Sādhanā (2020)45:62
https://doi.org/10.1007/s12046-020-1296-5

Order reduction mechanism for large-scale continuous-time systems using substructure preservation with dominant mode

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MS received 13 July 2019; revised 31 December 2019; accepted 1 January 2020

Abstract. This work is about a balanced truncation type order reduction method which is developed for stable and unstable large-scale continuous-time systems. In this method, a quantitative measure criterion for choosing the dominant eigenvalues helps in determining the steady-state and transient information of the dynamical system. These dominant eigenvalues are used to form a new substructure matrix that retains the dominant modes (or may desirable mode) of the original system. Retaining the dominant eigenvalues in the reduced mode assures stability and results in greater accuracy as the retained eigenvalues provides a physical link to the real system. In the quest to preserve the dominant eigenvalue of the real system, the proposed technique uses Sylvester equation for system transformation. Having obtained transformed model, the reduced model has been achieved by truncating the non-dominant eigenvalues using the singular perturbation approximation method. The efficiency and accuracy of the proposed method has been demonstrated by the benchmark test systems which were from the state-of-the-art models.

Keywords. Dominant eigenvalue; model transformation; order reduction; singular perturbation; Sylvester equation.

1. Introduction

Numerous applications in science and engineering are demonstrated by complex and high-order models which are hard to introspect, control and design. Models having simple designs are easier to look out for physical insights as compared to the complex ones and can lead to lower-order controllers that are easier to implement. The maiden reason behind various model reduction methods is to find a suitable low-order model which can preserve the input–output behavior along with other prerequisite features of the real system with as minimum error as possible.

A large number of investigations were done through which model order reduction methods are selected so as to maintain the system accuracy, simplicity [1–3]. Reduction methods found on eigenvalues of the system [4–8] maintain the interest owing to the less price of their computation while being the best choice for large-scale systems. The first reduction approach using state space was the modal analysis of Davison [4]. Generally, it retains the dominant eigenvalues along with corresponding eigenvectors of the system. Several modifications on this approach have been presented in [6, 9]. Skelton proposed component cost analysis method, which minimizes the cost function. The trace of the output covariance is referred as the cost function. Since this method requires computation of the cost components of the solution of Lyapnov equation which is not applicable to a large-scale system. Later, Lastman *et al* [10] introduced a similar method for the component cost analysis of Skelton which measures the impulse response energy of the system. However, the computation cost involved to solve the matrix is very complex and the eigen-solution of assigned metrics is different for the complex conjugate pair. Vishwakarma proposed a technique based on Hankel matrix, which changes this matrix into the normal form of Hermit; however, the main shortcoming of this technique is that it may lead to instability in the reduced model [11]. Varga [12] suggested a method that decomposes the original system into various sub-systems in the balanced form. Every sub-system has eigenvalues with largest Hankel singular values (square root of product of controllability and observability gramians), being consider as the dominant pole. Rommes deploying dominant poles of the system's transfer function where the corresponding residue over the real part of the eigenvalue measures the dominance [8, 13]. Saadvanti *et al* suggested modified

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Published online: 27 February 2020

dominant pole algorithm which can be used on the delay system of second order [14]. Tiwari *et al* [15, 16] using the concept of the dominant pole for the selection of poles in clustering technique, but the problem arises for the system having complex poles.

Order reduction for unstable system has gotten more attention; for overviews see [17–22]. Usually, unstable poles cannot be neglected while designing the dynamics of the systems. As Lyapunov equations are valid only in case of stable systems; hence, unstable poles are to be retained in the reduced order model. In case the number of unstable poles is high, the possibility of reducing the model is limited. The basic concept used for unstable systems first decomposes the real system into stable and unstable subsystem and reduction process is applied to retain the unstable system as it is while the reduction method is performed only on the stable system [18]. However, the disadvantages of the technique are (i) the degree of order reduction is proportional to the number of unstable poles and (ii) decomposition process may be ill-conditioned if poles of higher order model are close to the imaginary axis.

For dynamic system modeling for any application, it is important to achieve accuracy with minimal complexity. Many studies developed metrics to preserve the input–output behavior of the real system. Letting the real substructures in reduced model provides important physical characteristics as represented by [23, 24]. The various error minimization techniques use the concept of dominant pole retention [25–27]. In these methods, the poles closest to the $j\omega$ axis which means with the large time constants are mostly taken as dominant poles of the system. Although mostly these methods might provide better results, it has problems like the slow modes may not be dominant or all the poles may have similar time constants [28]. The difficulties faced by existing methods is to find structural non-minimal which means the eigenvalues that are controllable and observable. Aguirre [7] proposed a method based on the unit-step response to measure the model dominance for linear systems. The objective of this paper is to retain the dominant dynamic frequencies in reduced model to increase the accuracy in reduced model using singular perturbation approximation technique [29, 30].

This work is about a quantitative measure of model dominance for selecting the eigenvalues to form the new sub-system model. The proposed method is executed by transforming the state matrix in a structured form, so that it considers dominant eigenvalues in decreasing order. The remaining model transformation matrices are computed by solving the Sylvester equation. After obtaining the transformed model, the reduced model is computed by truncating the non-dominant eigenvalues with the help of singular perturbation approximation method. Owing to selection of the poles the accuracy of the reduced order model is improved, reduced model leads to good values in case of transient as well as steady-state responses. Due to selection of poles, the method is used to develop a new way of reducing unstable system. For finding the accuracy and efficiency of the developed technique, the results of this technique were compared with those of the existing techniques.

2. Problem formulation

Supposing the input–output relationship of a linear time-invariant system having order 'n' is given as

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t), \quad t > 0, \quad x(0) = x^0 \tag{1a}$$

$$y(t) = Cx(t) + Du(t), \quad t \geq 0 \tag{1b}$$

In this, the state x has dimension n, having m and p as number of inputs and outputs. That is, $x(t) \in R^{n\times n}$, $B \in R^{n\times m}$ and $C \in R^{p\times n}$ for all time instants $t \in R^n$.

Let P be an $n \times n$ nonsingular matrix with coefficients in the field of complex number C. Let $\underline{x}(t)$ be a column n-vector defined by

$$\underline{x}(t) = Px(t) \tag{2}$$

It can be clearly shown that $\underline{x}(t)$ obey the equations

$$\frac{dx(t)}{dt} = \underline{A}x(t) + \underline{B}u(t) \tag{3a}$$

$$y(t) = \underline{C}x(t) + Du(t) \tag{3b}$$

here $\underline{A} = PAP^{-1}$, $\underline{B} = PB$ and $\underline{C} = CP^{-1}$.

From Eq. (3) it concludes that the $\underline{x}(t)$ also qualifies as a state vector for a given system. State model (3) is said to be equivalent to the model (1) and P is an equivalent or similarity transformation. The crucial point to be noted is that the system model in Eqs. (1) and (3) have the identical input–output behavior, i.e. have the same eigenvalues (poles). For obtaining low order model, unnecessary dynamics are truncated from the transformed model.

In case of system order reduction, the goal is to obtain a reduced order model of LTI system,

$$\frac{dx_r(t)}{dt} = A_r x(t) + B_r u(t), \quad t > 0 \quad x_r(0) = x_0 \tag{4a}$$

$$y_r(t) = C_r x_r(t) + D_r u(t), \quad t \geq 0 \tag{4b}$$

of order $r, r \ll n$, which approximates $G(s)$, so that the necessary parameters of high order model can be preserved in low order model with minimum error.

3. Proposed reduction method

This section categorizes into two parts to describe the procedure in the reduced model. The first step is used to determine the dominant poles (eigenvalues) of the system mode and the second part consists of model transformation through the solution of Sylvester equation. The reduction method of LTI system advances through the following steps:

Step 1: *Quantitative measures of dominant eigenvalues of large-scale system*

The concept of pole dominance is used for the structural arrangement of system model and thereby to retain it in the reduced model. The relative dominance of all poles of high-order system is calculated using modal dominance index (MDI) [7]. The transfer function matrix of the system model (1) is defined as

$$G(s) = C(sI - A)^{-1}B + D \tag{5}$$

It has been supposed that $G(s)$ does not have repeated roots and can be elaborated in the form of partial expansion as

$$G(s) = \frac{N(s)}{D(s)} = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i} \tag{6}$$

$$\begin{aligned} GG(s) = & \frac{R_1}{(s-\lambda_1)} + \cdots + \frac{R_k}{(s-\lambda_k)} + \frac{R_{k+1}}{(s-\lambda_{k+1})} \\ & + \frac{R^*_{k+1}}{(s-\lambda^*_{k+1})} + \cdots + \frac{R_{k+q}}{(s-\lambda_{k+q})} + \frac{R^*_{k+q}}{\left(s-\lambda^*_{k+q}\right)} \end{aligned} \tag{7}$$

Or

$$\begin{aligned} G(s) = & \frac{R_1}{(s-\lambda_1)} + \cdots + \frac{R_k}{(s-\lambda_k)} \\ & + \frac{(R_{k+1} + R^*_{k+1})s - \left(R_{k+1}\lambda^*_{k+1} + R^*_{k+1}\lambda_{k+1}\right)}{(s-\lambda_{k+1})(s-\lambda^*_{k+1})} + \cdots \\ & + \frac{(R_{k+q} + R^*_{k+q})s - \left(R_{k+q}\lambda^*_{k+q} + R^*_{k+q}\lambda\lambda_{k+q}\right)}{\left(s-\lambda_{k+q}\right)\left(s-\lambda^*_{k+q}\right)} \end{aligned} \tag{8}$$

where R_i denotes the *ith* residue corresponding to pole λ_i while asterisks (*) denotes the complex conjugate. k, q are the number of real poles, conjugate pairs respectively, hence $n = k + 2q$. In addition, it is supposed that $Re(\lambda_i) < 0 \forall_i$.

The *ith* weighted residue is defined as ρ_i

$$\rho_i = -\frac{R_i}{\lambda_i} \quad i = 1, 2, \ldots, k \tag{9}$$

$$\rho_i = -\frac{\left(R_{k+p}\lambda^*_{k+p} + R^*_{k+p}\lambda_{k+p}\right)}{2\lambda_{k+p}\lambda^*_{k+q}} \tag{10}$$

The vital point to note in Eq. (10) is that the complex conjugate poles are having equal indices. A huge weighted residue 'ρ_i' denotes that the corresponding *ith* pole λ_i is having highest dominant pole of the system which means the pole that is well observable and controllable in the transfer function. In case of the Bode magnitude plot, the peaks are arising at the frequency which are close to the imaginary parts of the dominant pole [16, 31]. Let us assume a system having order n, if ρ_3 has highest magnitude value, then $\frac{R_3}{s-\lambda_3}$ is the most significant term and $-\lambda_3$ is taken as the most dominant pole. If ρ_2 has the second highest magnitude value, which means $\frac{R_2}{s-\lambda_2}$ is the second significant term and $-\lambda_2$ is considered the second dominant pole and hence it goes on in this similar pattern. While poles are arranged in decreasing weighted residue as

$$\rho_1 \geq \rho_2 \geq \rho_3 \geq \cdots \geq \rho_n \tag{11}$$

Let us say $\lambda_1, \lambda_2, \ldots, \lambda_n$ are corresponding n dominant poles of transfer function with their relative significance.

The model dominance index is described from the *ith* input to *jth* output which can be derived by computing

$$diag\left[\sigma^{ij}_1 \;\; \sigma^{ij}_2 \;\; \ldots \;\; \sigma^{ij}_n\right] = -Re\left[\underline{C}_j \underline{B}_i \underline{A}^{-1}\right] \tag{12}$$

where $Re[.]$ represents the real part and $i = 1, 2, 3, \ldots, v$ and $j = 1, 2, 3, \ldots, u$

$$\begin{aligned} & \underline{A} = V^{-1}AV \\ & \underline{C}_j = diag\left[\underline{c}^{-j}_1 \;\; \underline{c}^{-j}_2 \;\; \ldots \;\; \underline{c}^{-j}_n\right] \\ & c_j V = \left[c^{-j}_1 \;\; c^{-j}_2 \;\; \ldots \;\; c^{-j}_n\right] \\ & C = [c_1 \;\; c_2 \;\; \ldots \;\; c_n]^T \\ & \underline{B}_i = diag\left[\underline{b}^i_1 \;\; \underline{b}^i_2 \;\; \ldots \;\; \underline{b}^i_n\right] \\ & V = [v_1 \;\; v_2 \;\; \ldots \;\; v_n] \end{aligned} \tag{13}$$

where V is the modal matrix whose ith column is the eigenvector $v_i v_i$ of A and $diag[.]$ is the diagonal matrix with indicated element. Hence the following result holds.

The knowledge of dominant modes of the system is primitive as in some cases it is desirable to keep dominant eigenvalues in reduced model. This is advantageous as it assures the stability in the reduced model [32]. Additionally, various reduction methods used for multivariable systems usually require prior information of dominant eigenvalues to be retained [33].

Step 2: *Preservation of structure in reduced order model*

Main motivation of structure preservation in low order model is to maintain dominant frequencies of original model. Hence, in order to preserve dominant dynamic eigenvalues in the reduced model, transformed state matrix is formulated as:

$$\underline{A} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \tag{14}$$

where diagonal matrix is represented by Λ_1 which has n_r dominant and Λ_2 is n_{n-r} non-dominant eigenvalues of the high order system. The eigenvalues (real or complex) are arranged in diagonal, as $\lambda_i (i = 1, 2, \ldots k)$ real poles and $\alpha_i \pm \beta_i$ denotes complex poles, hence$n = k + 2q$. It is important to note that for the arrangement of eigenvalues in state matrix structure is done on the bases of weighted residue criterion. Therefore, based on Eq. (14) the truncation model is used so that non-dynamic dominant modes are removed.

To proceed with transformation equation (3), it holds the following condition:

$$\underline{A} = PAP^{-1} \tag{15}$$

The matrix P is computed using the Sylvester equation, written as [34]

$$AX + XB = C \tag{16}$$

where $A \in R^{n \times n}$, $B \in R^{m \times m}$ and $C \in R^{n \times m}$ are given matrices and the unique solution exists only if the eigenvalues $a_1, a_2, \ldots, a_n$ of A and $b_1, b_2, \ldots, b_m$ of B satisfy $a_i + b_i \neq 0$, $i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$. To compute the solution of P, Eq. (15) is written in the following format:

$$PAP^{-1} - \underline{A} = 0 \tag{17}$$

Now, by using the Sylvester equation in Eq. (16), Eq. (17) is modified in such a way that these two equations are exhibiting similarity; thus, Eq. (10) is rewritten:

$$PAP^{-1} - \underline{A} = \Delta \tag{18}$$

where Δ is matrix contains very small numbers. Multiplying matrix P in Eq. (18) both sides and written as

$$PA - (\underline{A} + \Delta)P = 0 \tag{19}$$

By comparing Eqs. (16) and (19) and fitting it into the Sylvester equation by assigning $X = P$, $A = -(\underline{A} + \Delta)$ and $B = A$. And matrix C is set zero, which fail to comply with the Sylvester equation restrains. To mitigate this problem, all the entries in C are tune to almost zero (*i.e.,* $\Delta = 3 \times 10^{-10}$), which has no visible effect on intended solution [35]. To find the solution of the Sylvester equation, the proposed method employs Schur reduction to triangular form using orthogonal similarity transformation [34]. Matrix A has been reduced to lower real Schur form A' using an orthogonal similarity transform U; that is A is reduced to real, block lower triangular form.

$$A' = U^T A U = \begin{bmatrix} A'_{11} & 0 & \cdots & 0 \\ A'_{21} & A'_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ A'_{p1} & A'_{p2} & \cdots & A'_{pp} \end{bmatrix} \tag{20}$$

here each matrix A'_{ii} can have order of at most two. Similarly, B is reduced to upper real Schur form by the orthogonal matrix V :

$$B' = V^T B V = \begin{bmatrix} B'_{11} & B'_{12} & \cdots & B'_{1q} \\ 0 & B'_{22} & \cdots & B'_{2q} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & B'_{qq} \end{bmatrix} \tag{21}$$

$$C' = U^T C V = \begin{bmatrix} C'_{11} & \cdots & C'_{1q} \\ \vdots & \cdots & \vdots \\ C'_{p1} & \cdots & C'_{pq} \end{bmatrix} \tag{22}$$

and

$$X' = U^T X V = \begin{bmatrix} X'_{11} & \cdots & X'_{1q} \\ \vdots & \cdots & \vdots \\ X'_{p1} & \cdots & X'_{pq} \end{bmatrix} \tag{23}$$

Then, Eq. (16) can be rewritten as

$$A'X' + X'B' = C' \tag{24}$$

which can be expanded to give

$$\begin{bmatrix} A'_{11} & 0 & \cdots & 0 \\ A'_{21} & A'_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ A'_{p1} & A'_{p2} & \cdots & A'_{pp} \end{bmatrix} \begin{bmatrix} X'_{11} & \cdots & X'_{1q} \\ \vdots & \cdots & \vdots \\ X'_{p1} & \cdots & X'_{pq} \end{bmatrix} + \begin{bmatrix} X'_{11} & \cdots & X'_{1q} \\ \vdots & \cdots & \vdots \\ X'_{p1} & \cdots & X'_{pq} \end{bmatrix} \begin{bmatrix} B'_{11} & B'_{12} & \cdots & B'_{1q} \\ 0 & B'_{22} & \cdots & B'_{2q} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & B'_{qq} \end{bmatrix} = \begin{bmatrix} C'_{11} & \cdots & C'_{1q} \\ \vdots & \cdots & \vdots \\ C'_{p1} & \cdots & C'_{pq} \end{bmatrix} \tag{25}$$

If the partition of A', B', C' and X' are conformal, then

$$A'_{kk}X'_{kl} + X'_{kl}B'_{ll} = C'_{kl} - \sum_{j=1}^{k-1} A'_{kj}X'_{jl} - \sum_{i=1}^{l-1} X'_{ki}B'_{il} \tag{26}$$

The above mentioned equations can be solved for $X'_{11}, X'_{21}, \ldots, X'_{p1}, X'_{12}, X'_{22}, \ldots$. After this the solution of Eq. (16) is then obtained by $X = UX'V^T$.

After finding solution of Sylvester equation (16), the transformation matrix ($X = P$) is solved and new transform system matrix is calculated as $\underline{A} = PAP^{-1}, \underline{B} = P^{-1}B, \underline{C} = CP$ and $\underline{D} = D$. This transform state matrix has eigenvalues place in accordance with their dominance in system characteristics.

Singular perturbation approximation technique is used in order to truncate non-dominant eigenvalues to obtain the reduced model. It performs on the basis of decomposition of fast and slow dynamic modes of the real system [36].

This approach can be applied to the systems that can be decoupled a priori into two subsystems: slow $(x_1(t))$ and fast $(x_2(t))$. The fast modes in singularly perturbed systems generally reach their steady state in small time as compared slow modes. Therefore, the slow modes "dominate" the response of the system [37]. This fact allows the system to be described in a state representation as:

$$[x_1(t)x_2(t)] = [A_{11}A_{12}A_{21}A_{22}][x_1(t)x_2(t)] + [B_1B_2]u(t) \tag{27a}$$

$$y(t) = [C_1C_2][x_1(t)x_2(t)] + Du(t) \tag{27b}$$

where $x_1 \in R^r$, $x_2 \in R^{n-r}$. The reduced model by SPA is calculated using this formula:

$$A_r = A_{11} - A_{12}A_{22}^{-1}A_{21} \tag{28a}$$

$$B_r = B_1 - A_{12}A_{22}^{-1}B_2 \tag{28b}$$

$$C_r = C_1 - C_2A_{22}^{-1}A_{21} \tag{28c}$$

$$D_r = D - C_2A_{22}^{-1}B_2 \tag{28d}$$

The final reduced model satisfies the absolute error bound

$$G(s) - G_r(s)_\infty \le 2\sum_{k=r+1}^{n} \sigma_k \tag{29}$$

Liu and Anderson [37] has shown that the same frequency error bound (29) can be obtained when a SPA of an internally balanced system is performed. The SPA allows for matching steady state gain of original model at the desired frequency $s = s_0$, and selecting $s_0 = 0$, corresponding to singular perturbation, whereas selecting $s_0 = \infty$ corresponds to direct truncation [36, 37].

4. Simulation and results

In this part, two test examples were given to evaluate efficiency of proposed method, and a detailed method is given for the first test system.

Test example 1 Let us consider 50th order unstable continuous-time linear system in which parameters for $A = diag(\lambda_1, \ldots, \lambda_{50})$, B, and C are taken as in [38]. Let $D = -0.335079697054298$, then transfer function is calculated by

$$GG(s) = \sum_{i=1}^{50} \frac{b_i c_i}{s - \lambda_i} + D$$

Using simulation results shown in figure 1, it is clear that frequency response of 20th order reduced model derived by this suggested method is very close to original system in accordance with optimal Hankel norm approximation

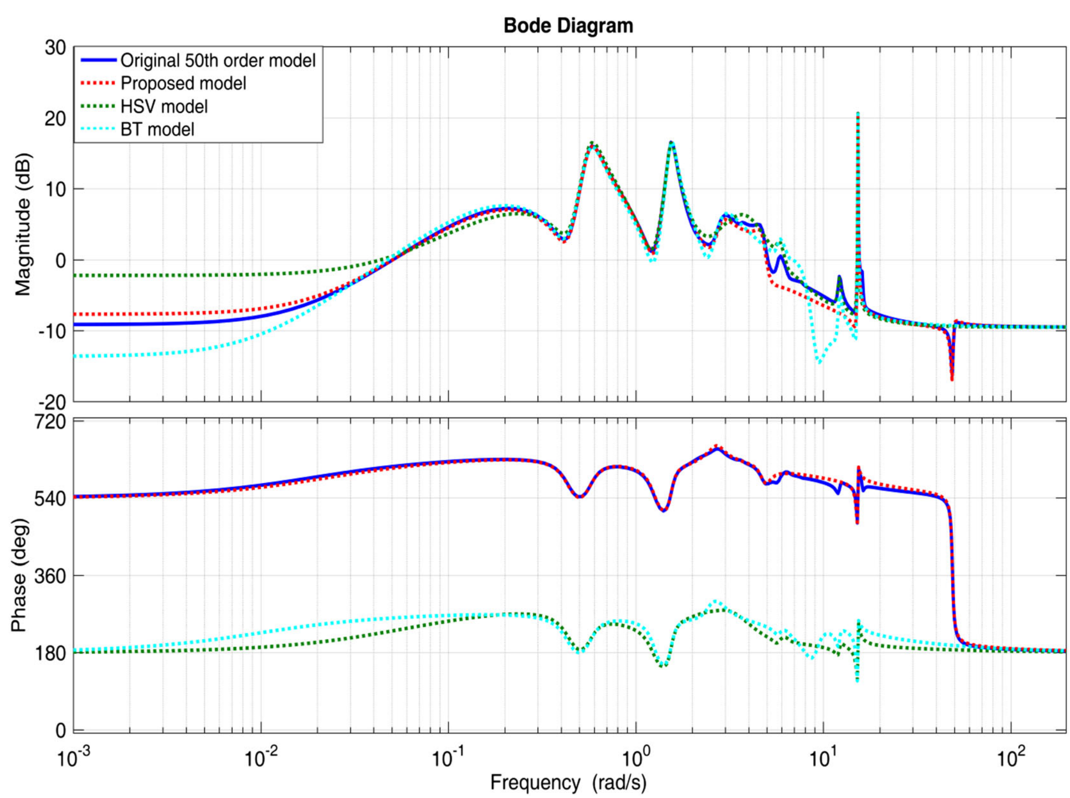

Figure 1. Bode diagram of real system and reduced models having 20th order.

Table 1. Detailed comparison of frequency domain computation of models for test example 2.

$\|G(s)\|_\infty = 10.7284$	Proposed method	BT method	HSV method
H_∞ norm $= \|G_r(s)\|_\infty$	10.6344	10.5701	10.8014
Actual H_∞ norm error bound $= \|G(s) - G_r(s)\|_\infty$	0.4481	0.6186	0.5131
Actual relative error bound $= \frac{\|G_r(s)-G(s)\|_\infty}{\|G(s)\|_\infty}$	0.0418	0.0577	0.0478

Table 2. Performance analysis of various model under different parameter changes.

Parameter variation	H_∞ norm $= \|G'(s)\|_\infty$	Actual H_∞ norm error bound $= \|G'(s) - G_r(s)\|_\infty$	Actual relative error bound $= \frac{\|G'_r(s)-G(s)\|_\infty}{\|G'(s)\|_\infty}$
+10% Variations in $G(s)$	10.728	5.7586	0.53678
+20% Variations in $G(s)$	10.728	5.7979	0.54045

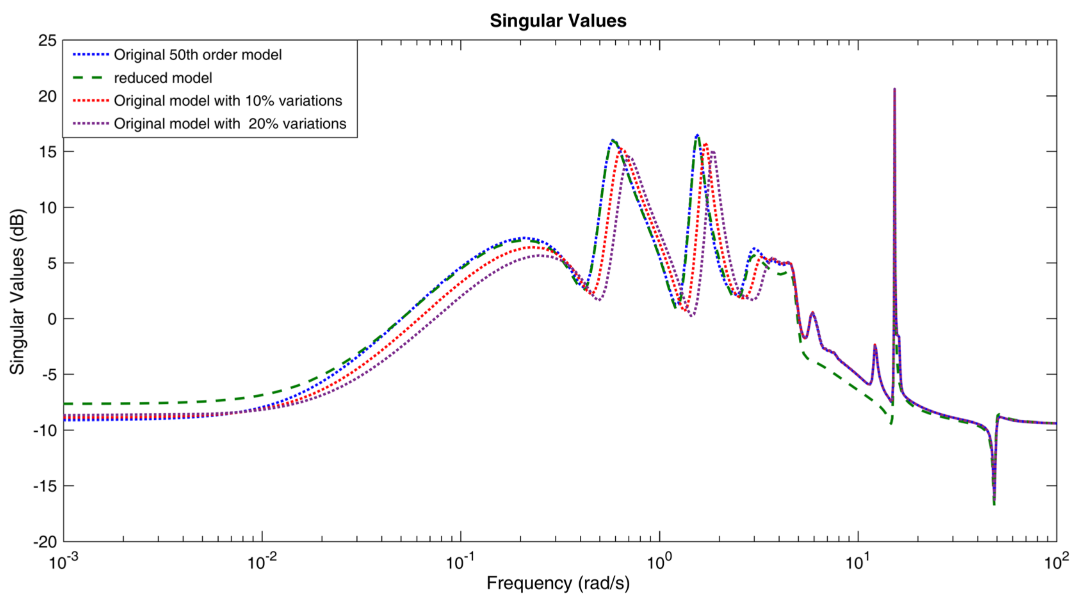

Figure 2. Comparative analysis between the Frequency response of the original model, reduced model and model with 10% and 20% variation in the randomly selected parameters.

(HSV) and balanced truncation (BT) [39] methods, respectively for the same input. Furthermore, the comparison of H_∞ error bounds of reduced models are mentioned in table 1. From table 2 and figure 1, it is distinctly ascertained that obtained model computed by suggested method is more accurate to the as compared to other available methods.

In order to check the sensitivity of the proposed method, variation analysis in the random parameters is performed. In the original system 15 randomly selected parameters are varied by 10% and 20%. Figure 2 shows the frequency response comparison of 50th order original model, 20th order reduced model, original model with the 10% and 20% variation in the 15 randomly selected parameters. It can be observed from figure 2 and table 2 that even if a few parameters of the original model are varied with 10% not much variation is observed while with 20% of the variation in the parameters of original model a slightly larger change is observed.

Test example 2 It has been assumed that building model of Los Angeles University Hospital of 48th order has 8 floors and every floor has 3 degrees of freedom, viz. displacements in x and y directions, and rotation [40].

Figure 3 shows that frequency response of real and reduced model and it depicts that suggested method is having good reduction error at entire frequency range. The

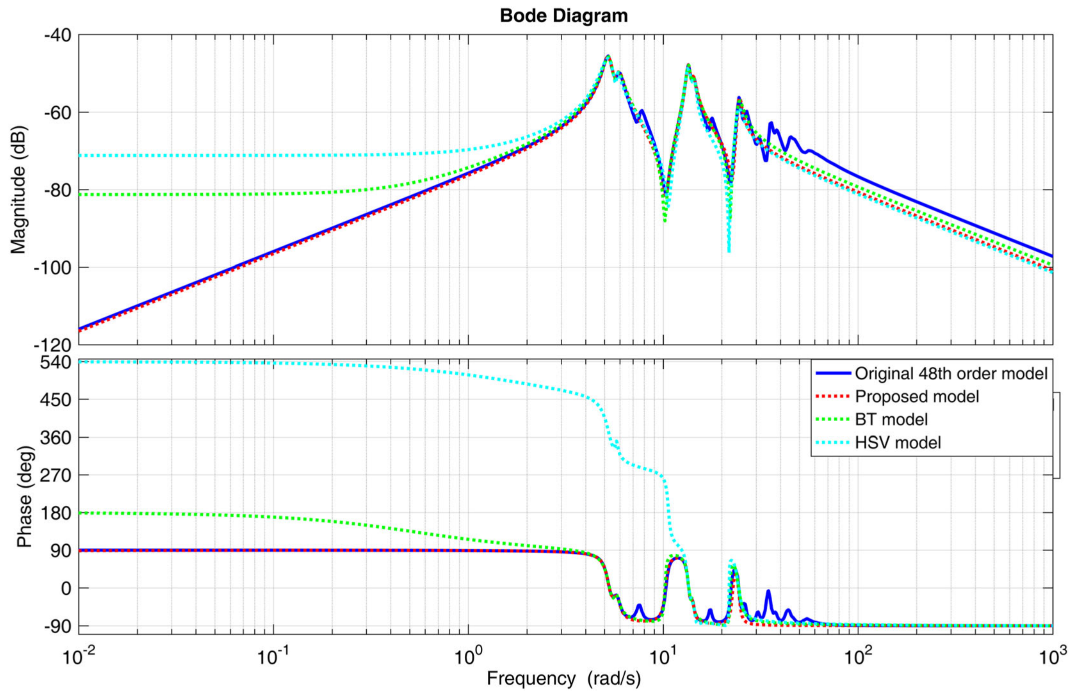

Figure 3. Bode diagram of real system and reduced models with 10th order.

Table 3. Comparison of frequency domain computation of models for test example 2.

$\|G(s)\|_\infty$= 0.0052	Proposed method	BT method	HSV method
H_∞ norm $= \|G_r(s)\|_\infty$	0.0052	0.0052054	0.0049
Actual H_∞ norm error bound $= \|G(s) - G_r(s)\|_\infty$	6.418×10^{-3}	6.004×10^{-4}	5.91×10^{-4}
Actual relative error bound $= \frac{\|G_r(s) - G(s)\|_\infty}{\|G(s)\|_\infty}$	0.1025	0.1146	0.1129

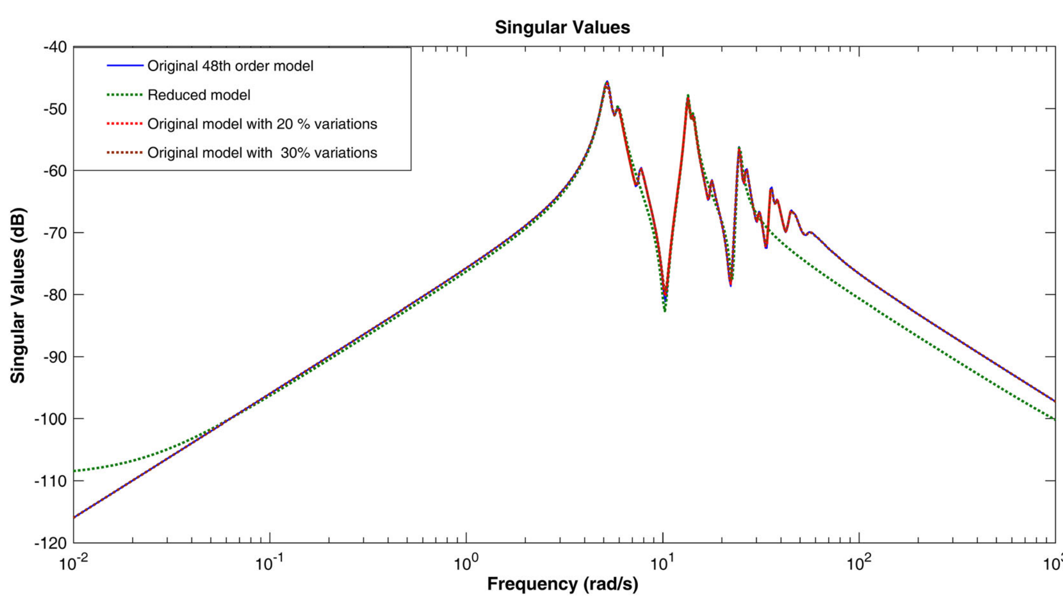

Figure 4. Comparative analysis between the Frequency response of the original model, reduced model and model with 20% and 30% variation in the randomly selected parameters.

Table 4. Performance analysis of various model under different parameter changes.

Parameter variation	H_∞ norm $= \lVert G'(s) \rVert_\infty$	Actual H_∞ norm error bound $= \lVert G'(s) - G_r(s) \rVert_\infty$	Actual relative error bound $= \frac{\lVert G'_r(s) - G(s) \rVert_\infty}{\lVert G'(s) \rVert_\infty}$
+20% Variations in $G(s)$	0.0050269	0.00061747	0.12283
+30% Variations in $G(s)$	0.0048345	0.00059403	0.11817

frequency error of BT (*balancemr*) and Hankel norm approximation *(hankelmr)* from MATLAB method is only good at mid-frequency ($when\ 0 \leq \omega \geq 10^2$). From the simulation result of output response shown in figure 3 and table 3 for Test Example 2, clearly demonstrated that the proposed method gives better accuracy than the BT and HSV reduction methods.

In order to check the sensitivity of the proposed method, variation analysis in the random parameters is performed. In the original system 196 randomly selected parameters are varied by 20% and 30%. Figure 4 shows the frequency response comparison of 48th order original model, 10th order reduced model, original model with the 20% and 30% variation in the 196 randomly selected parameters. It can be observed from figure 4 and table 4 that even if few parameters of the original model are varied with 20% not much variation is observed while with 30% of the variation in the parameters of original model a slightly larger change is observed.

5. Conclusion

In this paper, weighted residues were used to provide a consistent quantitative measure of the model dominance of a linear dynamical system. It is not similar to the classical approaches in which only time constant is used to specify dominant poles, the new weight takes into account of dominant information related to steady-state and transient response of the system. While determining the dominance of multivariable system, it is a prerequisite criterion to verify the dominant modes in each input–output path. The reduction method is based on the state-space model transformation into other state-space realization while taking into account weighted critical frequencies. Sylvester equation is employed to find transformation of system model. The developed reduction method is performed with substructure preservation, keeping weighted frequencies of real system in reduced model. The results exhibit that accuracy using proposed technique is significantly better than the existing methods.

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