Sādhanā (2019) 44:173
https://doi.org/10.1007/s12046-019-1164-3

Pre-stretch and frequency variation effect on the dielectric permittivity of a dielectric elastomer: an amended permittivity model

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MS received 23 March 2019; revised 10 May 2019; accepted 20 May 2019

Abstract. In the present letter, we model the pre-stretch and frequency parameter effect on the dielectric permittivity of a dielectric elastomer. The present work is the extension of our most recent work (Kumar and Sarangi in Mech Mater 128:1–10, 2019) in case of deformable smart material. In particular, pre-stretching as well as frequency parameter variables majorly affect the deformation mechanism of a dielectric elastomeric material. In line with this, we first develop a new amended permittivity model of a dielectric elastomeric material based on the fundamental laws of physics. This amended permittivity model successfully enrols the pre-stretch and frequency parameter effect on the dielectric permittivity of a dielectric elastomer. Additionally, the amended permittivity model also successfully resolves the existing numerical inaccuracies of the previously known permittivity models in large deformations. Next, the formulated amended permittivity model is calibrated with some available experimental data, and compared to a known permittivity model existing in the literature.

Keywords. Dielectric elastomer; permittivity; pre-stretch; frequency.

In recent years, dielectric elastomeric materials (DEs) have gained increasing attention of the research community working in the area of soft robotic materials [1–5]. DE is a smart material that produces a large strain with an application of electro-mechanical field [6–9]. In general, the dielectric material has very specific properties like lightweight, rapid response and, most important, low cost for the robotic application purposes. A well-known example of a DE is the electrically insulating material between two compliant electrodes of a capacitor. In addition, smart actuators made of a soft material in the shape of plate, cylindrical and spherical configurations are currently in popular demand for smart engineering as well as medical field applications.

In the literature, electro-mechanical instability phenomenon of a DE has been reviewed by different researchers [10–12]. However, the electro-mechanical instability phenomenon has long been recognized in the electrical power industry as a failure mode of polymer insulators. A well-known solution [4, 13–15] for this instability phenomenon is the pre-stretch, which was majorly accepted by the research community working on it. The pre-stretch resolves the electro-mechanical instability issue in the DE. However, at the same time, it has been unclear how the pre-stretch and frequency parameter variables affect the dielectric permittivity of the DE.

Recently, Zhu *et al* [16] studied the non-affine-model-based snap through instability performance of DEs. Herein, they [16] observed that the non-affine model may be reduced to the eight-chain model by ignoring the influence of the chain entanglements. Next, Sharma and Joglekar [17] studied the dynamic electro-mechanical instability through anisotropic effect by an energy-based approach. Herein, they [17] found a significant increase in the electro-mechanical field with an increase in the anisotropy parameter. Further, Sharma *et al* [18] examined the DC-dynamic pull in instability by the same energy approach. They [18] found an enhancement in the dynamic pull-in instability at electric fields that are lower than those corresponding to the static instability. Later, Sharma *et al* [19] reported electro-mechanical inhomogeneous deformation for both active and inactive regions in DEs. They [19] obtained a locus of localized thinning near the boundary of the active region only.

In line with the literature, we try to develop a new amended permittivity model based on the fundamental laws of physics. Herein, the proposed amended permittivity model successfully enrols the pre-stretch and frequency parameter effects on the dielectric permittivity of a DE. Additionally, the amended permittivity model successfully resolves the existing numerical inaccuracies of the previously known permittivity models in large deformations.

From Eq. $(25)_2$ in our recent article [7], we have the dielectric permittivity model as a function of stretch given as follows:

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$$\epsilon_r = 1.34\lambda^{-4} + 3.36. \tag{1}$$

Herein, we may introduce the additional pre-stretch and the activated stretching effects through the parameters λ_p and λ_a, respectively. These parameters are defined as follows:

$$\lambda_p = \frac{R_p}{R}, \quad \lambda_a = \frac{r}{R_p}, \quad \lambda = \frac{r}{R} = \lambda_p\lambda_a, \tag{2}$$

wherein R, R_p and r are the undeformed radius, pre-stretched radius and the deformed radius of the circular actuator, respectively. From Eqs. (1) and (2), we may obtain the permittivity model of a DE given as follows:

$$\epsilon_r = 1.34\lambda_p^{-4}\lambda_a^{-4} + 3.36. \tag{3}$$

Next, the electrostatic analysis of the dielectric constant begins with the electric dipole moment produced by an electron in a static electric field $\mathbf{E}$. The electron experiences a linear restoring force, $\mathbf{F} = -m{\omega_0}^2\mathbf{x}$:

$$e\mathbf{E} = m{\omega_0}^2\mathbf{x}, \tag{4}$$

wherein ω_0 represents the frequency parameter. The resulting displacement of charge, $\mathbf{x} = \frac{e\mathbf{E}}{m{\omega_0}^2}$, produces a molecular polarization as follows:

$$\mathbf{p}_{mol} = e\mathbf{x} = \frac{e^2\mathbf{E}}{m{\omega_0}^2}. \tag{5}$$

Then, if there are N molecules per unit volume with Z electrons per molecule, the dipole moment per unit volume is defined as follows:

$$\mathbf{P} = NZ\mathbf{p}_{mol} = \epsilon_0\chi_e\mathbf{E}. \tag{6}$$

Next, we have the standard relations $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$ and the relative dielectric permittivity $\epsilon_r = \frac{\epsilon}{\epsilon_0} = 1 + \chi_e$. Using these relationships with expression (6), we may obtain the relative dielectric permittivity expression in terms of the frequency parameter as follows:

$$\epsilon_{r-mod} = 1 + \frac{NZe^2}{m{\omega_0}^2\epsilon_0}. \tag{7}$$

Introducing a new constant $k = \frac{NZe^2}{m\epsilon_0}$, Eq. (7) may be re-written as follows:

$$\epsilon_{r-mod} = 1 + \frac{k}{[\omega_0(\lambda_a)]^2}. \tag{8}$$

Relation (8) represents the dielectric permittivity as a function of the frequency parameter $\omega(\lambda_a)$. The frequency parameter $\omega(\lambda_a)$ is also directly related to the activated stretch λ_a.

Now, for the quantification purpose herein we consider two cases given as follows:
Case 1: $\lambda_p = 1$ and $\lambda_a = 5$

$$\epsilon_{r-mod-1} = 1 + \frac{k}{[\omega_0(5)]^2}. \tag{9}$$

Case 2: $\lambda_p = 2$ and $\lambda_a = 2.5$

$$\epsilon_{r-mod-2} = 1 + \frac{k}{[\omega_0(2.5)]^2}. \tag{10}$$

Following the experimental and analytical observations by Wissler and Mazza [4, 20], we obtain $\omega_0 \propto \frac{1}{\lambda_a}$. Therefore, for $\frac{\omega_0(\lambda_a = 5)}{\omega_0(\lambda_a = 2.5)} = \frac{2.5}{5}$, we may analyse the comparison between them such that $(\epsilon_r)_{mod-2} < (\epsilon_r)_{mod-1}$. Now, we conclude that the pre-stretching effect as well as the activated stretching effect is directly linked to the dielectric permittivity.

Further, consider the fluctuation of the activated stretch λ_a in line with the experimental observations like $\omega_0 = k_1/\lambda_a$ in a circular actuator. We may modify the permittivity expression by substituting $\omega_0 = k_1/\lambda_a$ in Eq. (8) and by eliminating the λ_a variable from Eqs. (3) and (8) as follows:

$$\epsilon_{r-mod} = 1 + \frac{1.15k}{{k_1}^2}\frac{{\lambda_p}^{-2}}{\sqrt{\epsilon_r - 3.36}}. \tag{11}$$

Introducing a new constant $k_2 = \frac{k}{{k_1}^2}$ and substituting the value of ϵ_r from Eq. (3), Eq. (11) may be re-written as follows:

$$\epsilon_{r-mod} = 1 + k_2{\lambda_p}^{-2}\lambda^2. \tag{12}$$

Now, we may develop a new amended permittivity model $\epsilon_{r-amd} = f(\epsilon_r, \epsilon_{r-mod})$ by superimposing the ϵ_r and ϵ_{r-mod} expressions as follows:

$$\epsilon_{r-amd} = A\epsilon_r + B\epsilon_{r-mod}, \tag{13}$$

wherein A and B are constants. Using the expressions from Eqs. (1) and (12), a new amended permittivity model ϵ_{r-amd} may be obtained as follows:

$$\epsilon_{r-amd} = A.(1.34\lambda^{-4} + 3.36) + B.(1 + k_2{\lambda_p}^{-2}\lambda^2). \tag{14}$$

Model (14) is fitted to the Wissler and Mazza [5] experimental data using the least-square method, and plotted in figure 1. The best fit values of the constant parameters for $\lambda_p = 3$ are obtained as follows:

$$A = 0.74, \quad B = 1.4, \quad k_2 = -0.1. \tag{15}$$

In addition, Jimenez and McMeeking [21] also provided a permittivity model parallel to our proposed amended

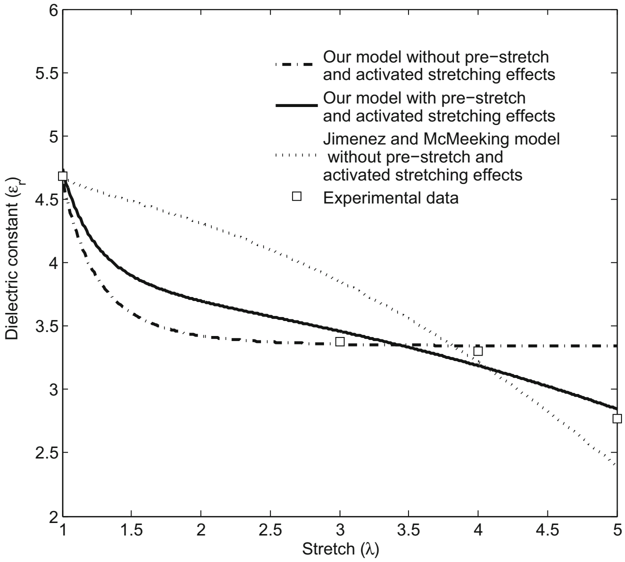

Figure 1. Comparison of three different theoretical models with the Wissler and Mazza [5] experimental data.

permittivity model (14) and recently published permittivity model (1) given as follows:

$$\epsilon_{r-JM} = \epsilon_0 - 2c_0(\lambda^2 - \lambda^{-4}). \tag{16}$$

For this, we again fit this Jimenez and McMeeking permittivity model model (16) to the Wissler and Mazza [5] experimental data using the least-square method shown in figure 1. The best fit values of the constant parameters $\epsilon_0 = 4.67$ and $c_0 = 0.0455$ are obtained.

Finally, in order to compare the accuracy of the proposed model (14), we compare and validate this (14) with the available theoretical models (16), (1) and the Wissler and Mazza [5] experimental data, as shown in figure 1. Herein, we may observe that the new amended permittivity model (14) fits the Wissler and Mazza [5] experimental data more accurately than the dielectric permittivity model (1) and the Jimenez and McMeeking permittivity model (16). This is because of the introduction of frequency variation effect through the activated stretch as well as the pre-stretch in the new amended permittivity model (14). These effects majorly affect the dielectric constant value at large deformation. However, the pre-stretch and activated stretching effects are not considered in the Jimenez and McMeeking permittivity model (16), also similar to our recently published permittivity model (1) without pre-stretch and activated stretching effects. In our recent theoretical model (1), we excluded the pre-stretching and frequency variation effects for simplicity in electro-magneto-elasticity. However, the formulation of the amended dielectric permittivity model (14) including these effects has been already reported herein through the present work.

In order to discuss the important parameters that greatly affect the phenomena of dielectric elastomers, we may think about pre-stretch, activated stretch and frequency parameters. However, the pre-stretch and activated stretch parameters collectively generate the frequency parameter effect. Whether we want to include this collectively or independently depends on ours assumptions. Herein, we use the collective linking between the activated stretch and the frequency parameters followed from various experimental observations. In addition, as previously discussed in the literature [4, 13–15], we may observe that the pre-stretching is the most adopted solution in order to control the pull-in instability of DEs. This also directly influences the deformation-dependent electric permittivity of a DE. In line with this, the proposed amended permittivity model (14) advances the existing thoughts with the consideration of an additional frequency parameter effect on the dielectric permittivity of DEs. This additional consideration may greatly affect the applicability of the presented amended permittivity model (14) in order to choose the best permittivity model for the analysis of various existing and non-existing phenomena of DEs.

In summary, the present letter describes the pre-stretch and the frequency parameter effect on the dielectric permittivity of a DE. In line with this, we formulated an amended permittivity model, which successfully enrolls these effects on the dielectric permittivity of the dielectrics. The amended permittivity model is calibrated with the Wissler and Mazza [5] experimental data, and compared to a known permittivity model existing in the literature. The results obtained through the proposed model show that as the pre-stretch and the frequency parameter effect increases, the permittivity value of the dielectrics decreases. At large deformation, these effects majorly affect the permittivity value of a DE. Finally, we believe that the developed permittivity model can be used to guide the design and fabrication of DE actuators.

Acknowledgements

Authors thank the anonymous referees of our recently published article *[Electro-magnetostriction under large deformation: modeling with experimental validation. Mechanics of Materials 128 (2019) 1–10]*, who gave valuable critical suggestions, due to which the present idea came to existence.

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