function.

1. Introduction

A series of solutions to deal with the pulse-width pulse-frequency (PWPF) modulator in line with the filter and the corresponding hard nonlinear block with the hysteresis are recently taken into real consideration. One of the new ideas in this area is to realize describing function. Referring to the aforementioned describing function, Vander *et al* consider multiple-input describing functions in the area of nonlinear system design. Krovel focuses on a traditional approach of adjusting PWPF modulator in the attitude control [1, 2]. Regarding the applications of the PWPF modulator, there are so many potential works. Tian *et al* present the multivariable finite time attitude control for quadrotor UAV, while Peng *et al* proposes state dependent model predictive control in the area of the orbital rendezvous through PWPF modulated thrusters. Lian *et al* consider liberation, Song *et al* present liberation suppression of flexible robots during attitude control and Leonangeli *et al* investigate experimental results in the area of on-off control modulation to deal with rigid and flexible free floating platforms [3–9].

The rest of the paper is organized as follows. The proposed control strategy is first given in section 2. The simulation results and concluding remarks are correspondingly presented in sections 3 and 4, respectively.

2. The proposed control strategy

2.1 *Parameter's adjustment through adaptation-based control strategy*

Adjustment of the parameters of the pulse-width pulse-frequency modulator through the proposed adaptation-based control strategy is now considered as k, τ, U_{off} and U_{on}, respectively. Regarding the describing function theory, it can be used to represent the model of the hard nonlinearity, whilst $A < U_{off}$ is taken as

$$\begin{cases} N_a = 0 \\ N_b = 0 \end{cases} \quad (1)$$

The results can then be represented in Eq. (2), whilst $U_{off} < A < U_{on}$ is taken as

$$\begin{cases} N_a = \frac{4U}{\pi A}\sqrt{1 - \frac{4U_{off}^2}{A^2}} \\ N_b = 0 \end{cases} \quad (2)$$

The results can finally be represented in Eq. (3), whilst $A > U_{on}$ is taken as

$$\begin{cases} N_a = \frac{2U}{\pi A}\left(\sqrt{1 - \frac{4U_{off}^2}{A^2}} + \sqrt{1 - \frac{4U_{on}^2}{A^2}}\right) \\ N_b = \frac{2U}{\pi A^2}\left(U_{on} - U_{off}\right) \end{cases} \quad (3)$$

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Correspondingly, by taking$\gamma = \max(u_c)$, the closed-loop outcomes can be calculated in Eq. (4), whilst $A < U_{off}$ is addressed

$$G_{c-l1} = 0 \tag{4}$$

And regarding $U_{off} < A < U_{on}$, the closed-loop outcomes can be acquired in this part of investigation as $\frac{b_0}{s+a_0}$, whilst the following parameters are taken

$$b_0 = \frac{4k\gamma U}{\pi A^2 \tau}\sqrt{A^2 - 4U_{off}^2} \tag{5}$$

$$a_0 = b_0 + \tau^{-1} \tag{6}$$

Regarding the models, there are

$$G_{mc-l1} = 0 \tag{7}$$

$$G_{mc-l2} = \frac{b_m}{a_{m1}s + a_{m2}} u_c; \quad a_{m1} = \tau, a_{m2} = \tau b_0 + 1 \tag{8}$$

$$G_{mc-l3} = k_m < \varphi_m \tag{9}$$

Now, the errors for realizing the adaptation rules can instantly be calculated through Eq. (10), i.e.,

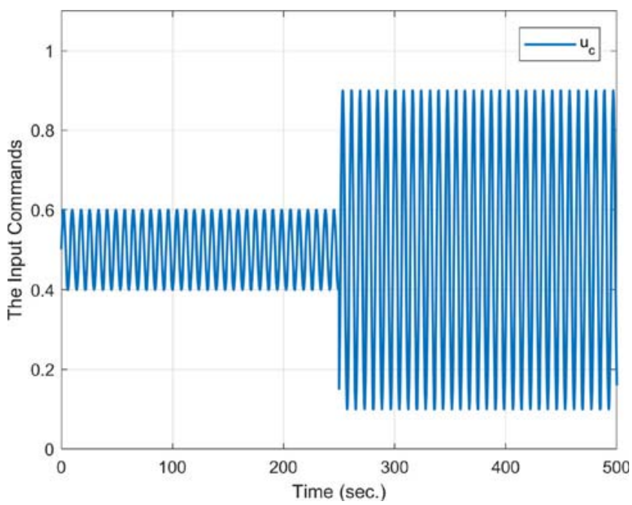

Figure 1. The profile of the input commands.

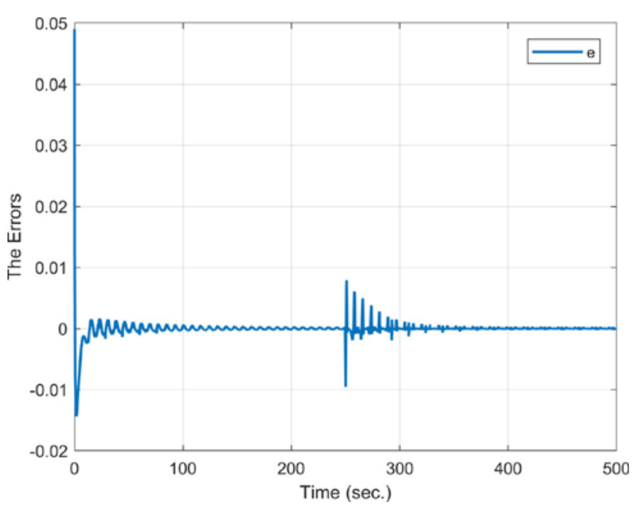

Figure 2. The profile of the errors.

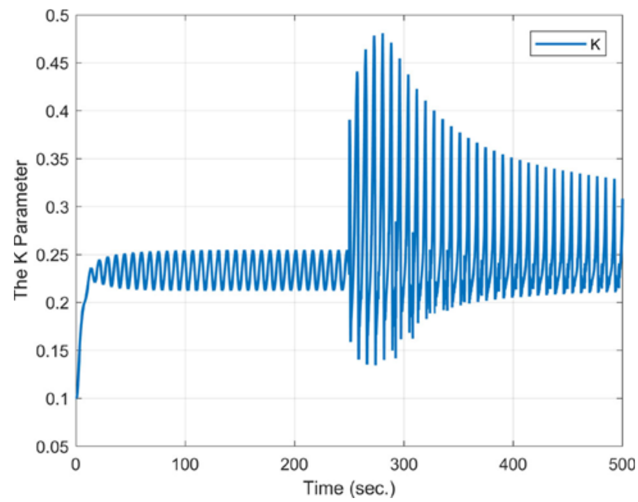

Figure 3. The profile of the k.

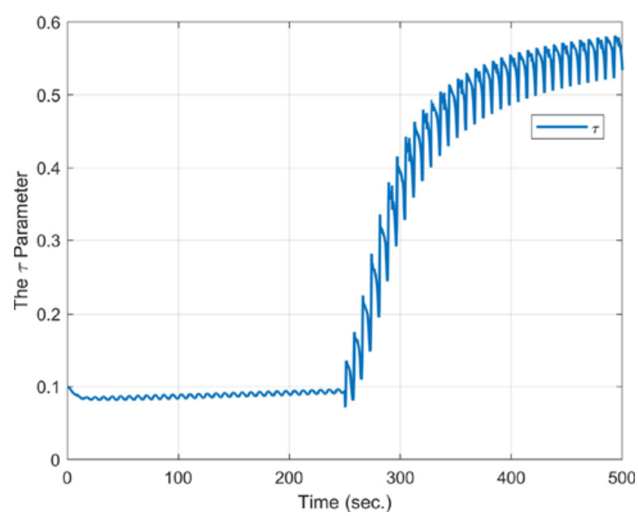

Figure 4. The profile of the τ.

$$e = \int y_p^2 dt - \int y_m^2 dt \tag{10}$$

Now, in case of developing the modulator parameter adjustment regarding the $U_{off} < A < U_{on}$, the following control law is designed

$$\tau \dot{u} + u - \tau\gamma^{-1} b_0 u_c + \tau\gamma^{-1} b_0 y_p = 0 \tag{11}$$

And, hereinafter, Eq. (12) results as

$$e = \frac{\tau^2 b_0^2 - b_m^2}{s(\tau s + \tau b_0 + 1)} u_c^2 \tag{12}$$

Subsequently, the modulator parameters adjustments by using the constant parameter ηcan acquire through MIT rule using sensitivity derivative by focusing on Eq. (12) based on the model referenced adaptation-based control algorithm by the following

$$\begin{cases} \dot{\tau} = -\eta e \dfrac{b_m^2 s^2 + 2\tau b_0^2(\tau b_0 + 1)s - \left(\tau^2 b_0^3 - b_m^2 b_0\right)}{s^2(\tau s + \tau b_0 + 1)^2} u_c^2 \\ \dot{b}_0 = -\eta e s \dfrac{2\tau^3 b_0 s + 2\tau^2 b_0(\tau b_0 + 1) - \tau^3 b_0^2 + \tau b_m^2}{s^2(\tau s + \tau b_0 + 1)^2} u_c^2 \end{cases} \tag{13}$$

The rest of the aforementioned parameters adjustments by addressing Eqs. (5) and (13) are correspondingly resulted as

$$\frac{\pi A^2 \tau}{4\gamma U\sqrt{A^2 - 4U_{off}^2}} \dot{b}_0 + \frac{4k}{A^2 - 4U_{off}^2} \dot{U}_{off} + \frac{k}{\tau}\dot{\tau} - \dot{k} = 0 \tag{14}$$

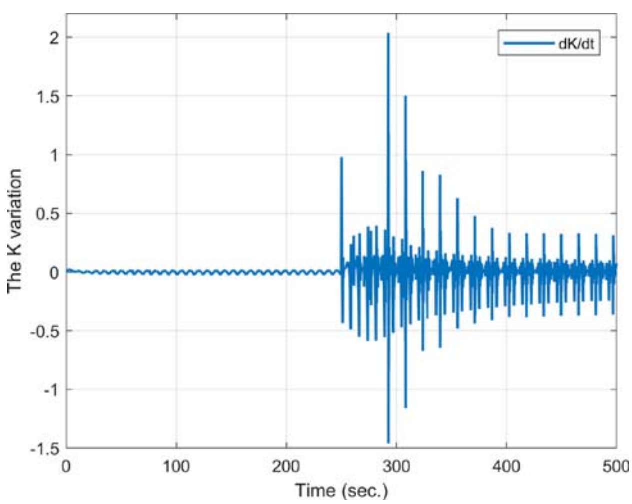

Figure 5. The profile of the $\dot{k}$.

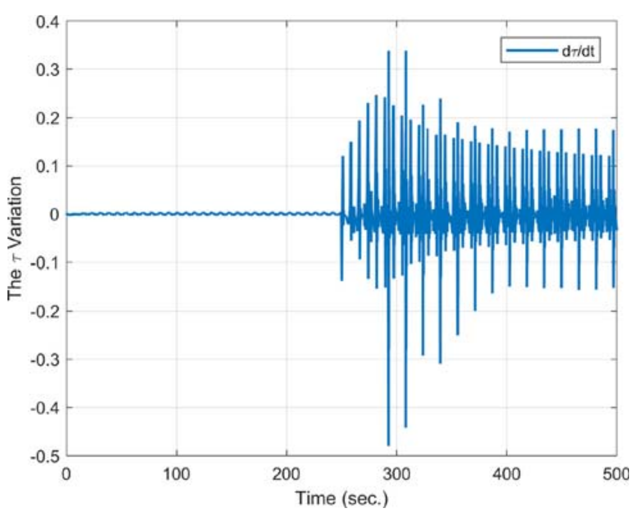

Figure 6. The profile of the $\dot{\tau}$.

3. Simulation results

The simulation results of the research are now carried out in this section. Hereafter, by supposing $u(t)$ as the step function, the input commands are taken by $u_c(t) = 0.5 + 0.1$ sin

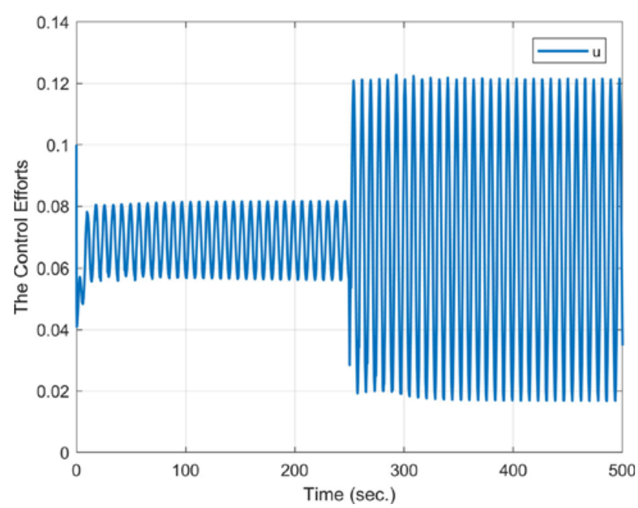

Figure 7. The control efforts.

Table 1. The initial values of the parameters.

	The parameters	The initial values
1	U_{on}	0.7
2	U_{off}	0.3
3	k	0.1
4	τ	0.1
5	η	0.1
6	γ	0.5

Table 2. The *ISE*, *ITSE*, *IAE* and the *ITAE* of the proposed approach and its benchmark.n

	The titles of approach	*ISE*	*ITSE*	*IAE*	*ITAE*	Running time (s)
1	The proposed approach	0.107	0.377	0.675	62.3	500
2	Krovel's approach [2]	0.115	0.718	0.965	94.0	500

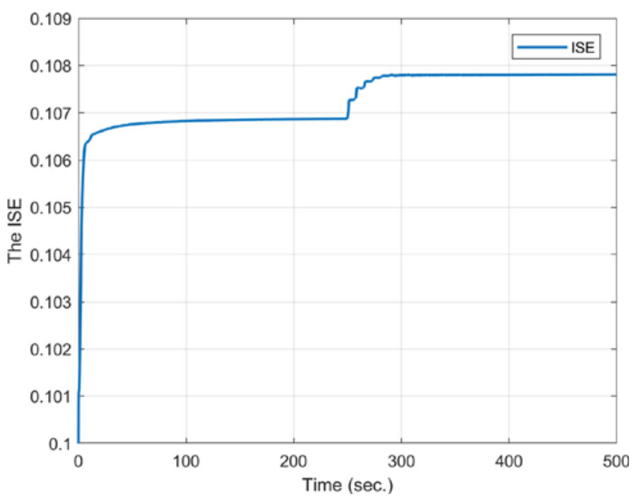

Figure 8. The profile of the *ISE*.

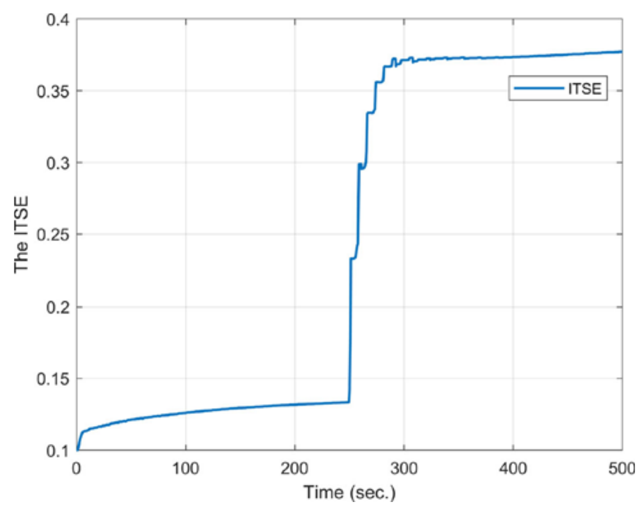

Figure 9. The profile of the *ITSE*.

$(\omega t + \theta)u(t) + 0.3\sin(\omega t + \theta))u(t - 250)$with $\omega = 0.8\,\frac{rad}{sec}$ and $\theta = 0$, where the corresponding profile is illustrated in figure1 and also the profile of the error is synchronously illustrated in figure 2. Refer to Eqs. (13)-(14), the profile of the $k$and the τ are first investigated in line with the variations of the $\dot{k}$and the $\dot{\tau}$with respect to time to address the state of $U_{off} < A < U_{on}$, as long as they are then considered to address the state of $A > U_{on}$, as well. The investigated results of the both states of the proposed approach regarding the $k$and the $\tau$are shown in figures 3 and 4, respectively, while the variations of the $\dot{k}$and $\dot{\tau}$ are correspondingly acquired based on the same outcomes in figures 5 and 6, respectively. The control efforts provided by the proposed control approach are illustrated in figure 7.

3.1 *The verification of the proposed control strategy performance*

In order to verify the investigated results regarding the parameter's adjustment of the PWPF, at first, the software, i.e., MATLAB is used to carry out the simulation programs. It should be noted that the proposed adaptation-based control approach needs to minimize the error, in the reasonable amount of time with the initial values of the parameters, tabulated in table 1.

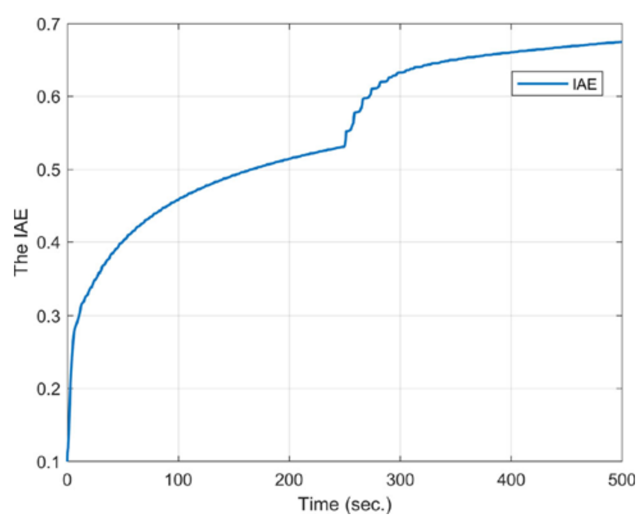

Figure 10. The profile of the *IAE*.

With this goal, the performance of the proposed control strategy is considered to be verified via the calculations of the integral square error (*ISE*), the integral time square error (*ITSE*), the integral absolute error (*IAE*) and finally

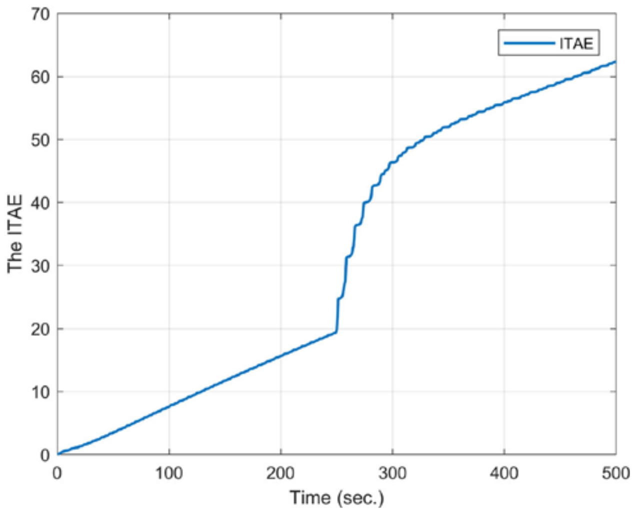

Figure 11. The profile of the *ITAE*.

the integral time absolute error (*ITAE*), all tabulated in table 2 and the corresponding outcomes are also illustrated in figures 8-11, respectively.

4. Concluding remarks

The pulse-width pulse-frequency modulator is considered in this research to adjust its parameters including k, τ, U_{off} and U_{on}, accurately, for the purpose of maintaining the closed-loop system performance in many applications including control, electronics, communications and so on. Despite the potential outcomes in the available literatures, the state-of-the-art solution through realizing the describing function is investigated.

It is to be noted that the filter's parameters and the corresponding hard nonlinearity with the hysteresis need to be first modeled. And then the proposed strategy is designed to handle the closed-loop adaptation-based control scheme concerning the above-referenced hard nonlinearity. It is shown through the established describing function. In other word, it aims us to present a new solution to adjust the entire parameters of the pulse-width pulse-frequency modulator through the model referenced adaptation-based control strategy under the state-of-the-art adaptation rules.

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