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Mehar approach for solving dual-hesitant fuzzy transportation problem with restrictions

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Abstract. Recently, a new type of transportation problems (named as dual-hesitant fuzzy transportation problem) as well as an approach to find the optimal solution of dual-hesitant fuzzy transportation problems have been proposed in the literature. In this paper, some dual-hesitant fuzzy transportation problems are considered to show that the existing approach is inappropriate as (i) The existing approach fails to find the optimal solution of dual-hesitant fuzzy transportation problems. (ii) On applying the existing approach different optimal transportation costs are obtained corresponding to alternative optimal solutions. Also, to resolve the inappropriateness of the existing approach, a new expression (named as Mehar score function) is proposed to transform a dual-hesitant fuzzy set into a real number. Furthermore, a new approach (named as Mehar approach), based upon the proposed Mehar score function, is proposed to find the optimal solution of dual-hesitant fuzzy transportation problems.

Keywords. Dual-hesitant fuzzy set; transportation problems; score function.

1. Introduction

In real-life problems, it may be easily observed that the price of the same product varies at different places. This variation may occur due to several factors. Transportation cost is one of the common factors for this variation. The price of a product is directly proportional to the transportation cost i.e., price of product will increase/decrease with the increase/decrease in the transportation cost. Due to the same reason, it is necessary to determine the optimal way of supplying the product from various sources to various destinations. In general, the classical methods (North west corner method, Least cost method, Vogel's approximation methods, etc.) are used to find one of the possible ways to transport the product and then the classical modified distribution method is applied to find the optimal way with the help of the obtained possible way to transport the product.

It is pertinent to mention that all the above mentioned classical methods can be used only if the precise value of all the transportation parameters (availability of the product at each source, demand of the product at each destination and the cost for supplying the one unit quantity of the product from each source to each destination) is known. However, in real-life situations these parameters are not precisely known.

For example,

1. The fair of a cab between two fixed places depends on the traffic jam or route followed by the cab or waiting time, etc.
2. The availability of a product depends on the various factors like weather condition, availability of transportation vehicle, etc.
3. The demand of a product depends upon various factors like weather conditions, fluctuation in price, etc.

Due to these facts, in the literature, fuzzy set [1] and its various extensions [2] have been used to represent various transportation parameters. Also, various methods have been proposed in literature to solve transportation problems under fuzzy environment and its various extensions.

Kaur and Kumar [3] proposed the fuzzy north-west corner method, fuzzy least cost method and fuzzy Vogel's approximation method to find the initial fuzzy basic feasible solution as well as fuzzy modified distribution method to solve such transportation problems in which the cost for transporting one unit quantity of the product from each source to each destination is represented by a generalized trapezoidal fuzzy number. While, all the remaining parameters are represented as non-negative real-numbers.

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Kaur and Kumar [4] proposed the fuzzy north-west corner method, fuzzy least cost method and fuzzy Vogel's approximation method to find the initial fuzzy basic feasible solution as well as fuzzy modified distribution method to solve such transportation problems in which all the transportation parameters are represented by generalized trapezoidal fuzzy numbers.

Kumar and Kaur [5] pointed out the drawbacks of existing methods for solving transportation problems under fuzzy environment. Also, to resolve the drawbacks, Kumar and Kaur [5] proposed method to solve such unbalanced transportation problems in which all the parameters are represented as trapezoidal fuzzy numbers. This method is based upon a fuzzy linear programming method which is obtained by replacing each parameter (represented by a non-negative real-number) of the transportation problem with a trapezoidal fuzzy number. In this method, firstly, the obtained fuzzy linear programming problem is transformed into its equivalent crisp linear programming problem. Then, the optimal solution of the transformed crisp linear programming problem is used to obtain the fuzzy optimal solution of the transportation problem under fuzzy environment.

Gupta and Kumar [6] extended Kumar and Kaur's method [5] to solve such multi-objective transportation problems in which all the parameters are represented by trapezoidal fuzzy numbers.

Ebrhaimnejad [7] pointed out to apply Kaur and Kumar's method [3], there is need to use arithmetic operations of fuzzy numbers and hence, much computational efforts are required to apply Kaur and Kumar's method [3]. To reduce the computational efforts, Ebrahimnejad [7] proposed a method to solve the same type of transportation problems. In this method, firstly, the considered generalized fuzzy transportation problem is transformed into its equivalent crisp transportation problem. Then, the optimal solution of the transformed crisp transportation problem is used to determine the optimal solution and the optimal generalized fuzzy transportation cost of the considered generalized fuzzy transportation problem.

Rani *et al* [8] proposed a method to reduce the computational efforts of Kumar and Kaur's method [6]. In this method, firstly, the fuzzy linear programming problem of a transportation problem under fuzzy environment is transformed into its equivalent four crisp linear programming problems. Then, the optimal solutions of these crisp linear programming problems are used to find the fuzzy optimal solution of the transportation problem under fuzzy environment.

Singh and Yadav [9] proposed the intuitionistic fuzzy north-west corner method, intuitionistic fuzzy least cost method and intuitionistic fuzzy Vogel's approximation method to find the initial intuitionistic fuzzy basic feasible solution as well as intuitionistic fuzzy modified distribution method to find the optimal solution of such transportation problems in which the cost for transporting one unit quantity of the product from each source to each destination is represented by a triangular intuitionistic fuzzy number. While, all the remaining parameters are represented as non-negative real-numbers.

Singh and Yadav [10] proposed the intuitionistic fuzzy north-west corner method, intuitionistic fuzzy least cost method and intuitionistic fuzzy Vogel's approximation method to find the initial intuitionistic fuzzy basic feasible solution as well as intuitionistic fuzzy modified distribution method to find the intuitionistic fuzzy optimal solution of such transportation problems in which the availability of the product at each source and the demand at each destination is represented by a triangular intuitionistic fuzzy number. While, the cost for transporting one unit quantity of the product from each source to each destination is represented by a non-negative real-number.

Kumar and Hussain [11] proposed a method for solving such balanced transportation problems in which each transportation parameter is represented as a triangular intuitionistic fuzzy number. This method is based on an intuitionistic fuzzy linear programming problem which is obtained by replacing each parameter (represented by a non-negative real-number) of the transportation problem with a triangular intuitionistic fuzzy number. In this method, firstly, the obtained intuitionistic fuzzy linear programming problem is transformed into its equivalent crisp linear programming problem. Then, the optimal solution of the transformed crisp linear programming problem is used to obtain the intuitionistic fuzzy optimal solution of the transportation problem under intuitionistic fuzzy environment.

Singh and Yadav [12] proposed the intuitionistic fuzzy-north-west corner method, intuitionistic fuzzy least cost method and intuitionistic fuzzy Vogel's approximation method to find the initial intuitionistic fuzzy basic feasible solution as well as intuitionistic fuzzy modified distribution method to find the intuitionistic fuzzy optimal solution of such transportation problems in which each parameter is represented by a triangular intuitionistic fuzzy number.

Ebrahimnejad [13] pointed out that more than one fuzzy optimal transportation cost is obtained on applying Kumar and Kaur's method [6], which is mathematically incorrect. Ebrahimnejad [13] also pointed out that this drawback is occurring due to using the inappropriate function for comparing trapezoidal fuzzy numbers. To resolve the drawback, Ebrhimnejad [13] proposed a method, based upon a different function for comparing trapezoidal fuzzy numbers, to solve such balanced transportation problems in which each parameter is represented by a trapezoidal fuzzy number.

Ebrahimnejad [14] proposed a method to transform such an unbalanced transportation problems into a balanced transportation problem in which each transportation parameter is represented by a generalized interval-valued trapezoidal fuzzy number. Ebrahimnejad [14] also proposed a method to solve this type of transportation problems. This method is based upon a generalized interval-valued fuzzy linear programming problem which is obtained by replacing each parameter (represented by a non-negative real-number) of the transportation problem with a generalized interval-valued trapezoidal fuzzy number. In this method, firstly, the obtained generalized interval-valued fuzzy linear programming problem is transformed into its equivalent crisp linear programming problem. Then, the optimal solution of the transformed crisp linear programming problem is used to obtain the generalized interval-valued fuzzy optimal solution of the transportation problem under generalized interval-valued fuzzy environment.

Ebrhimnejad and Vedegay [15] proposed a method for solving such balanced transportation problems in which each transportation parameter is represented as a trapezoidal intuitionistic fuzzy number. This method is based upon an intuitionistic fuzzy linear programming problem which is obtained by replacing each parameter (represented by a non-negative real-number) of the transportation problem with a trapezoidal intuitionistic fuzzy number. In this method, firstly, the obtained intuitionistic fuzzy linear programming problem is transformed into its equivalent crisp linear programming problem. Then, the optimal solution of the transformed crisp linear programming problem is used to obtain the intuitionistic fuzzy optimal solution of the transportation problem under intuitionistic fuzzy environment.

Kumar *et al* [16] proposed a method to solve such transportation problems in which the cost for supplying one unit quantity of the product from each source to each destination is represented by a Pythagorean fuzzy number. While, the remaining parameters are represented by non-negative real numbers. This method is based upon a Pythagorean fuzzy transportation table which is obtained by replacing the cost for supplying one unit quantity of the product (represented by a non-negative real-number) from each source to each destination of the transportation table with a Pythagorean fuzzy number. In this method, firstly, the obtained Pythagorean fuzzy transportation problem is transformed into a crisp transportation problem. Then, the classical methods (North west corner method, Least cost method, Vogel's approximation method, etc.) are used to obtain one of the possible solution of these transportation problems and hence, modified distribution method is applied to find the optimal solution of the transformed crisp transportation problem. Finally, the optimal solution of the transformed crisp transportation problem is used to find the optimal solution of the transportation problem under Pythagorean fuzzy environment.

Maity *et al* [17] pointed out that a supplier may have different types of vehicles to transport the product from each source to each destination. But, in general, it is assumed that the supplier will use that vehicle to transport the product corresponding to which the transportation cost will be minimum. However, this assumption is not realistic as in real-life situations, it is not always possible to transport the product with a vehicle having minimum transportation cost. Maity *et al* [17] also pointed out that if a vehicle having minimum transportation cost is used to supply the product. Then, the supplier will be fully satisfied. However, if a vehicle having minimum transportation cost is not used to transport the product. Then, the supplier will be partially satisfied and partially unsatisfied and hence, a degree of satisfaction and degree of dissatisfaction may be associated with the transportation cost. The degree of satisfaction will decrease with increase in the transportation cost. While, the degree of dissatisfaction will increase with the increase in the transportation cost.

To handle such real-life transportation problems, Maity *et al* [17] proposed the concept of dual-hesitant fuzzy transportation problems as well as a method to solve the dual-hesitant fuzzy transportation problems. In dual-hesitant fuzzy transportation problems, a degree of satisfaction and a degree of dissatisfaction is assigned with the transportation cost of each available vehicle e.g., let three vehicles be available to transport the product from the i^{th} source S_i to the j^{th} destination D_j and let the transportation cost corresponding to these vehicles be 30, 40 and 50. Furthermore, let the degree of satisfaction and the degree of dissatisfaction of the supplier corresponding to first, second and third vehicle be 0.5, 0.3, 0.2 and 0.4, 0.5, 0.7 respectively. Then, the transportation cost from the i^{th} source S_i to the j^{th} destination D_j may represented by the dual-hesitant fuzzy set $\{\{0.5, 0.3, 0.2\}, \{0.4, 0.5, 0.7\}\}(30, 40, 50)$.

It is pertinent to mention that as there does not exist any other approach except Maity *et al*'s approach [17] to solve the dual-hesitant fuzzy transportation problems. Therefore, in future, other researchers may use Maity *et al*'s approach [17] to find the optimal solution of real-life dual-hesitant fuzzy transportation problems. However, after a deep study, it is observed that Maity *et al*'s approach [17] is inappropriate. To validate this claim, in this paper, two dual-hesitant fuzzy transportation problems are solved by Maity *et al* [17] and shown that the obtained solutions are not appropriate. Also, it is pointed out that the inappropriateness in the obtained solutions is occurring due to using the inappropriate expression to transform a dual-hesitant fuzzy set into a real-number. Furthermore, to resolve the inappropriateness of Maity *et al*'s approach [17], a new expression (named as Mehar score function) and an appropriate approach (named as Mehar approach), based upon the proposed Mehar score function, is proposed to find the optimal solution of dual-hesitant fuzzy transportation problems.

This paper is organized as follows: In section 2, Maity *et al*'s approach [17] to find the optimal solution of a dual-hesitant fuzzy transportation problem is discussed. In section 3, some dual-hesitant fuzzy transportation problems are considered to point out the inappropriateness of Maity *et al*'s approach [17]. In section 4, the reason for the inappropriateness of Maity *et al*'s approach [17] is pointed out as well as a new expression (named as Mehar score function) is proposed to transform a dual-hesitant fuzzy set into a real number. In section 5, an appropriate approach (named as Mehar approach), based upon the proposed Mehar score function, is proposed to find the optimal solution for dual-hesitant fuzzy transportation problems. In section 6, the optimal solutions of the dual-hesitant fuzzy transportation problems, considered in section 5, are obtained with the help of the proposed Mehar approach. In section 7, the conclusions of the study are provided. .

2. Maity *et al's* approach to find the optimal solution of dual-hesitant fuzzy transportation problems

Maity *et al* [17] proposed the following approach to find the optimal solution of the dual-hesitant fuzzy transportation problem (represented by table 1).

Step 1 Transform the dual-hesitant fuzzy transportation problem (represented by Table 1) into its equivalent crisp transportation problem (represented by Table 2).

Step 2 Find the optimal solution $\{x_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\}$ of the transformed crisp transportation problem (represented by table 2). The obtained optimal solution represents the optimal solution of the dual-hesitant fuzzy transportation problem (represented by table 1).

Step 3 Using the optimal solution $\{x_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\}$, obtained in Step 2, find the optimal transportation cost $\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{(c_{ij1}+c_{ij2}+\cdots+c_{ijk}+\cdots+c_{ijp})x_{ij}}{p}$.

Table 1. Dual-hesitant fuzzy transportation problem.

	D_1	D_2	$\cdots$	D_n	
S_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\cdots$	$\tilde{c}_{1n}$	a_1
S_2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\cdots$	$\tilde{c}_{2n}$	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	$\cdots$	$\tilde{c}_{mn}$	a_m
	b_1	b_2	$\cdots$	b_n	

Where,

(i) The dual-hesitant fuzzy set $\tilde{c}_{ij} = \{\{\gamma_{ij1}, \gamma_{ij2}, \ldots, \gamma_{ijk}, \ldots, \gamma_{ijp}\}, \{\eta_{ij1}, \eta_{ij2}, \ldots, \eta_{ijk}, \ldots, \eta_{ijp}\}\}$ $(\{c_{ij1}, c_{ij2}, \ldots, c_{ijk}, \ldots, c_{ijp}\})$ represents the cost for supplying the unit quantity of the product from the i^{th} source S_i to the j^{th} destination D_j.

(ii) γ_{ijk} and η_{ijk} represents the degree of satisfaction and degree of dissatisfaction respectively of the decision maker with respect to the cost c_{ijk} required for transporting one unit quantity of the product from the i^{th} source S_i to the j^{th} destination D_j by the k^{th} vehicle and satisfies the conditions $0 \le \gamma_{ijk} \le 1, 0 \le \eta_{ijk} \le 1, \gamma_{ijk} + \eta_{ijk} \le 1$.

(iii) The real-number a_i represents the availability of the product at the i^{th} source S_i.

(iv) The real-number b_j represents the demand of the product at the j^{th} destination D_j.

(v) The natural number m represents the number of available sources.

(vi) The natural number n represents the number of available destinations.

Table 2. Transformed crisp transportation problem.

	D_1	D_2	$\cdots$	D_n	Av
S_1	$Score(\tilde{c}_{11})$	$Score(\tilde{c}_{12})$	$\cdots$	$Score(\tilde{c}_{1n})$	a_1
S_2	$Score(\tilde{c}_{21})$	$Score(\tilde{c}_{22})$	$\cdots$	$Score(\tilde{c}_{2n})$	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$Score(\tilde{c}_{m1})$	$Score(\tilde{c}_{m2})$	$\cdots$	$Score(\tilde{c}_{mn})$	a_m
	b_1	b_2	$\cdots$	b_n	

Where,

$$Score(\tilde{c}_{ij}) = Score\left(\left(\left\{\begin{matrix}\{\gamma_{ij1}, \gamma_{ij2}, \ldots, \gamma_{ijk}, \ldots, \gamma_{ijp}\}, \\ \{\eta_{ij1}, \eta_{ij2}, \ldots, \eta_{ijk}, \ldots, \eta_{ijp}\}\end{matrix}\right\}(\{c_{ij1}, c_{ij2}, \ldots, c_{ijk}, \ldots, c_{ijp}\})\right)\right) = \left|\frac{1}{p}\sum_{k=1}^{p}\gamma_{ijk} - \frac{1}{p}\sum_{k=1}^{p}\eta_{ijk}\right|.$$

3. Inappropriateness of Maity *et al*'s approach

It is inappropriate to use Maity *et al*'s approach [17] to solve dual-hesitant fuzzy transportation problems due to the following reasons:

1. Maity *et al*'s approach [17] fails to find the optimal solution of a dual-hesitant fuzzy transportation problem. To validate this claim, the dual-hesitant fuzzy transportation problem, considered in Example 1, is solved by Maity *et al*'s approach [17] and shown that the obtained solution is not an optimal solution.

Example 1. A supplier needs to supply the milk from two plants S_1 and S_2 to two places D_1 and D_2. The availability $a_i(i = 1, 2)$ of the milk at sources $S_i(i = 1, 2)$, the demands $b_j(j = 1, 2)$ of the milk at destinations $D_j(j = 1, 2)$ and the dual-hesitant fuzzy transportation cost for supplying one unit (100 L) quantity of milk from sources $S_i(i = 1, 2)$ to destinations $D_j(j = 1, 2)$ are mentioned in Table 3. The supplier is interested to find the optimal way for supplying the milk and the corresponding associated minimum transportation cost.

Using Maity *et al*'s approach [17], the optimal way for supplying the milk and the corresponding associated minimum transportation cost for the dual-hesitant fuzzy transportation problem (represented by table 3) can be obtained as follows:

Step 1 Using Step 1 of Maity *et al*'s approach [17], the dual-hesitant fuzzy transportation problem (represented by table 3) can be transformed into its equivalent crisp transportation problem (represented by table 4).

Step 2 On solving the crisp transportation problem (represented by table 4), the following optimal solution is obtained.

$$x_{11} = 20, x_{12} = 0, x_{21} = 0, x_{22} = 20.$$

Step 3 Using Step 3 of Maity *et al*'s approach [17], the optimal transportation cost is $\frac{(70+80+90)(20)}{3} + \frac{(10+20+30)(0)}{3} + \frac{(5+15)(0)}{2} + \frac{(80+90)(20)}{2} = 3300$.

It is obvious that according to Maity *et al*'s approach [17], the optimal solution of the dual-hesitant fuzzy transportation problem (represented by table 3) is $x_{11} = 20, x_{12} = 0, x_{21} = 0, x_{22} = 20$.

While, it is not the optimal solution as the total transportation cost of the crisp transportation problem (represented by table 4) corresponding to the feasible solution $x_{11} = 0, x_{12} = 20, x_{21} = 20, x_{22} = 0$ i.e., $\frac{(70+80+90)(0)}{3} + \frac{(10+20+30)(20)}{3} + \frac{(5+15)(20)}{2} + \frac{(80+90)(0)}{2} = 600$ is less than 3300.

This clearly indicates that Maity *et al*'s approach [17] fails to find the optimal solution of the considered dual-hesitant fuzzy transportation problem.

2. On applying Maity *et al*'s approach [17] different optimal transportation cost is obtained corresponding to alternative optimal solutions. This contradicts the well-known fact that the optimal transportation cost corresponding to all the possible alternative optimal solutions should be same. To validate this claim, the dual-hesitant fuzzy transportation problem, considered in Example 2, is solved by Maity *et al*'s approach [17].

Table 3. Dual-hesitant fuzzy transportation problem.

	D_1	D_2	Av
S_1	$\left\{\begin{matrix}\{0.5,0.4,0.3\},\\ \{0.1,0.2,0.3\}\end{matrix}\right\}$ $(70,80,90)$	$\left\{\begin{matrix}\{0.8,0.7,0.6\},\\ \{0.1,0.2,0.3\}\end{matrix}\right\}$ $(10,20,30)$	20
S_2	$\left\{\begin{matrix}\{0.8,0.6\},\\ \{0.1,0.3\}\end{matrix}\right\}$ $(5,15)$	$\left\{\begin{matrix}\{0.6,0.5\},\\ \{0.2,0.3\}\end{matrix}\right\}$ $(80,90)$	20
	20	20	

Example 2. A supplier needs to supply the milk from two plants S_1 and S_2 to two places D_1 and D_2. The availability $a_i(i = 1, 2)$ of the milk at sources $S_i(i = 1, 2)$, the demands $b_j(j = 1, 2)$ of the milk at destinations $D_j(j = 1, 2)$ and the dual-hesitant fuzzy transportation cost for supplying one unit (100 L) quantity of milk from sources $S_i(i = 1, 2)$ to destinations $D_j(j = 1, 2)$ are mentioned in table 5. The supplier is interested to find the optimal way for supplying the milk and the corresponding associated minimum transportation cost.

Using Maity *et al*'s approach [17], the optimal way for supplying the milk and the corresponding associated minimum transportation cost for the dual-hesitant fuzzy transportation (represented by table 5) can be obtained as follows:

Step 1 Using Step 1 of Maity *et al*'s approach [17], the dual-hesitant fuzzy transportation problem (represented by table 5) can be transformed into its equivalent crisp transportation problem (represented by table 6).

Step 2 On solving the crisp transportation problem (represented by table 6), the following two optimal basic feasible solutions are obtained:

(i) $x_{11} = 20, x_{12} = 0, x_{21} = 0, x_{22} = 20$.
(ii) $x_{11} = 0, x_{12} = 20, x_{21} = 20, x_{22} = 0$.

Step 3 Using Step 3 of Maity *et al*'s approach [17], the optimal transportation cost corresponding to

(i) The first optimal basic feasible solution $x_{11} = 20, x_{12} = 0, x_{21} = 0, x_{22} = 20$ is $\frac{(70+80+90)(20)}{3} + \frac{(10+20+30)(0)}{3} + \frac{(5+15)(0)}{2} + \frac{(80+90)(20)}{2} = 3300$.
(ii) The second optimal basic feasible solution $x_{11} = 0, x_{12} = 20, x_{21} = 20, x_{22} = 0$ is $\frac{(70+80+90)(0)}{3} + \frac{(10+20+30)(20)}{3} + \frac{(5+15)(20)}{2} + \frac{(80+90)(20)}{2} = 600$.

Table 4. Transformed crisp transportation problem.

	D_1	D_2	Av
S_1	$Score\left(\begin{matrix}\left\{\begin{matrix}\{0.5,0.4,0.3\},\\ \{0.1,0.2,0.3\}\end{matrix}\right\}\\ (70,80,90)\end{matrix}\right) = \left\lvert\frac{0.5+0.4+0.3}{3} - \frac{0.1+0.2+0.3}{3}\right\rvert = 0.2$	$Score\left(\begin{matrix}\left\{\begin{matrix}\{0.8,0.7,0.6\},\\ \{0.1,0.2,0.3,\}\end{matrix}\right\}\\ (10,20,30)\end{matrix}\right) = \left\lvert\frac{0.8+0.7+0.6}{3} - \frac{0.1+0.2+0.3}{3}\right\rvert = 0.5$	20
S_2	$Score\left(\begin{matrix}\left\{\begin{matrix}\{0.8,0.6\},\\ \{0.1,0.3\}\end{matrix}\right\}\\ (5,15)\end{matrix}\right) = \left\lvert\frac{0.8+0.6}{2} - \frac{0.1+0.3}{2}\right\rvert = 0.5$	$Score\left(\begin{matrix}\left\{\begin{matrix}\{0.6,0.5\},\\ \{0.2,0.3\}\end{matrix}\right\}\\ (80,90)\end{matrix}\right) = \left\lvert\frac{0.6+0.5}{2} - \frac{0.2+0.3}{2}\right\rvert = 0.3$	20
	20	20	

It is obvious that, on applying Maity *et al*'s approach [17], different optimal transportation cost is obtained corresponding to alternative optimal solutions, which is mathematically incorrect.

3. Maity *et al* [17] claimed that as the optimal transportation cost of a dual-hesitant fuzzy transportation problem with score value i.e., by their proposed approach will lie between the optimal transportation cost of dual-hesitant fuzzy transportation problem with minimum hesitant fuzzy cost and the optimal transportation cost of a dual-hesitant fuzzy transportation problem with maximum hesitant fuzzy cost. Therefore, the optimal solution, obtained by their proposed approach, is the best optimal solution. However, in actual case, this condition will not necessarily be satisfied. To validate this claim, the optimal transportation cost of the dual-hesitant fuzzy transportation problem, considered in Example 1, is obtained by considering the minimum hesitant fuzzy cost and maximum hesitant fuzzy cost.

The optimal transportation cost of the dual-hesitant fuzzy transportation problem, considered in Example 1, by considering the minimum hesitant fuzzy cost and maximum hesitant fuzzy cost can be obtained as follows:

Step 1 The crisp transportation problem (represented by table 7) represents the dual-hesitant fuzzy transportation problem with minimum hesitant fuzzy cost and the crisp transportation problem (represented by table 8) represents the dual-hesitant fuzzy transportation problem with maximum hesitant fuzzy cost corresponding to the dual-hesitant fuzzy transportation problem (represented by table 3).

Step 2 On solving the crisp transportation problem with minimum hesitant fuzzy cost (represented by table 7), the obtained optimal solution is $x_{11} = 0, x_{12} = 20, x_{21} = 20, x_{22} = 0$ and the corresponding optimal transportation cost is 300. Also, on solving the crisp transportation problem with maximum hesitant fuzzy cost (represented by table 8), the obtained optimal solution is $x_{11} = 0, x_{12} = 20, x_{21} = 20, x_{22} = 0$ and the corresponding optimal transportation cost is 900.

Table 5. Dual-hesitant fuzzy transportation problem.

	D_1	D_2	Av
S_1	$\left\{ \begin{array}{c} \{0.5, 0.4, 0.3\}, \\ \{0.1, 0.2, 0.3\} \end{array} \right\}$ (70, 80, 90)	$\left\{ \begin{array}{c} \{0.6, 0.5, 0.4\}, \\ \{0.2, 0.3, 0.4\} \end{array} \right\}$ (10, 20, 30)	20
S_2	$\left\{ \begin{array}{c} \{0.6, 0.4\}, \\ \{0.2, 0.4\} \end{array} \right\}$ (5, 15)	$\left\{ \begin{array}{c} \{0.6, 0.5\}, \\ \{0.3, 0.4\} \end{array} \right\}$ (80, 90)	20
	20	20	

Table 6. Transformed crisp transportation problem.

	D_1	D_2	Av
S_1	$Score\left(\begin{array}{c} \left\{ \begin{array}{c} \{0.5, 0.4, 0.3\}, \\ \{0.1, 0.2, 0.3\} \end{array} \right\} \\ (70, 80, 90) \end{array} \right) = \left| \frac{0.5+0.4+0.3}{3} - \frac{0.1+0.2+0.3}{3} \right| = 0.2$	$Score\left(\begin{array}{c} \left\{ \begin{array}{c} \{0.6, 0.5, 0.4\}, \\ \{0.2, 0.3, 0.4\} \end{array} \right\} \\ (10, 20, 30) \end{array} \right) = \left| \frac{0.6+0.5+0.4}{3} - \frac{0.2+0.3+0.4}{3} \right| = 0.2$	20
S_2	$Score\left(\begin{array}{c} \left\{ \begin{array}{c} \{0.6, 0.4\}, \\ \{0.2, 0.4\} \end{array} \right\} \\ (5, 15) \end{array} \right) = \left| \frac{0.6+0.4}{2} - \frac{0.2+0.4}{2} \right| = 0.2$	$Score\left(\begin{array}{c} \left\{ \begin{array}{c} \{0.6, 0.5\}, \\ \{0.3, 0.4\} \end{array} \right\} \\ (80, 90) \end{array} \right) = \left| \frac{0.6+0.5}{2} - \frac{0.3+0.4}{2} \right| = 0.2$	20
	20	20	

Table 7. Transportation problem with minimum hesitant fuzzy cost.

	D_1	D_2	Av
S_1	70	10	20
S_2	5	80	20
	20	20	

Table 8. Transportation problem with maximum hesitant fuzzy cost.

	D_1	D_2	Av
S_1	90	30	20
S_2	15	90	20
	20	20	

Furthermore, it is obvious from Step 3 of Example 1 that on solving the crisp transportation problem with score value (represented by Table 4), the obtained optimal transportation cost is 3300.

It is obvious that the optimal transportation cost of the transportation problem with score value i.e., 3300 does not lie between the optimal transportation cost of the dual-hesitant fuzzy transportation problem with minimum hesitant fuzz cost i.e., 300 and the optimal transportation cost of the dual-hesitant fuzzy transportation problem with maximum hesitant fuzzy cost i.e., 900.

4. Proposed Mehar score function

It is obvious that the expression

$$Score(\tilde{c}_{ij}) = Score\left(\left(\begin{matrix}\{\gamma_{ij1},\gamma_{ij2},\ldots,\gamma_{ijk},\ldots,\gamma_{ijp}\},\\ \{\eta_{ij1},\eta_{ij2},\ldots,\eta_{ijk},\ldots,\eta_{ijp}\}\end{matrix}\right)(\{c_{ij1},c_{ij2},\ldots,c_{ijk},\ldots,c_{ijp}\})\right) = \left|\frac{1}{p}\sum_{k=1}^{p}\gamma_{ijk} - \frac{1}{p}\sum_{k=1}^{p}\eta_{ijk}\right|,$$

used by Maity *et al* [17], is independent from the values of $c_{ij1}, c_{ij2}, \ldots, c_{ijk}, \ldots, c_{ijp}$. Due to the same reason, Maity *et al*'s approach [17] fails to find the appropriate solution of the considered dual-hesitant fuzzy transportation problems. This expression is proposed by considering the assumption that the decision maker would like to maximize the value of $\frac{1}{p}\sum_{k=1}^{p}\gamma_{ijk}$ and to minimize the value of $\frac{1}{p}\sum_{k=1}^{p}\eta_{ijk}$ simultaneously. While, in actual case, the decision maker would like to minimize the value of $\frac{1}{p}\sum_{k=1}^{p}c_{ijk}$, to maximize the value of $\frac{1}{p}\sum_{k=1}^{p}\gamma_{ijk}$ and to minimize the value of $\frac{1}{p}\sum_{k=1}^{p}\eta_{ijk}$ simultaneously. Therefore, it is appropriate to use the following expression (named as Mehar score function) to transform a dual-hesitant fuzzy set into a real number instead of using the existing expression:

$$MScore(\tilde{c}_{ij}) = MScore\left(\left\{\begin{matrix}\{\gamma_{ij1},\gamma_{ij2},\ldots,\gamma_{ijk},\ldots,\gamma_{ijp}\},\\ \{\eta_{ij1},\eta_{ij2},\ldots,\eta_{ijl},\ldots,\eta_{ijp}\}\end{matrix}\right\}(\{c_{ij1},c_{ij2},\ldots,c_{ijk},\ldots,c_{ijp}\})\right) = \frac{1}{p}\sum_{k=1}^{p}c_{ijk} + \frac{1}{p}\sum_{k=1}^{p}\eta_{ijk} - \frac{1}{p}\sum_{k=1}^{p}\gamma_{ijk}$$

5. Proposed Mehar approach

It is obvious from Section 3 that it is not appropriate to use Maity *et al*'s approach [17] is to find the optimal solution of dual-hesitant fuzzy transportation problems.

Table 9. Transformed crisp transportation problem.

	D_1	D_2	$\cdots$	D_n	Av
S_1	$MScore(\tilde{c}_{11})$	$MScore(\tilde{c}_{12})$	$\cdots$	$MScore(\tilde{c}_{1n})$	a_1
S_2	$MScore(\tilde{c}_{21})$	$MScore(\tilde{c}_{22})$	$\cdots$	$MScore(\tilde{c}_{2n})$	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$MScore(\tilde{c}_{m1})$	$MScore(\tilde{c}_{m2})$	$\cdots$	$MScore(\tilde{c}_{mn})$	a_m
D	b_1	b_2	$\cdots$	b_n	

In this section, an appropriate approach (named as Mehar approach) is proposed to find the optimal solution of the dual-hesitant fuzzy transportation problems.

The steps of the proposed Mehar approach are as follows:

Step 1 Transform the dual-hesitant fuzzy transportation problem (represented by table 2) into the crisp transportation problem represented by table 9.where

$$MScore(\tilde{c}_{ij}) = \frac{1}{p}\sum_{k=1}^{p}c_{ijk} + \frac{1}{p}\sum_{k=1}^{p}\eta_{ijk} - \frac{1}{p}\sum_{k=1}^{p}\gamma_{ijk}$$

Step 2 Find the optimal solution $\{x_{ij}, i = 1,2,\ldots,m; j = 1,2,\ldots,n\}$ and the optimal transportation of the transformed crisp transportation problem (represented by Table 9). The obtained optimal solution and the obtained optimal transportation cost represents the optimal solution and optimal transportation cost respectively of the dual-hesitant fuzzy transportation problem (represented by Table 2).

Remark 1 In the proposed Mehar approach, the considered dual-hesitant fuzzy transportation problem is firstly transformed into its equivalent crisp transportation problem. Then, the optimal solution of the transformed crisp transportation problem is obtained. Since, there exist several methods in the literature to solve a crisp transportation problem and hence, different researchers may use different methods to solve the transformed crisp transportation problem. Therefore, the computational time complexity and accuracy performance of the proposed Mehar approach will be same as the computational time complexity and accuracy performance of that approach which will be used to solve the transformed crisp transportation problem.

6. Optimal solutions of the considered dual-hesitant fuzzy transportation problems

In section 3, two dual-hesitant fuzzy transportation problems (represented by table 3 and table 5) are solved by Maity *et al*'s approach [17] and pointed out Maity *et al*'s approach [17] fails to find the optimal solutions of the considered dual-hesitant fuzzy transportation problems. In this section, the appropriate solutions of these problems are obtained by the proposed Mehar approach.

6.1 *Optimal solution of the first dual-hesitant fuzzy transportation problem*

Using the proposed Mehar approach, the optimal solution of the first dual-hesitant fuzzy transportation problem (represented by table 3) can be obtained as follows:

Step 1 Using Step 1 of the Mehar approach, proposed in section 5, the dual-hesitant fuzzy transportation problem (represented by table 3) can be transformed into its equivalent crisp transportation problem (represented by table 10).

Step 2 On solving the crisp transportation problem (represented by table 10), the obtained optimal solution is $x_{11} = 0, x_{12} = 20, x_{21} = 20, x_{22} = 0$ and the obtained optimal transportation cost is 580.

6.2 *Optimal solution of the second dual-hesitant fuzzy transportation problem*

Using the proposed Mehar approach, the optimal solution of the second dual-hesitant fuzzy transportation problem (represented by table 5) can be obtained as follows:

Step 1 Using Step 1 of the Mehar approach, proposed in section 5, the dual-hesitant fuzzy transportation problem (represented by table 5) can be transformed into its equivalent crisp transportation problem (represented by table 11).

Step 2 On solving the crisp transportation problem (represented by table 11), the obtained optimal solution is $x_{11} = 0, x_{12} = 20, x_{21} = 20, x_{22} = 0$ and the obtained optimal transportation cost is 592.

Table 10. Transformed crisp transportation problem.

	D_1	D_2	
S_1	$MScore\left(\begin{array}{c}\left\{\begin{array}{c}\{0.5,0.4,0.3\},\\ \{0.1,0.2,0.3\}\end{array}\right\}\\ (70,80,90)\end{array}\right) = \frac{70+80+90}{3}+\frac{0.1+0.2+0.3}{3}-\frac{0.5+0.4+0.3}{3} = 79.8$	$MScore\left(\begin{array}{c}\left\{\begin{array}{c}\{0.8,0.7,0.6\},\\ \{0.1,0.2,0.3,\}\end{array}\right\}\\ (10,20,30)\end{array}\right) = \frac{10+20+30}{3}+\frac{0.1+0.2+0.3}{3}-\frac{0.8+0.7+0.6}{3} = 19.5$	20
S_2	$MScore\left(\begin{array}{c}\left\{\begin{array}{c}\{0.8,0.6\},\\ \{0.1,0.3\}\end{array}\right\}\\ (5,15)\end{array}\right) = \frac{5+15}{2}+\frac{0.1+0.3}{2}-\frac{0.8+0.6}{2} = 9.5$	$MScore\left(\begin{array}{c}\left\{\begin{array}{c}\{0.6,0.5\},\\ \{0.2,0.3\}\end{array}\right\}\\ (80,90)\end{array}\right) = \frac{80+90}{2}+\frac{0.2+0.3}{2}-\frac{0.6+0.5}{2} = 84.7$	20
	20	20	

Table 11. Transformed crisp transportation problem.

	D_1	D_2	
S_1	$MScore\left(\begin{array}{c}\left\{\begin{array}{c}\{0.5,0.4,0.3\},\\ \{0.1,0.2,0.3\}\end{array}\right\}\\ (70,80,90)\end{array}\right) = \frac{70+80+90}{3}+\frac{0.1+0.2+0.3}{3}-\frac{0.5+0.4+0.3}{3} = 79.8$	$MScore\left(\begin{array}{c}\left\{\begin{array}{c}\{0.6,0.5,0.4\},\\ \{0.2,0.3,0.4,\}\end{array}\right\}\\ (10,20,30)\end{array}\right) = \frac{10+20+30}{3}+\frac{0.2+0.3+0.4}{3}-\frac{0.6+0.5+0.4}{3} = 19.8$	20
S_2	$MScore\left(\begin{array}{c}\left\{\begin{array}{c}\{0.6,0.4\},\\ \{0.2,0.4\}\end{array}\right\}\\ (5,15)\end{array}\right) = \frac{5+15}{2}+\frac{0.2+0.4}{2}-\frac{0.6+0.4}{2} = 9.8$	$MScore\left(\begin{array}{c}\left\{\begin{array}{c}\{0.6,0.5\},\\ \{0.3,0.4\}\end{array}\right\}\\ (80,90)\end{array}\right) = \frac{80+90}{2}+\frac{0.3+0.4}{2}-\frac{0.6+0.5}{2} = 84.8$	20
	20	20	

7. Conclusions

It is pointed out that Maity *et al*'s approach [17] is not appropriate. Also, it is pointed out that the inappropriateness of score function, used by Maity *et al* [17], is the reason for the inappropriateness of Maity *et al*'s approach [17]. Furthermore, to resolve the inappropriateness of Maity *et al*'s approach [17], a new expression (named as Mehar score function) and an appropriate approach (named as Mehar approach), based upon the proposed Mehar score function, is proposed. In future, the proposed Mehar approach may be extended to solve generalized dual-hesitant intuitionistic fuzzy multi-objective transportation problems which are the generalization of the existing intuitionistic fuzzy multi-objective transportation problems [18].

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