Sādhanā (2020)45:99
https://doi.org/10.1007/s12046-020-1291-x

Robust control of single-machine infinite bus system: a novel approach

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MS received 8 December 2018; revised 10 December 2019; accepted 11 December 2019

Abstract. This work proposes a novel robust controller in Linear Matrix Inequality framework for a single-machine infinite bus system. The mathematical equations defining the system are highly nonlinear. The novelty of the paper is the linear control law, which is obtained without linearizing the system. Along with the nonlinearity here, another two contingencies are also taken into account—(i) uncertainties in terms of parameter variations and losses and (ii) time delay in the feedback control law. The proposed controller is expected to accommodate the parameter uncertainties and the time delay. Simulation results support the analytical findings.

Keywords. Single-machine infinite bus system; robust delay independent control; LMI; LSVF controller.

1. Introduction

The challenges in controlling power systems are increasing day by day. The new aspects not yet considered in the system dynamics are now taken into account [1–5]. The nonlinearity and uncertainty become inevitable while considering stabilization of the system. The existence of time delay in system's states and input is often found [6–9]. There are two conventional practices to stabilize the nonlinear system. The first and the older one is designing a controller based on its linearized model and the second one is to use some nonlinear controllers like sliding mode control, back stepping, feedback linearization, etc. However, the techniques have some difficulties associated with them. The jacobian linearization is found to be inefficient when the system dynamics is highly nonlinear or the operating point is not close to the equilibrium points; on the other hand, the advanced nonlinear controllers may have some application-related difficulties in some cases.

In this work, the stabilization of a well-known model of single-machine infinite bus (SMIB) system is considered. Some available literature considers the linear model. However, a more accurate model with nonlinearity is considered in [9, 10]. Moreover, as discussed earlier, the model also contains uncertainty in terms of parameter variations and losses along with delay in input. The traditional practice to stabilize an SMIB system was using an Automatic Voltage Regulator (AVR) and the Power System Stabilizer (PSS) control structure. However, now a days, the excitation control for the system is found to be more efficient.

Here the newly proposed controller is designed in such a way that the original system does not need to be linearized. It removes the difficulties arising during operating point variation. Moreover, the uncertainty in terms of some important parameters (discussed in detail in section 4) and time delay in feedback control is considered here. The designed linear controller is found to be efficient to handle all the permissible uncertainties and time delay.

2. Preliminaries

Consider a nonlinear single-input single-output system, with parametric uncertainty a, in the form

$$\begin{aligned} \dot{x} &= f(x,a) + g(x,a)u(t-\tau) \\ y &= h(x) \end{aligned} \tag{1}$$

where all the terms have their usual significance.

2.1 *Uncoupling of uncertainty and time delay*

The standard techniques of nonlinear control theory cannot be applied to (1) because of the input delay and the uncertainty. Here, a novel approach is introduced to handle the uncertainty and time delay. Define a nonlinear function $\chi(x,a)$ as

$$\chi(x,a) = \begin{bmatrix} \chi_1(x,a) \\ \chi_2(x,a) \\ \vdots \\ \chi_n(x,a) \end{bmatrix} \tag{2}$$

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Published online: 30 April 2020

$$= \underbrace{\begin{bmatrix} h(x) \\ L_{f_n}h(x) \\ L_{f_n}^2 h(x) \\ \vdots \\ L_{f_n}^{r-1}h(x) \end{bmatrix}}_{Nominal\ part} + \underbrace{\begin{bmatrix} 0 \\ L_{\tilde{f}}h(x) \\ L_{\tilde{f}}^2 h(x) \\ \vdots \\ L_{\tilde{f}}^{r-1}h(x) \end{bmatrix}}_{Uncertain\ part} \tag{3}$$

where $L_{f_n}^i h(x)$ and $L_{\tilde{f}_n}^i h(x)$ $(i = 1, 2, \ldots, n-1)$ are the Lie derivatives [11] of the output function $h(x)$ along the vector fields $f_n(x)$ and $\tilde{f}(x)$, respectively. Now

$$\dot{\chi}(x, a) = \underbrace{\begin{bmatrix} L_{f_n}h(x) \\ L_{f_n}^2 h(x) \\ \vdots \\ L_{f_n}^{r-1}h(x) \\ L_{f_n}^r h(x) + L_{g_n}L_{f_n}^{r-1}h(x)u(t) \end{bmatrix}}_{Nominal\ part} + \underbrace{\begin{bmatrix} L_{\tilde{f}}h(x) \\ L_{\tilde{f}}^2 h(x) \\ \vdots \\ L_{\tilde{f}}^{r-1}h(x) \\ L_{\tilde{f}}^r h(x) + \theta \end{bmatrix}}_{Uncertain\ part}. \tag{4}$$

The uncertain part in (4) is defined as

$$\bar{\Omega}(x) = \begin{bmatrix} \bar{\Omega}_1 \\ \bar{\Omega}_2 \\ \vdots \\ \bar{\Omega}_n \end{bmatrix} = \begin{bmatrix} L_{\tilde{f}}h(x) \\ \vdots \\ L_{\tilde{f}}^r h(x) + \theta \end{bmatrix} \tag{5}$$

where

$$\theta = L_g L_f^{r-1} h(x)u(t-\tau) - L_g L_f^{r-1} h(x)u(t).$$

The main aim of the work is to design a robust LSVF controller in the form $u(t) = kx(t)$ for a system having both uncertainties and time delay, where k is the controller's gain. In case of input delay, the bounds on the current as well as delayed input are also needed. To obtain the bounds on delayed state, a well-known assumption that the current and delayed states lies within the same bounds is made here following some standard literature [12]. The assumption can further be extended for the input too. With a $\phi \in [-\tau, 0]$

$$\|u(t+\phi)\| \le q\|u(t)\| \quad \forall t \in [0\, \infty). \tag{6}$$

It may be noted there may be an error between the system input $u(t-\tau)$ and the manipulated input $u(t)$. To minimize that error here, a suitable tuning parameter μ has been chosen such that

$$|u(t-\tau) - u(t)| \le \mu. \tag{7}$$

With larger value of μ, the error will be greater. Similarly, small value of μ denotes less error. In this work, μ is left for online adjustment.

In this work a method introduced in [13] is used to model the uncertain parts in (4) by computing their respective upper and lower bounds using different optimization techniques. The bounds on each uncertain term of (4) are denoted here as m_i

Finally the following structured form considering the maximum effect of delay and uncertainty is obtained.

$$\chi(x) = \underbrace{\begin{bmatrix} h(x) \\ L_{f_n}h(x) \\ L_{f_n}^2 h(x) \\ \vdots \\ L_{f_n}^{r-1}h(x) \end{bmatrix}}_{Nominal\ part} + \underbrace{\sum_{i=1}^{n} Z_i I_l \begin{bmatrix} h(x) \\ L_{f_n}h(x) \\ L_{f_n}^2 h(x) \\ \vdots \\ L_{f_n}^{r-1}h(x) \end{bmatrix}}_{Uncertain\ part} \tag{8}$$

and

$$\dot{\chi}(x) = \begin{bmatrix} L_{f_n}h(x) \\ L_{f_n}^2 h(x) \\ \vdots \\ L_{f_n}^{r-1}h(x) \\ 0 \end{bmatrix} + \sum_{i=1}^{n} F_i \Delta_i Z_i \begin{bmatrix} h(x) \\ L_{f_n}h(x) \\ L_{f_n}^2 h(x) \\ \vdots \\ L_{f_n}^{r-1}h(x) \end{bmatrix} + \left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} + \Upsilon_n \sum_{i=1}^{n} \Delta_i Z_i \right) (L_{f_n}^r h(x) + L_g L_{f_n}^{r-1} h(x) u) \tag{9}$$

where F_i is an $r \times 1$ matrix whose i^{th} entry is 1 and other entries are 0 and Υ_n is an $r \times 1$ matrix whose r^{th} entry is 1 and other entries are 0. Z_i denotes a $1 \times n$ matrix with all entries except the i^{th} entry equal to 0 and i^{th} entry equal to m_i and $\|\Delta_i\| < 1$. I_l is the lower shift identity matrix.

2.2 *Bounds on Lie derivatives*

The aim of this work is to construct a set of Linear Matrix Inequalities (LMIs) to find out the desired linear controller. To formulate the LMIs, we need the affine upper bounds of the terms in (8) and (9), and the Lie derivatives in (3) and (4) play an important role. Here, using the theorem in [14] the bounds are obtained.

Finally the affine upper bounds of the Lie derivatives $h(x)$,..., $L_{f_n}^{n-1}h(x)$ and the product terms $h(x)L_{g_n}L_{f_n}^{r-1}h(x)$, $L_{f_n}h(x)L_{g_n}L_{f_n}^{r-1}h(x)$,..., $L_{f_n}^{r-1}h(x)\ L_{g_n}L_{f_n}^{r-1}h(x)$ are obtained using *TheoremB*.

$$\begin{aligned} \overline{h}(x; X, \varphi) &= p_0 x + q_0, \\ \overline{L_{f_n}h}(x; X, \varphi) &= p_1 x + q_1, \\ &\vdots \\ \overline{L_{f_n}^{r-1}h}(x; X, \varphi) &= p_{n-1}x + q_{n-1}, \\ \overline{L_{f_n}^{r}h}(x; X, \varphi) &= p_n x + q_n. \end{aligned} \tag{10}$$

The bounds on the product terms $h(x)L_{g_n}L_{f_n}^{r-1}h(x)$, $L_{f_n}h(x)L_{g_n}\ L_{f_n}^{r-1}h(x), \ldots, L_{f_n}^{r-1}h(x)L_{g_n}L_{f_n}^{r-1}h(x)$ are found as follows:

$$\begin{aligned} \overline{h(x; X, \varphi)L_{g_n}L_{f_n}^{r-1}h(x; X, \varphi)} &= \xi_0 x + \rho_0, \\ \overline{L_f h(x; X, \varphi)L_{g_n}L_{f_n}^{r-1}h(x; X, \varphi)} &= \xi_1 x + \rho_1, \\ &\vdots \\ \overline{L_{f_n}^{r-1}h(x; X, \varphi)L_{g_n}L_{f_n}^{r-1}h(x; X, \varphi)} &= \xi_{n-1}x + \rho_{n-1}. \end{aligned} \tag{11}$$

Define the symbols p_i, q_i, ξ_i and ρ_i in a specified order.

$$\begin{aligned} A_0 &= \begin{bmatrix} p_0 \\ p_1 \\ \cdots \\ p_{n-1} \end{bmatrix}, A_1 = \begin{bmatrix} p_1 \\ p_2 \\ \cdots \\ p_n \end{bmatrix}, B_0 = \begin{bmatrix} q_0 \\ q_1 \\ \cdots \\ q_{n-1} \end{bmatrix} \\ B_1 &= \begin{bmatrix} q_1 \\ q_2 \\ \cdots \\ q_n \end{bmatrix}, \Lambda = \begin{bmatrix} \xi_0 \\ \xi_1 \\ \cdots \\ \xi_{n-1} \end{bmatrix}, \Gamma = \begin{bmatrix} \rho_0 \\ \rho_1 \\ \cdots \\ \rho_{n-1} \end{bmatrix} \end{aligned} \tag{12}$$

The next section describes the controller design for the original uncertain system (1) using vector $\chi(x)$ (as in (3)) to construct the CLF.

3. Main result

The following theorem is newly introduced to get the linear robust controller for the nonlinear system having uncertainty and time delay in input.

Theorem 1 *The uncertain system in (1) will be stabilized by a control law $u(t) = kx(t)$, if there exists a controller 'k' and the matrices $\bar{P}$, M, F and N such that*

$$\begin{bmatrix} Q & \dfrac{R + S^T}{2} \\ \dfrac{S + R^T}{2} & T - E_{min_1} \end{bmatrix} \prec 0 \tag{13}$$

and

$$\begin{bmatrix} 2\bar{P} & k^T L^T + P^T \\ Lk + P & 2I - 2\epsilon I \end{bmatrix} \prec 0 \tag{14}$$

$$\begin{bmatrix} 2M & k^T L^T + P^T \hat{I}^T Z^T \\ Lk + Z\hat{I}P & 2I - 2\epsilon_1 I \end{bmatrix} \prec 0 \tag{15}$$

$$\begin{bmatrix} 2F & m_r k^T L^T + P^T \\ m_r kL + P & 2I - 2\epsilon_2 I \end{bmatrix} \prec 0 \tag{16}$$

$$\begin{bmatrix} 2N & m_r k^T L^T + P^T \hat{I}^T Z^T \\ m_r kL + Z\hat{I}P & 2I - 2\epsilon_3 I \end{bmatrix} \prec 0 \tag{17}$$

$$\epsilon > 0, \quad \epsilon_1 > 0, \quad \epsilon_2 > 0, \quad \epsilon_3 > 0 \tag{18}$$

holds. where

$$\begin{aligned} Q &= A_0^T P A_1 + \Lambda^T \bar{P} + A_0^T P \bar{Z} A_0 \\ &\quad + A_0^T \bar{Z} \hat{I} P \bar{Z} A_0 + A_0^T \bar{Z} \hat{I} P A_1 + A_0^T P I_n \bar{Z} A_1 \\ &\quad + A_0^T \bar{Z} \hat{I} P I_n \bar{Z} A_1 + \Lambda^T M + \Lambda^T N + \Lambda^T F \\ R &= A_0^T P B_1 + A_0^T P \bar{Z} B_0 + A_0^T \bar{Z} \hat{I} P \bar{Z} B_0 \\ &\quad + A_0^T \bar{Z} \hat{I} P B_1 + A_0^T P I_n \bar{Z} B_1 + A_0^T \bar{Z} \hat{I} P I_n \bar{Z} B_1 \\ S &= B_0^T P A_1 + B_0^T P \bar{Z} A_0 + B_0^T \bar{Z} \hat{I} P \bar{Z} A_0 \\ &\quad + B_0^T \bar{Z} \hat{I} P A_1 + B_0^T P I_n \bar{Z} A_1 + B_0^T \bar{Z} \hat{I} P I_n \bar{Z} A_1 \\ &\quad + \Gamma^T M + \Gamma^T N + \Gamma^T F \\ T &= B_0^T P B_1 + B_0^T P \bar{Z} B_0 + B_0^T \bar{Z} \hat{I} P \bar{Z} B_0 \\ &\quad + B_0^T \bar{Z} \hat{I} P B_1 + B_0^T P I_n \bar{Z} B_1 + B_0^T \bar{Z} \hat{I} P I_n \bar{Z} B_1 \end{aligned} \tag{19}$$

A_j, B_j $(j = 0, 1)$, Λ and Γ are as defined in (12) and

$$P \succ 0.$$

L is an $n \times 1$ matrix with all entries except r^{th} entry as 0, r^{th} entry as 1 and $\hat{I}$ is the upper shift identity matrix. I_n denotes an $n \times n$ sparse matrix having all entries except $(r, r)^{th}$ entry 0 and $(r, r)^{th}$ entry 1. E_{min} is the minimum error due to the overestimation of the functions while replacing them *with their upper bounds. M is an unknown matrix with appropriate dimension* $(n \times n)$.

The matrix Z is defined as follows:

$$\bar{Z} = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & m_n \end{bmatrix}$$

where $m_i\,(i = 1, 2, \ldots, n)$ *are the bounds of uncertainty as defined in section 2.1.*

Proof Let $\chi : \Re^n \rightarrow \Re^n$ be the mapping or the diffeomorphism from x coordinate to z coordinate defined as in (8) and (9).

Now, starting from the transformed coordinates, the Lyapunov function $V(z)$ is chosen as

$$V(z) = \frac{1}{2} z(t)^T P z(t) dt.$$

V is assumed to be continuous and differentiable and $P \succ 0$ is a symmetric positive definite matrix. For stability, the requirement [15] is

$$\dot{V}(z(t)) = z(t)^T P \dot{z}(t) = \chi(x)^T P \dot{\chi}(x) < 0. \tag{20}$$

Let (13) be true; then

$$\begin{aligned}
&\Rightarrow \begin{bmatrix} x \\ 1 \end{bmatrix}^T \begin{bmatrix} Q & \frac{R + S^T}{2} \\ \frac{S + R^T}{2} & T \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} \\
&\quad - E_{min} < 0 \Rightarrow x^T (A_0^T P A_1 + \Lambda^T \bar{P} + A_0^T P \bar{Z} A_0 \\
&\quad + A_0^T \bar{Z} \hat{I} P \bar{Z} A_0 + A_0^T \bar{Z} \hat{I} P A_1 + A_0^T P I_n \bar{Z} A_1 \\
&\quad + A_0^T \bar{Z} \hat{I} P I_n \bar{Z} A_1 + \Lambda^T M + \Lambda^T N + \Lambda^T F) x \\
&\quad + x^T (A_0^T P B_1 + A_0^T P \bar{Z} B_0 + A_0^T \bar{Z} \hat{I} P \bar{Z} B_0 \\
&\quad + A_0^T \bar{Z} \hat{I} P B_1 + A_0^T P I_n \bar{Z} B_1) \\
&\quad + (B_0^T P A_1 + B_0^T P \bar{Z} A_0 + B_0^T \bar{Z} \hat{I} P \bar{Z} A_0 + B_0^T \bar{Z} \hat{I} P A_1 \\
&\quad + B_0^T P I_n \bar{Z} A_1 + B_0^T \bar{Z} \hat{I} P I_n \\
&\quad \bar{Z} A_1 + \Gamma^T M + \Gamma^T N + \Gamma^T F) x + B_0^T P B_1 \\
&\quad + B_0^T P \bar{Z} B_0 in + B_0^T \bar{Z} \hat{I} P \bar{Z} B_0 + B_0^T \bar{Z} \hat{I} P B_1 \\
&\quad + B_0^T P I_n \bar{Z} B_1 + B_0^T \bar{Z} \hat{I} P I_n \bar{Z} B_1 - E_{min} < 0 \\
&\Rightarrow \bar{\chi}(x)^T P \bar{\dot{\chi}}(x) - E_{min} < 0 \\
&\Rightarrow (\chi(x) + E_1)^T P (\dot{\chi}(x) + E_2) - E_{min} < 0 \\
&\Rightarrow \chi(x)^T P \dot{\chi}(x) + E - E_{min} < 0.
\end{aligned} \tag{21}$$

As $E - E_{min} \geq 0$, the following can be obtained:

$$\chi(x)^T P \dot{\chi}(x) < 0. \tag{22}$$

Hence, the controller numerically obtained by solving the LMIs in *Theorem* 1 can stabilize the transformed coordinate system in (4). □

Remark 1 $\bar{\chi}(x)$ and $\bar{\dot{\chi}}(x)$ are, respectively, the upper bounds of $\chi(x)$ and $\bar{\chi}(x)$.

Remark 2 $E_1 = \bar{\chi}(x) - \chi(x)$.

Remark 3 $E_2 = \bar{\dot{\chi}}(x) - \dot{\chi}(x)$.

Remark 4 $E = E_1^T P \dot{\chi} + \chi^T P E_2 + E_1^T P E_2$.

Remark 5 $E_{min} = \underbrace{min}_{x} E$.

Note that this discussion ensures asymptotic stability in the new co-ordinates z. Having defined stability for the system in the new coordinate, the final objective is to stabilize the original system (1) in $x(t)$. It is known that under a nonlinear transformation like (3), global asymptotic stability (GAS) of the transformed states does not automatically imply that of the original one. Hence, further investigation is needed here.

At first, consider the system in the transformed coordinate

$$z(t) = \chi(x(t))$$

It is established in [11] that if a diffeomorphism $\chi(x)$ from x coordinates to z coordinates can be defined, then there also exists an inverse mapping $\chi^{-1} = \tilde{\chi}$ from z to x, where χ and $\tilde{\chi}$ are well defined and continuous. Now, consider

$$x(t) = \tilde{\chi}(z(t)). \tag{23}$$

The following two functions are defined as in [16]:

$$\underline{\alpha}(\varrho) = min_{|z| \geq \varrho} |\tilde{\chi}(z)| \tag{24}$$

and

$$\overline{\alpha}(\varrho) = max_{|z| \leq \varrho} |\tilde{\chi}(z)| \tag{25}$$

where $\overline{\alpha}$ and $\underline{\alpha}$ are class K_∞ [16] functions and

$$\underline{\alpha}(z) \leq |\tilde{\chi}(z)| \leq \overline{\alpha}(z) \tag{26}$$

with

$$z(t, z_0) = \tilde{\chi}(x(t, x_0)) \tag{27}$$

x_0 and z_0 being the initial states of the corresponding systems. As the controller k solved from the LMIs in *Theorem* 1 makes the closed loop system stable in transformed coordinate z, it follows that

$$|z(t, z_0)| \leq c|z_0| e^{-\lambda t} \quad \forall t \geq 0 \tag{28}$$

for c, $\lambda > 0$.

Putting (23) in (28) yields

$$|z(t, z_0)| = |\tilde{\chi}(x(t, x_0))| \leq c|z_0| e^{-\lambda t}. \tag{29}$$

Using (26) in (29), the following can be written:

$$\underline{\alpha}(|x(t,x_0)|) \le |\tilde{\chi}(x(t,x_0))| \le c|z_0|e^{-\lambda t} \Rightarrow \underline{\alpha}(|x(t,x_0)|) \le c|z_0|e^{-\lambda t}. \quad (30)$$

Again, using (27) in (23) and later (26)

$$|z_0| = |\tilde{\chi}(x_0)| \le \overline{\alpha}|x_0|. \quad (31)$$

Now, replacing (31) in (29) gives

$$\begin{aligned} &\underline{\alpha}(|x(t,x_0)|) \le c\overline{\alpha}|x_0|e^{-\lambda t} \\ &\Rightarrow |x(t,x_0)| \le \underline{\alpha}^{-1}(c\overline{\alpha}|x_0|e^{-\lambda t}) \\ &\qquad = \beta(|x_0,t|)e^{-\lambda t}. \end{aligned} \quad (32)$$

Thus it leads to the fact that under a continuous and invertible mapping χ, as in (26), if the states in the transformed coordinates z approach their equilibrium point as $t \to \infty$, then the original state x will also tend to corresponding equilibrium point as $t \to \infty$. GAS of the transformed states $z(t)$, here, implies GAS of (1). Or in other words GAS of z implies the GAS of x, under a continuous and invertible mapping χ, obeying (26). This concludes the proof. □

Here in this paper, unlike the feedback linearization technique where the control law also changes with the change in coordinate, no change is made in the control law while changing the coordinate or in other words the control law remains the same for both original and transformed systems. Hence, it can be concluded that any nonlinear system (1) can be stabilized if the solution of the LMIs in *Theorem* 1 exists.

4. Example

The SMIB model [19] can be written as follows:

$$\begin{aligned} \dot{E}'_q &= -\frac{1}{T'_d}E'_q + \frac{1}{T_{d0}}(x_d - x'_d)V_s\cos\delta \\ &\quad + (\frac{1}{T_{d0}} + \Delta T_{d0})V_f, \\ \dot{\omega} &= \frac{\omega_0}{H}P_m - \frac{D}{H}(\omega - \omega_0) \\ &\quad - \frac{\omega_0 E'_q V_s \times \sin\delta}{Hx'_{d\Sigma}}, \\ \dot{\delta} &= \omega - \omega_0, \\ y(t) &= \omega. \end{aligned} \quad (33)$$

All the parameters and constants have the same significance as in [19].

It is quite difficult to find out the exact value of the synchronous generator in an SMIB system due to its highly nonlinear dynamics. Hence, one needs to approximate the values, which subsequently paves way for uncertainty. In this work, we consider the following factors to make the controller designing more realistic [17, 18].

- The damping coefficient D is not always known at the first instance. Hence, it can be considered as a source of uncertainty if an approximate value of D is chosen. Let

$$\alpha_1 = \frac{D}{H}. \quad (34)$$

- The continuous change in load affects the stability of an SMIB system. There must be a continuous balancing of load and generation or the mechanical power input (P_m). Hence, the mathematical model should also consider the variation in P_m as an source of uncertainty. Let

$$\alpha_2 = \frac{P_m}{H}. \quad (35)$$

- Different types of faults are very common in power systems. In the SMIB system, they have a huge impact on transmission line's reactance x_{TL}. This is again a source of uncertainty and defined as $\alpha_{pe} = \Delta a V_s E'_q \sin\delta$ and $\alpha_{id} = \Delta a(E'_q - V_s\cos\delta)$, where $\Delta a = \frac{1}{\Delta x'_{d\Sigma}}$, $\Delta x'_{d\Sigma}$ being the change in $x'_{d\Sigma}$.
- The terms T'_d and T_{d_0} also depend on x_{TL}; the uncertainty associated with these two parameters are defined as $\Delta T'_d$ and ΔT_{d_0}, respectively.
- Most of the models available in the literature assume that there is no time delay in feedback. However, it should be taken into account that the feedback control may not be instantaneous. This makes the model more realistic. However, the time delayed feedback may have adverse impacts on the stability issue.

Finally the SMIB system with uncertainty and time delay is written as

$$\begin{aligned} \dot{E}'_q &= \left(-\frac{1}{T'_d} + \Delta T'_d\right)E'_q \\ &\quad + \frac{1}{T_{d0}}(x_d - x'_d)V_s\cos\delta \\ &\quad + (\frac{1}{T_{d0}} + \Delta T_{d0})V_f(t-\tau) + \alpha_{id}, \\ \dot{\omega} &= \alpha_1\omega_0 - \alpha_2(\omega - \omega_0) \\ &\quad - \frac{\omega_0 E'_q V_s \times \sin\delta}{Hx'_{d\Sigma}} + \alpha_{pe}, \\ \dot{\delta} &= \omega - \omega_0. \end{aligned} \quad (36)$$

The uncertain power system model (33) can be expressed as

$$\dot{x} = f_n(x) + \tilde{f}(x) + g_n(x)u(t-\tau) + \tilde{g}(x)u(t-\tau)$$

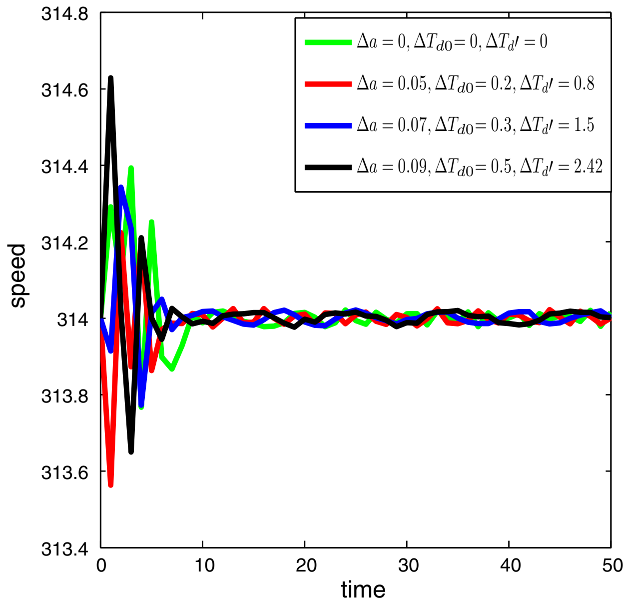

Figure 1. Plots for Example 2.

where

$$f_n(x) = \begin{bmatrix} -\frac{1}{T'_d}E'_q + \frac{1}{T_{d0}}(x_d - x'_d)V_s\cos\delta + \frac{1}{T_{d0}}V_f \\ -\frac{\omega_0 E'_q V_s \sin\delta}{Hx'_{d\Sigma}} \\ \omega - \omega_0 \end{bmatrix},$$

$$\tilde{f}(x) = \begin{bmatrix} \Delta T'_d E'_q + \alpha_{id} \\ \alpha_1\omega_0 - \alpha_2(\omega - \omega_0) + \alpha_{pe} \\ 0 \end{bmatrix},$$

$$g_n(x) = \begin{bmatrix} \frac{1}{T_{d0}} \\ 0 \\ 0 \end{bmatrix} \quad \tilde{g}(x) = \begin{bmatrix} \Delta T_{d0} \\ 0 \\ 0 \end{bmatrix}.$$

It may be noted that with the output ω the relative degree of system (33) [11] is not equal to the order of the system. However, with an arbitrary output $w(t) = \delta - \delta(0)$ ([11, 18, 19]) system (33) will be of full relative degree and can achieve the assumption made in section 2. The individual bounds for the uncertain terms are found using PSO. The mapping $\chi(x)$ is constructed with the arbitrary output $w(t)$ and its Lie derivatives instead of the Lie derivatives of the actual output $y(t)$ [18]. Finally, the controller for the uncertain system in (33) is designed by solving the LMIs in *Theorem* 1.

The parameters used for the SMIB system are the same as in [18] and the uncertainties are defined as $1 \le D \le 4$, $0.36 \le P_m \le 1$, $\Delta T'_d \le 2.42$, $\alpha_{id} \le 0.09$, $\Delta T_{d0} \le 0.5$ and $\alpha_{pe} \le 0.5$. Time delay in input τ is assumed to be 5 s.

Simulation results in figure 1 show the effectiveness of the proposed controller.

5. Conclusion

The main result centres around stabilization of nonlinear systems containing norm-bounded uncertainty and time delay. The proposed controller is linear, which may have advantages in terms of real life applications. Finally, the controller is designed for an SMIB system. The SMIB model considered here includes different types of parameter variations, changes in line reactance, etc. as uncertainty. The delay in the feedback is also considered. Finally the proposed controller is found to be efficient to accommodate all the permissible input delays and uncertainties in the range.

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