Inlet swirl decay of non-Newtonian fluid in laminar flows through tubes

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Abstract. Use of swirling velocity at the inlet has been a classical way to enhance the transport properties of flow within pipes. Owing to the laminar nature of flow of fluids in small tubes, enhancing transport within such tubes is very important for applications involving fluid mixing or heat transfer. Non-Newtonian fluids have varied applications in small pipes ranging from heat pipes to micro heat exchangers. To enhance the transport characteristics, we adopt the method of including swirling of fluid at the inlet of the pipe. Considering this, we investigate the effect of the power-law index of the fluid on the decay of the inlet swirl. We find that the length through which swirl decays to 1% of its value at inlet is strongly dependent on the power-law index value. A correlation for the decay length as a function of Reynolds number and power-law index is developed.

Keywords. Swirl decay; inlet swirl; non-Newtonian; power-law; laminar flow.

1. Introduction

With the ever-expanding and ever-diversifying medical industry, medical research involving bio-fluids have become very important in the modern world. Since such fluids are often available in very small quantities, such fluids are typically transported using small channels such as mini-channels and mini-tubes. Although mixing of such bio-fluids is very important in medical industry, small hydraulic diameter of such channels makes flows within them laminar. For example, mixing of bio-fluids such as blood [1] or dental slurry [2] has been often studied in the literature because of their significance to enhancement of human life albeit their availability in small quantities. Therefore, laminar flows have been the subject of intense investigation over the recent years due to their applications in small devices [3–5]. Such laminar flows have a decreased overall thermal and mass transport characteristics [6]. Inducing a swirl in the fluid flow is a classical way to enhance the transport characteristics of such flows [7–10]. Swirling of fluid in a pipe has also been found to have other numerous important applications like in micro-mixers, cyclone separators and micro-combustion chambers [11, 12]. Pharmaceutical materials, which are non-Newtonian in nature are also mixed in small quantities. It is well known that swirling flows tend to enhance transport, however, due to the viscous effect of the fluid in contact with

pipe walls there is decay of the swirl which hampers further mixing. Therefore, it becomes very important to study and understand the nature of decay of swirl. In applications mentioned above, where the flow will be predominantly laminar and fluid will be essentially non-Newtonian, the understanding of swirl decay becomes critical.

It is important to mention that the decay of swirl has been extensively studied in the literature for Newtonian fluids such as water. Examples of the research on the decay of swirl in flows can be found with Keith and Sonju [13], who were the first to discuss the swirl decay in a turbulent flow through pipes. Reader-Harris [14] found an approximate solution of the Navier-Stokes equation to predict the swirl decay as a function of the pipe's length. The decay of swirl in turbulent swirling flow in an annular pipe was studied by Scott and Bartelt [15]. Numerical experiments were conducted by Ayinde [16] to study the correlation of decay of inlet swirl in a laminar pipe flow. Yao and Feng [17] found a generalized analytical solution to find the rate of swirl decay in a laminar pipe flow; here the authors modeled the swirl velocity profile as a combination of a forced vortex in the core and free vortex in the annulus. Effect of wall slip on swirl decay, heat transport properties and thermodynamic properties [18–20] was studied and it was found that increase in wall slip reduces the rate of swirl decay. The decay of swirl was studied for gas–liquid two-phase flow in a pipe [21]. Inviscid swirling flows also have been studied numerically [22, 23] along with their stability analyses [24]. In a recent study, the effect of variation of properties with

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temperature on the swirl decay in micro-tubes was studied keeping in mind the heat transfer applications [25]. Experiments on the decay of swirl in laminar pipe flow were conducted by Rocha *et al* [26] and the results were compared with their numerical results. The area of interest is found to be the augmentation of the flow transport properties, which occurs due to the inlet swirl velocity in a laminar pipe flow. The decay of the inlet swirl phenomenon is found to occur close to the inlet.

Non-Newtonian fluids are increasingly used in many chemical processing industries, such as the ones dealing with paints, slurries or polymer solutions. Augmentation of the transport properties of these fluids may be necessary when they are subjected to mixing as well as heat transfer applications. Although we see a large amount of recent literature published on swirl flow, to the best of our knowledge, we have not found any literature for swirl decay studies based on non-Newtonian fluids. Although, experimental heat transfer study was conducted for laminar swirling in pipe flow generated by insertion of twisted tapes for a non-Newtonian power-law fluid and an augmentation in heat transfer was found [27–29], there has not been any study conducted on the effect of non-Newtonian behavior on the swirl decay. The area of interest is found to be the augmentation of the flow transport properties, which occurs due to the inlet swirl velocity in a laminar pipe flow. The decay of the inlet swirl phenomenon is found to occur close to the inlet. Therefore, in the present study, we computationally analyze the effect of inlet swirl in a non-Newtonian power-law fluid on the swirl decay. We take laminar flow behavior and we use a swirl velocity profile as a combination of a forced vortex in the core and free vortex in the annulus [17]. We take an effort to match our results with the results of Pati and Kumar [25]. We plot the rate of swirl decay close to the inlet of the pipe for various parameters such as Reynolds number and power-law index. We also make an effort to find a correlation for the swirl decay as a function of other parameters, which could aid in the design of such systems.

2. Theoretical formulation

2.1 Model and assumptions

In order to investigate the swirl decay characteristics of a power-law fluid, the following assumptions have been made:

- The flow is steady, laminar and incompressible
- The fluid behaves as a non-Newtonian power law fluid
- The thermo-physical properties of the fluid have been assumed to be invariant with the position as well as temperature.
- Flow is assumed to be axisymmetric
- Body force has been neglected

At the inlet, we impose a swirl direction velocity along with an axial flow velocity as shown in figure 1. The diameter of the pipe is considered to be $D(= 2R)$, where R is the radius of the pipe. The length of the pipe is represented by L as shown in figure 1. We assume the cylindrical coordinate system for describing the flow with the coordinates being r , θ and z and corresponding velocities being V_r , V_θ and V_z respectively.

The computational domain is chosen to be axis-symmetric for ease of conducting the numerical study, without loss of general mechanism of swirl decay as shown in figure 2. We also assume that the thermophysical properties of the fluid are temperature-independent.

The governing equations for the liquid region ($0 \leq r < R, 0 \leq z \leq L$) of a power law fluid, expressed in cylindrical coordinates for the present axis-symmetric study, are written as [16, 17],

Continuity Equation:

$$\frac{1}{r} \frac{\partial}{\partial r} (rV_r) + \frac{\partial V_z}{\partial z} = 0 \quad (1)$$

r - momentum equation:

$$\rho \left(V_r \frac{\partial V_r}{\partial r} - \frac{V_\theta^2}{r} + V_z \frac{\partial V_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \left[\frac{1}{r} \frac{\partial}{\partial r} \left(\eta r \frac{\partial V_r}{\partial r} \right) + \frac{\partial}{\partial z} \left(\eta \frac{\partial V_r}{\partial z} \right) \right] \quad (2)$$

θ - momentum equation:

$$\rho \left(V_r \frac{\partial V_\theta}{\partial r} + \frac{V_r V_\theta}{r} + V_z \frac{\partial V_\theta}{\partial z} \right) = \left[\frac{1}{r} \frac{\partial}{\partial r} \left(\eta r \frac{\partial V_\theta}{\partial r} \right) + \frac{\partial}{\partial z} \left(\eta \frac{\partial V_\theta}{\partial z} \right) \right] \quad (3)$$

z - momentum equation:

$$\rho \left(V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \left[\frac{1}{r} \frac{\partial}{\partial r} \left(\eta r \frac{\partial V_z}{\partial r} \right) + \frac{\partial}{\partial z} \left(\eta \frac{\partial V_z}{\partial z} \right) \right] \quad (4)$$

Where power law viscosity η is given by $\eta = \kappa \left(\sqrt{\frac{1}{2} \mathbf{D} : \mathbf{D}} \right)^{n-1}$ and $\mathbf{D}$ is the deformation tensor given by:

Figure 1. Schematic diagram describing the problem.

Figure 2. Schematic diagram of the computational domain.

$$\mathbf{D} = \frac{1}{2} \begin{bmatrix} 2 \frac{\partial V_r}{\partial r} & \frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r} & \frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \\ \frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r} & 2 \frac{V_r}{r} & \frac{\partial V_\theta}{\partial z} \\ \frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} & \frac{\partial V_\theta}{\partial z} & 2 \frac{\partial V_z}{\partial z} \end{bmatrix} \quad (5)$$

Here, κ is defined as the flow consistency index and n is the power law index. In the present study, we assume small values of inlet swirl number, assuming that the vane angle of the theoretical swirl generator as being small [26]. This assumption makes the shear stress values in our study moderate and does not approach extreme values.

We assume a plug flow in the axial direction at the fluid inlet section (see figure 1), the swirl flow velocity is given in the form of a Rankine vortex [17]. Mathematically, the boundary conditions for the flow may be written as:

$$\begin{aligned} \text{At fluid inlet } (z = 0) : V_z = U, V_r = 0, V_\theta &= \begin{cases} V_{\theta, \max} \frac{r}{r_t}, & r \leq r_t \\ V_{\theta, \max} \frac{r_t}{r} \left(\frac{R-r}{R-r_t} \right), & r > r_t \end{cases} \end{aligned} \quad (6)$$

$$\text{At fluid outlet } (z = L) : \text{Gauge pressure } p = 0 \quad (7)$$

$$\text{At the pipe axis } (r = 0) : \frac{\partial V_z}{\partial r} = 0, \frac{\partial V_r}{\partial r} = 0, \frac{\partial V_\theta}{\partial r} = 0 \quad (8)$$

$$\begin{aligned} \text{At the pipe wall } (r = R = D/2) : V_z = 0, \\ V_\theta = 0 \text{ (No - slip)}, V_r = 0 \text{ (no - penetration)} \end{aligned} \quad (9)$$

In Eq. (6), $V_{\theta, \max}$ is the value of maximum swirl velocity and r_t is the transition radius, where the swirl velocity changes from a forced to free vortex.

2.2 Dimensionless parameters

The results of the present study have been presented using dimensionless parameters that have been briefly described in this section. In order to quantify the intensity of swirl at a cross-section of the pipe, we use the definition of swirl number as the ratio of the axial flux of the swirl momentum to the axial flux of the axial momentum [16]. Swirl number is written mathematically as:

$$S = \frac{2\pi \int_0^R V_z (r V_\theta) r dr}{\pi R^3 U^2} \quad (10)$$

The swirl number at the inlet to the pipe, calculated using Eq. (10) is denoted by S_0 and is known as the inlet swirl number. In order to interpret our results, we use the Reynolds number based on the average axial velocity of the fluid, which may be defined as:

$$\text{Re} = \frac{\rho U^{(2-n)} (2R)^n}{\kappa} \quad (11)$$

where U is the magnitude of the plug profile of the inlet axial velocity.

The dimensionless axial distance is obtained by non-dimensionalization using the diameter of the pipe $D (= 2R)$ and is given by z/D . The dimensionless radial distance is given as r/R . Accordingly the dimensionless transition radius becomes r_t/R .

3. Numerical solution methodology and model validation

We solve computationally Eqs. (1)–(4) with boundary conditions (6)–(9) using Ansys Fluent within the computational domain shown in figure 2. Second order discretization is used for spatial coordinate and SIMPLE algorithm [30] is used to resolve the pressure–velocity coupling. The mesh of the computational domain is made in such a way that it is finer closer to the walls and coarser away from the walls (near the axis). We first perform grid independence studies in order to make our results free from the effect of the finiteness of the mesh. We plot the grid independence studies as shown in figure 3(a) and we find that changing the number of the mesh from 40000 to 60000 does not change the results significantly and hence we use the mesh with 40000 grids for our present study. Our computational model is further validated using existing data for a Newtonian fluid from the work of Pati and Kumar [25] for invariant thermophysical properties and it can be observed from figure 3(b), that a good match is obtained between our present study and the existing data in the literature for the parameters considered.

Figure 3. (a) Grid independence test: Using a plot of wall shear stress $\tau_w = \eta \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right)$ vs the dimensionless length of the channel for $n = 1.1$, $Re = 1000$, $S_0 = 0.2$ and $r_t/R = 0.75$. (b) Our present results validated with the results of Pati and Kumar [25] for the laminar flow of a Newtonian fluid with $Re = 500$, $S_0 = 0.5$ and $r_t/R = 0.75$.

4. Results and discussions

We describe the results and discussion by first having a discussion on the effect of the power-law index on the development of axial velocity profile near the channel entrance and the decay of the swirl velocity profile along the channel. Then we describe the effect of various parameters considered here, along with the power-law index on the swirl decay. The decay of swirl has been quantified by the ratio of the swirl number at a cross-section to the swirl number at the inlet. We also develop a correlation for predicting the swirl decay of laminar pipe flow of non-Newtonian fluids.

4.1 Velocity distribution

The development of the axial velocity profile close to the inlet region is shown in figure 4(a) and (b) for a shear-thinning type and a shear-thickening type of fluid respectively. It can be observed from the figure that the axial velocity profile develops along the length till it reaches a fully developed state when the swirl velocity has decayed significantly [see figure 5(a) and (b)]. The centerline value of the fully developed axial velocity profile varies with the power-law index as established by classical Hagen-Poiseuille flow of a power-law fluid. The development of the flow velocity to its fully developed state takes place after the fluid has traversed through a longer channel length because of the swirl velocity affecting the development of the axial flow velocity. The effect of the swirl velocity profile plays a significant role in altering the axial velocity profile's development for both shear thickening and shear thinning fluids.

With the main focus of this work being the understanding the decay of swirl, it is important to understand the changes in the swirl velocity profile along the length of the channel close to the inlet, which is shown in figure 5(a) and (b) for a shear-thinning type and a shear-thickening type of fluid respectively. We observe that for the same value of swirl velocity given at the inlet of the channel, a more significant decay of swirl takes place closer to the inlet for a shear-thickening fluid as compared to a shear thinning fluid. For example, for the shear thickening fluid, the maximum value of the swirl velocity is about 0.15, when $z/D = 20$ as opposed to the value being about 0.27 for a shear thinning fluid [see figure 5(a) and (b)]. This can be directly attributed to the effective viscosity becoming higher for a shear-thickening fluid when the velocity gradients are large. We also observe that the Rankine vortex profile input at the inlet of the channel smoothens along the length of the channel due to diffusive effects of the viscous fluid flow. It is also important to observe the shift in the maxima of the swirl velocity profile as z/D increases in both cases of shear thinning and shear thickening fluids shown in figure 5(a) and (b) respectively. This may be attributed to the effect of high shear stress close to the tube wall ($r/R = 1$) causing the swirl velocity maxima close to it to reduce more significantly in value as compared to the swirl velocity farther away from the tube wall. This observation corresponds to what was observed by Ayinde [16] for Newtonian fluid flow. It is also important to note that for shear thickening fluids, the shear stress closer to the wall is higher and hence the shift of the maxima away from the wall is larger as seen in figure 5(b).

4.2 Effect of power-law index on swirl decay

In order to study the effect of power-law index on swirl decay, we choose to represent two cases of shear thinning

Figure 4. Axial velocity distribution along the axial direction in the pipe with $Re = 1000$, $S_0 = 0.2$ and $r_i/R = 0.75$ for (a) $n = 0.8$ and (b) $n = 1.2$.

Figure 5. Swirl velocity distribution along the axial direction in the pipe with $S_0 = 0.2$, $Re = 1000$ and $r_i/R = 0.75$ for (a) $n = 0.8$ and (b) $n = 1.2$.

fluids viz. $n = 0.6$ and $n = 0.8$ and two cases of shear thickening fluids viz. $n = 1.2$ and $n = 1.4$. The value of $n = 1$ represents a Newtonian fluid. The value of $n \sim 0.63$ corresponds to the power-law index commonly used for studying flow of blood [31].

The ratio of the local swirl number (S) at a cross-section to the inlet swirl number (S_0) is used to depict the swirl decay along the length of the channel as shown for different values of the power-law index n in figure 6(a) and (b) for two different Reynolds numbers of 1000 and 400 respectively. It can be observed from the figures that the decay of swirl is larger at even small distances from the inlet of the pipe for a smaller value of Reynolds number depicted in figure 6(b). The value of swirl number reaches 1% of its value at the inlet at a much shorter distance for $Re = 400$ as compared to the case with $Re = 1000$ irrespective of the

value of n . Reason for this is due to two aspects, one being that the viscous forces being higher for smaller values of Reynolds number and the other being the for larger values of Reynolds number (i.e. larger axial velocities), the swirl velocity is transported (advected) through larger distances from the inlet. Furthermore, higher viscous effect tends to aid in quicker decay of swirl. We also observe that the decay of swirl along the axial distance is slower for smaller values of power-law index irrespective of the Reynolds number. A smaller value of power-law index means that the effective value of viscosity is much lower. Lower effective viscosity makes the fluid less viscous and thereby slowing down the dissipation process for the swirl velocity. For smaller power-law indices, the rate at which swirl would decay along the length diminishes as evident from figure 6(a) and (b). The direct consequence of this result is

Figure 6. Decay of swirl (described by swirl number) along the length of the pipe for various values of power-law index n with $S_0 = 0.2$, and $r_i/R = 0.75$ for (a) $Re = 1000$ and (b) $Re = 400$.

that enhancement of transport properties would be higher for shear-thinning fluids as compared to shear-thickening fluids.

4.3 Effect of Reynolds number on swirl decay

The swirl decay along the length of the channel is shown for different Reynolds numbers for shear-thinning fluid and shear thickening fluid respectively in figure 7(a) and (b). We observe from the figures that the swirl decay close to the inlet of the pipe is faster for lower Reynolds numbers irrespective of the power-law index. The swirl decay is slower for a shear thinning fluid as compared to a shear thickening fluid when the Reynolds number is kept constant as seen from figure 7(a) and (b). This observation is consistent with the observations of Yao and Fang [17] for Newtonian liquids. Reynolds number, being a measure of

the inertia of the flowing fluid, implies higher inertia of fluid flow with higher Reynolds numbers. Higher Reynolds number increases the transport of the swirl velocity into the pipe. Therefore, as an observation, one may see that the value of the swirl number is higher farther from the inlet of the pipe for larger Reynolds numbers. This data obtained and shown in figure 7 is consistent with the observation from figures 4, 5 and 6. It is important to note that the lower power-law index coupled with higher Reynolds number permits the swirl velocity to penetrate further into the pipe before getting dissipated.

4.4 Effect of inlet swirl number on swirl decay

The swirl decay along the length of the channel is shown for different inlets swirl number S_0 for shear-thinning fluid and shear thickening fluid respectively in figure 8(a) and

Figure 7. Decay of swirl (described by swirl number) along the length of the pipe for various values of Reynolds number Re with $S_0 = 0.2$, and $r_i/R = 0.75$ for (a) $n = 0.8$ and (b) $n = 1.2$.

Figure 8. Decay of swirl (described by swirl number) along the length of the pipe for various values of inlet swirl number S_0 with $Re = 1000$ and $r_t/R = 0.75$ for (a) $n = 0.8$ and (b) $n = 1.2$.

(b). We observe from figure 8(a) and (b) that for both the cases of shear thickening fluid and shear thinning fluid, the swirl decays slightly faster for higher values of inlet swirl number (S_0). Broadly, increase in the inlet swirl number (S_0) is affected by increasing swirl velocity value at the inlet; higher swirl velocity at the inlet leads to larger velocity gradients close to the inlet of the channel and thereby increasing the decay rate of the swirl velocity as seen from figure 8(a) and (b). We further observe from figure 8 that, in general, irrespective of the power-law index, the decay of swirl is a very weak function of the inlet swirl number. For the smaller values of the inlet swirl number studied in this work, the effect of inlet swirl number on the decay of swirl is quite small. At this juncture it would be interesting to mention that, for Newtonian fluids also the effect of inlet swirl number on swirl decay was found to be quite weak, which was reported by Ayinde [16]

and Kaushik *et al* [18]. Furthermore, in the present study, the effects of inlet swirl number on the decay of swirl are more pronounced in case of shear thinning fluid as compared to the shear thickening fluid case. Therefore, we infer that the effect of the inlet swirl number on swirl decay is not as significant as the effect of Reynolds number and power-law index.

4.5 Effect of transition radius (r_t) on swirl decay

The effect of transition radius on the decay of swirl is shown in figure 9(a) and (b). As the value of transition radius increases, the decay of swirl is faster as seen from figure 9(a) and (b). We may attribute this to the increase in swirl velocity gradient close to the pipe wall with increase in the transition radius. Larger gradient close to the pipe wall tends to make the shear stress higher at the wall and

Figure 9. Decay of swirl (described by swirl number) along the length of the pipe for various values of transition radius r_t/R with $Re = 1000$ and $S_0 = 0.2$ for (a) $n = 0.8$ and (b) $n = 1.2$.

Table 1. Effect of various parameters on the decay of swirl.

Sl. no.	Parameter	Effect on swirl decay	
		Weak/strong	Increase decay/decrease decay
1.	Power-law Index (n)	Strong	Increase in n increases swirl decay
2.	Reynolds Number (Re)	Strong	Increase in Re decreases swirl decay
3.	Inlet swirl number (S_0)	Weak	Increase in S_0 increases swirl decay
4.	Transition radius (r_t)	Weak	Increase in r_t increases swirl decay

thereby increasing the rate of decay of the swirl velocity. We further observe from figure 9(a) and (b) that irrespective of the power-law index, the decay of swirl is weakly dependent on the dimensionless transition radius (r_t/R). Similar to the case of inlet swirl number [see figure 8(a)], the effect of transition radius is pronounced in case of shear thinning fluid as compared to the shear thickening fluid case as seen from figure 9(a).

4.6 Summary of parameters affecting swirl decay

As per the discussions in the earlier subsections, we summarize the effect of each of the parameters on the decay of swirl in table 1.

4.7 Development of correlation for predicting swirl decay

We have seen from the earlier discussion that the swirl decay is a strong function of Reynolds number and power-law index. It is important to predict the distance at which the swirl velocity would vanish for different values of Reynolds number and power law indices. It can be seen from figures 6 to 9 that the decay of swirl is a straight line in the semi-log plots shown. We also observe that the swirl decay weakly depends on the inlet swirl number and

Figure 10. Contour of the variation of m , given by Eq. (12).

log plots given by figures 6–9) is suggested for fitting the data between S/S_0 and z/D , given by:

$$\frac{S}{S_0} = e^{m(\frac{z}{D})} \quad (12)$$

The fitting equation given by Eq. (12) has a term m , which is a function of the Reynolds number as well as the power law index. The variation of the value of m is given in figure 9. A polynomial fit of term m , a function of n and Re , may be written as:

$$m = \begin{cases} -0.39725 + 0.74770n^3 - 1.87434n^2 + 1.49696n - 0.000004Re + 0.00000888n \times Re & \text{for } n \leq 1 \\ -3.9714 + 1.62333n^3 - 6.67238n^2 + 8.9514n - 0.0000264Re + 0.00002697n \times Re & \text{for } n > 1 \end{cases} \quad (13)$$

dimensionless transition radius. Therefore, it may be safe to choose an exponential profile for the decay of swirl. In order to find the distance at which swirl would have almost completely decayed based on the Reynolds number and the power-law index, an exponential fit (considering the semi-

The above correlation given by Eqs. (12) and (13) is valid for $0.6 \leq n \leq 1.4$ and $400 \leq Re \leq 1000$. We observe from figure 10 that the value of m is higher for higher values of power law index n . The value of m varies significantly with Reynolds number for power law index n

Figure 11. The values of m predicted from the fit equation ($m_{\text{predicted}}$) vs the values obtained from simulation (m_{obtained}).

between 1.1 and 1.3. The value of m can be easily calculated using the data given in Eq. (13) for different Re and n and the value of swirl intensity can be found at any axial location z/D from the inlet. When $z = 0$, we get back the inlet swirl number value S_0 .

The robustness of the fit obtained using Eq. (13) is to be studied before being used directly to get the rate of the decay for different Reynolds numbers and power-law indices. Therefore, we obtain the accuracy of the fit equation and plot it in figure 11. We observe from figure 11 that most of the m values predicted by our correlation are within the $\pm 10\%$ error limit of the values obtained from our simulations. We may safely use the correlation given by Eqs. (12) and (13) for predicting swirl decay within a reasonable tolerance.

5. Conclusion

We have executed the numerical analysis on the effect of swirl velocity at the inlet on its decay characteristics in laminar flow of a non-Newtonian power-law fluid through a pipe. We have found that the effect of change in the power-law index of the fluid plays a vital role in understanding the decay of the swirl. We have shown that, in general, the length of swirl decay increases significantly with reduction of the power-law index value. We also found that the length of decay of swirl strongly depends upon the Reynolds number of the flow as well as the power-law index. The dependence on the inlet swirl number and the profile shape (transition radius) is weak for the decay of swirl. We also found a correlation for the decay length as a function of Reynolds number and power-law index which gives a good fit for most of our simulation data.

Nomenclature

S	swirl number
n	power law index
D	diameter of the pipe, m
r_t	transition radius of Rankine Vortex, m
z	axial coordinate, m
r	radial coordinate, m
Re	Reynolds number
U	Inlet axial velocity, m/s
V_z	axial velocity, m/s
V_θ	swirl Velocity, m/s
V_r	radial Velocity, m/s

Greek symbols

η	viscosity, kg/ms
κ	consistency index of the fluid, kg/ms^{1-n}
θ	azimuthal coordinate, <i>radians</i>
ρ	density, kg/m^3

Subscripts

0 value at inlet

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