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Development of a Markov chain based tool for studying effectiveness of Vendor Managed Inventory and result analysis from a pilot study

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Abstract. Vendor Managed Inventory is looked upon as a reliable method for inventory management. There is still a scope for improving the process. In this work, we presented a report on the pilot setting and its testing for Vendor Managed Inventory in an auto parts manufacturing company in Northern India. The pilot study was conducted by switching supplies of six components of an assembly from conventional to VMI mode. The switch was carried out in January 2018 and a time series plotting of five performance indicators nine months prior to and nine months after the switch was done and analysed. Further, transition probabilities of inventory performance index were estimated based upon the correlations with four most influential performance indicators. Sixteen state switching scenarios were modelled and Markov chain analysis of Inventory Performance Index for two of them was carried out to test the VMI performance.

Keywords. Vendor Managed Inventory (VMI); Time series; Markov chain analysis.

1. Introduction

Vendor Managed Inventory (VMI) is a strategic engagement between manufacturers and their suppliers/vendors [1]. Under this engagement, the vendors manage the flow of materials and information, and hold stocks based on direct assessment of demands generated at the coupling points with the customers [2]. It is a complex strategy that cannot be generalised as it incorporates uniqueness of strategic and operations objectives and goals, resources, structures, capabilities, processes, products, integration, policies, laws, regulations, and technology of a specific industry [3, 4]. The traditional management practices assisting VMI are just in time (JIT), total quality management (TQM), materials requirements planning (MRP), total productive maintenance (TPM), enterprise resources planning (ERP), and business process reengineering (BPR) [5].

VMI rides on closely integrated communication, collaboration, and information sharing structures and the technologies enabling those [6, 7]. In traditional industries, organisational, cultural, financial, and operational barriers are key hurdles to VMI [8–10]. Yet, VMI strives on its efficiency and effectiveness promises and ability to deliver riding on technology enhancements and automation as evident in the modern supply chains [11, 12]. Based on timely and accurate information collection, VMI supports a manufacturer to effectively face uncertainties in demands through appending of additional capacities, reducing ordering costs, reducing inventory carrying costs, and improving just-in-time replenishment ability by responding to demand uncertainties dynamically [13, 14]. A detailed learning for replenishing inventory using consignment inventory in firms using VMI is studied, researchers proposed methodology rooted in Bellman equation [15]. In a recent study, a single supplier multi-retailer setting where supplier is authorized to take all inventory-related decisions, they developed a non-linear mixed integer programming model to compare the optimal replenishment frequency and quantity for each of the retailers, they found that the model is more stable than the previous models [16].

Tilokavichai *et al* [17] developed a Markov model for a logistics network where the stochastic parameters chosen with known distribution are: demand of outbound logistics, inbound logistics, and products returned from customers. The model was used for logistics activity problems having high holding cost and uncertain demand pattern. Markov chain is applied by researchers for providing estimates to managers and obtaining estimates for future. They carried out a case study in Thailand to show the applications of the proposed model and yielded a very effective logistics performance network at very high level of customer satisfaction.

The Markov analysis gives a probabilistic information regarding a particular situation [18]. Researchers Xia *et al* [19] observed various time sequences and grouped it differently into various groups and within each group they formulated relevance among multivariate longitudinal data. They applied Markov Chains Monte Carlo for posterior

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analysis and then used Gibbs sampling to draw the conclusions, Mathew *et al* [20] have studied the public-sector undertaking data of the year 2016–2018 in Cochin, Kerala (India) and made manpower recruitment plans and movement of employees within organization. Markov analysis proved to be an effective tool for HR decision making. Markov Analysis gives reliable probabilistic results, which inspired us to use this technique for calculating inventory performance index in our pilot study.

In this article, the report of a pilot project conducted in an auto parts manufacturing company in India is presented. The pilot was conducted by introducing a VMI agreement with one of the vendor producing six different sub-components for the manufacturing company through an old outsourcing contract for car seat structure assembly. The contract was modified to enable vendor to manage the local inventory of the supplied sub-components (Railings, Mounting brackets, Frames) in an old warehouse of the manufacturing company (Customer). This article presents details of the pilot setting, the longitudinal test design [21], the data set collected, test results, and the final discussions and conclusions on the pilot study.

2. The pilot setting

An auto parts manufacturer in Northern India was selected for the pilot testing of VMI. The manufacturer (customer) had old contracts with multiple vendors for producing sub-components. One of the vendors was selected for this pilot. The purchase ordering process between the manufacturer (customer) and the vendor was suspended and the vendor was allowed managing the onsite inventory of their components directly. For this purpose, the vendor assigned an onsite coordinator, who was trained on a module of the materials requirements planning (MRP) version II on SAP software for tracking the production usage and buffer stocks of their components only. Based on the requirements generated, the onsite coordinator generated weekly consignment requests for the vendor to ship the sub-components as per the requirement specifications. This process replaced the earlier monthly purchase ordering system managed by an employee of the customer.

This process retained the delivery challans (formal document that is sent along with shipment and contain details of the items and quantity of good delivered) being issued to claim the payments. The inspection process was shifted upstream from the delivery stage to the buffer feed stage. All delivery inspection processes were suspended for this pilot. The inspections were carried out by the vendor, and no separate bills were raised to the customer. Instead, the scope of inspections at the buffer feeding was increased.

As the consignments were shipped weekly instead of monthly, the vendor started using small delivery vehicles instead of mini-trucks. The packaging was also changed from large metallic containers to small wooden/ply-board boxes fit for handling smaller consignments. The docking and unloading, unpacking, internal movement, and storing processes were managed by the vendor onsite with the help of workers on their payrolls. Earlier, these tasks were handled by contract workers hired by the customers; those were replaced by the workers hired by the vendor. In the new VMI arrangement, the vendor billed for these services to the customer. The inventory codes used by the customers in the MRP II were used by the vendor although they were not delivered against purchase orders. The payment cycles shifted from on-delivery (based on delivery challans) to on-buffer-feed (based on pre-feed inspection clearances) [22]. However, transportation costs were billed by the vendor on delivery challans as per the older agreement because they did not agree to eliminate it in the new arrangement. The earlier arrangement of taking back unused materials applied to this pilot as well.

3. The test design

The test design composed of collection of financial and performance data and four different metrics were chosen in agreement with both the customer and vendor, the financial and performance data is related to the following heads:

(a) Transportation cost (actual costs billed to the customer)
(b) Material handling cost (sum of actual services costs billed to customer for docking and unloading, unpacking, internal movements within the warehouse, and storing)
(c) Cost of the sub-components (actual costs billed to the customer after the materials were passed by inspection officer at the buffer entry point of the assembly line/machine, where the six sub-components were consumed)
(d) Lead times (time difference between the requirement generation timestamp in MRP II and the inspection approval timestamp at the buffer entry point at the assembly unit)

The miscellaneous costs, such as electricity, general warehouse maintenance costs, cleaning and housekeeping costs, etc., were kept out of this test design as they were not billed by the vendor and were presumed to remain unchanged in the VMI arrangement. A fifth metric was captured from SAP application used by the manufacturing company to run the MRP II. SAP means System Applications and Products in Data Processing; it is a commercial producer of MRP II system used here. SAP generates an internal inventory performance index (IPI) for each item code stored in the inventory, which is based on more than 50 variables captured internally within SAP and indicates a value in the scale 1 to 10 (higher is better). For example, a value at 7.3 indicates that the IPI score is at 73% of an internally estimated SAP benchmark for inventory performance, and there is a 27% scope of improvement.

The data sets for the pilot after introducing the VMI were collected for three quarters (9 months), and integrated with three quarters of data collected prior to introducing the VMI on a time series. The overall data table included data collected from April 2017 to September 2018. The first analysis conducted was a basic time series plotting of the data sets and observing the probability relationships between the variables. The second analysis was done using the Markov chain analysis by assuming correlation coefficients as transition probabilities. Correlation coefficients were used for estimating the initial reachability matrix and evolving the final reachability matrix by considering transitive relationships. By completing the iterative steps using interpretive structural modelling technique (ISM), the most relevant variables influencing VMI were obtained in the eighth iteration: reduced costs and improved services [23]. This method, however, was not having any prediction ability. This is because the transitive relationships do not indicate any predictive probability of the VMI performance in MICMAC analysis.

Markov chain analysis was chosen for the pilot study because:

(a) It has an ability of predictive analytics based on transition probabilities;
(b) It does not require direct application of historical data if there is a method followed to estimate the transition probabilities between the current state to the next state in the graph; that is, Markov chain does not require a memory of transition probabilities such that the next state transition can be estimated based on the conditions at the current state only;

A review of Markov chain analysis is presented as follows:

Markov chain analysis is used for a variable that is not influenced by its past values [24, 25]. It works by forming a state space 0 to S over a time domain that is finite and in which, there are discrete jumps from one state to another represented by the Markov probability matrix presented below [24]:

$$A = \begin{bmatrix} P_{00} & P_{01} & P_{02} & \cdots & P_{0s} \\ P_{10} & P_{11} & P_{12} & \cdots & P_{1s} \\ P_{20} & P_{21} & P_{22} & \cdots & P_{2s} \\ P_{s0} & P_{s1} & P_{s2} & \cdots & P_{ss} \end{bmatrix} \tag{1}$$

where 1, 2, 3…s are states, P_{ss} is the conditional probability of transition from state ‘s’ to ‘s’ itself.

The Markov probability matrix is always M X M and the sum of probabilities in either a row or a column is always unity [24]. In a simple experimentation, the values of P_{00} to P_{SS} can be estimated by running multiple trials and carefully examining the change of state of a variable and noting the different states of the variable [24, 25]. Then, the transition probability between any two states is the ratio of number of times the state transition between these two states was observed to the total number of trials conducted.

When a variable is complex (that is, influenced by many other factor variables) then the simple experimentation setting will not be helpful [24]. To study the Markov chain of a complex variable, a controlled experimentation setup needs to be formed in which, probabilities of occurrence of each factor variable need to be taken into the sample. Sampling of multiple random variables in a controlled experimentation setting for Markov chain analysis of a complex variable was discussed by Walsh [26]. The sampling equation for “m” events realisations is presented as follows:

$$E[f(x)]_m = \frac{1}{m}\sum_{i=1}^{m} f(x_i) \tag{2}$$

where $\quad integral\ of\ f(x) = \frac{1}{m}\sum_{i=1}^{m} f\left(\frac{x}{y_i}\right) = \int f\left(\frac{x}{y_i}\right) f(y_i)dy_i$, where y_i are draws from m, f(x) is generated by all $f(x_i)$ and $E[f(x)]_m$ is expectation of f(x) over m.

If the factor variables and the influenced variable are all occurring on a common time series (that is, they are overlapping), then the next state probability of the influenced variable can be estimated by its correlation with other overlapping variables on the time series. The correlation may be estimated using auto- or cross-correlation, or Pearson correlation. This is not a pure Markov Chain albeit uses some historical knowledge based on Gibbs sampling [26]. It works for sufficiently long chain length. If the chain length tends to infinity, the above equation becomes a Monte Carlo estimate using Gibbs sampling space.

The individual transition probabilities of the influencing variables shall be between 0 and 1, and their sum shall be unity. One more aspect to be kept in mind is about the direction of transition of the influenced variable. For example, if X is a variable influencing Y, then if Y increases based on a transition probability “p” of X, it will decrease based on a transition probability “(1–p)” of X. If there is a chance that Y may increase without any change in X, then the probability “p” needs to be split into two transition probabilities. Similarly, if there is chance that Y may reduce without any change in X, then the probability “(1–p)” needs to be split into two transition probabilities.

In this research, the Pearson correlation coefficients between the four state variables and the IPI variable are taken as transition probabilities. An increase or decrease of IPI is investigated based on increase or decrease of the state variables. Total 16 scenarios are possible as there are four state variables. The test results are presented in subsequent section. The next section presents the test data.

3.1 *Data set*

The data set comprised of monthly values recorded of transportation cost, material handling cost, cost of the sub-components, lead times, and internal inventory performance index (IPI) of SAP from April 2017 to September 2018. The VMI agreement was signed on 14 December 2017 with its commencement date as 1st January 2018. table 1 presents monthwise data collected from the manufacturer. Prior to VMI introduction in January 2018, the data was collected from internal costing journals maintained by the cost accountant (that were later fed to the internal financial application formed by a module in SAP). From January 2018, onwards, the data was collected from the bills raised by the vendor assigned the VMI contract.

The data set was entered into SPSS for further testing and analysis in three steps: times series analysis, Pearson correlation analysis, and Markov chain predictive modelling. The time series analysis was conducted to study the variations in discrete values of the variables over the experimentation period longitudinally (for eighteen months). The Pearson correlation coefficients were obtained at p = 0.01 (99% confidence interval). The correlation coefficient reflects the extent to which, two different variables transition together. For example, a Pearson correlation coefficient of 0.5 indicates that two variables are transitioning together 50% of the time. Hence, the Pearson correlation can be used as the transition probability between two variables. When multiple variables are influencing the variable under study, and their events are sampled separately, then the following equation can be used to calculate the combined transition probabilities of the variables influencing the main variable using Gibbs sampling [26] this may lead to Monte Carlo approximation when the time series chain of the overlapping stochastic variables is infinite:

$$P(E)_i = \frac{1}{m}\sum_{i=1}^{m} P(x_i) \tag{3}$$

where m = number of samples; $P(E)_i$ = transition probability of the variable studied;

$P(x_i)$ = transition probability of each influencing variable.

The Markov chains are analysed accordingly to predict the variables causing the IPI to increase or reduce. The test results and analysis are discussed in the next section.

3.2 *Test results*

To evaluate the test results, the month of January 2018 should be remembered as the month of change as the inventory management system was switched from internally managed to VMI in this month. In all the time series charts the change occurring in and after January 2018 needs to be studied carefully.

The first variable studied on the time series is transportation cost (TC). TC exhibited a reducing trend from the first month of VMI itself, in our case it is billed to customer. The per-kilometre costs of smaller vehicles are lower than trucks. The vendor used smaller vehicles as the materials were shipped on weekly internal demands raised instead of monthly orders. This also reduced road taxes and tollgate expenses as per the government slabs. In April 2018, the transportation costs dipped significantly, and then exhibited an increasing trend thereafter. Although, one may safely assume that the transportation cost during VMI will be lower as compared to when inventories were managed internally. However, the actual picture will be clearer after collecting data for a longer period. An increasing trend was evident after a sudden dip in April 2018 but remained lower than the pre-VMI TC (figure 1).

The materials handling cost (MHC) shot up significantly in the first month of the VMI. Thereafter, MHC exhibited a sharp declining trend to costs close to pre-VMI period. The MHC reduced below pre-VMI levels only in the month of September 2018, which is the last month of this pilot test study. In January 2018, the vendor had to deploy more resources during the takeover process, and later the resources were withdrawn when the internal processes streamlined. Given that the vendor decided to follow a weekly replenishment schedule, new internal storage bays were constructed for stocking smaller wooden packs instead of the large metallic containers used earlier. In addition, new machines for unpacking from wooden boxes and internal movement of smaller packages were acquired. This may explain the sudden rise of MHC in January and February 2018 (figure 2).

The cost of the components (COTC) comprises the six sub-components supplied by the vendor for the assembly line selected by the manufacturer/customer to test VMI. The COTC showed clear reducing trend after introduction of VMI (figure 3). The vendor reported reduced wastage of materials because of using smaller and easy-to-open packages, shorter storage inventory holding, and lighter internal movement in small volumes. Further, the vendor also reduced over ordering to some extent by carefully analysing the materials consumption in MRP II and switching to a weekly replenishment schedule. These adjustments helped in achieving a reducing trend of COTC. During the pilot test period, the reducing trend has not stopped. Hence, further observations will be needed.

The lead-time (LDT) increased sharply in the months of January and February 2018. The vendor reported some initial delays when the switchover to VMI was in progress. The delays were primarily because of the time taken in delivery of the new materials-handling equipment to manage the weekly replenishment schedule (instead of monthly schedule) and to handle smaller consignments coming in wooden or ply wood boxes instead of metal containers. Thereafter, LDT exhibited a sharp decline to levels lower than the pre-VMI period. At the end of the pilot period, LDT exhibited an increasing trend again (figure 4). Further sampling is needed to study if the trend has increased further or has stabilised after some time to values lower than the pre-VMI period.

Table 1. Data collected from the pilot implementation and testing of VMI.

Month	Transportation cost (TC) INR inclusive of taxes (Indian rupees)	Material handling cost (MHC) INR inclusive of taxes (Indian rupees)	Cost of the sub-components INR inclusive of taxes (Indian rupees)	Lead times (LDT) (hours)	Inventory performance index (IPI) (captured from SAP) (higher is better)
Sept 2018	1,50,305	1,91,570	74,98,990	74	8.3
Aug 2018	1,46,764	1,94,478	75,99,978	73	8.4
July 2018	1,48,115	1,95,395	75,78,745	78	8.3
June 2018	1,51,980	1,97,899	77,67,448	81	8.3
May 2018	1,53,450	1,97,576	79,76,939	83	8.1
Apr 2018	1,44,676	1,99,879	81,37,494	84	8.0
Mar 2018	1,57,358	2,08,056	85,49,224	89	8.0
Feb 2018	1,61,000	2,29,787	87,48,492	86	7.6
Jan 2018	1,67,455	2,34,445	87,48,944	84	7.5
Dec 2017	1,78,760	1,97,643	87,67,658	82	7.8
Nov 2017	1,74,285	1,96,381	87,33,483	83	7.6
Oct 2017	1,75, 976	1,97,948	89,63,743	83	7.9
Sept 2017	1,74,008	1,98,983	88,75,491	80	7.5
Aug 2017	1,76,280	1,95,294	88,93,284	79	7.8
July 2017	1,77,224	1,94,209	89,56,923	77	7.7
June 2017	1,79,512	1,93,492	89,46,385	83	7.9
May 2017	1,78,330	1,95,836	88,24,953	79	7.7
Apr 2017	1,74,560	1,97,653	87,50,591	78	7.8

The inventory performance index (IPI) reduced to a low value in January 2018 as presented in figure 5 (higher values of IPI reflect better inventory management performance). The IPI gained slightly in the month of February 2018. Thereafter, it increased sharply and maintained an increasing trend until August 2018. After August 2018, a declining trend had started. Further sampling is needed to observe if the declining trend proceeds or IPI stabilizes at values lower than the pre-VMI period. The IPI clearly reflects a good initial performance of the vendor in managing the VMI. Much longer trends will reflect if the performance sustained. As described by Walsh (2004) [26], an infinite time series comprising of multiple stochastic variables (number of events tending to infinity) follows Monte Carlo approximation of Gibbs sampling (Eq. 2) and may exhibit a cooling period before stabilising. Because of stochastic nature of the variables, it is difficult to estimate how long the cooling period would continue. In this pilot testing period, it is difficult to estimate if the cooling period is over or will continue. Initial shocks were evident in MHC, LDT, and IPI. However, it is very difficult to ascertain if the VMI system has cooled down after the initial shocks.

Hence, a more realistic approach to evaluate the performance of VMI is to study Markov chains with Monte Carlo approximation of Gibbs sampling (Eq. 2). As studied earlier in this chapter, Markov chains only require the observations at the current state to predict the future state (that is,

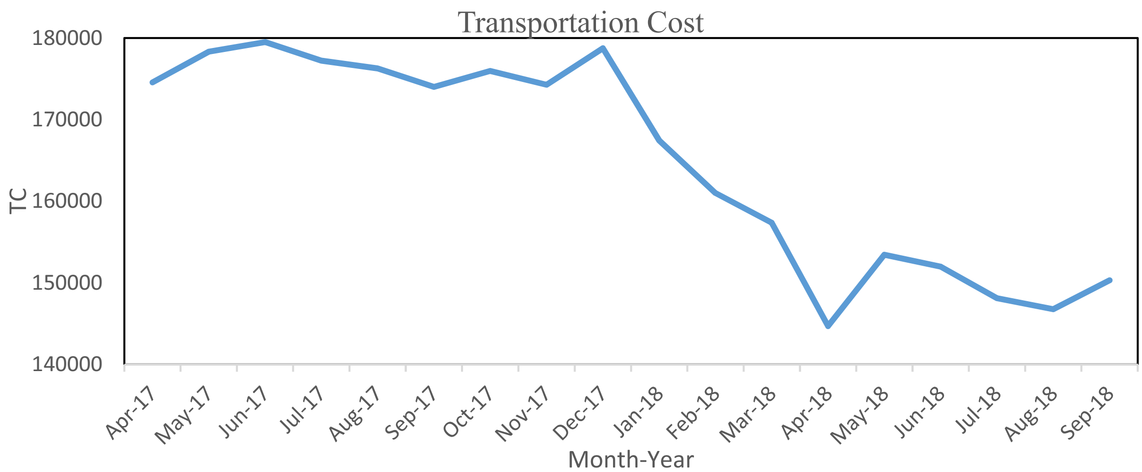

Figure 1. Time series plotting of the transportation costs (TC).

Figure 2. Time series plotting of the materials handling costs (MHC).

Markov chains have no memory). However, Monte Carlo approximation of Gibbs sampling helps in deriving the transition probabilities from the historical sampling of the time series having overlapping variables.

The IPI is used as the reference variable because it reflects inventory management performance and is estimated internally in the SAP application based on more than 50 variables. The correlations between IPI and the rest of the variables (TC, MHC, COTC, and LDT) are presented in table 2. Although multiple correlation coefficients can be calculated using SPSS, the Pearson correlation coefficient has been chosen as it was earlier used in our previous articles.

The correlations are captured at 0.01 level for ensuring 99% confidence interval of the sampled events. The correlation coefficients are negative because reduction in TC, MHC, COTC, and LDT causes increase of IPI. The Markov chain is formed to predict three states of IPI: IPI remains the same, IPI shall increase, and IPI shall decrease. Taking the Pearson correlation coefficients as transition probabilities, the following transition probabilities of IPI are estimated:

(a) IPI shall increase with reduction in TC (or IPI shall reduce with increase in TC) at a probability $P_{IPI/TC} = 0.739$.
(b) IPI shall reduce with reduction in TC (or IPI shall increase with increase in TC) at a probability closer to $(1 - P_{IPI/TC}) - 0.01 = 0.251$ (although it might appear unlikely, a small probability of 0.01 should be assumed that IPI will not change with either increase or decrease of the influential variables).
(c) IPI shall remain unchanged with either reduction or increase of TC = 0.01
(d) IPI shall increase with reduction in MHC (or IPI shall reduce with increase in MHC) at a probability $P_{IPI/TC} = 0.462$.

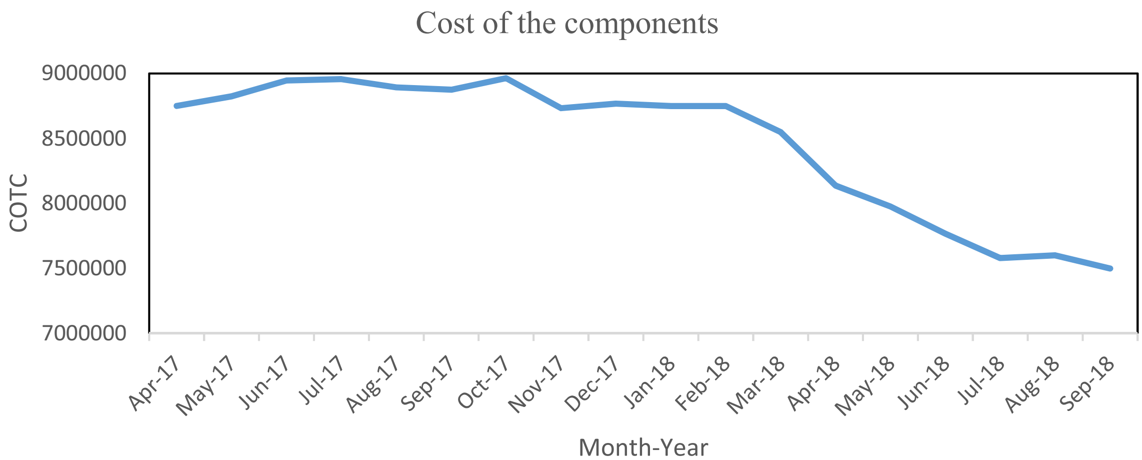

Figure 3. Time series plotting of the costs of the components (COTC).

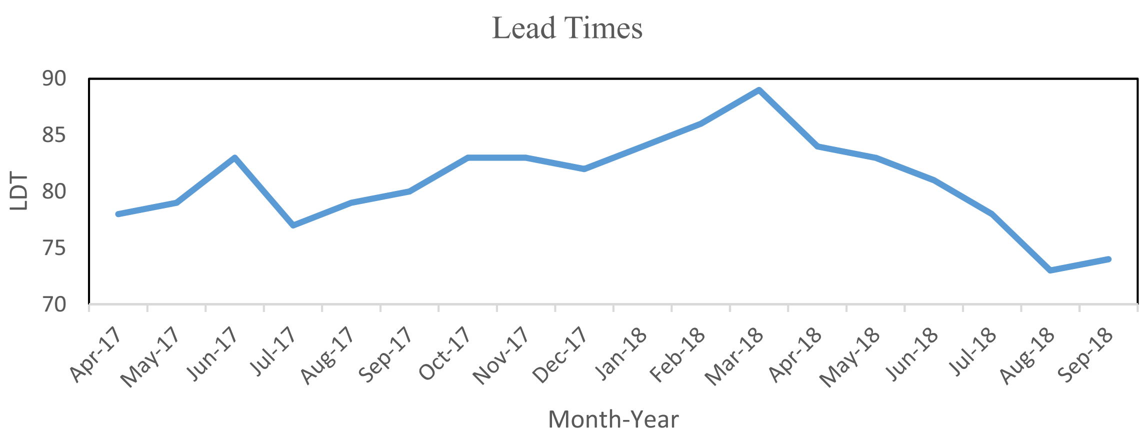

Figure 4. Time series plotting of the Lead Times (LDT).

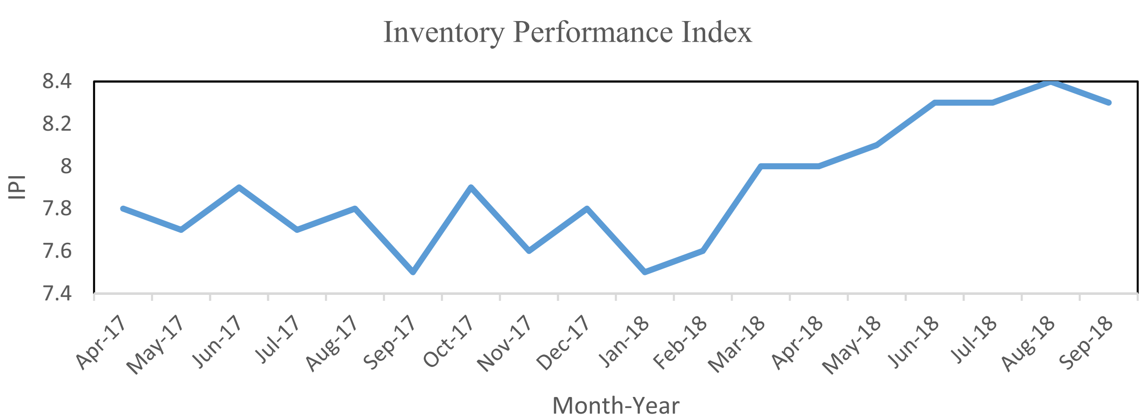

Figure 5. Time series plotting of the Inventory Performance Index (IPI).

(e) IPI shall reduce with reduction in MHC (or IPI shall increase with increase in MHC) at a probability closer to $(1 - P_{IPI/MHC}) - 0.01 = 0.528$.

(f) IPI shall remain unchanged with either reduction or increase of MHC = 0.01

(g) IPI shall increase with reduction in COTC (or IPI shall reduce with increase in COTC) at a probability $P_{IPI/COTC} = 0.873$.
(h) IPI shall reduce with reduction in COTC (or IPI shall increase with increase in COTC) at a probability closer to $(1 - PIPI/COTC) - 0.01 = 0.117$.
(i) IPI shall remain unchanged with either reduction or increase of COTC = 0.01
(j) IPI shall increase with reduction in LDT (or IPI shall reduce with increase in LDT) at a probability $P_{IPI/LDT} = 0.394$.
(k) IPI shall reduce with reduction in LDT (or IPI shall increase with increase in LDT) at a probability closer to $(1 - P_{IPI/LTD}) - 0.01 = 0.596$.
(l) IPI shall remain unchanged with either reduction or increase of LDT = 0.01

With an estimate of all the individual transition probabilities accounted, the Markov chains can be used to predict the change of state of IPI by assessing all possible scenarios of states of the four influencing variables: TC, MHC, COTC, and LDT. The scenarios are presented in table 3 with the transition probabilities of IPI estimated in each scenario as presented in the last column. With four variables evaluated, there are $2^4 = 16$ scenarios possible. Each scenario can result in a three-state Markov Chain:

State A = Current state of IPI;

State B = Next state of IPI witnessing an increase in its value;

State C = Next state of IPI witnessing a reduction in its value;

Given that there are three states, Eq. 1 for Markov chain shall reduce to the following:

$$A = \begin{bmatrix} P_{AA} & P_{AB} & P_{AC} \\ P_{BA} & P_{BB} & P_{BC} \\ P_{CA} & P_{CB} & P_{CC} \end{bmatrix} \tag{4}$$

The probabilities indicated in the matrix are the following:

P_{AA} = Probability of IPI to remain in current state A;

P_{AB} = Probability of IPI to transition from current state A to the state B resulting in an increase in its value;

P_{AC} = Probability of IPI to transition from current state A to the state C resulting in a reduction in its value;

P_{BB} = Probability of IPI to remain in current state B;

P_{BA} = Probability of IPI to transition from current state B to the state A resulting in a reduction in its value;

P_{BC} = Probability of IPI to transition from current state B to the state C resulting in an increase in its value;

P_{BB} = Probability of IPI to remain in the state B;

P_{BA} = Probability of IPI to transition from current state B to the state A resulting in a reduction in its value;

P_{BC} = Probability of IPI to transition from current state B to the state C resulting in a reduction in its value;

P_{CC} = Probability of IPI to remain in the state C;

P_{CA} = Probability of IPI to transition from current state C to the state A resulting in an increase in its value;

P_{CB} = Probability of IPI to transition from current state B to the state C resulting in an increase in its value;

The table 3 is valid as in September 2018. It may be noted that all these probability estimates are dynamic and stochastic. These values will change continuously as the time series progresses carrying forward these overlapping variables.

However, Markov chain can provide a prediction of improvement or reduction of inventory performance in any month based on the current state of scenarios in that month.

Table 2. Correlations' table.

Correlations		TC	MHC	COTC	LDT	IPI
TC	Pearson correlation	1	− .050	.897**	.100	− .739**
	Sig. (2-tailed)		.845	.000	.694	.000
	N	18	18	18	18	18
MHC	Pearson correlation	− .050	1	.233	.552*	− .462
	Sig. (2-tailed)	.845		.352	.017	.053
	N	18	18	18	18	18
COTC	Pearson correlation	.897**	.233	1	.413	− .873**
	Sig. (2-tailed)	.000	.352		.089	.000
	N	18	18	18	18	18
LDT	Pearson correlation	.100	.552*	.413	1	− .394
	Sig. (2-tailed)	.694	.017	.089		.106
	N	18	18	18	18	18
IPI	Pearson correlation	− .739**	− .462	− .873**	− .394	1
	Sig. (2-tailed)	.000	.053	.000	.106	
	N	18	18	18	18	18

**Correlation is significant at the 0.01 level (2-tailed).

*Correlation is significant at the 0.05 level (2-tailed).

Table 3. Probability estimates.

Scenario no.	State of TC	State of MHC	State of COTC	State of LDT	Probability estimates for IPI (using Eq. 3)
1	Reduces	Reduces	Reduces	Reduces	IPI remains unchanged = 0.01; IPI increases = 0.617; IPI reduces = 0.373;
2	Reduces	Reduces	Reduces	Increases	IPI remains unchanged = 0.01; IPI increases = 0.6675; IPI reduces = 0.3225;
3	Reduces	Reduces	Increases	Reduces	IPI remains unchanged = 0.01; IPI increases = 0.428; IPI reduces = 0.562;
4	Reduces	Reduces	Increases	Increases	IPI remains unchanged = 0.01; IPI increases = 0.4785; IPI reduces = 0.5115;
5	Reduces	Increases	Reduces	Reduces	IPI remains unchanged = 0.01; IPI increases = 0.6335; IPI reduces = 0.3565;
6	Reduces	Increases	Reduces	Increases	IPI remains unchanged = 0.01; IPI increases = 0.684; IPI reduces = 0.306;
7	Reduces	Increases	Increases	Reduces	IPI remains unchanged = 0.01; IPI increases = 0.4445; IPI reduces = 0.5455;
8	Reduces	Increases	Increases	Increases	IPI remains unchanged = 0.01; IPI increases = 0.495; IPI reduces = 0.495;
9	Increases	Reduces	Reduces	Reduces	IPI remains unchanged = 0.01; IPI increases = 0.495; IPI reduces = 0.495;
10	Increases	Reduces	Reduces	Increases	IPI remains unchanged = 0.01; IPI increases = 0.5455; IPI reduces = 0.4445;
11	Increases	Reduces	Increases	Reduces	IPI remains unchanged = 0.01; IPI increases = 0.306; IPI reduces = 0.684;
12	Increases	Reduces	Increases	Increases	IPI remains unchanged = 0.01; IPI increases = 0.3565; IPI reduces = 0.6335;
13	Increases	Increases	Reduces	Reduces	IPI remains unchanged = 0.01; IPI increases = 0.5115; IPI reduces = 0.4785;
14	Increases	Increases	Reduces	Increases	IPI remains unchanged = 0.01; IPI increases = 0.562; IPI reduces = 0.428;
15	Increases	Increases	Increases	Reduces	IPI remains unchanged = 0.01; IPI increases = 0.3225; IPI reduces = 0.6675;

To illustrate some scenarios, the scenario nos. 5 and 12 in table are analysed using Markov chain.

Equation 4 for scenario 12 shall be as presented below taking the probability values estimated in table 3:

$$A = \begin{bmatrix} P_{AA} & P_{AB} & P_{AC} \\ P_{BA} & P_{BB} & P_{BC} \\ P_{CA} & P_{CB} & P_{CC} \end{bmatrix} = \begin{bmatrix} 0.01 & 0.6335 & 0.3565 \\ 0.3565 & 0.01 & 0.3565 \\ 0.6335 & 0.6335 & 0.01 \end{bmatrix} \tag{5}$$

Based on this Markov matrix, the Markov chain for predictions can be plotted as presented in (figure 6).

This Markov chain presents a plot of state transitions if there are chances of reduction of TC, increase of MHC, reduction in COTC, and reduction in LDT. Similarly, the Markov chain for Scenario 15 in table 3 can be plotted as shown in figure 7. In this scenario, there are chances of increase of TC, increase of MHC, increase in COTC, and reduction in LDT. The state transition probabilities are

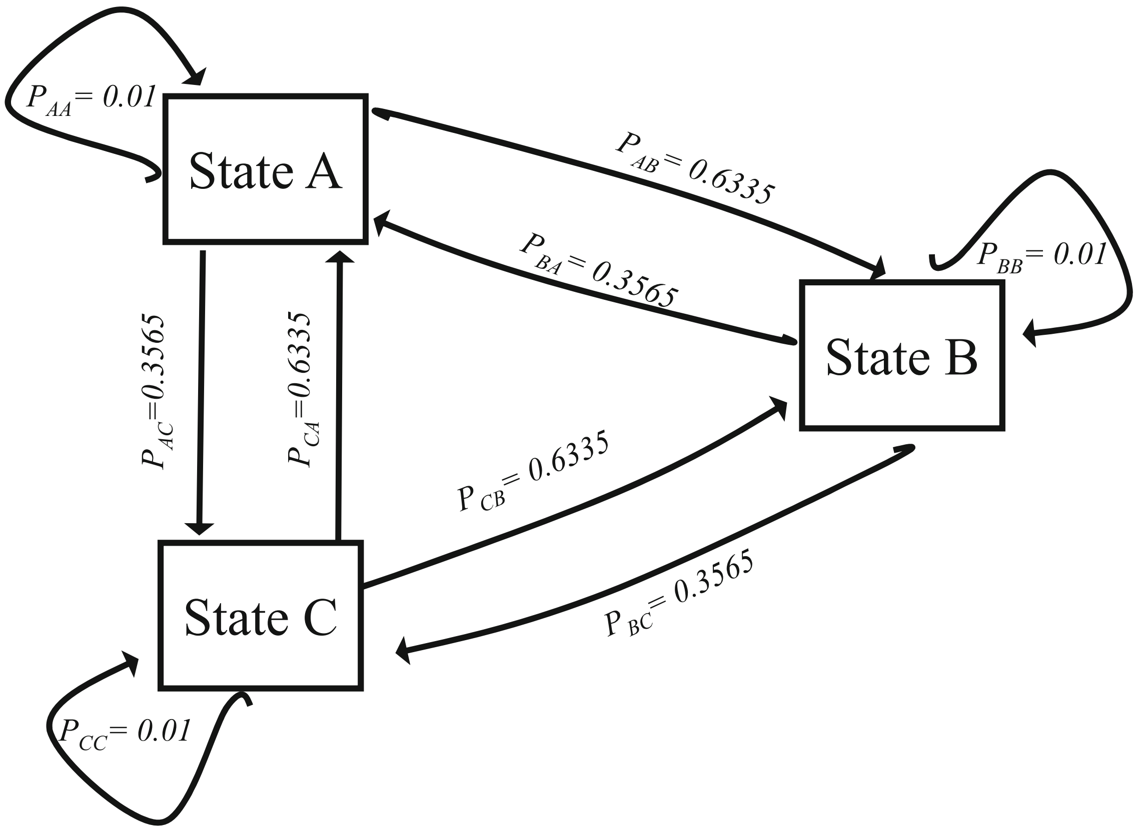

Figure 6. Markov chain analysis for scenario 5 of table 3.

based on the current state of the variables that are derived from correlation analysis of the overlapping variables on the time series prior to the current state. A small finite probability of 0.01 is assumed that the current state in the Markov chain will be retained. This means that the exact values of TC, MHC, COTC, LDT, and IPI in the current state will be replicated in the next state. Although such a scenario may not happen at all, Markov chain allows keeping a small finite probability that it may occur. As stated above, the probabilities will keep changing as the time series progresses. January 2018 was the month of introduction of VMI for six sub-components manufactured by a vendor. The sample of nine months after the introduction does not reveal whether the system has stabilised or the cooling period is still progressing. However, Markov chain presents a definitive answer about the next state if the current state can be modelled as per probabilities based on the time series before and after introduction of VMI. It also provides a view into the probabilities that the current state will be retained again after the next state. To get more accurate results, the sampling period prior to VMI introduction was kept equal to the sampling period after VMI was introduced.

With all these analyses, the question on effectiveness of VMI for improving inventory management performance in the pilot setting can now be answered. The next section presents a detailed discussion and key conclusions about this question.

4. Discussion of test results

From the findings of the pilot study, the time series analysis, and estimation of transition probabilities for Markov chain plotting, it is clear that VMI is not an achievement albeit is a journey. VMI should not be considered as a failure-proof strategy returning definitive benefits. VMI is highly complex with multiple overlapping variables on a time series playing their respective roles. To demonstrate the VMI phenomena, four critical variables were studied during the pilot study and their impact on the inventory performance index generated in SAP was assessed. While IPI is a complex metric influenced by more than 50 variables in the SAP system, the correlations of TC, MHC, COTC, and LDT with IPI were found to be significant at 99% confidence interval. A model of transition probabilities comprising $2^4 = 16$ scenarios was formed. There can be more scenarios when the number of stochastic variables is increased. For N no. of variables, the number of scenarios will be 2^{N}. TC, MHC, COTC, and LDT were chosen by the logistics head of the auto parts manufacturer as they are most critical for them,

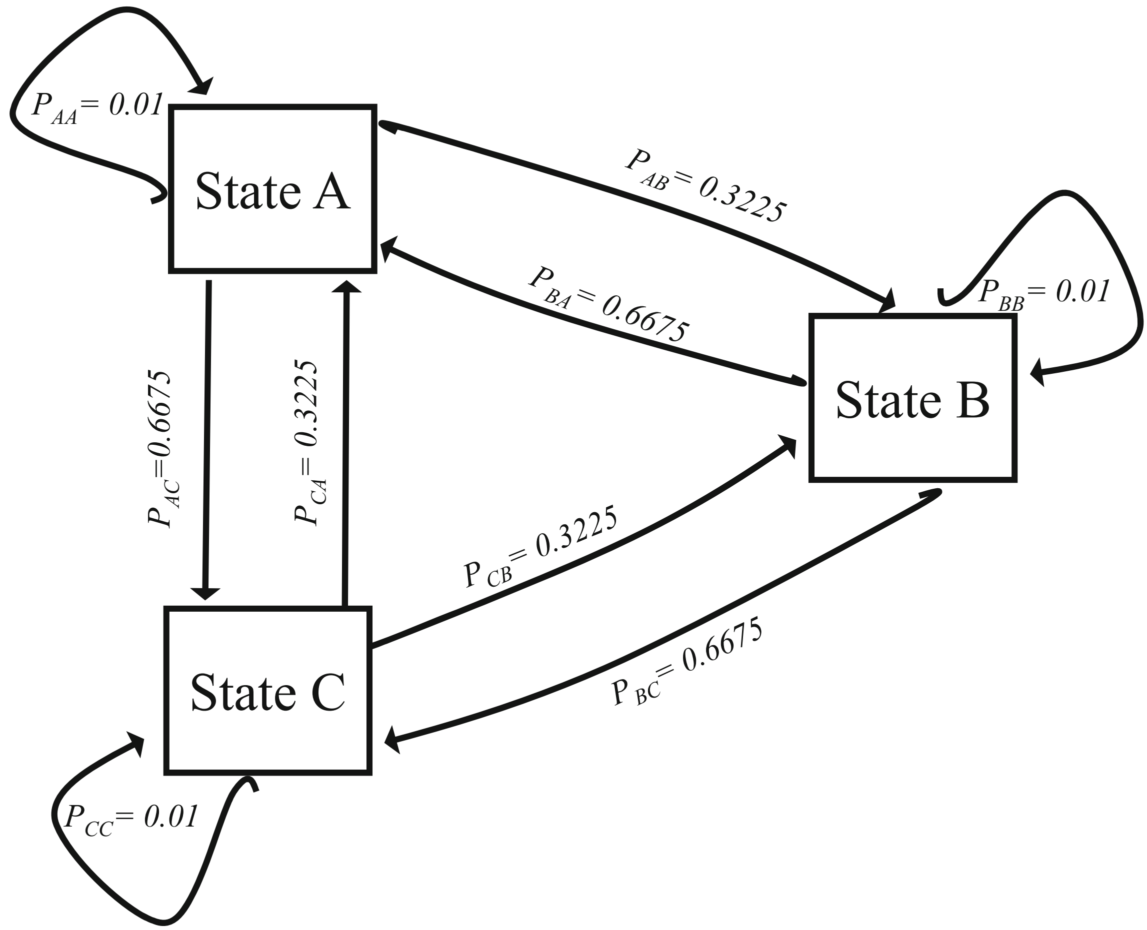

Figure 7. Markov chain plotting for Scenario 15 of table 3.

they vary stochastically, and they influence their key variable IPI significantly.

Now the question is whether VMI was beneficial for the manufacturing organisation. From the time series analysis of nine months, VMI appears to be returning positive results. However, Walsh (2004) [26] warned that it is risky to trust a time series system of overlapping stochastic variables if the sample is not long enough. Surely, 18 months is not a long enough time series. If all the observations are merely part of a cooling period, then anything can happen beyond this period. The best approach is to revisit VMI every month and evolve an empirical model only when it has existed for a sufficiently long period (like, five years). Markov chain is a powerful empirical method to revisit the performance of VMI every month. The probabilities will change every month with the change in correlation coefficients (as new data shall be added every month). Hence, Markov chain predictions will change every month and table 3 will be updated.

The manufacturer may follow the steps listed below to build and operate an inventory performance monitoring and control system using VMI:

(a) Define a period sufficiently long enough before introduction of VMI;
(b) Define a cut-off date when the inventory management system shall fully switch to VMI; preferably, this date should be the first day of a month;
(c) The month of transition from internal inventory management system may be ignored in the time series as the shocks in this month may affect the accuracy of the analysis;
(d) The inventory manager may wait for a few months more allowing the VMI to cool down;
(e) The month of introduction (the month full of shocks) and the months of stabilising period (also facing some shocks) may be eliminated from the analysis framework to get better results;
(f) In the month after the stabilising period, the correlation coefficients of all the key inventory performance variables with the inventory performance index should be calculated using SPSS or any statistical tool. The VMI introduction and stabilising months may be excluded from this calculation.
(g) The top four variables influencing the inventory performance index significantly may be selected;

(h) The transition probabilities between the four influencing variables and the inventory performance index may be estimated using the correlation coefficients;
(i) The scenario table comprising of 16 different scenarios of the four variables (similar to table 3) may be formed; the transition probabilities for each scenario may be entered in the last column;
(j) Now the inventory manager is ready for testing VMI performance; the switch between the previous and current states of the four variables may be noted (excluding the VMI introduction and stabilising months). It should be noted that VMI may have stabilised bit the cooling period may be still in progress;
(k) The Markov chain-switching plot may be constructed based on the scenario of the four variables and the state switching probabilities of IPI taken from the table of all scenarios.
(l) The Markov chain analysis needs to be conducted every month based on the switching states of the four variables recorded between the previous and current states. The state switching probabilities of IPI will also be updated every month, as the correlation coefficients will change with progress of the time series and addition of monthly data.

5. Conclusion

If monitored closely following this process, VMI can result in better performance than internal inventory management. The state changes of variables in the Markov switching chain should be favourable for positive VMI performance (that is, the inventory performance index should be contained within pre-defined bounds). The time series analysis in this chapter provides this confidence. However, VMI can cause losses if not monitored closely and corrective actions causing unfavourable state changes of the variables are not taken every month. The future researchers may like to test Markov chain analysis in a setting having seasonal fluctuations and shocks caused by them. The shocks will be reflected in the state changes of the four variables: Transportation cost (TC), Material Handling Cost (MHC), Cost of the components (COTC), and Lead time (LDT). It is suggested that the shock zones are plotted on a separate time series to keep the usual time series clean and stable.

Nomenclature

MRP	Manufacturer resource planning
VMI	Vendor Managed Inventory
JIT	Just in time
RM	Reachability Matrix
IPI	Inventory Performance Index
MHC	Materials Handling Cost
COTC	Cost of the components
LDT	Lead Time
ISM	Interpretive Structural Modelling
TQM	Total Quality Management
BPR	Business Process Engineering
SPSS	Statistical Package for Social Sciences
m	Number of samples
$P(E)_i$	Transition probability of the variable studied
$P(x_i)$	Transition probability of each influencing variable
Pss	Conditional probability of transition from state s to s itself

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