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# Measuring congestion by anchor points in DEA

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**Abstract.** One of the most important issues in microeconomics is congestion. In general, an increase in inputs will result in an increase in outputs. However, in some cases, it does not happen. Hence, in these situations congestion occurs. The existence of the congestion reduces efficiency of Decision Making Units (DMUs), so determination of congestion is highly regarded. Some studies suggested methods to determine the congestion via solving conventional Data Envelopment Analysis (DEA) models, in which first an inefficient unit was depicted on the BCC frontier. However, sometimes, some optimal projections are obtained, where some previous models encounter problems. In this paper, according to S-shape form of the production function and with respect to the geometric features of anchor points, we have developed an algorithm by the connection between the anchor points and congestion definition. In this algorithm, with no need for efficiency value and projecting the inefficient DMUs on BCC efficiency frontier, only by determining the anchor point with the largest output and comparing inefficient units with it, with an easier calculation, and solving conventional DEA models, congested DMUs and their status of congestion are obtained and their values are calculated. At the end, the proposed algorithm is illustrated by some examples and the results are compared to those of the existing methods.

**Keywords.** Production function; congestion; Data Envelopment Analysis; anchor points.

## 1. Introduction

The production function in microeconomics is highly regarded and indicates the possible maximum output in terms of a combination of existing inputs. Performance of units based on its inputs and outputs can be determined by production function. This function represents the best performance of all units, which is named the efficiency frontier, but determining the criterion of the production function is difficult. There are various methods to find it. One of the most effective methods is Data Envelopment Analysis (DEA). DEA is a non-parametric method for measuring the relative efficiency of peer Decision Making Units (DMUs) based on Linear Programming (LP), first proposed by Charnes *et al* [1] and named as CCR model. Banker *et al* [2] proposed the BCC model with Variable Returns to Scale (VRS) assumption to determine efficient and inefficient units and compare their efficiencies.

In the microeconomics, the function production form is S-shape and has a maximum point. In the usual occasion, the increase in input leads to an increase in output, but under conditions an increase in one or more inputs leads to a decrease in one or more outputs; in this situation, congestion occurs. Congestion is the phenomenon in the production process where the increase in input does not increase output. Hence, the part of the function production that moves downwards is the starting point of congestion, and a production function to determine the congestion is essential.

According to the definition, congestion occurs when increase in some inputs results in decrease in some outputs, so congestion can be considered as shortfalls in outputs. Fare and Svensson [3] introduced three forms of congestion to distinguish different strengths. These forms are related to each other under assumptions commonly made in production theory. Fare and Grosskopf [4] considered an implementable form for analysing congestion in content DEA. Later, Fare *et al* [5] proposed an operationally implemental model via DEA (called FGL approach) for production efficiency evaluation. Their model is a radial approach that calculates the congestion impacts as the ratio of the observed amounts to expected amounts. Their models show only existence or non-existence of congestion and its amount cannot be computed. Another approach suggested by Cooper *et al* [6] is a slack-based approach (called CTT model), which not only evaluates the congestion impact but also calculates

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its amount in each input as the difference between the observed amounts and expected amounts. Bracket *et al* [7] extend the CTT model and examine the trade-off between employment and output, which can be used to increase employment or increase output in the Chinese production. Cooper *et al* [8] expanded a unified model for both congestion and inefficiency analysis. Later, Cooper *et al* [9] integrated the two stages of CTT approach into a single model to determine congestion. Jahanshahloo and Khodabakhshi [10] introduced a model to determine input congestion based on relax input combination for improving outputs. Later, Khodabakhshi [11] provided a method to detect the input congestion in the stochastic DEA. Wei and Yan [12] and Tone and Sahoo [13] separately rebuilt the Production Possibility Set (PPS) and explored the congestion impact in terms of immoderate inputs.

Returns to Scale (RTS) are related important economic implications of DEA efficiencies. From economic theory, issues of RTS and congestion are closely related to each other. Hence, Wei and Yan [12] proposed a necessary and sufficient condition and used four conventional output-oriented DEA models with a modified model to study congestion and RTS with respect to DEA efficiency, simultaneously.

Tone and Sahoo [13] developed a new slack-based approach to evaluate the scale elasticity in the presence of congestion under the assumption of convexity and strong output disposability. The DEA models to determine the congestion use the projection of an inefficient DMU on the BCC efficiency frontier. They proposed new concepts of strong and weak congestion. These models considered an optimal solution in the investigation on DEA-based congestion. Since different projections of an inefficient DMU may produce different results on its congestion status, choosing one of them is not arbitrary. Therefore, Sueyoshi and Sekitiani [14] explored how to deal with the occurrence of multiple solutions in DEA-based congestion measurement. They proposed wide congestion, conceptually covering both strong and weak congestion, and suggested an approach that produces a unique optimal solution and a unique projection. Two DEA models are proposed, which can uniquely determine the status of wide congestion. Later, Mehdiloozad *et al* [15] proposed a method to recognize the evidence of congestion. To cope effectively with the occurrence of multiple projections, they defined the congestion of an inefficient DMU at its Max-projection that lies in the relative interior of a corresponding minimum face. Later, by showing that the global reference set spans the minimum face, they proposed an LP model for identifying the Max-projection. They demonstrated that the congestion status of an inefficient DMU facing multiple projections could be determined using its reference DMUs.

The first and important aims of DEA are characterizing a PPS and in particular determining the extreme efficient points of PPS, which are considered as benchmarks. The anchor points are extreme points of the PPS, which define the transition from the efficient to the free disposability portion. Thanssoulis and Allen [16] first used the concept of anchor points. Bougnel and Dula [17] proposed an algorithm for determining anchor points according to the geometry of these points. Mostafaee and Soleimani-damaneh [18] suggest utilizing sensitivity analysis techniques. Two necessary and sufficient conditions accompanying four sufficient conditions are provided by Mostafaee and Soleimani-damaneh [19] to identify the anchor points with respect to VRS assumption.

In this paper, according to S-shape form of the production function with maximum point, from which the slope is negative, and the properties of anchor points, the connection between the anchor points and congestion definition is created. Later, an algorithm for identifying the congested units and congestion status via applying DEA models is suggested. Since, for inefficient units, multiple projections may occur on BCC efficient frontier, different types of congestion for them can be obtained. We present an algorithm with no need for efficiency value; only by comparing the inefficient units with the anchor points with the largest output as the benchmark and easier calculation, the status of congestion and its amount can be explored.

The rest of this paper is organized as follows. Section 2 contains some preliminaries. Section 3 proposes a new algorithm based on the relationship between anchor points and congestion. Section 4 presents the results of the mentioned algorithm on examples adopted from Tone and Sahoo [13], Cooper *et al* [9] and Sueyoshi and Sekitiani [14]. Section 5 concludes the research.

## 2. Preliminaries

### 2.1 *DEA models*

Assume that there exist a set of DMUs $\{DMU_j, j = 1, \ldots, n\}$ such that each $DMU_j$ produces multiple nonnegative outputs $y_{rj}(r = 1, \ldots, s)$ utilizing multiple nonnegative inputs $x_{ij}(i = 1, \ldots, m)$. It is assumed that $x_j = (x_{1j}, \ldots, x_{mj})^T \neq \mathbf{0}$ and $y_j = (y_{1j}, \ldots, y_{sj})^T \neq 0$ for each $j$. The superscript "$T$" stands for transpose. Moreover, suppose that each $DMU_j$, $j \in J = \{1, \ldots, n\}$ can be expressed as its input and output vectors in the following manner: $D_j = (x_j, y_j)^T$. We assume that there is no duplicate DMU.

The PPS is represented as follows:

$$\text{PPS} = \{(X, Y) : Y \text{ can be produced by } X\}.$$

Banker *et al* [2] introduced the following PPS, which is denote by $T_V$, regarding the VRS assumption of production technology:

$$T_V = \{(X, Y) | \lambda x \leq X, \ \lambda y \geq Y, \ \mathbf{1}\lambda = 1, \ \lambda \geq \mathbf{0}\}$$

Banker *et al* [2] proposed the following output-oriented model (BCC model) to evaluate the performance of $DMU_p$ (DMU under evaluation) under VRS assumption:

$$
\begin{aligned}
& Z^* = Max \quad \phi \\
& s.t \\
& \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{ip} \quad \forall i \\
& \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \phi y_{rp} \quad \forall r \\
& \sum\nolimits_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \quad \forall j \\
& s_i^-, s_r^+ \geq 0 \quad \forall i, r
\end{aligned} \tag{1}
$$

DMU$_p$ is called BCC extreme efficient if and only if $\phi^* = 1$ and all optimal slacks are zero, i.e. $s_i^{-*} = 0, \quad i = 1, \ldots, m$ and $s_r^{+*} = 0, \quad r = 1, \ldots, s$.

Superscript "*" indicates optimality.

The input–output-oriented model for evaluating DMU$_p$ = ($x_p$, $y_p$) is as follows:

$$
\begin{aligned}
& Min \quad \eta = \quad \theta - \phi \\
& s.t \\
& \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{ip} \quad \forall i \\
& \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = \phi y_{rp} \quad \forall r \\
& \sum\nolimits_{j=1}^{n} \lambda_j = 1 \\
& 0 < \theta \leq 1 \\
& \phi > 1 \\
& \lambda_j \geq 0 \quad \forall j \\
& s_i^-, s_r^+ \geq 0 \quad \forall i, r
\end{aligned} \tag{2}
$$

Let $(\eta^*, \phi^*, \theta^*, s^{-*}, s^{+*}, \lambda^*)$ be the optimal values of model (2).

The optimal value of model (2) is equal to or less than zero.

**Definition 1** DMU$_p$ is called extreme efficient by model (2) if and only if $\theta^* = 1$, $\phi^* = 1$ and all optimal slacks are zero, i.e. $s_i^{-*} = 0, \quad i = 1, \ldots, m$ and $s_r^{+*} = 0, \quad r = 1, \ldots, s$.

**Definition 2** DMU$_p$ is input-oriented efficient if and if only if $\theta^* = 1$ and $s_i^{-*} = 0, \quad i = 1, \ldots, m$; else, it is inefficient.

**Definition 3** DMU$_p$ is output-oriented efficient if and if only if $\phi^* = 1$ and $s_r^{+*} = 0, \quad r = 1, \ldots, s$; else, it is inefficient.

For an inefficient DMU$_p$, its reference set is constructed by the optimal $\lambda_j$s in model (2), which is positive, as follows:

$$E_p = \left\{ j \middle| \lambda_j^* > 0, j = 1, \ldots, n \right\}.$$

## 2.2 *Congestion*

Since the presence of congestion reduces efficiency, measuring congestion is particularly important to assess the performance of DMUs. Congestion is presented in the performance of a DMU when some outputs that are maximally possible are reduced by increasing one or more input without improving any other input or output.

According to the input disposability postulate, we have

$$(X, Y) \in PPS, \; \bar{X} \geq X \Leftrightarrow (\bar{X}, Y) \in PPS$$

When there exists congestion, input disposability postulate is violated. Hence, in this situation, Tone and Sahoo [13] modified the PPS of $T_v$ as follows and named it $T_{convex}$:

$$T_{convex} = \{(X, Y) | X = \lambda X, \; Y \leq \lambda Y, \; \mathbf{1}\lambda = \mathbf{1}, \; \lambda \geq \mathbf{0}\}.$$

By considering output disposability, Fare *et al* [5], in a two-stage evaluation of congestion, introduced FGL model as follows:

$$
\begin{aligned}
& \hat{\beta}^* = Min \quad \hat{\beta} \\
& s.t \\
& \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} = \tau x_{ip} \quad \forall i \\
& \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} = \hat{\beta} y_{rp} \quad \forall r \\
& \sum\nolimits_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \quad \forall j
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
& \phi^* = Max \phi \\
& s.t \\
& \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} \leq x_{ip} \quad \forall i \\
& \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{rp} \quad \forall r \\
& \sum\nolimits_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \quad \forall j
\end{aligned} \tag{4}
$$

$\tau$ is applied to the proportional scale of the convex compounds of observed inputs and outputs. Presence and non-presence of congestion are defined as follows:

(a) $c(\phi^*, \hat{\beta}^*) = \frac{\phi^*}{\hat{\beta}^*} > 1$ indicates presence of congestion in input.

(b) $c(\varphi^*, \hat{\beta}^*) = \frac{\varphi^*}{\hat{\beta}^*} = 1$ indicates non-presence of congestion in input.

One of the disadvantages of their model is that only the existence or non-existence of congestion is determined and its amount is not obtained. Cooper *et al* [9] replaced two models approach with a sing LP model to determine congestion and calculate its value as follows:

$$
\begin{aligned}
& \phi^* = Max \quad \phi + \varepsilon(\sum_{r=1}^{s} s_r^+ - \sum_{i=1}^{m} s_i^c) \\
& s.t \\
& \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} + s_i^c = x_{ip} \quad \forall i \\
& \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \phi y_{rp} \quad \forall r \\
& \sum\nolimits_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \quad \forall j \\
& s_i^c, s_r^+ \geq 0 \quad \forall i, r
\end{aligned} \tag{5}
$$

where $\varepsilon$ is a non-Archimedean element.

According to their definition, congestion is present if and only if, in an optimal solution of the model, at least one of the following conditions holds:

(a) $\phi^* > 1$ and there is at least one $s_i^{c*} > 0$.
(b) There is at least one $s_i^{c*} > 0$, $i = 1, \ldots, m$ and at least one $s_r^{+*} \neq 0$ $(r = 1, \ldots, s)$

where $(\phi^*, \lambda^*, s^{+*}, s_i^{c*})$ is an optimal solution of this model.

The amount of congestion in $i$th input ($i$ = 1, …, $m$) is recognized by $s_i^{c*}$.

Tone and Sahoo [13] have investigated a relation between congestion and RTS. These two economic concepts are strictly related to each other. For the first time, the concepts of strong and weak congestion were proposed by them. DMU$_p$ is strongly congested if it is strongly efficient with respect to $T_{convex}$ and there exists an activity$(\bar{x}, \bar{y}) \in T_{convex}$ such that $\bar{x} = \alpha x_p (0 < \alpha < 1)$ and$\bar{y} \geq \beta y_p (\beta > 1)$. That is, if proportional reduction in all inputs of a DMU warrantees an increase in all output then strong congestion occurs.

DMU$_p$ is weakly congested if there exists an activity $(\bar{x}, \bar{y}) \in T_{convex}$ that uses fewer resources in one or more inputs for making one or more outputs. It implies $\hat{x} \leq x_p, \hat{y} \geq y_p, \hat{y} \neq y_p$. In their method, first, inefficient DMUs were projected onto the efficient frontier of $T_{convex}$ and whether each DMU is strongly or weakly congested was recognized [13]. Therefore, their approach needs to solve output-oriented model (6) to evaluate DMU$_p$ with respect to $T_{convex}$:

$$\begin{aligned} & Max\ \tau = \phi + \varepsilon\left(\sum_{r=1}^{s} s_r^+\right) \\ & s.t \\ & \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} = x_{ip} \quad \forall i \\ & \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \phi y_{rp} \quad \forall r \\ & \sum\nolimits_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0 \quad \forall j \\ & s_r^+ \geq 0 \quad \forall r \end{aligned} \tag{6}$$

DMU$_p$ is called strongly efficient with respect to $T_{convex}$ if and only if $\varphi^* = 1$ and all optimal slacks are zero.

Project point for inefficient DMU$_p$ can be obtained as $\hat{x}_p = x_p$, $\hat{y}_p = \phi^* y_p + s^{+*}$ where $(\phi^*, s_r^{+*})$ is optimal solution of model (6); $(\hat{x}_p, \hat{y}_p)$ is strongly efficient DMU under $T_{convex}$.

Tone and Sahoo [13] proposed the following procedure in two stages to find out status of congestion.

*Step 1*: Let $(\phi^*, s^{-*}, s^{+*})$ be the optimal solution of model (1).

(a) If $\phi^* = 1$, $s^{-*} = 0$ and $s^{+*} = 0$ then $(\hat{x}_p, \hat{y}_p)$ is BCC extreme efficient and does not evidence congestion under VRS assumption and stop.
(b) If $\phi^* = 1$, $s^{-*} \neq 0$ and $s^{+*} = 0$ then $(\hat{x}_p, \hat{y}_p)$ is BCC inefficient and stop.
(c) If $\varphi^* = 1$ and $s^{-*} \neq 0$ or $\phi^* > 1$ then$(\hat{x}_p, \hat{y}_p)$ is congested and go to step 2.

*Step 2*: Solve the following model.

$$\begin{aligned} & Max \quad \rho = v\hat{x}_p \\ & s.t \\ & -vx_j + uy_j + w \leq 0 \quad \forall j \neq p \\ & -v\hat{x}_p + u\hat{y}_p + w = 0 \\ & u\hat{y}_p = 1 \\ & u \geq 0, v \geq 0 \end{aligned} \tag{7}$$

Let $\bar{\rho}$ be the optimal value of this model. Then:

(a) If $\bar{\rho} < 0$ then $(\hat{x}_p, \hat{y}_p)$ is strongly congested.
(b) If $\bar{\rho} \geq 0$ then $(\hat{x}_p, \hat{y}_p)$ is weakly but not strongly congested.

Previous methods consider a unique solution in assessing the DEA-based congestion measurement.

In dealing with the occurrence of multiple projections for identifying congestion, the previous studies are all troublesome theoretically and in application. To cope up with this problem Sueyoshi and Sekitian [14] suggested a procedure in two steps, each step contains one LP problem, and presented wide congestion as a new definition. They defined that a DMU$_p$ is widely congested if it lies on boundary of $T_{convex}$ and there exists an activity in $T_{convex}$ that uses less resources in one or more inputs to produce more products in one or more outputs.

They proposed a two steps procedure as follows.

Step 1 – Solve the following LP:

$$\begin{aligned} & Max \quad \varepsilon + \sum_{r=1}^{m} d_r^y \\ & s.t \\ & \sum\nolimits_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \sigma \leq 0 \quad \forall j \\ & \sum\nolimits_{r=1}^{s} u_r y_{rp} = 1 \\ & \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} == x_{ip} \quad \forall i \\ & \sum\nolimits_{r=1}^{s} \lambda_j y_{rj} - d_r^y = \beta y_{rp} \quad \forall r \\ & \sum\nolimits_{i=1}^{m} v_i x_{ip} - \sigma = \beta \\ & v_i x_{ip} - \varepsilon \geq 0 \quad \forall i \\ & \varepsilon \leq \delta \\ & \sum\nolimits_{j=1}^{n} \lambda_j = 1 \\ & d_r^y \geq 0 \quad \forall r \\ & \lambda_j \geq 0 \quad \forall j \\ & u_r,\ v_i \geq 0 \quad \forall r, i \\ & \sigma, \beta, \varepsilon : URS \end{aligned} \tag{8}$$

where an arbitrary real number $\sigma$ guarantees the existence of an optimal solution of model (8).

(a) If $\beta^* < 0$ then the project point of $DMU_p$, i.e. $(x_p, \beta^* y_p)$, is widely congested.
(b) If $\beta^* = 0$ and $\sum_{r=1}^{s} d_r^{y*} > 0$ then $(x_p, \beta^* y_p)$ is widely congested.
(c) If $\beta^* = 0$ and $\sum_{r=1}^{s} d_r^{y*} = 0$ then go step 2.

Step 2 – Solve the following LP:

$$\begin{aligned}
&Max \quad \alpha \\
&s.t \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \sigma \le 0 \quad \forall j \\
&\sum_{r=1}^{s} u_r y_{rp} = 1 \\
&\sum_{i=1}^{m} v_i x_{ip} - \sigma = \beta^* \\
&u_r y_{rp} - \alpha \ge 0 \quad \forall r \\
&v_i,\ u_r \ge 0 \quad \forall i, r \\
&\alpha \ge 0 \\
&\sigma : URS
\end{aligned} \tag{9}$$

where $\beta^*$ is an optimal solution of model (8).

Based on the optimal objective value of (9), the wide congestion of $DMU_p$ is identified as follows:

(a) If $\alpha^* > 0$ then $(x_p, \beta^* y_p)$ is not widely congested.
(b) If $\alpha^* = 0$ then $(x_p, \beta^* y_p)$ is widely congested.

Next, Mehdiloozad *et al* [15] suggested a method to recognize the evidence of congestion. To cope effectively with the occurrence of multiple projections, all DMUs are evaluated by solving model (6), and divided into three categories: (1) efficient DMUs, (2) inefficient DMUs with zero output slack vector and (3) inefficient DMUs with non-zero output slack vector. To explore the congestion status for first and second categories, model (10) is solved:

$$\begin{aligned}
&Max \sum_{i=1}^{m} t_i^- + \sum_{r=1}^{s} t_r^+ \\
&s.t \\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- + t_i^- = (1+w) x_{io} \quad \forall i \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ - t_r^+ = (1+w) y_{ro} \quad \forall r \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&s_i^-, s_r^+ \ge 0 \quad \forall i, r \\
&\lambda_j \ge 0 \quad \forall j \\
&w \ge 0 \quad 0 \le t_i^- \le 1 \quad \forall i \quad 0 \le t_r^+ \le 1, \quad \forall r
\end{aligned} \tag{10}$$

Let $(\lambda^*, s^{-*}, s^{+*}, t^{-*}, t^{+*}, w^*)$ be optimal solution to this model. Put $\begin{pmatrix} \alpha^{\max} \\ \beta^{\max} \end{pmatrix} = \frac{1}{1+w^*} \begin{pmatrix} s^{-*} + t^{-*} \\ s^{+*} + t^{+*} \end{pmatrix}$. $DMU_o$ is weakly congested if and only if $\beta^{\max} \ne 0_s$ and is strongly congested if and only if $\alpha^{\max} > 0_m$ and $\beta^{\max} > 0_s$.

Also, for DMUs belonging to third categories, multiple projections may occur; model (11) must be solved to find out the Max-projection:

$$\begin{aligned}
&Max \quad \sum_{j=1}^{n} \lambda_j^2 \\
&s.t \\
&\sum_{j=1}^{n} (\lambda_j^1 + \lambda_j^2) x_{ij} = x_{io}(1+v) \quad \forall i \\
&\sum_{j=1}^{n} (\lambda_j^1 + \lambda_j^2) y_{rj} - s_r^{+*} = \phi^* y_{ro}(1+v) \quad \forall r \\
&\sum_{j=1}^{n} (\lambda_j^1 + \lambda_j^2) = 1 \\
&\lambda_j^1 \ge 0, \quad 0 \le \lambda_j^2 \le 1 \quad \forall j \\
&v, s_r^{+*} \ge 0 \quad \forall r
\end{aligned} \tag{11}$$

Let $(\lambda_j^{1*}, \lambda_j^{2*}, v^*)$ be the optimal solution to model (11) and $(\phi^*, s_r^{+*})$ the optimal solution to model (6). Then $\lambda^{\max} = \frac{1}{1+v^*}(\lambda^{1*} + \lambda^{2*})$ and $\begin{pmatrix} \mathbf{x}_o^{\max} \\ \mathbf{y}_o^{\max} \end{pmatrix} = \begin{pmatrix} \mathbf{x}_o \lambda^{\max} \\ \mathbf{y}_o \lambda^{\max} \end{pmatrix}$. By replacing $\begin{pmatrix} X_o \\ Y_o \end{pmatrix}$ with $\begin{pmatrix} X_o^{\max} \\ Y_o^{\max} \end{pmatrix}$ and solving model (10), the status of congestion is determined similar to first and second categories, which are mentioned earlier.

## 2.3 *Anchor point*

The anchor points are extreme points of the PPS that define the transition from the efficient frontier to the free disposability portion [19]. Mostafaee *et al* [9] provided the necessary and sufficient condition as the following theorem to determine the anchor points:

**Theorem 1** *Let $\varepsilon \in (0, \min\{2y_{rp}\})$ be given. The extreme efficient point $DMU_p = (x_p, y_p)$, with $x_p > 0$ and $y_p > 0$, is an anchor point if and only if the optimal value of the objective function of the following problem is zero for some $k \in \{1, \ldots, m\}$:*

$$\begin{aligned}
&\sigma_k^I = Max \quad \sigma \\
&s.t \\
&\sigma \le x_{ip} - \sum_{j=1}^{n} \lambda_j x_{ij} \quad \forall i \ne k \\
&\sigma \le (x_{kp} + \frac{\varepsilon}{2}) - \sum_{j=1}^{n} \lambda_j x_{kj} \\
&\sigma \le \sum_{j=1}^{n} \lambda_j y_{rj} - y_{rp} \quad \forall r \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\sigma \ge 0 \\
&\lambda_j \ge 0 \quad \forall j
\end{aligned} \tag{12}$$

Later, they proposed the following theorem to characterize the anchor points among extreme BCC efficient DMUs:

**Theorem 2** *Let $DMU_p = (x_p, y_p)$ be BCC extreme efficient. If one of the super efficiency models (13) or (14) is infeasible then $DMU_p$ is an anchor point.*

$$\phi_o = Max \quad \phi \tag{13}$$

$$\theta_o = Min \quad \theta \tag{14}$$

## 3. Proposed algorithm

In this paper, given the form of the production function and anchor point attributes, we provide an algorithm for determining congestion of inefficient units. Consider an S-shape form of the production function, as in figure 1; the point $A$ is a critical point of the function. In fact, the slope of the tangent line of the curve at this point is zero. Therefore, point $A$ is a maximum point of the production function. In figure 1, from the point $A$ the slope of the curve is descending; hence, with respect to the geometric properties of the anchor point, point $A$ is an anchor point with $X$-intercept that is infinitive negative; from this point, DMUs with higher inputs and lower outputs have congestion.

To identify congested units, first solve model (2) to identify the efficient and inefficient units. Depending on the results, we define the following sets.

$E$: Index set of extreme DMUs.
$E^c$: Index set of inefficient DMUs in input–output oriented.
$E_{in}$: Index set of efficient DMUs in input oriented.
$E^c_{in}$: Index set of inefficient DMUs in input oriented.
$E_o$: Index set of efficient DMUs in output oriented.
$E^c_o$: Index set of inefficient DMUs in output oriented.
$E^R_j$: Index set of reference set of DMU$_j$ $(j \in E^c_{in})$.

Now, by solving model (12) for units in $E$, the anchor points are obtained.

Two cases may occur:

(I) Obtained anchor points are unique.

If the selected anchor point is the reference of input oriented inefficient units, these units are efficient in output oriented (these units can be used as a criterion for determining congested units; among them the DMU with the largest slack is called congested-specific unit). Thus, all the inefficient DMUs whose input, in at least one component, is greater than input of the congested-specific unit have congestion. Otherwise, all inefficient DMUs whose inputs are, at least in one component, greater than the input of anchor point have congestion.

(II) The anchor points are not unique.

Models (13) and (14) should be solved for obtaining anchor points. If both are infeasible for the DMU under consideration, then outputs of this unit are greater than the outputs of the other anchor points. We call it the deterministic anchor point. If it is the reference of congested-specific unit, then all the inefficient units whose inputs, in at least one component, are greater than the input value of the congested-specific unit have congestion. Else, all inefficient DMUs whose inputs are, in at least one component, greater than the input of deterministic anchor point have congestion.

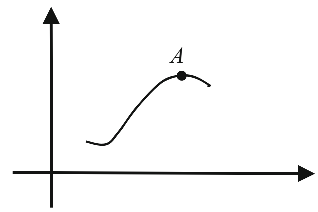


**Figure 1.** S-shape production function.

These contents can be summarized as the following algorithm to determine and compute congestion:

1. Solve the model (2) to measure efficiency of DMUs and identify efficient and inefficient DMUs.
2. For the extreme efficient DMUs, model (12) should be solved for determining the anchor points, and define the following set:

$E_{AN}$ = Index set of anchor points, which is not an empty set.

Two conditions may occur:

2-1. If $E_{AN}$ has a unique member as $o$, go to step 3.
2-2. If $E_{AN}$has multiple members then solve models (13) and (14) for points belonging to set $E_{AN}$.

If for each $o \in E_{AN}$ models (13) and (14) are infeasible, then it will be an anchor point with the larger output, which we name as the deterministic anchor point and go to step 3.

3. Two situations may be occur:

3-1. If $E_o \cap E^c_{in} \neq \varphi$ then the unit belonging to this set with the largest slack variable is named as the congested-specific unit and it is put in DMU$_t$.

Now, if $E_{AN} \subseteq E^R_t$ and $x_{ij} \geq x_{it}$ for each $j \in E^c(j = ,\ldots,n)$ $(i = 1,\ldots,m)$ then DMU$_j$ is named weakly congested and if $x_{ij} > x_{it}$ for each $j \in E^c(j =,\ldots,n)$, $i = 1,\ldots,m$ then DMU$_j$ is named strongly congested. The value of congestion for $i$th input is

$$x^c_{ij} = x_{ij} - x_{it} \quad i = 1,\ldots,m, \quad j \in E^c(j = 1,\ldots,n).$$

3-2. If $E_o \cap E^c_{in} = \varphi$ and $x_{ij} \geq x_{io}$ ($x_{io}$ is $i$th input of deterministic anchor point) for each $j \in E^c(j = ,\ldots,n)$ $(i = 1,\ldots,m)$ then DMU$_j$ is named weakly congested and if $x_{ij} > x_{io}$ for each $j \in E^c(j = ,\ldots,n)$ $(i = 1,\ldots,m)$ DMU$_j$ is named strongly congested and the amount of congestion for $i$th input is

$$x^c_{ij} = x_{ij} - x_{io} \quad i = 1,\ldots,m, \quad j \in E^c\,(j = 1,\ldots,n).$$

## 4. Numerical examples

Now we apply the proposed algorithm to numerical examples of Cooper *et al* [9], Tone and Sahoo [13] and Sueyoshi and Sekitiani [14] and compare the results to previous ones.

**4.1.** Consider the example of Cooper *et al* [9]. The dataset and results of their method are presented in table 1. As can be seen, by solving model (5) and the definition given by them, units *D–G* have congestion.

Now above example resolve by our presented algorithm. First, the model (2) is solved to measure efficiency of all DMUs. Results are shown in table 2. According to the results, DMUs are classified as follows:

$$E = \{A, B\} \quad E_o = \{A, B, C\} \quad E_o^c = \{D, E, F, G, H\}$$
$$E^c = \{C, D, E, F, G, H\} \quad E_{in} = \{A, B\}$$
$$E_{in}^c = \{D, C, E, F, G, H\}$$

Later, Model (12) for DMUs in $E$ is solved to determine anchor points. $DMU_B$ is specified as an anchor point. Let $E_{AN} = \{B\}$. Obviously, the anchor point is unique. Since $E_o \cap E_{in}^c = \{c\}$ and $B \in E_c^R$ according to the definition, $DMU_C$ is called congested-specific unit. $DMU_B$ is an anchor point and reference of $DMU_C$. Therefore, all inefficient units whose inputs, in at least one component, are greater than the inputs of the congested-specific unit) $DMU_C$) have congestion. Therefore, $DMU_D$, $DMU_E$, $DMU_F$ and $DMU_G$ have strong congestion among inefficient DMUs because their input is strictly greater than the input of the congested-specific unit. As seen, the congested units identified by the proposed algorithm are similar to the selected DMUs by Cooper *et al* [9]. The value of input congestion obtained is as follows:

$$x_D^c = x_D - x_C = 5 - 3 = 2 \quad x_E^c = x_E - x_C = 4 - 3 = 1$$
$$x_F^c = x_F - x_C = 4 - 3 = 1 \quad x_G^c = x_G - x_C = 4.5 - 3 = 1.5$$

**Table 1.** Dataset of Cooper *et al* [9] and results of the method for existence or non-existence of congestion.

| DMU | Input | Output | Congestion |
|---|---|---|---|
| A | 1 | 0.5 | × |
| B | 2 | 2 | × |
| C | 3 | 2 | × |
| D | 5 | 1 | ✔ |
| E | 4 | 1 | ✔ |
| F | 4 | 1.2 | ✔ |
| G | 4.5 | 1.2 | × |
| H | 3 | 1 | × |

**4.2.** Consider the second example of Tone and Sahoo [13] as shown in table 3. They compared the FGL model and model proposed by Sueyoshi and Sekitiani [14] to their model. The results are given in table 4. For this example, our presented algorithm is implemented. First, model (2) is solved to calculate efficiency of all DMUs. Results are shown in table 5. DMUs are classified as follows:

$$E = \{A, B\} \quad E_o = \{A, B\} \quad E_{in} = \{A, B\}$$
$$E_o^c = \{C, D\} \quad E_{in}^c = \{C, D\}$$

Model (12) was run for $DMU_A$ and $DMU_B$, which are extreme efficient units, to obtain anchor points. $DMU_B$ was identified as an anchor point. Hence, we put $E_{AN} = \{B\}$. Since $E_o \cap E_{in}^c = \varphi$ all units among inefficient DMUs (belonging to set$E^c$) whose inputs, in at least one component, are greater than the inputs of the anchor point ($DMU_B$) have congestion. For inefficient $DMU_C$, the first input is equal to and its second one is larger than the input of anchor point ($x_{1C} \geq x_{1B}$ and $x_{2C} > x_{2B}$). Therefore, by definition, it has weak congestion. Also, $DMU_D$ has strong congestion because both inputs are strictly larger than the inputs of anchor point ($x_{1D} > x_{1B}$ and $x_{2D} > x_{2B}$).

The congestion value for $DMU_C$ and $DMU_D$ are as follows:

$$x_{1C}^c = 2 - 2 = 0 \quad x_{2C}^c = 3 - 2 = 1$$
$$x_{1D}^c = 3 - 2 = 1 \quad x_{2D}^c = 3 - 2 = 1$$

Results obtained from our procedure are shown in 5th column table 4 and compared with results of [13]. $DMU_C$ has no congestion by FGL model, but by our procedure and [13, 14] it has congestion. In addition, identified congested units are similar with our procedure and [13, 14]. One cannot claim that one method is better because all four

**Table 2.** Results of model (2) and the proposed algorithm on [9] for existence or non-existence of congestion.

| DMU | $\theta^*$ | $\varphi^*$ | $s^{+*}$ | $s^{-*}$ | $E_{AN}$ | Congested-specific unit | Congestion |
|---|---|---|---|---|---|---|---|
| A | 1 | 1 | 0 | 0 | × | × | × |
| B | 1 | 1 | 0 | 0 | ✔ | × | × |
| C | 0.66 | 1 | 0 | 0 | × | ✔ | × |
| D | 0.40 | 2 | 0 | 0 | × | × | ✔ |
| E | 0.50 | 2 | 0 | 0 | × | × | ✔ |
| F | 0.50 | 1.66 | 0 | 0 | × | × | ✔ |
| G | 0.44 | 1.66 | 0 | 0 | × | × | ✔ |
| H | 0.67 | 2 | 0 | 0 | × | × | × |

**Table 3.** Dataset of second example of Tone and Sahoo [13].

| DMU | Input 1 | Input 2 | Output 1 | Output 2 |
|---|---|---|---|---|
| A | 1 | 1 | 1 | 1 |
| B | 2 | 2 | 2 | 2 |
| C | 2 | 3 | 2 | 1 |
| D | 3 | 3 | 1 | 1 |

models are presented with different definitions. In this paper, the proposed algorithm, like two other methods [13, 14] with greater sensitivity, determine congestion.

**4.3.** Consider example 2 of Sueyoshi and Sekitiani [14]. They consider seven DMUs with two inputs and four outputs that are shown in table 6. To determine the congestion of inefficient DMUs in the presence of multiple efficient projections.

Sueyoshi and Sekitiani [14] and Mehdiloozad *et al* [15] used their own methods for this example. The results are shown in the last two columns of this table. For inefficient $\mathrm{DMU}_A$ with non-zero output slack and $\phi^* = 1$ via model (6), $\mathrm{DMU}_B$, $\mathrm{DMU}_C$ and $\mathrm{DMU}_D$ are efficient projections for $\mathrm{DMU}_A$.

Now run our algorithm for the data of table 6. First, by solving model (2), we obtain the efficiency of all DMUs. Results demonstrated in table 7 and DMUs are categorized as follows:

$$E = \{B, E, F, G\} \quad E_o = E_{in}\{B, E, F, G\}$$
$$E_o^c = E_{in}^c = \{A, C, D\}$$

Units $B$ and $E$–$G$ are extreme efficient units in the input–output oriented. To determine the anchor points, model (12) is solved for these units. The anchor points set is $E_{AN} = \{B, E, F, G\}$. Obviously, the anchor point set has no unique member. Hence, models (13) and (14) should be run for DMUs belong to $E_{AN}$. The optimal value of models (13) and (14) is infeasible for $\mathrm{DMU}_E$. Therefore, $\mathrm{DMU}_E$ is an anchor point with larger output, which we call a deterministic anchor point.

Since $E_o \cap E_{in}^c = \varphi$, DMUs whose inputs are at least one component larger than the inputs of $\mathrm{DMU}_E$ are congested. Obviously, as inputs of $\mathrm{DMU}_A$, $\mathrm{DMU}_C$ and $\mathrm{DMU}_D$ are larger than the inputs of $\mathrm{DMU}_E$, according to the definition, they are strangely congested.

The values of inputs congestion are calculated as follows:

$$x_{1A}^c = x_{1A} - x_{1E} = 2 - 1 = 1 \quad x_{2A}^c = x_{2A} - x_{2E} = 2 - 1 = 1$$
$$x_{1C}^c = x_{1C} - x_{1E} = 2 - 1 = 1 \quad x_{2C}^c = x_{2C} - x_{2E} = 2 - 1 = 1$$
$$x_{1D}^c = x_{1D} - x_{1E} = 2 - 1 = 1 \quad x_{2D}^c = x_{2D} - x_{2E} = 2 - 1 = 1$$

Unlike the Sueyoshi and Sekitiani method [14] and Mehdiloozad *et al* method [15], the proposed algorithm does not need the efficiency values, projections of $\mathrm{DMU}_A$ and other inefficient DMUs. Only by identifying efficient and inefficient units, and anchor point with the largest output as benchmark, and comparing the inefficient units with it and easier computations, the status and amount of congestion are determined. Steps 1, 2-1 and 3-1 of our algorithm were implemented for the example proposed by Cooper *et al* [9].

On comparing the results, the congested units are similar in the two papers. For the example of Tone and Sahoo [13], steps 1, 2-1 and 3-2 of our algorithm were run. Results show that our procedure, like the model of Tone and Sahoo [13], is the same. For the example of Sueyoshi and Sekitian [14], steps 1, 2-2, 3-2 were run; by comparing inefficient units, via model (2), to the anchor point with the largest output, we were able to obtain congested DMUs. The results are similar to the results of Sueyoshi and Sekitiani method [14] and Mehdiloozad *et al* method [15].

**Table 4.** Results of methods for existence and non-existence of congestion.

| DMU | FGL model | TS model | Sueyoshi model | Proposed model |
|---|---|---|---|---|
| A | × | × | × | × |
| B | × | × | × | × |
| C | × | Weak congestion | Wide congestion | Weak congestion |
| D | Congestion | Strong congestion | Wide congestion | Strong congestion |

**Table 5.** Results of model (2) and the proposed algorithm for [13]. Existence or non-existence of congestion.

| DMU | $\theta^*$ | $\varphi^*$ | $s_1^{-*}$ | $s_2^{-*}$ | $s_1^{+*}$ | $s_2^{+*}$ | $E_{AN}$ | Congested-specific unit | Congestion |
|---|---|---|---|---|---|---|---|---|---|
| A | 1 | 1 | 0 | 0 | 0 | 0 | × | × | × |
| B | 1 | 1 | 0 | 0 | 0 | 0 | ✔ | × | × |
| C | 1 | 1 | 0 | 1 | 0 | 1 | × | × | ✔ |
| D | 0.66 | 2 | 0 | 0 | 0 | 0 | × | × | ✔ |

**Table 6.** Dataset and results of Sueyoshi and Sekitiani method [14] and Mehdiloozad *et al* method [15].

| DMU | Input 1 | Input 2 | Output 1 | Output 2 | Output 3 | Output 4 | Sueyoshi and Sekitiani method | Mehdiloozad *et al* method |
|---|---|---|---|---|---|---|---|---|
| A | 2 | 2 | 2 | 2 | 2 | 2 | Wide congestion | Weak congestion |
| B | 2 | 2 | 2 | 3 | 2 | 2 | × | × |
| C | 2 | 2 | 2 | 2 | 3 | 2 | Wide congestion | Weak congestion |
| D | 2 | 2 | 2 | 2 | 2 | 3 | Wide congestion | Weak congestion |
| E | 1 | 1 | 2 | 2.5 | 3 | 2 | × | × |
| F | 1 | 3 | 2 | 2 | 2 | 4 | × | × |
| G | 2 | 1 | 2 | 2.5 | 2.25 | 3 | × | × |

**Table 7.** Results of model (2) and the proposed algorithm for [14].

| DMU | $\theta^*$ | $\varphi^*$ | $s_1^{-*}$ | $s_2^{-*}$ | $s_1^{+*}$ | $s_2^{+*}$ | $s_3^{+*}$ | $s_4^{+*}$ | $E_{AN}$ | Congested-specific unit | Determinist anchor point | Congestion |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 0.50 | 1 | 0 | 0 | 0 | 0.50 | 1 | 1 | × | × | × | Strong congestion |
| B | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | ✔ | × | × | × |
| C | 0.50 | 1 | 0 | 0 | 0 | 0.50 | 0 | 0 | × | × | × | Strong congestion |
| D | 0.75 | 1 | 0 | 0 | 0 | 0.37 | 0.37 | 0 | × | × | × | Strong congestion |
| E | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | ✔ | × | ✔ | × |
| F | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | ✔ | × | × | × |
| G | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | ✔ | × | × | × |

## 5. Conclusion

In the available papers, the presence and non-presence of congestion was discussed or computed by depicting inefficient DMUs on the efficiency frontier. In this paper, with respect to S-shape form of the production function, the congestion definition and anchor points geometric properties, we have developed the connection between congestion definition and anchor points. Later, an algorithm has been suggested for identifying congested DMUs, computing its value and determining the status of congested DMUs. Without specifying the projection inefficient DMUs on the BCC efficient frontier, first by solving conventional DEA model, the efficiency of DMUs is measured and later among the extreme efficient units the anchor points are identified. If the anchor point is unique and it is the reference of congested-specific unit (inefficient input-oriented and efficient output-oriented DMU with the largest slack among DMUs) then all inefficient units whose inputs are, at least in one component, greater than the input value of the congested-specific unit have congestion; else all inefficient DMUs whose inputs are, at least in one component, greater than the input of anchor point are congested. If the anchor point is not unique, by solving two LP models for all anchor points, the deterministic anchor point is obtained (anchor point with largest output). Like the previous one, if the deterministic anchor point is the reference of congested-specific unit, then all the inefficient units whose inputs are, in at least one component, greater than the input of congested-specific unit, have congestion. Otherwise, all inefficient DMUs whose inputs are, in at least one component, greater than the input of the deterministic anchor point have congestion.

Therefore, due to the occurrence of any problems in determining the congested units in the presence of the optimal multiple projections for inefficient units, one of the advantages of our proposed method is the lack of the need to determine the projection for inefficient units and efficiency value. In addition, by easier calculation and only by comparing inefficient units with an anchor point with the largest output, congested units and their amount can be obtained.

Finally, our algorithm is implemented for examples of Cooper *et al* [9], Tone and Sahoo [13] and Sueyoshi and Sekitiani [14]. Comparing the results, we find that congested DMUs obtained by our procedure are similar to the results obtained in the papers [9, 13, 14].

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