Sādhanā (2020) 45:105
https://doi.org/10.1007/s12046-020-01356-8

Control of the inverted pendulum system: a Smith fractional-order predictive model representation

M R ZANGENEH-MADAR[1] and A H MAZINAN[2,*]

[1]Department of Control Engineering, Science and Research Branch, Islamic Azad University (IAU), Tehran, Iran
[2]Department of Control Engineering, Faculty of Electrical Engineering, South Tehran Branch, Islamic Azad University (IAU), No. 209, North Iranshahr St., P.O. Box 11365/4435, Tehran, Iran
e-mail: ahmazinan@gmail.com; mazinan@azad.ac.ir

MS received 12 April 2019; revised 19 March 2020; accepted 3 April 2020

Abstract. The inverted pendulum system in the presence of uncertainties and external disturbances with delay is considered as one of the most applicable nonlinear systems to be used in real environments and industrial domains, extensively. The subject behind the research is to design a state-of-the-art fractional-order sliding mode control approach to stabilize the inverted pendulum angle. Subsequently, a consideration of uncertainties and external disturbances in the pendulum dynamic's parameters such as pendulum length and chariot's mass aims us to find an efficient technique to decrease chattering and correspondingly increase system's performance as well. In addition, the Smith predictive model representation along with the control approach is realized as a solution of eliminating time delay. In a word, the realization of sliding mode control to deal with uncertainties and external disturbances in line with an application of fractional-order sliding surfaces for decreasing chattering under the aforementioned Smith predictive model representation to eliminate delay is taken into real consideration as remarkable novelty of research proposed. Finally, the effectiveness of the approach analyzed here is verified in the form of numerical simulations through MATLAB software, tangibly.

Keywords. Smith fractional-order predictive model representation; sliding mode control; inverted pendulum system.

1. Introduction

The control theory provides a basis for dealing with the parameters of the system to be adjusted carefully in the specific conditions and constraints of a set of processes under consideration. The appropriate design of the control approach is related to technological advances and its real-time implementation, in general. With regard to engineering systems, many robotic applications have been a favorite in control investigations. They are often used to implement empirical models, evaluate the effectiveness of new control techniques and validate their practical capabilities. The most common robotic systems including Pendubot system [1], Furuta pendulum [2], inverted pendulum, wheel reaction pendulum [3], bicycle [4], VTOL aircraft [5] can be considered in this area. For at least fifty years, the inverted pendulum has been the most popular application in nonlinear control theory. This application has a lot of utilization in industry, which has increased its popularity and attracted a lot of attention. In spite of its simple structure, the inverted pendulum is considered to be the most basic system amongst the system proposed. There are various types of this system that have been investigated by the control researchers. The most known types of inverted pendulum include single arm swirling pendulum [6], inverted swirling pendulum [7] and double inverted pendulum [8]. The versions that have been researched less about their control are: double-edged swirling pendulum [9], double inverted pendulum [10], triple swirling pendulum [11], quadruple swirling pendulum [12] and the three-dimensional spherical pendulum [13]. The inverted pendulum system used includes a pendulum mounted on a chariot, so that the pendulum can easily rotate on the chariot. The chariot moves by a direct current motor. The chariot is on a limited rail attached to the engine by a belt and gear. The rails in this system have a length of one meter. The overall and primary goal of the inverted pendulum problem is to move the pendulum from the static state by the engine's order, and then in the shortest possible time, bring the pendulum to its unstable point and keep it constant without the slightest vibration. Concerning the control of this type of systems the related potential materials are given in [14–19].

*For correspondence
Published online: 06 May 2020

The rest of the paper is organized as follows. "The proposed control approach" is given in section 2. "The numerical simulations" and "concluding remarks" are presented in sections 3 and 4, respectively.

2. The proposed control approach

2.1 *The system modeling*

The second-order dynamic model for inverted pendulum is as follow [17]:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f(x) + \Delta f(x) + b(x)u + d(t) \\ y(t) = x_1(t - \Delta) \end{cases} \quad (1)$$

In which $x_1(t) = \theta(t)$ is the pendulum angle and $x_2(t) = \dot{\theta}(t)$ shows the gradient of this angle. Also $f(x)$ and $b(x)$ are defined as the following equations:

$$f(x) = \frac{g sin(x) - \frac{m l x_2^2 \cos(x_1) sin(x_1)}{m_c + m}}{l\left(\frac{4}{3} - \frac{m cos^2(x_1)}{m_c + m}\right)}$$
$$b(x) = \frac{\frac{cos(x_1)}{m_c + m}}{l\left(\frac{4}{3} - \frac{m cos^2(x_1)}{m_c + m}\right)} \quad (2)$$

Figure 1 illustrates the overview of the laboratory used inverted pendulum system.

2.2 *The fractional-order sliding mode control approach with delay*

We consider, first, the fractional-order sliding surface as follows:

$$s = K_p e + K_i D^{-\lambda} e + K_d D^{\lambda} e \quad (3)$$

which k_d and k_p are constant and $e = x_1 - x_1^d$ is the signal of state's error. It is to note that λ $(0 < \lambda < 1)$ expresses the derivation and fractional-order integral. In addition, D_t^{α} and $D_t^{-\alpha}$ express the derivation operator and the Caputo fractional-order integral and for $0 < \alpha < 1$ is defined as follows [19]:

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha}} d\tau, \quad 0 < \alpha \le 1$$
$$D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \quad 0 < \alpha \le 1 \quad (4)$$

By choosing the sliding surface similar to (3), after the zeroing, the governing equation should be as follows:

$$k_p e + k_i D^{-\lambda} e + k_d D^{\lambda} e = 0 \quad (5)$$

That in fact by choosing positive gains, a stable differential equation should be obtained to zero the error. In this case, the derivative of the sliding surface according to (3) is obtained as follows:

$$\dot{s} = k_p \dot{e} + k_i D^{1-\lambda} e + k_d D^{\lambda-1} \ddot{e} \quad (6)$$

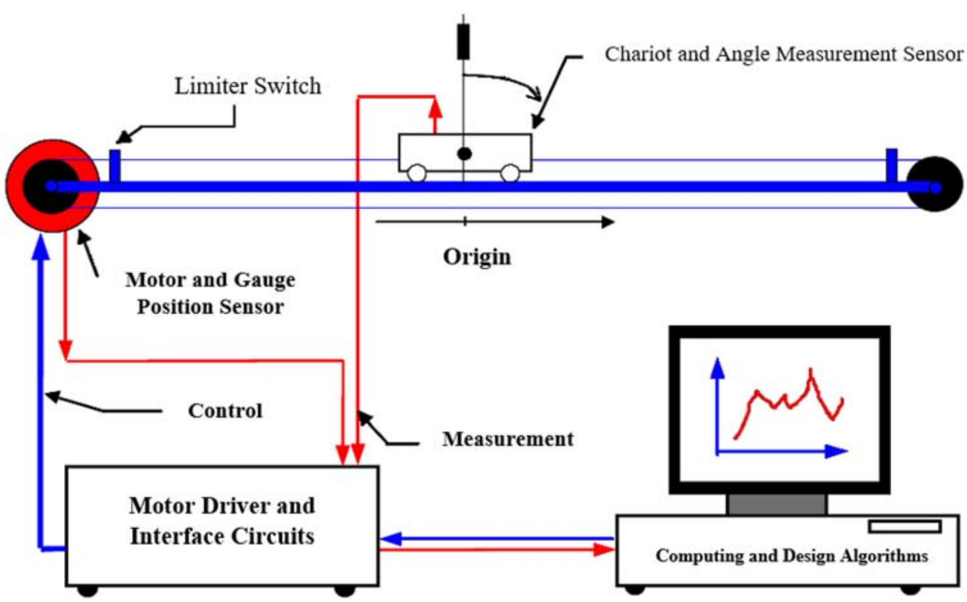

Figure 1. An overview of a laboratory for inverted pendulum system.

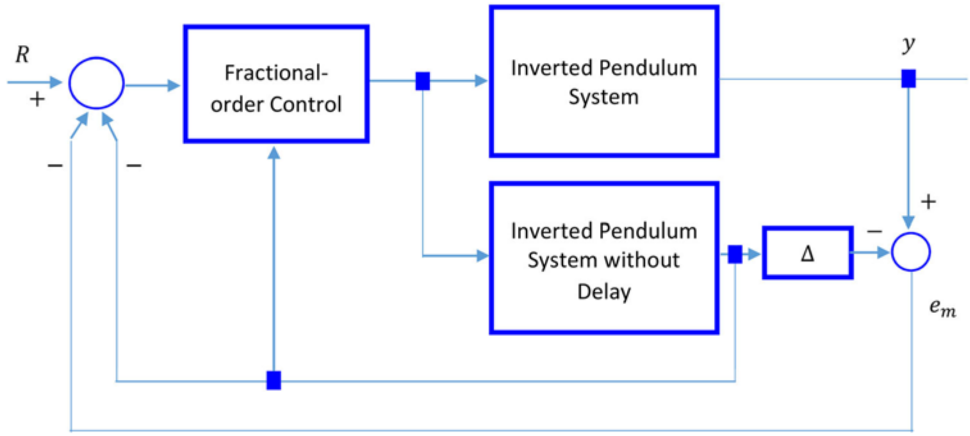

Figure 2. The schematic diagram of Smith predictive fractional-order sliding mode control approach with delay.

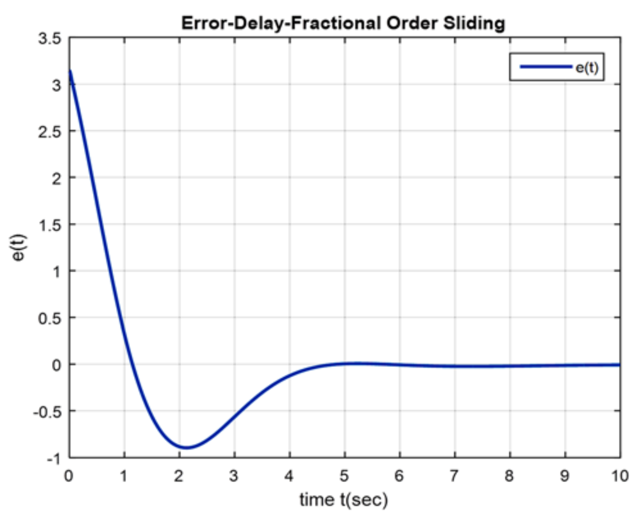

Figure 3. The states error through fractional-order sliding mode control approach under the Smith predictive method for the inverted pendulum system with delay.

In the following, the design of the new control law and its stability study in the inverted pendulum model with uncertainty and turbulence are presented in the form of the theorem and proved to be stable by the use of the Lyapunov technique:

Theorem Consider the inverted pendulum system in the presence of uncertainty and disturbance with the definition of $E(t) = \Delta f(x) + d(t)$, by the choice of control input as:

$$u = \frac{-1}{k_d b(x)} D^{1-\lambda}\left[\mu s + \rho singn(s) + k_p\left(x_2 - \dot{x}_1^d\right) + k_i D^{1-\lambda}\left(x_1 - x_1^d\right) + k_d D^{\lambda-1}\left(f(x) - \ddot{x}_1^d\right)\right] \quad (7)$$

In which $\mu > 0$ and $\rho > k_d\gamma$, the sliding surface in (3) and, as the result, the error value of $e = x_1 - x_1^d$ converges to zero.

Proof In order to prove the stability, the Lyapunov function considers similar to the preceding as follows:

$$V = \frac{1}{2}s^2 > 0 \quad (8)$$

By derivation from Lyapunov function, we have:

$$\dot{V} = s\dot{s} = s\left\{k_p\dot{e} + k_I D^{1-\lambda}e + k_d D^{\lambda-1}\ddot{e}\right\} \quad (9)$$

Now by substitution dynamic model of the system from (1) in (9), the differential form of Lyapunov function should be like this:

$$\begin{aligned}\dot{V} &= s\left\{k_p\left(x_2 - \dot{x}_1^d\right) + k_i D^{1-\lambda}\left(x_2 - x_1^d\right) + k_d D^{\lambda-1}\left(f(x) + b(x)u + \Delta f(x) + d(t) - \ddot{x}_1^d\right)\right\} \\ &= S\left\{k_p\left(x_2 - \dot{x}_1^d\right) + k_i D^{1-\lambda}\left(x_1 - x_1^d\right) + k_d D^{\lambda-1}\left(f(x) + b(x)u + E(t) - \ddot{x}_1^d\right\}\right.\end{aligned} \quad (10)$$

By substitution of the proposed control input in (7) in (10), after simplification, the differential form of Lyapunov function should be as follows:

$$\dot{V} = s\left(-\mu s - \rho sign(s) + k_d D^{\lambda-1}(E(t))\right) \quad (11)$$

Now by the consideration of $D^{\lambda-1}(E(t)) < \gamma$, we have:

$$\dot{V} \le -\mu s^2 - \rho|s| + k_d|s|\left|D^{\lambda-1}(E(t))\right| \le -\mu s^2 - \rho|s| + k_d\gamma|s| = -\mu s^2 - (\rho - k_d\gamma)|s| \quad (12)$$

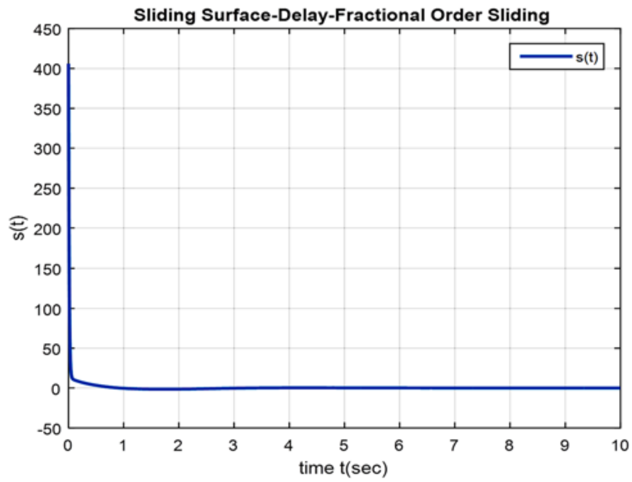

Figure 4. The sliding surface through fractional-order sliding mode control approach under the Smith predictive method for the inverted pendulum system with delay.

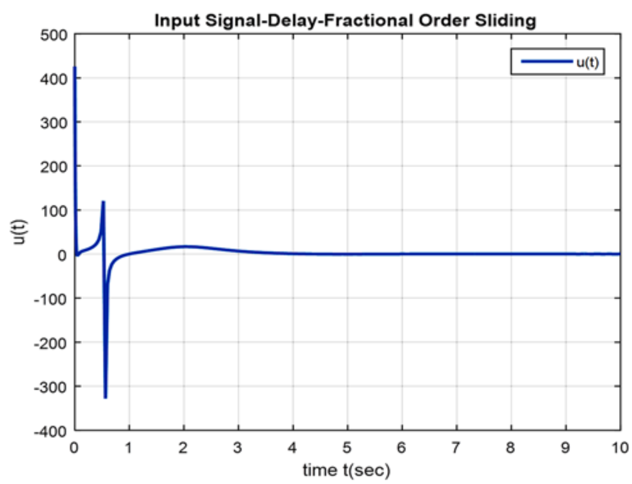

Figure 5. The control effort through fractional-order sliding mode control approach under the Smith predictive method for the inverted pendulum system with delay.

With the choice of $\mu > 0$ and $\rho > k_d\gamma$ the value of V is negative for all values of s and the asymptotical stabilization of s to zero is guaranteed and thus according to the definition of the sliding surface in (3) and (5), the error becomes zero.

2.3 *The Smith predictive fractional-order sliding mode control approach with delay*

The inverted pendulum system, like many other practical ones has a time delay and with reference to the dynamics of the model system in [17], has a delay time of 0.2 s. This small amount of delay, if not considered in the controller design, can cause system oscillation and even system instability. To prevent such problems, the delay time should be considered in the system dynamics and the controller is designed in such a way as to be able to resolve the problems caused by the delay. In the proposed schematic, illustrated in figure 2, the fractional-order sliding mode control (FOSMC) is given concerning the system and by knowing the delay Δ, effect of this delay can be removed by using the Smith predictive method. In fact, the structure of this one deals with delay under modeling uncertainties and disturbances.

3. The numerical simulations

The simulations of the proposed control approach is carried out for the inverted pendulum system to increase the controller's efficiency and simulation results to be analyzed. It should conclude that the sliding mode control has realized well alongside Smith predictive method in dealing with uncertainties and delays. The parameters of controllers are first considered in the theorem of this research as follows:

$$k_p = 3, k_i = 4, k_d = 5, \rho = 3, u = 1.5 \tag{13}$$

It is to note that in the simulation, uncertainty and disturbance are considered in the pendulum model. The applied disturbance to the system is $d(t) = 0.1\sin(5t)$ and uncertainty is considered in line with the percent of the dynamic parameters ($\Delta f = 0.1f$). In fact, the sum of disturbance and uncertainty can be considered as Eq. (14).

$$E = d(t) + \Delta f = 0.1F + 0.1\sin(5t) \tag{14}$$

In addition, in the Smith predictive method, illustrated in figure 2, time delay, which equals to 0.2 s for the system is taken. The outcome in the presence of uncertainties and disturbances is illustrated in figure 3. It should be noted that Figure 3 to 5 clearly illustrate that the convergence of the error to zero in the presence of uncertainty and turbulence also works well. The only significant difference is the increase in the input domain in Figure 5, which means more control efforts to eliminate the disturbances and uncertainties.

4. Conclusion

The design of a novel sliding mode control approach in the second-order inverted pendulum nonlinear system has been considered. For this, at first, the new fractional-order sliding mode control is designed and proved to be stable through Lyapunov theorem. Subsequently, the effect of eliminating delay concerning the stability of inverted pendulum through Smith predictive method in the proposed one has been considered. At the end, the simulations verify well the effectiveness of the proposed control approach in stabilizing and omitting chattering in the pendulum's angle.

References

[1] Fantoni I, Lozano R, and Spong M W 2000 Energy based control of the pendubot. *IEEE Trans. Automat. Contr.* 45 (4): 725–729

[2] Acosta J A 2010 Furuta's Pendulum: A conservative nonlinear model for theory and practice. *Math. Probl. Eng.* 2010 Article ID 742894

[3] Block D J, Åström K J and Spong M W 2007 The reaction wheel pendulum. *Synth. Lect. Control mechatronics* 1 (1): 1–105

[4] Åström K J, Klein R E and Lennartsson A 2005 Bicycle dynamics and control. *IEEE Control Syst. Mag.* 25 (4): 26–47

[5] Martin P, Devasia S and Paden B 1996 A different look at output tracking: control of a VTOL aircraft. *Automatica* 32 (1): 101–107

[6] Åström K J and Furuta K 2000 Swinging up a pendulum by energy control. *Automatica* 36 (2): 287–295

[7] Yoshida K 1999 Swing-up control of an inverted pendulum by energy-based methods. *American Control Conference* 6: 4045–4047

[8] Furuta K, Okutani, T and Sone H 1978 Computer control of a double inverted pendulum. *Comput. Electr. Eng.* 5 (1): 67–84

[9] Bortoff S A 1996 Robust swing-up control for a rotational double pendulum. *IFAC Proc.* 29 (1): 2810–2815

[10] Tsai M C and Shen B H 2007 Synchronisation control of parallel dual inverted pendulums driven by linear servomotors. *IET Control Theory Appl.* 1 (1): 320–327

[11] Furut K, Ochiai T and Ono N 1984 Attitude control of a triple inverted pendulum. *Int. J. Control.* 39 (6): 1351–1365

[12] Li H, Zhihong M and Jiayin W 2002 Variable universe adaptive fuzzy control on the quadruple inverted pendulum. *Sci. China Ser. E Technol. Sci.* 45 (2): 213–224

[13] Shen J, Sanyal A K, Chaturvedi N A, Bernstein D and McClamroch H 2004 Dynamics and control of a 3D pendulum. *IEEE Conference on Decision and Control* 1: 323–328

[14] Lai C L and Hsu P L 2010 Design the remote control system with the time-delay estimator and the adaptive Smith predictor. *IEEE Trans. Ind. Informatics* 6 (1): 73–80

[15] Huzmezan M, Gough W A, Dumont G A and Kovac S 2002 Time delay integrating systems: a challenge for process control industries. A practical solution. *Control Eng. Pract.* 10 (10):1153–1161

[16] Normey-Rico J E, Bordons C and Camacho E F 1997 Improving the robustness of dead-time compensating PI controllers. *Control Eng. Pract.* 5 (6): 801–810

[17] Wu T S and Karkoub M 2014 $H\infty$ fuzzy adaptive tracking control design for nonlinear systems with output delays. *Fuzzy Sets Syst.* 254: 1–25

[18] Aghababa M P and Akbari M E 2012 A chattering-free robust adaptive sliding mode controller for synchronization of two different chaotic systems with unknown uncertainties and external disturbances. *Appl. Math. Comput.* 218 (9): 5757–5768

[19] Li H S, Luo Y and Chen Y Q 2010 A fractional-order proportional and derivative (FOPD) motion controller: Tuning rule and experiments. *IEEE Trans. Contr. Syst. Tech.* 18 (2): 516–520