Sādhanā (2020)45:16
https://doi.org/10.1007/s12046-019-1256-0

Dynamic tail re-assignment model for optimal line-of-flight breakages

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MS received 5 December 2018; revised 4 September 2019; accepted 24 November 2019

Abstract. The literature in aircraft routing focuses on cyclic rotation with the planned maintenance being assigned to the aircraft at the end of every rotation. The rotations are a set of flights provided with sufficient Maintenance Opportunity (MO) such that the planned maintenance could be carried out for the aircraft. In this research, a novel mathematical model has been introduced to the operational aircraft route assignment which considers both planned and ad hoc maintenances of the aircraft. A line-of-flight is defined as the set of geographic and time feasible flights being assigned to the hypothetical aircraft without any actual operational constraints. The model is formulated for the scenario where commercial planning department independently makes the line-of-flights and the maintenances have to be incorporated in those line-of-flights with minimal perturbations. In addition to the exact solution, the problem has also been solved using two heuristic solution approaches for the tailored module which is called the Tail Re-assignment, a problem dealt with by many airlines. The Tail Re-assignment problem can be considered as an optimization as well as feasibility problem. The objective of this research is to provide a quick solution that is feasible and near-optimal which can help in the managerial decisions in the tactical horizon. The model is tested with eight schedules with flights varying from 45 to 314, and additionally with multiple maintenance hubs and planning horizon of 20 days. The solution has all the hard constraints satisfied with the total number of onward flight rule breakages difference being minimal. The computation result shows that heuristic solutions solve the schedule for a medium-sized airline in quick time with less than 2% deviation from the exact solution.

Keywords. Aircraft routing; line-of-flights; tail Re-assignment; multiple maintenance hubs.

1. Introduction

The airline industry is immensely supported by the techniques used in operations research in a broad spectrum of areas such as strategic planning, schedule generation, fleet assignment, maintenance routing, crew pairing and rostering, passenger recovery and daily operational tasks. There are large scale data and information involved in solving each module of real-time airline operational planning. India has one of the fastest-growing aviation industry where over 13 million international passengers and 60 million domestic travel with one national carrier, Air India, and six other private-owned airline groups, in 2013–14 (Centre for Asia Pacific Aviation). Indian airports are one of the top ten global markets handling around 170 million passengers. The growth is significantly dependent on the costs and if that could be brought down, it would trigger new demand from the unexplored domain of the market that includes the young and vibrant population of the middle class India. Though the market is rapidly growing, the domestic on-time performance of the industry is low at 78.1% for departures and 77.7% for arrivals (Directorate General of Civil Aviation, Government of India) compared to the foreign markets such as Australia were it is 85.8% for departures and 84.3% for arrivals in 2014 (Department of Infrastructure and Regional Development, Australian Government). This is due to the lack of robust system in the planning phase which leads to umpteen uncertainties during operations. This work depicts modelling and analysis of solution methodologies for Tail Re-assignment which is a significant task in real-world airline operations, and on optimizing, will bring immense savings as well as efficiency to the airlines. Before the above-stated problem is addressed, a generic overview of the different components of airline planning and scheduling is provided.

A sequential process with partial feedback is used by most of the airline companies to plan and solve the strategic and operational tasks. The schedule design is created essentially 1 year prior to operation and is concerned with designing flight schedules which include decisions regarding the airports to be served, origin, destination, and timing of every flight based on the market behavior. Once the flight schedule is generated, a flight network is constructed, and the flight legs are assigned to various aircraft types, and

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this process is called a fleet assignment. According to Barnhart *et al* [1], the fleet assignment problem is the process of assigning a fleet type to each flight leg such that the expected profit contribution is maximized. Once the fleet has been decided, every individual crew member gets their schedule based on complex job-related rules and regulations known as crew scheduling and rostering. While some airlines carry out the process of aircraft routing after crew scheduling, others perform this task beforehand. Aircraft routing is the process of allotting individual aircraft to each of the flights such that all the operational constraints of the aircraft are satisfied. Though the modules are tightly coupled with each other, these problems are modelled and solved piece by piece due to the magnitude of the problem and the variables involved. The optimal solution of one module is used as input to the subsequent problems. Brio *et al* [2] have considered the manual intervention in obtaining the optimal solution for maintenance scheduling. These approaches are innovative, but more importance is given to the intuitive ability of scheduler and cannot be utilized in the current state of affairs due to the scalability of the problem.

This paper presents a mathematical model, and two other solution approaches for a tailored module which is called Tail Re-assignment, a problem dealt with in many airlines in India. In real time, the optimization criterion in aircraft routing can be more related to the quality and robustness of the solution, rather than cost functions. This paper suggests an alternative methodology of airline tail assignment which does not use the cost function, but uses the onward flight breakage rule, to take advantage of real-time data from strategic as well as an operational aspect of airline operations. The heuristic algorithms can be classified into two groups: constructive and improvement algorithms. A constructive algorithm builds a solution from the very beginning by assigning values to one or more decision variables in each iteration. An improvement algorithm starts with an initial feasible solution and focusses on achieving a better solution after each iteration. The heuristic techniques are applied based on the problem on hand, and there are different principles that govern the design of the same.

Thus, this paper contributes to the literature of Airline scheduling in the following ways:

- To develop a novel approach for solving Tail Re-assignment that is dynamic and non-cyclic in nature by optimizing the line of flight breakage while taking all operational constraints into consideration.
- To establish a mathematical model as well as two other approaches, one based on a stochastic greedy algorithm and the other based on Min-conflict algorithm, thus considerably reducing the solution time.
- To show how repair-based heuristic comprehensively solves even large size problems with more than 1000 flights.
- To validate the proposed solution, methodologies using real-world schedules obtained from major Indian aviation IT specialists, to generate routes without details regarding the cost structure of the individual flight legs.

Rest of this paper is organized as follows: Section 2 gives a concise summary of aircraft routing. Section 2.1 deals with the Tail Re-assignment and section 3 describes the formulation of a mathematical model for the problem considered. Section 4 presents the solution methodologies adopted in this work. Section 5 discusses the computational results. Finally, section 6 concludes the paper.

2. Literature review

The review of the relevant literature is provided in the following sections. The emphasis is given on the tactical planning aspects in the airline industry, specifically in aircraft routing and tail assignment. Aircraft Routing

The process of time table generation and aircraft planning are fairly similar in most of the airlines, while there is a significant difference in the process of maintenance routing. According to Cordeau *et al* [3], Maintenance Opportunity (MO) is a stoppage of aircraft at any maintenance station that can satisfy the minimum time requirement for maintenance of that aircraft. Federal Aviation Administration (FAA) in the US has established maintenance regulations which are described in Gopalan and Talluri [4]. According to the FAA, airlines should carry out four types of aircraft maintenance, generally known as A, B, C and D checks. The research states that the actual maintenance activity performed during a MO depends on the operational schedule. The specific aircraft maintenance constraint is considered that establishes that any aircraft route must provide a MO at least every certain number of days. A simplification of this constraint has been considered in the research done by Díaz-Ramírez [5], with the assumption that all overnight deadheads satisfy this maintenance duration requirement, and there is only one maintenance station, which is known as the base.

Different authors have provided different names for the process of aircraft routing based on the nature of the problem they have dealt with. The terms such as Through Assignment and Maintenance Routing have been coined to denote the problems where consideration of through values and maintenance are done separately. According to Gopalan and Talluri [6], maintenance routing is the assignment of aircraft to a set of flights, such that the aircraft will fly without maintenance for at most three days and return back to the maintenance hub for one night to undergo the stipulated maintenance. Desaulniers [7] reports set partitioning and time-constrained multi-commodity network flow formulation to solve aircraft scheduling and routing problem. Barnhart *et al* [8] define the concept of augmented string which is the string of flights with a maintenance flight that requires minimum time attached to the end. In the research, simultaneous fleet assignment and aircraft routing are modelled and solved using the Branch and Price technique.

A rigorous literature review on the modern techniques used for airline scheduling is done by Cohn and Barnhart [9]. Research has been done to integrate fleet assignment and aircraft routing by Zeghal *et al* [10], where the total net profit is maximized by using optimization-based heuristics. An attempt is made by Karaoglan *et al* [11] to solve aircraft routing in the cargo sector. Liang *et al* [12, 13] solve the daily maintenance routing problem using a novel compact-network representation of the time–space network in which the objective is the maximization of through values, and the short connections are penalized. In the earlier studies in aircraft routing by Feo and Bard [14], Kabbani and Patty [15] and Gopalan and Talluri [6], a significant assumption imposed on the model is that the maintenance occurs only during the night, while the flights have ensued during the daytime. This work further solves the integrated Aircraft Routing and Fleet Assignment problem using a diving heuristic. Most approaches to aircraft routing consider generic maintenance checks and not individual Ad hoc aircraft restriction constraints, thus making the process dependent on further manual intervention, which is an important aspect of the model presented in this paper.

2.1 *Tail re-assignment*

Tail assignment, as defined by Grönskvist [16], is the problem of assigning individual aircraft, identified by the tail number, to the flight legs taking into consideration the maintenance schedule for each aircraft. In light of the heavy competition, it is mandatory that airline companies keep their operating costs to a minimum by effective management of their flights, aircraft, and crew. According to Ageeva and Clarke [17], safety is an indispensable element for all airline operations and cannot be waived at any cost to meet other goals. Therefore, it is essential to provide each type of aircraft in the fleet with a separate maintenance-routing plan. As there are uncertainties involved, all routing plans must be impeccably coordinated to provide the best overall service. Currently, most airline companies do planning and operations in a sequential manner. The literature review is summarized in table 1.

The schedule design process begins 12 months prior to implementation and involves designing flight schedules, i.e., the origin, destination, and timing of each flight. In this research, a new version of the tail assignment problem is formulated, with the purpose of avoiding a number of shortcomings with conventional models used by airlines today. The maintenance scheduling after aircraft assignment has been done by Sriram and Haghani [22] in which aircraft shuffling for flights is done between different fleets with the penalization of such assignments. The seven-day cyclic schedule is considered with maintenance checks at night, and re-assignment of one-day trips is executed instead of individual flight legs. Integration of aircraft routing, crew pairing and tail assignment is done in work by Ruther *et al* [23], where the problem is solved closer to the Day of Operation (DOO) to obtain a significantly accurate solution.

In this work, a novel concept of Tail re-assignment is introduced, which can be defined as the process of allocating flights that are pre-assigned to hypothetical non-cyclic line-of-flights in a time horizon to the aircraft such that all operational constraints are satisfied with the minimization of line-of-flight breakages. The change in the original line-of-flights due to the operational constraints are called the line-of-flight breakages. The input schedule used in this work adopts a hub-and-spoke network as there are immense possibilities in cost-saving due to centralized operations that are pushing many airlines in India, traditionally working with point-to-point networks. The model discussed in the paper optimizes with respect to variable criterion and is valid for the different planning horizon. Also, it does not create a cyclic route which will not be meaningful for such varying environment.

In addition to the operational and planned maintenance constraints, this work also considers the ad hoc maintenance constraints of the aircraft, which does not come into picture during planning. The process of re-fleeting is done by Jarrah *et al* [24] who reports it being used in United Airlines. Winterer [25] solves the flight swapping with the retiming problem, i.e. the problem of retiming and swapping flights between fleets close to the day of operation, as a special case of a general requested resource reallocation with the retiming problem (R4P). A specialized algorithm is presented, which can produce multiple solutions, and is shown to be more efficient than a MIP model for the tested instances. The instances come from two European airlines, and contain 20–107 aircraft and 95–644 flights, for 2–8 fleets (fig. 1).

An example space–time network that involves in the Tail Re-assignment process is shown in figure 2. The set of line-of-flights is used along with availability and maintenance details of the aircraft. The flight which is supposed to be flown after a particular flight is its onward flight; for example, flight 103 is the onward flight of flight 102. The data will be dynamic based on the planning horizon and will differ in aspects such as flight timings and location, maintenance schedules and aircraft that should undergo maintenance. The maintenance schedule may or may not overlap with the flights assigned to the aircraft.

The last flight of each line-of-flights need not end at the starting location, and the schedule for the next planning horizon is chosen from the point where the current ends. The scope of planning horizon extends from a few days to weeks for the proposed model. While tail swapping is handled by the proposed model, there is no re-fleeting involved. This is due to the fact that though the cockpit

Table 1. Summary of literature.

Author and year	Modeling technique	Solution methodology	Characteristics
Kabbani and Patty [15]	Set partitioning model	Two state solution approach	Overnight maintenance, three day cyclic scheduling
Clarke *et al* [18]	Time space Network	Lagrangian relaxation	Two types of maintenance
Barnhart *et al* [8]	Flight string	Branch and Price	Daily schedule, maintenance at night
Gopalan, and Talluri [4]	Time space Network	Two stage iterative process	Daily routing, overnight maintenance
Gopalan and Talluri [6]	Time space network	Euler Tour Based Heuristics	Single maintenance station, four day routing
Liang *et al* [19]	Time space network	Heuristic Solution	Aircraft rotation, maintenance performed only during night
Liang *et al* [20]	Time space network	Heuristic Solution	Weekly scheduling
Haouari *et al* [21]	Connection Network	Heuristic Solution	Single type of maintenance check, daily schedule

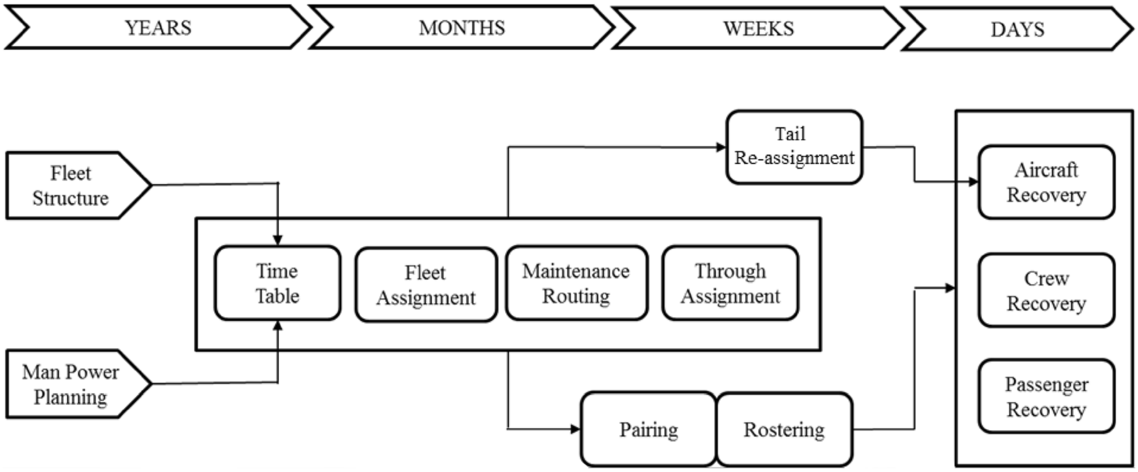

Figure 1. Airline scheduling modules.

configuration and crew requirement may be similar, it may not be cost-effective to swap between different aircraft types.

3. Mathematical modelling approach

The major difference in the existing literature and the proposed work is that the line-of-flights are pre-assigned and the maintenance schedule is known in the later stage. The solution is provided for the scenario where commercial department independently makes the line-of-flights, and the maintenance has to be incorporated in those line-of-flights with minimal disturbances. The airport and available time for the aircraft is known. Let F the set of flights including the maintenance slots, which are assumed as flights from and to the maintenance hub with the corresponding start and end times. There is no cost included for each flight. The set of maintenance flights alone are denoted by $M \subseteq F$. A is the set of aircraft for which the available time and the arrival station from the last flight of the previous planning horizon are provided. The availability of the aircraft is also represented as flights, for example, if an aircraft AC_{01} is available from 00:00 at Airport A (as in figure 2), it is assumed as a flight starting from A at 00:00 and ends in A at 00:00.

The objective of the proposed model is to assign line-of-flights to aircraft such that it maximizes the similitude between the initial schedule and the routing after including individual operational constraints of the aircraft. To enforce this, the onward flight rule is applied for every flight in the time horizon, except the last flights of each line-of-flight. The number of aircraft in the fleet is represented by *a,* and the total number of flights, including the maintenance flights undertaken by these aircraft, is n. These aircraft are allocated to a set of K line-of-flights which has the starting and ending flights in set U.

For each flight pair, $i, j \in F$ and $k \in K$, x_{ij}^{k} denotes the connection between the flights. x_{ij}^{k} is equal to 1 if i^{th} flight and j^{th} flight are connected by k^{th} line-of-flight in the new

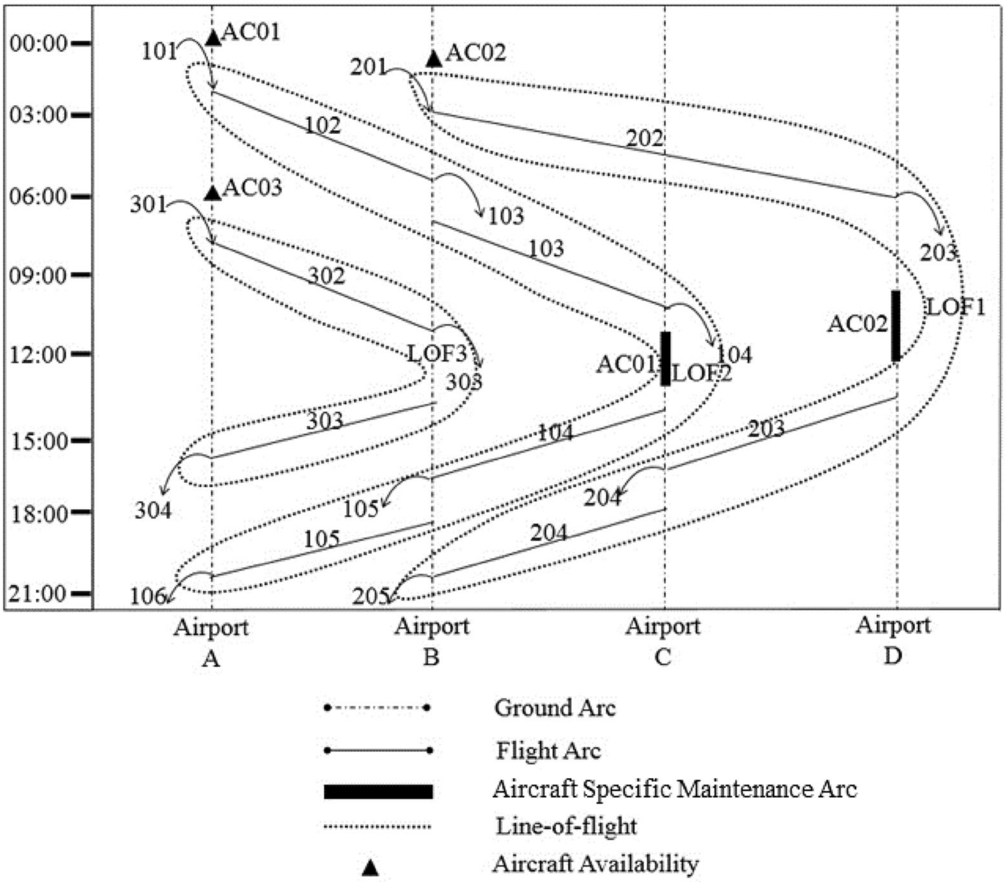

Figure 2. Space–time network with four airports.

schedule, and equal to 0 otherwise. For every flight i ∈ F, we represent its starting time and ending time as ST_i and ET_i respectively. The starting location of flight i is denoted by SL_i and ending location by EL_i. The maintenance flights are shown as m ∈ M and the aircraft that undergoes the corresponding maintenance is denoted by a_m. For each flight pair, i, j ∈ F and k ∈ K, y_{ij}^k denotes the connection between the flights in the original schedule as created during line-of-flight generation. y_{ij}^k is equal to 1 if i^th^ flight and j^th^ flight are connected by k^th^ line-of-flight in the original schedule, and equal to 0 otherwise.

The decision variables used are,

$$x_{ij}^k = \begin{cases} 1 & \textit{if flight i is connected to flight j by lof of k} \\ 0 & \textit{otherwise} \end{cases}$$

The parameters for the model are

$$t_{ij}^k = \begin{cases} 1 & \textit{if flight i is connected to flight j by lof of k} \\ 0 & \textit{otherwise} \end{cases}$$

ST_j – Start Time for Flight j
ET_i – End Time for Flight i
SL_j – Start Location for Flight j
EL_i – End Location for Flight i

$$Maximize \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} x_{ij}^k t_{ij}^k$$

$$\sum_{i=1}^{n}\sum_{k=1}^{a} x_{ij}^k \le 1 \quad \forall j \tag{1}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{a} x_{ij}^k \le 1 \quad \forall i \tag{2}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{a} x_{ij}^k = \mathrm{n} - \mathrm{a} \quad \mathrm{i} \ne \mathrm{j} \tag{3}$$

$$x_{ij}^k (ST_j - ET_i) \ge 0 \quad \forall i, j, k \tag{4}$$

$$x_{ij}^k (SL_j - EL_i) = 0 \quad \forall i, j, k \tag{5}$$

$$\sum_{i=1}^{n} x_{ij}^k = \sum_{l=1}^{n} x_{jl}^k \quad \forall j \notin U, K \tag{6}$$

$$\sum_{j=1}^{n} x^k_{a_{mj}} = \sum_{j=1}^{n} x^k_{mj} \quad \forall k, m \in M, a_m \subset \mathrm{A} \tag{7}$$

$$x^k_{ij} \in \{0,1\}, \forall i,j \in F, \forall k \in K \tag{8}$$

The objective of the model is to maximize the similarity of flight connections with that of the initial schedule. The first and second set of constraints represent the flight limiting constraints with the first set enforcing the incoming flights to one or none per flight pair. Similarly, the second set of constraints limits the outgoing flights from a flight pair to less than or equal to one. The third set of constraints, the flight-pair count constraints, handles the flight-pair variables and limits its count to the difference the total number of flights and aircraft. The fourth set of constraints represents the flight timing constraints and it ensures that in case the flight pairs are allotted, then the timing of the flights does not overlap. Similarly, the fifth set of constraints denotes the geographic constraints whereby the model ensures that the successive flight departure airport is the same as arrival airport of the current flight. The sixth set of constraints intends the flight pair flow balance constraints with the exceptions of flights at the extreme end of the line of flights. The seventh sets of constraints ensure that the maintenance flight and the aircraft requiring that maintenance get allotted to the same line of flight. The final set of constraints enforces the flight assignment to be 0 or 1 in the original as well as generated schedule.

The solution to the Tail Re-assignment model provides assignment of pre-assigned line-of-flights to the aircraft for the specific and dynamic time horizon. The minimum ground time is also considered with the flight time so that realistic solution for aircraft routing is obtained. The model is solved using branch and cut technique and the total number of flight pair variables for this model is the product of flight pair and the number of aircraft.

The number of variables in the problem is depended on the number of flights as well as the number of aircraft. It is found that as the number of flights is increased, the solution time for branch and bound method increase exponentially as presented in figure 3.

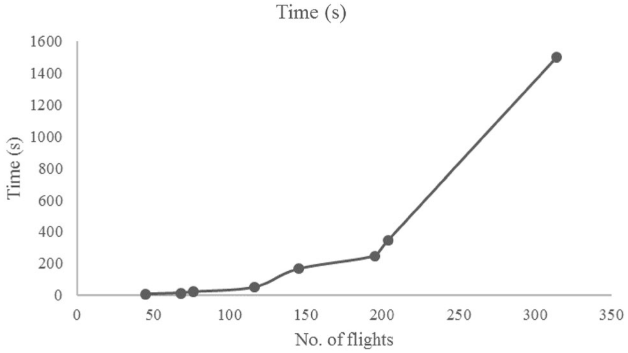

Figure 3. Time dependency on number of flights.

4. Solution methodologies

For the tail re-assignment problem considered, the following algorithms are developed as solution methodologies:

(1) Stochastic Greedy Algorithm – The proposed stochastic greedy algorithm is a constructive algorithm which does a stochastic reallocation resulting in candidate solutions which are built based on the single sweep through the data. This approach builds up the solution set, one element at a time starting from scratch, and terminates with the first complete solution.

(2) Airline Tail Re-assignment Algorithm – This algorithm is a repair-based heuristic that works only in the space where disruptions occur, which in this work are the ad hoc maintenance slots. In this approach, the concept of an interval graph is used to model the problem.

4.1 *Interval graph*

A graph $G = (N, A)$ is defined by a set $N = \{1,\ldots, n\}$ of vertices (also known as nodes), and a set A of connecting arcs called edges, where each edge in A joins exactly a pair of nodes in N and has no orientation. If an edge joins nodes i and j, it is denoted by (i, j). A pair of nodes in N are said to be adjacent, if there is an edge joining them in A. The degree of a node is the number of edges containing it. An interval graph is an undirected graph formed from a family of intervals S_i, $i = 0, 1, 2 \ldots, n$

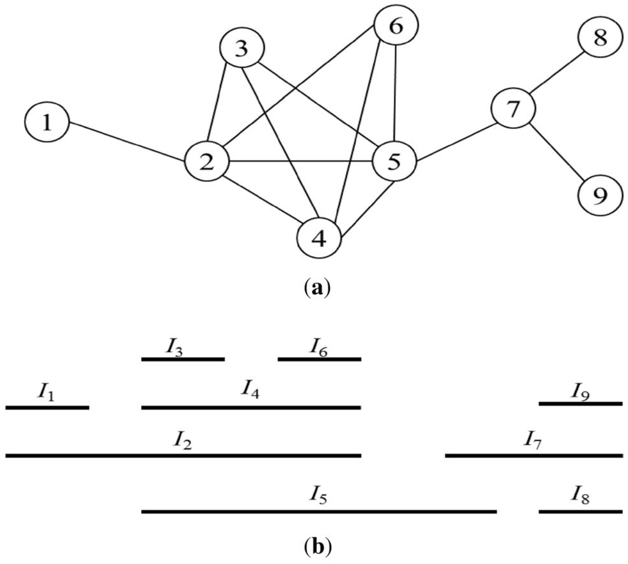

Figure 4. Interval graph (a) and timeline (b).

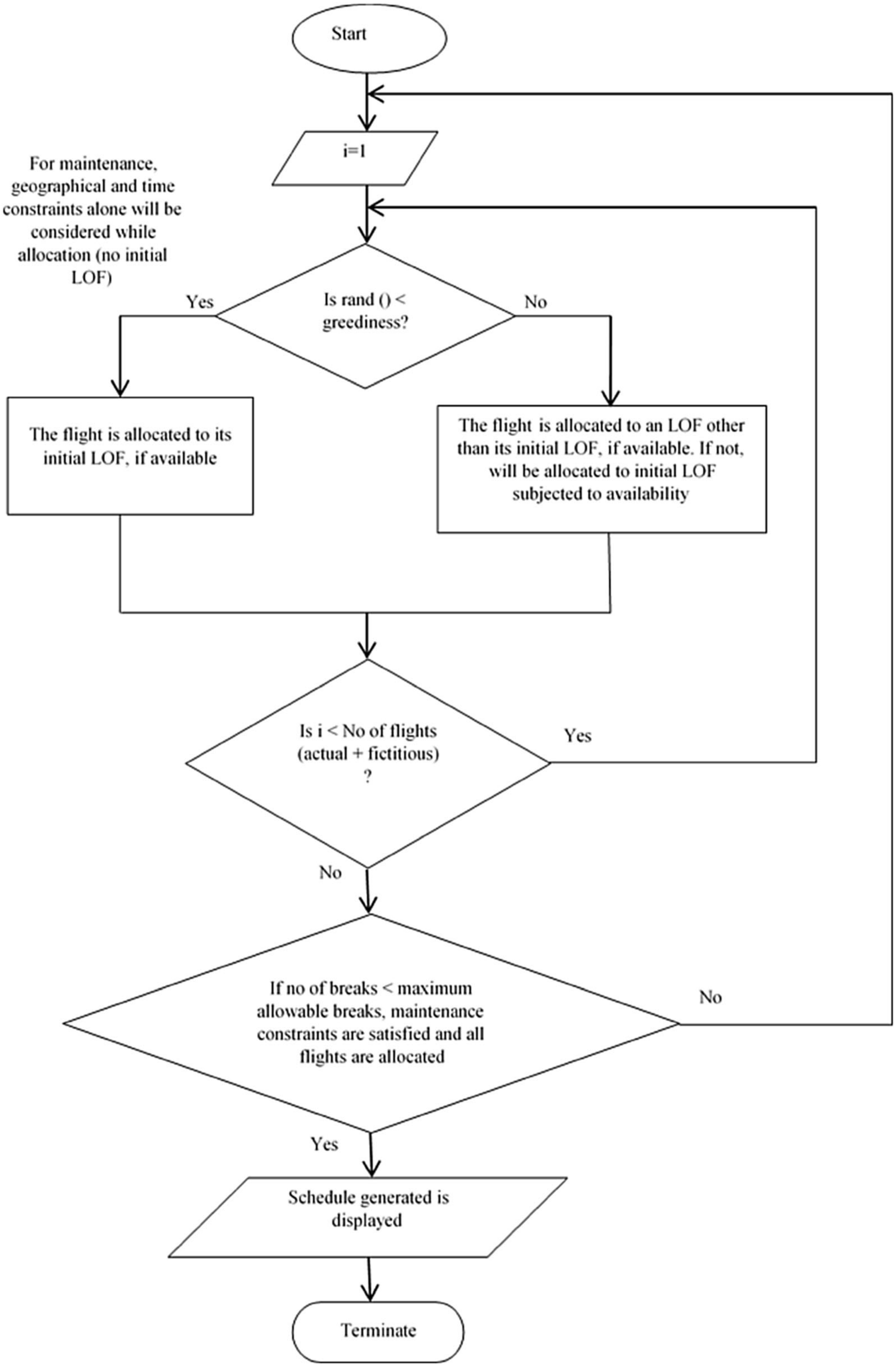

Figure 5. Stochastic Greedy Algorithm.

For each interval S_i, one vertex v_i is created and two vertices v_i and v_j are connected by an edge whenever the corresponding two sets have a nonempty intersection, that is,

$$E(G) = \{\{v_i, v_j\} | S_i \cap S_j \neq \emptyset\}.$$

A graph G is an interval graph if and only if the maximal cliques of G can be ordered M_1, M_2, …, M_k such that for

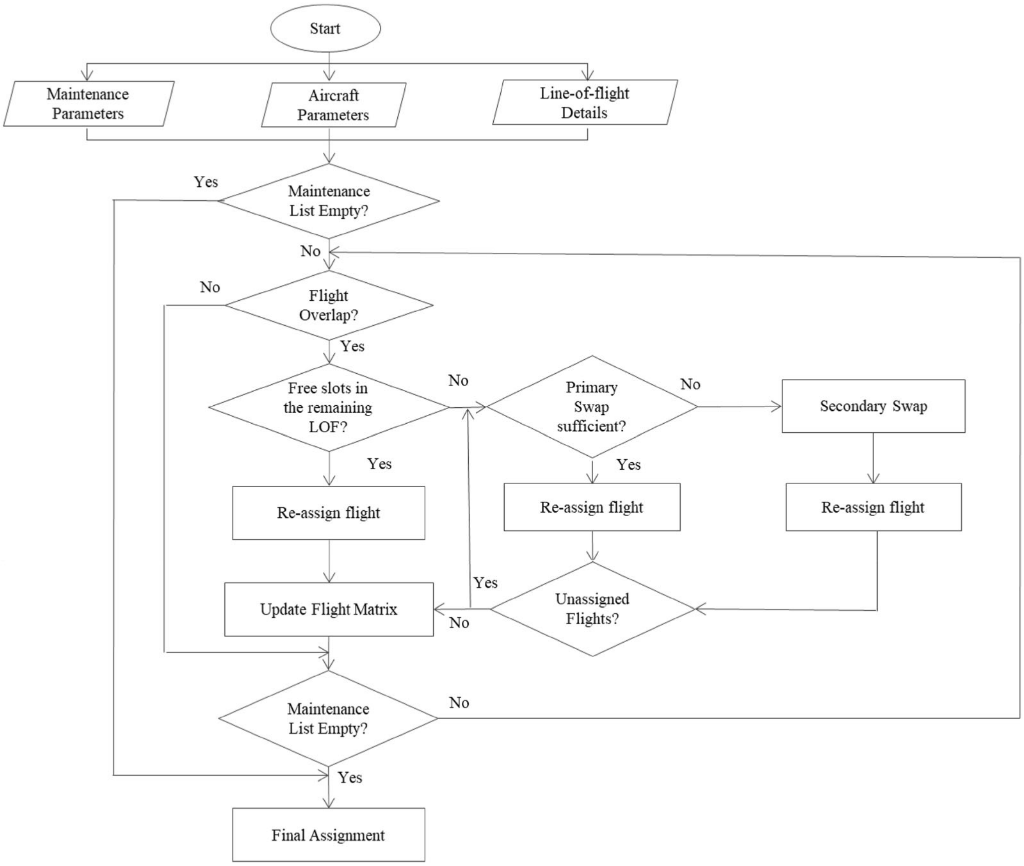

Figure 6. Airline Tail Re-assignment Algorithm (ATRA).

any $v \in M_i \cap M_k$, where $i < k$, it is also the case that $v \in M_j$ for any M_j, $i \leq j \leq k$, which is known as interval graph isomorphism as stated in the paper by Peter [26]. For instance, the following interval graph represents the corresponding interval in timeline as given in figure 4 (a) and (b) respectively.

4.2 *Stochastic Greedy Algorithm (SGA)*

The Tail Re-assignment problem is a variation of the activity selection problem. For an activity selection problem, the greedy algorithm is known to be the best algorithm. Hence it is only logical to assume that a variation of the greedy algorithm is used to solve the problem. Stochastic nature is introduced to account for reallocations that might occur during scheduling to facilitate maintenance activities. Moreover, greedy algorithms are faster compared to population-based algorithms like genetic algorithm. The quality of the solution obtained might not be as good as the genetic algorithm, but considering the operational nature of the problem, the greedy algorithm provides an acceptable solution in a shorter duration of time. The algorithm flow chart is represented in figure 5.

In SGA, the decisions made are based on a probability. A uniform random number generated is checked against the set randomness value. A purely greedy decision is made if it has a lower value. If it is greater than randomness, then a non-greedy option that best satisfies the constraints is selected.

If a spectrum of solution methodologies is considered, the greedy algorithm will be providing solutions in the quickest time. The other end of the spectrum is the exact methods with the highest quality of solution but with the slowest solution times. Meta-heuristics like genetic algorithms occupy a section around the middle with not so bad solution quality and reasonably faster solution times. SGA will occupy a section closer to the greedy algorithm but is not deterministic as traditional greedy algorithms. All these features make SGA a suitable algorithm for this problem with operational importance and were a good enough solution that satisfies the constraints would suffice.

4.3 *Airline Tail Re-Assignment Algorithm (ATRA)*

This work proposes a two-phase algorithm, Airline Tail Re-assignment Algorithm (ATRA). It is inspired by a minconflicts algorithm which is a heuristic technique used to solve constraint satisfaction problems. The algorithm identifies the flight schedule for a single fleet and simultaneously allocates the aircraft routes considering both maintenance constraints and fleet size for a given planning horizon. ATRA, as an assignment problem with side constraints, can be viewed as a repair heuristic thus, making it a fast solution methodology for even large-scale problems with more than 1000 flights. While the mathematical model and SGA are build heuristic which generates the solution as a whole from the inputs. The solution constitutes the aircraft assigned to different flights satisfying the maintenance constraints. An iterative process is used for the identification of routes for each aircraft. Once assigned, all the variables related to the legs covered in the routing solution are set to one, and the maintenance conditions are checked one by one.

The solution of the model identifies the route that assigns all the flights in the given horizon with geographical continuity in the network. The solution begins with identifying a large number of possible flights in the planning horizon and related parameters such as time and location of the flight. This information is assumed to be given, known, and fixed for a flight with a specific departure time. This method starts with a base solution from the data provided. The solution is found out in an incremental manner in the sense that the final schedule is obtained by the routes found one at the time.

The tail assignment problem is usually solved after the fleet assignment has been completed. The first phase of this work is allotment of the equipment, in which candidate flight segments are linked to a specific aircraft tail number within a given sub-fleet of the airline based on maintenance constraints. The data obtained sheets are initial flight assignment to the specific line-of-flight holder, which is prepared by the commercial department. Also, the maintenance data is obtained from the maintenance department. The first phase of the algorithm is the assignment of the various line-of-flight holder to the aircraft based on initial location and available time of the aircraft.

There are primarily three stages involved in this algorithm: free swap, primary swap and secondary swap. These are local search in the area near the maintenance slots of the aircraft so that the flights do not overlap with the maintenance. The free swap is the allocation of the flights that are overlapped with the maintenance schedule of the given aircraft to other aircraft, in case any of the aircraft is free. If there are overlapping flights for the flights to be re-allocated, then the algorithm does a mutual swap between the

Table 2. Inter-flight times frequency distribution.

Class	Interval	Frequency
1	0–2	175
2	2–4	36
3	4–6	31
4	6–8	19
5	8–10	13
6	10–12	20
7	12–14	3
8	14–16	1
9	16–18	3
10	18–20	2
11	20–22	2
12	22–24	0
13	24–26	0
14	26–28	1
15	28–30	1
16	30–32	0
17	32–34	1

Table 3. Schedule generator analysis.

Scale parameter 0.5			Scale parameter 1.0			Scale parameter 1.5			Scale parameter 2.0		
Flights/ LOF	LOF	Generated status	Flights/ LOF	LOF	Generated status	Flights/ LOF	LOF	Generated status	Flights/ LOF	LOF	Generated status
10	5	No	10	5	No	10	5	Yes	10	5	Yes
20	5	No	20	5	Yes	20	5	Yes	20	5	Yes
30	5	No	30	5	Yes	30	5	Yes	30	5	Yes
40	5	No	40	5	Yes	40	5	Yes	40	5	Yes
50	5	No	50	5	Yes	50	5	Yes	50	5	Yes
60	5	No	60	5	Yes	60	5	Yes	60	5	Yes
70	5	Yes	70	5	Yes	70	5	Yes	70	5	Yes
10	6	No	10	6	No	10	6	Yes	10	6	Yes
20	6	No	20	6	Yes	20	6	Yes	20	6	Yes
30	6	No	30	6	Yes	30	6	Yes	30	6	Yes
40	6	Yes	40	6	Yes	40	6	Yes	40	6	Yes
50	6	Yes	50	6	Yes	50	6	Yes	50	6	Yes
60	6	Yes	60	6	Yes	60	6	Yes	60	6	Yes
70	6	Yes	70	6	Yes	70	6	Yes	70	6	Yes

Table 4. Analysis of model robustness.

							Branch & Bound		SGA			ATRA		
Problem instance	Days	Flights	Flights/LOF	LOF	Maintenance	No. of stations	No change	Time	No change	Time	% Deviation	No change	Time	% Deviation
D01	14	75	15	5	3	8	64	23	–	–	–	–	–	–
D02	14	100	20	5	3	8	87	52	–	–	–	–	–	–
D03	14	150	25	6	3	8	133	188	–	–	–	124	0.2	0.07
D04	14	200	40	5	3	8	186	412	184	1.2	0.01	175	0.4	0.06
D05	14	250	50	5	3	8	239	964	234	1.8	0.02	236	0.3	0.01
D06	14	300	50	6	3	8	291	1470	291	3.9	0	286	0.3	0.02
E01	14	75	15	5	3	8	68	26	–	–	–	–	–	–
E02	14	100	20	5	3	8	89	44	–	–	–	81	0.3	0.09
E03	14	150	25	6	3	8	138	172	136	1.1	0.01	130	0.4	0.06
E04	14	200	40	5	3	8	185	396	182	1.5	0.02	184	0.3	0.01
E05	14	250	50	5	3	8	246	981	245	2.6	0	238	0.2	0.03
E06	14	300	50	6	3	8	293	1520	291	4.8	0.01	287	0.2	0.02
F01	14	75	15	5	3	8	71	21	69	1.0	0.03	69	0.2	0.03
F02	14	100	20	5	3	8	94	53	92	1.0	0.02	89	0.2	0.05
F03	14	150	25	6	3	8	150	194	150	1.2	0	150	0.3	0
F04	14	200	40	5	3	8	196	419	194	2.0	0.01	194	0.3	0.01
F05	14	250	50	5	3	8	244	948	244	3.1	0	244	0.3	0
F06	14	300	50	6	3	8	297	1430	297	4.3	0	297	0.3	0

Table 5. Output summary.

Problem set	No. of flights	No. of service locations	Maintenance slots	No. of LOFs	Branch & Cut		SGA (best of 100 iterations)			ATRA		
					Flights unchanged	Time (s)	Flights unchanged	Time (s)	% Deviation	Flights unchanged	Time (s)	% deviation
P01	45	6	2	5	44	6	44	0.402	0	44	0.373	0
P02	68	12	3	4	65	12	65	0.588	0	64	0.402	1.538
P03	76	8	2	4	73	19	73	0.78	0	72	0.392	1.369
P04	116	6	2	4	114	48	114	0.812	0	114	0.385	0
P05	145	6	3	5	141	164	141	1.034	0	140	0.426	0.709
P06	195	8	3	5	193	247	193	0.978	0	192	0.407	0.518
P07	204	12	3	6	201	447	201	1.436	0	201	0.419	0
P08	314	11	3	6	306	1500	305	4.644	0.327	302	0.371	1.307

flights that are feasible. This is based on the fact that if the flights are connected in the interval graph, then there is an overlap between the flights. In case there are no connections between the flight assigned to an aircraft and any other flights of aircraft a, then it means there are no flights that overlap during that flight duration under consideration, and a free swap occurs.

If the flights of all other aircraft are connected to the flight at maintenance, then there is a mutual swap between the flight during maintenance and the flight such that there is no overlap of the new flight with the maintenance slot and this process is called the primary swap. If there are no flights feasible for primary swap, then the algorithm steps into the secondary swap, wherein the flights of the swapped aircraft are checked with all the remaining aircraft, other than the maintenance aircraft for free and primary swap, as shown in figure 6.

This process is repeated for all the maintenance checks stops when all the flights are assigned the aircraft. This solution algorithm is a heuristic one, in the sense that it does not guarantee an optimal solution with minimum breakages to the line-of-flight while covering all flight legs in the network. Thus, the final flight schedule to be covered by the fleet will be, in fact, defined by the routing solutions satisfying maintenance constraints and fleet size capacity. The definition of flight legs, departure times and maintenance schedule is given by the routes provided. There is no need for a specific or additional formulation for the flight scheduling problem.

The output is the re-assigned flights to aircraft based on the maintenance constraints so that the onward flight rule is not broken beyond a limit. The final flight schedule is identified together with the aircraft routes while satisfying maintenance requirements. The methodology used is advantageous while dealing with some of the problems that arise in the flight scheduling process such as feasibility: the inclusion of any flight leg in a route establishes its feasibility. The heuristic procedure provides fairly good solutions (near-optimal) with high savings in computational time.

5. Computational results

The input data for airline scheduling problems required for the formulation of the model has been generated based on real-time data obtained from one of the leading Indian companies that specializes in navigational information and optimization solutions for airline services. The Commercial Department of the airlines prepares the fleet assignment without taking into consideration the operational constraints, mainly maintenance. The maintenance information is obtained from the Operations team. Both this information is used as the input for the system built.

The various parameters involved are categorised as Flight Parameters, Aircraft Parameters and Maintenance

Table 6. Tail Re-assignment sample schedule.

Aircraft line	Air operator certificate	Flight no.	Departure station	Arrival station	Departure time	Arrival time	Flight time	Aircraft type
4	AA	323	BLR	DEL	9:30	11:30	2:00	A330
4	AA	324	DEL	BLR	12:30	14:30	2:00	A330
4	AA	325	BLR	MAA	15:00	16:00	1:00	A330
4	AA	326	MAA	BLR	17:00	18:00	1:00	A330
4	AA	327	BLR	PNQ	09:30 (2)	10:30 (2)	1:00	A330
4	AA	328	PNQ	BLR	11:00 (2)	12:00 (2)	1:00	A330
4	AA	329	BLR	HYD	12:30 (2)	13:30 (2)	1:00	A330
4	AA	M1	BLR	BLR	14:15	14:45	00:30	A330
6	AA	180	BLR	DEL	9:00	11:00	2:00	A330
6	AA	181	DEL	BLR	12:00	14:00	2:00	A330
6	AA	182	BLR	MAA	15:30	16:30	1:00	A330
7	AA	531	BLR	GOI	5:30	6:30	1:00	A330
7	AA	532	GOI	BLR	7:00	8:00	1:00	A330
7	AA	533	BLR	CCU	8:30	10:30	2:00	A330
7	AA	534	CCU	BLR	11:00	13:00	2:00	A330
7	AA	535	BLR	CCJ	13:30	14:30	2:00	A330
7	AA	536	CCJ	BLR	15:00	16:00	1:00	A330
7	AA	537	BLR	MAA	17:00	18:00	1:00	A330
7	AA	M2	BLR	BLR	15:30	16:30	1:00	A330
11	AA	722	BLR	HYD	12:30	13:30	1:00	A330
11	AA	723	HYD	BLR	14:00	15:00	1:00	A330
11	AA	724	BLR	PNQ	16:30	17:30	1:00	A330
11	AA	725	PNQ	BLR	18:30	19:30	1:00	A330
11	AA	726	BLR	DEL	20:00	22:00	2:00	A330

Parameters. The variables included in Flight Parameters are Flight ID, Flight Date, Departure Station, Arrival Station, Standard Time Departure (STD), Standard Time Arrival (STA), Minimum Ground Time, Line of Flight (LOF) Holder and Onward Flight. The various Aircraft Parameters are Aircraft Type, Total Seats, Aircraft Registration, Aircraft Available Station and Availability Time. The various Maintenance Parameters includes Activity ID, Maintenance Start Time, Maintenance End Time and Location. The generic classification of constraints is Geographical Continuity, Ground Time, No Overlap, Aircraft Type Match and Onward Flight Rule. The Geographical Continuity constraint states that the sequence of flight assigned against an aircraft should always be continuous, i.e., if two flights F1 and F2 are assigned against an aircraft, F2 following F1 then the arrival airport of F1 should be same as departure airport of F2.

It has been found from the real-time data that the inter flight times fits the Weibull distribution with parameters $\alpha = 0.46877$, $\beta = 1.8061$, and $\gamma = 0$ (table 2). The robustness of the data generator algorithm, mathematical model and the different algorithms has been analyzed for four different levels of scale parameter (β), mainly 0.5, 1.0, 1.5 and 2.0. It has been found that the schedule generator fails for lower number of flights and line of flights when $\beta = 0.5$. The results for the schedule generator have been provided in table 3.

The behavior of the mathematical model and the algorithms has been studied for three variations of β value, mainly, 1.0, 1.5 and 2.0. Six problem instances have been solved for each scale parameter and are represented as D01–D06 (for $\beta = 1.0$), E01–E06 (for $\beta = 1.5$) and F01–F06 (for $\beta = 2.0$). With the reduction of β, the inter flight times decreases and thus increases the aircraft utilization.

It is found that for lower values of β, SGA and ATRA give infeasible solution when the number of flights is below 100, as shown in table 4. Though ATRA provides solution for problem instance with 150 flights, SGA fails to do so. This can be attributed to the fact that the initial assignment occurs on a probabilistic basis in SGA.

The Tail Re-assignment problem is mathematically modeled in C# using Concert Technology and solved using branch and cut technique in CPLEX solver. The SGA and ATRA presented in the previous section have been implemented using C#. The solution is run on a HP Pavilion M6 Notebook PC with Core i5-3210M 2.5GHz Intel processor and 6GB RAM running Windows 7 Professional. Variability in run times is experienced even when running the same algorithm. Therefore, the algorithm is run five times and the average of these results is used when evaluating the performance of runtime. The model is tested on real-world instances of a major airline company in India. The output is obtained as the flights that are initially assigned to each aircraft as well as the re-allocated flights based on the

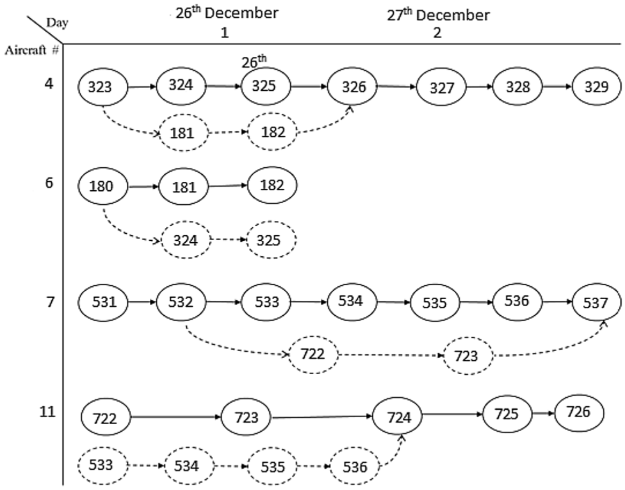

Figure 7. Sample Tail Re-assignment Model.

maintenance schedule of the aircraft and is summarized in table 5. All the flights are assigned to the aircrafts with geographical continuity constraint satisfied.

The schedules provided are domestic schedules which include flights varying from 45 to 314, with multiple maintenance hubs and planning horizon of 20 days. The problem instances P01 to P08 represent the real-time flight schedule with Weibull distributed inter-flight times with scale parameter β as 1.8061. The summary of results is shown in table 5. All the hard constraints are satisfied while the total number of onward flight rule breakages difference is minimal.

The input to the model is the aircraft position, availability and maintenance details along with the line-of-flight which is obtained from aircraft routing. The schedule contains the flight details with the arrival and departure station as well as chocks on and chocks off time (as in table 6). In addition to the flights, the maintenance is also scheduled for individual aircraft which is shown in the end of the line-of-flights. The output of Tail Re-assignment model will be allocation of all flights to aircraft that satisfies all the maintenance schedules. The model swaps flights that overlap with maintenance checks assigned to the aircraft with other aircraft of the fleet, thus creating minimal breakage in the line of flights.

The example provided in figure 7 shows that the Line of Flight 0323-0324-0325-0326-0327-0328-0329 is initially assigned to aircraft number 4. If the aircraft 4 has to undergo maintenance M1 at BLR from 14:15 to 14:45, the flights 0323-0324-0325-0326 have to be cancelled, as they overlap with the maintenance. The cancellation start point for the upward pass will be 0323, which overlaps with the maintenance. It propagates to 0323 as the arrival station (DEL) does not match with the maintenance station. For the downward pass, the flight 0325 is cancelled first. And this propagates to 0326 as the departure station (MAA) of 0326 is different from the maintenance station. The model finds the best set of flights to be re-assigned to this line-of-flight such that the maintenance constraint is satisfied. Thus, there will be swap of flights 0324-0325 and 0181-0182 between Line of Flights 4 and 6 respectively. The Tail Re-assignment model changes the line-of-flight to 0323-0181-0182-0326-0327-0328-0329. Similarly, the flight sequence of aircraft 7 changes from 0531 – 0532 – 0533 – 0534 – 0535 – 0536 – 0537 to 0531 – 0 532 – 0722 – 0723 – 0537 and aircraft 11 changes from 0722 – 0723 – 0724 –0 725 – 0726 to 0533 – 0534 – 0535 – 0536 – 0724 – 0725 – 0726 due to the inclusion of maintenance M2 in aircraft 7.

6. Conclusions

In this research, a new paradigm for tail assignment, which has the routes and pairings for aircraft generated based on the initial line-of-flights and the onward flight allocation is maximum followed. Routes are generated specifically for each aircraft in accordance with the operational and maintenance constraints, thereby eliminating the need for planners to solve it manually. These routes consider the location, maintenance and flying history of the individual aircraft. Unlike the maintenance routing, all the planned and unplanned maintenance requirements for the aircraft during the planning horizon is taken into consideration. The Tail Re-assignment plays an important role in the mitigation of crew pairing disruption on the inclusion of operational constraints of the aircraft. Crew pairing disruption can end up in delays in the flights due to unavailability of the crew as Flight Duty Period (FDP) Limitation is exceeded on re-assignment of flights with the original line-of-flight.

As the solving time for exact solutions increases exponentially with the problem size, two heuristic algorithms are used to solve the problem to obtain a good feasible solution in real time. This includes the Stochastic Greedy Algorithm (SGA) and Airline Tail Re-assignment Algorithm (ATRA), based on Min-conflict approach with the reduction in the computational time, with minimal variation from the optimal solution, so that swift operational decisions could be taken by the planner. Also, the robustness of the models is analyzed with various data sets. It is found that ATRA and SGA are unable to solve the problem when the alternatives are less. The results show that although the optimal is obtained with the branch and bound method, the time taken to solve increases exponentially with an increase in the number of variables. Also, it is noted that the variations in the run time for ATRA are minimal with the increase in the number of flights, locations and aircraft as it works only on the repair sites of the problem which are the maintenance slots. Further research can be done on integrating the aspects of crew pairing and aircraft routing. The base and flying hour constraint of the crew, as well as aircraft can be included in the current problem, thus eliminating the time-consuming sequential process of planning.

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