Deep echo state network with reservoirs of multiple activation functions for time-series prediction

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Abstract. In this paper, an improved deep echo state network is proposed, named as multiple activation functions deep echo state network (MAF-DESN), where states are activated by multiple activation functions. A sufficient condition for MAF-DESN is given to guarantee that MAF-DESN possesses the echo state property. Finally, the MAF-DESN is applied to chaotic time-series predictions and compared to other ESN deformation models and popular LSTM. Simulation results show that under same network size condition, MAF-DESN possesses stronger explanatory power in chaotic far-infrared laser predictions (R -square=0.9537, others $\leq$ 0.6487), and better fitting ability in daily foreign exchange rates (MAE=0.0040, others $\geq$ 0.0047) and chaotic far-infrared laser (MAE=3.4042, others $\geq$ 4.9021). In high-dimension-input task, MAF-DESN improved the performance when the results were compared (R -square=0.4274, others $\leq$ 0.3975 and MAE=5.2221, others $\geq$ 7.6876), while the train time of MAF-DESN did not increase when compared to DESN.

Keywords. Deep echo state network; reservoir computing; multiple activation functions; time-series prediction.

1. Introduction

In machine learning and computational intelligence communities, an increasing number of neural network models, especially echo state network (ESN), are being applied to time-series prediction tasks. Over the last decades, the classic ESN has been recognized as the most efficient solution for training recurrent neural network (RNN), possessing many merits such as simplified training process and less time consumption. However, the performance of ESN suffers due to the inherent ill-posed problem. For the sake of refining the classic ESN, researchers attempted many optimizing approaches to improve the performance of ESN. Optimizing approaches of ESN are divided into two groups approximately. The first group of optimizing approaches vary the details of original ESN, such as varying gain and bias of activation function [1], replacing normalized root mean square error (NRMSE) by correntropy [2, 3] and adding Laplacian algorithm [4] to reduce the dimension of reservoir states. The second group of optimizing approaches mainly combine the ESN with other complementary neural networks or methods, including but not limited to echo state queueing networks (ESQN) [5],

robust echo state networks (RESN) [2], combination structure of reservoir computing and support vector machines (RCSVM) [6], deep belief echo state network (DBESN) [7], deep recurrent neural network (DRNN) [8], optimized echo state network (OESN) by binary particle swarm optimal algorithm [9], stacked denoising autoencoders (SDA) [10], dynamical regularized echo state network (DRESN) [11], adaptive lasso echo state network (ALESN) [12], recursive Bayesian echo state network (RBESN) [13], polynomial echo state network [14], multi-layered echo state network (MLESN) [15] and stacked deep echo state network (DESN) [16].

Aforementioned DESN is similar to MLESN, which is a deep structure composed of multiple reservoirs. Like other deep structures [17–28], DESN is a network with more than three layers, and has been applied to multiple simulation tasks [15]. However, to our best knowledge, there is little literature about sufficient condition of DESN and directly applying the sufficient condition of ESN is not suitable since the input weights of hidden layer to layer are similar to reservoir weights of ESN, which also need constraints. Hence, we propose the sufficient condition of DESN as well as the calculation process in the main body to guarantee convergence. However, in traditional ESN or DESN, the hidden neurons are activated by a single

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$$E(\mathbf{d}, \mathbf{y}) = \sqrt{\frac{\langle \|\mathbf{d} - \mathbf{y}\|^2 \rangle}{\langle \|\mathbf{d} - \langle \mathbf{d} \rangle\|^2 \rangle}} \quad (8)$$

where $\mathbf{y}$ is the output, $\mathbf{d}$ is the teacher output, $\|\cdot\|$ denotes the Euclidean distance and $\langle \cdot \rangle$ denotes the average function.

According to [30], if ESN network possesses echo state property (ESP), then $\|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\| \leq \|\mathbf{x}(n) - \mathbf{x}'(n)\|$ ($\|\cdot\|$ denotes Euclidean distance) is satisfied, where $\mathbf{x}(n+1), \mathbf{x}'(n+1)$ are the states generated by the same input $\mathbf{u}(n+1)$. Generalized to DESN, ESP is described as $\|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\| \leq \|\mathbf{x}(n) - \mathbf{x}'(n)\|$ where $\mathbf{x}(n+1), \mathbf{x}'(n+1)$ are the gathering of reservoir states generated by the same input $\mathbf{u}(n+1)$. To our best knowledge, this is the first time of ESP of DESN has been proposed as here. According to ESP of DESN, we obtain following inequations:

$$\begin{cases} \frac{(1 - a^{(l)} + a^{(l)} A^{(l)})^2}{1 - (a^{(l+1)} A_{\text{in}}^{(l+1)})^2} \leq 1, l = 1, \dots, L-1 \\ (1 - a^{(l)} + a^{(l)} A^{(l)})^2 \leq 1, l = L \end{cases} \quad (9)$$

where $A^{(l)}$ is reservoir spectral radius of l th layer and $A_{\text{in}}^{(l+1)}$ is input weight matrix spectral radius of l th layer. Proof of Inequation (9) is given in Appendix A.

Since the sufficient condition of one reservoir is $0 < A < 1$, this implies that every reservoir of DESN should meet the following inequation:

$$0 < A^{(l)} < 1, l = 1, \dots, L. \quad (10)$$

Substituting Inequation (10) into Inequation (9), we obtain sufficient condition of DESN as follows:

$$\begin{cases} 0 < A^{(l)} < 1, l = 1, \dots, L-1 \\ 1 - 2/a^{(l)} < A^{(l)} < 1, l = L \\ A_{\text{in}}^{(l)} \leq \sqrt{1 - (1 - a^{(l-1)} + a^{(l-1)} A^{(l-1)})^2} / a^{(l)}, l = 2, \dots, L \end{cases} \quad (11)$$

Now, the sufficient condition of DESN is calculated. The next section addresses modified DESN and its sufficient condition.

3. The modified DESN

Based on DESN, to solve different learning tasks, to explore suitable activation function, utilizing the weighted sum of multiple activation functions to take the place of original single activation function in Eq. (1), one can obtain modified hidden layer update equation as follows:

$$\mathbf{x}^{(l)}(n+1) = (1 - a^{(l)})\mathbf{x}^{(l)}(n) + \sum_{i=1}^{N_{\lambda}^{(l)}} \left(\lambda_i^{(l)} f_i^{(l)}(\mathbf{x}_{\text{in}}^{(l)}(n+1)) \right) \quad (12)$$

where $N_{\lambda}^{(l)}$ denotes the number of activation functions of l th layer and $\lambda_i^{(l)}$ denotes the coefficient of activation function $f_i^{(l)}$. In other words, every layer of this modified DESN is a linear superposition of multiple reservoirs.

For the sake of ESP, the sufficient conditions are given as follows:

1. $f_i^{(l)}$ is a sigmoid function or a linear function that satisfies $\|f_i^{(l)}(x) - f_i^{(l)}(x')\| \leq \|x - x'\|$;
2. $0 < a^{(l)} \leq 1, l = 1, \dots, L$;
3. $\sum_{i=1}^{N_{\lambda}^{(l)}} |\lambda_i^{(l)}| \neq 0$;
4. Following sufficient conditions are for aforementioned models, where $A^{(l)}$ denotes the maximal singular value of $\mathbf{W}^{(l)}$ and $A_{\text{in}}^{(l)}$ denotes the maximal singular value of $\mathbf{W}_{\text{in}}^{(l)} (l \geq 2)$:

$$\begin{cases} 0 < A^{(l)} < 1, l = 1, \dots, L-1 \\ 0 < A^{(l)} < a^{(l)} / \sum_{i=1}^{N_{\lambda}^{(l)}} |\lambda_i^{(l)}|, l = L \\ A_{\text{in}}^{(l)} \leq \sqrt{1 - (1 - a^{(l-1)} + \sum_{i=1}^{N_{\lambda}^{(l-1)}} |\lambda_i^{(l-1)}| A^{(l-1)})^2} / a^{(l)}, l = 2, \dots, L \end{cases} \quad (13)$$

If aforementioned sufficient conditions are satisfied, then discrete models (12) has the ESP. For a proof of sufficient condition, see Appendix B.

According to the ESP sufficient condition, one can obtain the constraints of the coefficients of activation function as follows:

$$\begin{cases} \sum_{i=1}^{N_{\lambda}^{(L)}} |\lambda_i^{(L)}| < a^{(L)} / A^{(L)}, l = L \\ \sum_{i=1}^{N_{\lambda}^{(l)}} |\lambda_i^{(l)}| < \left(\sqrt{1 - (a^{(l+1)} A_{\text{in}}^{(l+1)})^2} + a^{(l)} - 1 \right) / A^{(l)}, l = 1, \dots, L-1 \end{cases} \quad (14)$$

When MAF-DESN is a two-layer model, activated by double activation functions, the constraints of the coefficients of activation function are simplified as

$$\begin{cases} |\lambda_1^{(2)}| + |\lambda_2^{(2)}| < a^{(2)} / A^{(2)} \\ |\lambda_1^{(1)}| + |\lambda_2^{(1)}| < \left(\sqrt{1 - (a^{(2)} A_{\text{in}}^{(2)})^2} + a^{(1)} - 1 \right) / A^{(1)}, \end{cases} \quad (15)$$

which are the constraints used in section 4.

In conclusion, the ESP sufficient condition is a Lipschitz condition to ensure that the reservoir activation function is a linear combination of contractive mapping.

Table 1. Activation function influence (test score).

f_1f_1	f_1f_2	f_1f_3	f_1f_4	f_2f_2	f_2f_3	f_2f_4	f_3f_3	f_3f_4	f_4f_4
0.0621	0.0308	0.0479	0.0853	0.0455	0.0619	0.0559	0.0813	0.0530	0.0957

4. Simulation and analysis

4.1 MAF-DESN model experimental analysis

In this section, numerical simulation experiments are conducted to verify the performance of MAF-DESN. In the first simulation, the dataset is downloaded from web (<https://datamarket.com/>), which is daily foreign exchange rates (German/US) from 31 December 1979 to 31 December 1998, scaled to range [0,1] by minmax method before feeding into network. These data are processed as the teacher signal pair <input history data, target data>, where data at time t are used to predict data at time $t+1$. The dataset contains 4775 samples, divided into 2 parts: 3199 samples for training and 1576 samples for testing.

The activation functions considered here include

$$f_1(x) = (e^x - e^{-x}) / (e^x + e^{-x}) \quad (16)$$

$$f_2(x) = (1 - e^{-x}) / (1 + e^{-x}) \quad (17)$$

Figure 2. Satisfied vs dissatisfied (activation functions are f_1 and f_2 , reservoir size is 3).

$$f_3(x) = (2/\pi) \arctan((2/\pi)x) \quad (18)$$

$$f_4(x) = x / \sqrt{1 + x^2} \quad (19)$$

which are nonlinear sigmoid functions satisfying $\|f(x) - f(x')\| \leq \|x - x'\|$.

In this simulation, two-layer MAF-DESN is adopted; every layer with two activation functions is updated according to Inequation (12) with $a^{(l)} = 1$, in order to obtain activation functions' influences (table 1), spectral radius with activation function coefficient influence (figure 2) and overfitting problem (figure 3). For sufficient conditions, spectral radius and activation function coefficient must satisfy

$$\begin{cases} |\lambda_1^{(2)}| + |\lambda_2^{(2)}| < 1/A^{(2)} \\ |\lambda_1^{(1)}| + |\lambda_2^{(1)}| < \sqrt{1 - (A_{in}^{(2)})^2} / A^{(1)} \end{cases} \quad (20)$$

In this simulation, $A^{(1)} = 0.5884$, $A^{(2)} = 0.5227$ and $A_{in}^{(2)} = 0.9444$. Hence, sufficient conditions are calculated as

$$\begin{cases} |\lambda_1^{(2)}| + |\lambda_2^{(2)}| < 0.5588 \\ |\lambda_1^{(1)}| + |\lambda_2^{(1)}| < 1.9131 \end{cases} \quad (21)$$

Output evaluation score is calculated by root mean square error (RMSE), namely

Figure 3. Overfitting curves (when neurons of reservoir increase to 3, test score reaches the minimal RMSE, after which test score increases).

$$\text{RMSE}(\mathbf{d}, \mathbf{y}) = \sqrt{(1/k) \sum_{n=1}^k (d(n) - y(n))^2} \quad (22)$$

where k denotes sample size, $d(n)$ denotes target output and $y(n)$ denotes network prediction output.

Table 1 demonstrates that different activation functions slightly influence network training and testing based on same network settings (reservoir size is 3). Figure 2 shows the differences of satisfying sufficient condition with dissatisfying sufficient condition (reservoir size is fixed as 3); case 1 denotes that first layer sufficient condition is violated and second layer sufficient condition is satisfied; case 2 denotes that first layer sufficient condition is adhered and second layer sufficient condition is violated; case 3 denotes that first layer and second layer sufficient conditions are violated. When away from the sufficient condition constraint, test score tends to be higher and performance of MAF-DESN tends to be worse. Figure 3 is an example of overfitting problem, where reservoir neurons increase from one to ten. Before reservoir size reaches three, test score gradually approaches training score. When reservoir size is bigger than three, test score gradually gets bigger and drifts away from training score, which implies that MAF-DESN model is over-trained. When neuron size is beyond eight, train score gets bigger, which might be because the higher dimensional space of one-dimensional input map might have upper dimensional limit; in other words, the state matrix is ill-posed. In next simulations, test score is close to train score to avoid overfit problem.

4.2 MAF-DESN vs other models

In this section, MAF-DESN model is compared to ESN, ESN-IP [1], ESN-PSO [9], DESN [16] and LSTM.

Evaluation methods are R -square (coefficient of determination, closer to 1 denotes stronger explanatory power of the model to output), R (correlation coefficient), RMSE (root mean square error), SSE (the sum of squares due to error) and MAE (mean absolute error):

$$r(\mathbf{d}, \mathbf{y}) = \text{cov}(\mathbf{d}, \mathbf{y}) / (\sigma_d \sigma_y) \quad (23)$$

$$\text{SSE}(\mathbf{d}, \mathbf{y}) = \sum_{n=1}^k (d(n) - y(n))^2 \quad (24)$$

$$\text{MAE}(\mathbf{d}, \mathbf{y}) = (1/k) \sum_{n=1}^k |d(n) - y(n)| \quad (25)$$

where RMSE, SSE and MAE closer to zero denote better fitting ability. Prediction tasks are samples of daily foreign exchange rates (German/US) (<https://datamarket.com/>), IBM common stock closing prices (<https://datamarket.com/>) and chaotic far-infrared laser (<http://www.weigend.com/>). These data are processed as the teacher signal pair <input history data, target data>, where data at time t are used to predict data at time $t+1$. In these one-dimension-input tasks, model size is 3, and train sample rate is 0.67. Two-layer MAF-DESN is activated by a combination of f_1 and f_2 with best possible guess coefficients, ESN and its variants are activated by f_1 , and LSTM is optimized by Adam optimizer.

From tables 2, 3 and 4, R -square of MAF-DESN is closer to 1 than those of other models; meanwhile, SSE, MAE and RMSE of MAF-DESN are closer to 0 than those of other models with similar time consumption, which indicates that the performance of MAF-DESN is better than those of other models considered here. From figures 4–7, peak region (black frame) of MAF-DESN is better fitted than other models, which demonstrates that MAF-DESN has better local fitting ability in one-dimension-input task.

Table 2. Daily foreign exchange rates (German/US).

	ESN	ESN-IP	ESN-PSO	LSTM	DESN	MAF-DESN
R -square	0.9902	0.9906	0.9898	0.9938	0.9866	0.9900
R	0.9951	0.9953	0.9949	0.9969	0.9933	0.9950
SSE	0.6823	2.6878	0.8434	0.5022	0.5120	0.3811
MAE	0.0058	0.0129	0.0066	0.0047	0.0049	0.0040
RMSE	0.0208	0.0413	0.0232	0.0141	0.0180	0.0156
Train time (s)	0.0156	0.1233	1.5160	6.7440	0.0312	0.0938

Table 3. IBM common stock closing prices.

	ESN	ESN-IP	ESN-PSO	LSTM	DESN	MAF-DESN
R -square	0.9742	0.9740	0.9742	0.9738	0.9742	0.9720
R	0.9870	0.9869	0.9870	0.9868	0.9870	0.9859
SSE	5765.57	6065.75	6194.59	7801.07	6710.41	3928.24
MAE	2.0312	2.1002	2.1145	2.3658	2.2101	1.7313
RMSE	8.3345	8.2492	8.6391	9.6948	8.9915	6.8796
Train time (s)	0.0012	0.0059	1.1725	6.0889	0.0026	0.0156

Table 4. Chaotic far-infrared laser.

	ESN	ESN-IP	ESN-PSO	LSTM	DESN	MAF-DESN
R-square	0.3799	0.3449	0.3889	0.3279	0.6487	0.9537
R	0.6164	0.5873	0.6236	0.5726	0.8054	0.9766
SSE	368673	401739	373945	297059	164087	66188
MAE	8.0786	8.4742	8.2035	7.7368	4.9021	3.4042
RMSE	33.6200	35.1077	33.8669	30.1535	22.3931	14.3494
Train time (s)	0.3700	0.0313	1.3734	15.2609	0.0156	0.0156

Figure 4. Daily foreign exchange rates (German/US) (target and network prediction outputs). **Figure 5.** IBM common stock closing prices (target and network prediction outputs).

Figure 6. Chaotic far-infrared laser (target and network prediction outputs).

Figure 7. The local image (black frame) of figure 6 enlarged (target and network prediction outputs).

4.3 MAF-DESN dealing with high-dimension-input task

In this section, high-dimension-input task is downloaded from web (<http://lib.stat.cmu.edu/datasets/>). The input is an eight-dimensional vector consisting of temperature, relative humidity, carbon monoxide, sulphur dioxide, nitrogen dioxide, hydrocarbons, ozone and particulates,

which is a quantitative digital number describing the degree of environmental pollution. The output is a three-dimensional vector consisting of total mortality, respiratory mortality and cardiovascular mortality, which is disease mortality caused by environmental pollution. Since environmental variation is related to time, models with cyclic structures are used to make disease mortality prediction.

Table 5. Pollution-caused mortality (Y1, Y2 and Y3, respectively, correspond to total mortality, respiratory mortality and cardiovascular mortality).

		ESN	ESN-IP	LSTM	DESN	MAF-DESN
Y1	<i>R</i> -square	0.2682	0.2612	0.2557	0.2841	0.2923
	<i>R</i>	0.5179	0.5111	0.5056	0.5330	0.5406
	SSE	18470	18286	25578	17976	14808
	MAE	8.1169	8.0349	9.7860	8.1141	7.2830
	RMSE	10.4853	10.4330	12.3391	10.3441	9.3885
Y2	<i>R</i> -square	0.1556	0.1510	0.1048	0.1423	0.1665
	<i>R</i>	0.3945	0.3886	0.3238	0.3773	0.4080
	SSE	668	682	718	560	714
	MAE	1.5772	1.5925	1.5862	1.4127	1.6500
	RMSE	1.9935	2.0149	2.0672	1.8260	2.0609
Y3	<i>R</i> -square	0.3862	0.3869	0.3830	0.3975	0.4274
	<i>R</i>	0.6214	0.6220	0.6189	0.6305	0.6538
	SSE	11850	11510	18155	13383	6663
	MAE	7.1160	6.9702	8.7862	7.6876	5.2221
	RMSE	8.3984	8.2771	10.3956	8.9254	6.2977
Train time (s)		0.0156	0.0156	20.4088	0.0313	0.313

Figure 8. Total mortality. **Figure 9.** Respiratory mortality.

Figure 10. Cardiovascular mortality.

In this simulation, model size is 20 because the disease mortality prediction task is more complicated than the aforementioned tasks. From table 5, R -square and R of MAF-DESN in total mortality, respiratory mortality and cardiovascular mortality prediction are slightly closer to 1 than those of other models, and SSE, MAE and RMSE of MAF-DESN model are closer to 0 than those of other models, which demonstrate that MAF-DESN in high-dimension-input task has advantage over other models considered here. Figures 8–10 show the comparison details of prediction mortality

5. Conclusions

In this paper, we introduced a novel model named MAF-DESN, which uses two or more activation functions to substitute the original reservoir activation function in one layer, and the scaling factor of activation functions varied layer by layer. The simulation shows that MAF-DESN possesses stronger explanatory power and better fitting ability with little increase of time consumption. However, in our simulations, the parameters are tuned by experience, which may not be the best fitting parameters; besides, the ill-posed problem of DESN is not solved in the proposed model. In the future, we attempt to apply the steepest descent method to find the best fitting parameters, as well as apply some dimensionality reduction algorithm (such as Laplacian eigenmap [4], Principal Component Analysis, singular value decomposition, Empirical Mode Decomposition, etc.) to refine the ill-posed problem

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Appendix A

Before proving that $\|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\| \leq \|\mathbf{x}(n) - \mathbf{x}'(n)\|$, we describe the relationship between $\|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\|$ and $\|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|$.

“(a)”: According to aforementioned gathering of reservoir states $\mathbf{x}(n)$ and reservoir states of l th layer $\mathbf{x}^{(l)}(n)$, we unfold $\mathbf{x}(n)$ as the following vector:

$$\mathbf{x}(n) = \begin{bmatrix} \underbrace{(\mathbf{x}^{(1)}(n))^T}_{x_1^{(1)}(n) \cdots x_{N_1}^{(1)}(n)} \cdots \underbrace{(\mathbf{x}^{(L)}(n))^T}_{x_1^{(L)}(n) \cdots x_{N_L}^{(L)}(n)} \end{bmatrix} \quad (\text{A.1})$$

where superscript denotes reservoir location, the maximum number of layers is L , subscript denotes neuron location of current reservoir and maximum number of reservoir neurons is N_l ($l = 1, \dots, L$). Then, substituting Eq. (A.1) into $\|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\|$, we obtain following equations:

$$\begin{aligned} \|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\| &= \left\| \begin{bmatrix} x_1^{(1)}(n+1) - x_1'^{(1)}(n+1) \\ \vdots \\ x_{N_1}^{(1)}(n+1) - x_{N_1}'^{(1)}(n+1) \\ \vdots \\ x_1^{(L)}(n+1) - x_1'^{(L)}(n+1) \\ \vdots \\ x_{N_L}^{(L)}(n+1) - x_{N_L}'^{(L)}(n+1) \end{bmatrix} \right\| \\ &= \sqrt{\sum_{l=1}^L \sum_{j=1}^{N_l} (x_j^{(l)}(n+1) - x_j'^{(l)}(n+1))^2}, \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned}
& \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\| \\
&= \left\| \begin{array}{c} x_1^{(l)}(n+1) - x_1'^{(l)}(n+1) \\ \vdots \\ x_{N_l}^{(l)}(n+1) - x_{N_l}'^{(l)}(n+1) \end{array} \right\| \quad (\text{A.3}) \\
&= \sqrt{\sum_{j=1}^{N_l} \left(x_j^{(l)}(n+1) - x_j'^{(l)}(n+1) \right)^2}.
\end{aligned}$$

According to Eqs. (A.2) and (A.3), we obtain the relationship between $\|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\|$ and $\|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|$ as follows:

$$\begin{aligned}
& \|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\|^2 \\
&= \sum_{l=1}^L \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2. \quad (\text{A.4})
\end{aligned}$$

“(b)”: Now, the proof of Inequation (9) is described here. According to Eq. (A.4), proof of $\|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\| \leq \|\mathbf{x}(n) - \mathbf{x}'(n)\|$ is equivalent to that of

$$\begin{aligned}
& \sum_{l=1}^L \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\
&\leq \sum_{l=1}^L \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2. \quad (\text{A.5})
\end{aligned}$$

Hence, we prove the latter inequation as follows:

$$\begin{aligned}
& \sum_{l=1}^L \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\
&= \sum_{l=1}^L \left\| \left((1 - a^{(l)}) \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right. \right. \\
&\quad \left. \left. + a^{(l)} \left(f^{(l)} \left(\mathbf{x}_{in}^{(l)}(n+1) \right) - f^{(l)} \left(\mathbf{x}_{in}'^{(l)}(n+1) \right) \right) \right) \right\|^2 \\
&\leq \sum_{l=1}^L \left\| \left((1 - a^{(l)}) \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right. \right. \\
&\quad \left. \left. + a^{(l)} \left(\mathbf{x}_{in}^{(l)}(n+1) - \mathbf{x}_{in}'^{(l)}(n+1) \right) \right) \right\|^2 \\
&= \sum_{l=1}^L \left\| \left((1 - a^{(l)}) \mathbf{E}^{(l)} + a^{(l)} \mathbf{W}^{(l)} \right) \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right. \\
&\quad \left. + a^{(l)} \mathbf{W}_{in}^{(l)} \left(\mathbf{i}^{(l)}(n+1) - \mathbf{i}'^{(l)}(n+1) \right) \right\|^2 \\
&\leq \sum_{l=1}^L \left(\left\| \left((1 - a^{(l)}) \mathbf{E}^{(l)} + a^{(l)} \mathbf{W}^{(l)} \right) \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right\| \right. \\
&\quad \left. + \left\| a^{(l)} \mathbf{W}_{in}^{(l)} \left(\mathbf{i}^{(l)}(n+1) - \mathbf{i}'^{(l)}(n+1) \right) \right\| \right)^2 \\
&\leq \sum_{l=1}^L \left\| \left((1 - a^{(l)}) \mathbf{E}^{(l)} + a^{(l)} \mathbf{W}^{(l)} \right) \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right\|^2 \\
&\quad + \sum_{l=1}^L \left\| a^{(l)} \mathbf{W}_{in}^{(l)} \left(\mathbf{i}^{(l)}(n+1) - \mathbf{i}'^{(l)}(n+1) \right) \right\|^2. \quad (\text{A.6})
\end{aligned}$$

According to Eq. (2), we know that

$$\begin{cases} \mathbf{i}^{(l)}(n+1) - \mathbf{i}'^{(l)}(n+1) = 0, l = 1 \\ \mathbf{i}^{(l)}(n+1) - \mathbf{i}'^{(l)}(n+1) = \mathbf{x}^{(l-1)}(n+1) - \mathbf{x}'^{(l-1)}(n+1), l > 1 \end{cases} \quad (\text{A.7})$$

Then

$$\begin{aligned}
& \|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\|^2 = \sum_{l=1}^L \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\
&\leq \sum_{l=1}^L \left\| \left((1 - a^{(l)}) \mathbf{E}^{(l)} + a^{(l)} \mathbf{W}^{(l)} \right) \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right\|^2 \\
&\quad + \sum_{l=2}^L \left\| a^{(l)} \mathbf{W}_{in}^{(l)} \left(\mathbf{x}^{(l-1)}(n+1) - \mathbf{x}'^{(l-1)}(n+1) \right) \right\|^2 \\
&\leq \sum_{l=1}^L \left\| \left((1 - a^{(l)}) \mathbf{E}^{(l)} + a^{(l)} \mathbf{W}^{(l)} \right) \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right\|^2 \\
&\quad + \sum_{l=1}^{L-1} \left\| a^{(l+1)} \mathbf{W}_{in}^{(l+1)} \left(\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1) \right) \right\|^2 \\
&\leq \sum_{l=1}^L \left(1 - a^{(l)} + a^{(l)} \Lambda^{(l)} \right)^2 \left\| \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right\|^2 \\
&\quad + \sum_{l=1}^{L-1} \left(a^{(l+1)} \Lambda_{in}^{(l+1)} \right)^2 \left\| \left(\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1) \right) \right\|^2 \\
&\Rightarrow \sum_{l=1}^L \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\
&\quad - \sum_{l=1}^{L-1} \left(a^{(l+1)} \Lambda_{in}^{(l+1)} \right)^2 \left\| \left(\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1) \right) \right\|^2 \\
&\leq \sum_{l=1}^L \left(1 - a^{(l)} + a^{(l)} \Lambda^{(l)} \right)^2 \left\| \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right\|^2 \\
&\Rightarrow \sum_{l=1}^{L-1} \left(1 - \left(a^{(l+1)} \Lambda_{in}^{(l+1)} \right)^2 \right) \left\| \mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1) \right\|^2 \\
&\quad + \left\| \mathbf{x}^{(L)}(n+1) - \mathbf{x}'^{(L)}(n+1) \right\|^2 \\
&\leq \sum_{l=1}^L \left(1 - a^{(l)} + a^{(l)} \Lambda^{(l)} \right)^2 \left\| \left(\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right) \right\|^2. \quad (\text{A.8})
\end{aligned}$$

Unfolding of Inequation (A.8) is as follows: when $l=1, 2, \dots, L-1$

$$\begin{aligned}
& \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\
&\leq \frac{\left(1 - a^{(l)} + a^{(l)} \Lambda^{(l)} \right)^2}{1 - \left(a^{(l+1)} \Lambda_{in}^{(l+1)} \right)^2} \left\| \mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right\|^2. \quad (\text{A.9})
\end{aligned}$$

When $l=L$

$$\begin{aligned}
& \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\
&\leq \left(1 - a^{(l)} + a^{(l)} \Lambda^{(l)} \right)^2 \left\| \mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right\|^2. \quad (\text{A.10})
\end{aligned}$$

According to Inequations (A.9) and (A.10), when following Inequations (A.11) and (A.12)

$$\begin{aligned}
& \frac{\left(1 - a^{(l)} + a^{(l)} \Lambda^{(l)} \right)^2}{1 - \left(a^{(l+1)} \Lambda_{in}^{(l+1)} \right)^2} \left\| \mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right\|^2 \\
&\leq \left\| \mathbf{x}^{(L)}(n) - \mathbf{x}'^{(L)}(n) \right\|^2, l = 1, 2, \dots, L-1 \quad (\text{A.11})
\end{aligned}$$

$$\begin{aligned}
& \left(1 - a^{(l)} + a^{(l)} \Lambda^{(l)} \right)^2 \left\| \mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n) \right\|^2 \\
&\leq \left\| \mathbf{x}^{(L)}(n+1) - \mathbf{x}'^{(L)}(n+1) \right\|^2, l = L \quad (\text{A.12})
\end{aligned}$$

are satisfied, then

$$\|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \leq \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2 \quad (\text{A.13})$$

are satisfied, which means that

$$\sum_{l=1}^L \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \leq \sum_{l=1}^L \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2 \quad (\text{A.14})$$

are satisfied, which also means that $\|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\|^2 \leq \|\mathbf{x}(n) - \mathbf{x}'(n)\|^2$ are satisfied. We transform Inequalities (A.11) and (A.12) into the following form:

$$\begin{cases} \frac{(1 - a^{(l)} + a^{(l)} A^{(l)})^2}{1 - (a^{(l+1)} A_{\text{in}}^{(l+1)})^2} \leq 1, l=1, 2, \dots, L-1 \\ (1 - a^{(l)} + a^{(l)} A^{(l)})^2 \leq 1, l=L \end{cases} \quad (\text{A.15})$$

Then, the proof process is completed.

Appendix B

According to Eq. (A.4), proof of Inequality (13) is equivalent to that of (A.14). Hence, we prove the latter Inequality (A.14) as follows:

$$\begin{aligned} & \sum_{l=1}^L \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\ &= \sum_{l=1}^L \left\| (1 - a^{(l)}) (\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)) \right. \\ & \quad \left. + \sum_{i=1}^{N_i^{(l)}} \left(\lambda_i^{(l)} (f_i^{(l)}(\mathbf{x}_{\text{in}}^{(l)}(n+1)) - f_i^{(l)}(\mathbf{x}'_{\text{in}}^{(l)}(n+1))) \right) \right\|^2 \\ &\leq \sum_{l=1}^L \left\| (1 - a^{(l)}) (\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)) \right. \\ & \quad \left. + \sum_{i=1}^{N_i^{(l)}} \left(\lambda_i^{(l)} (\mathbf{x}_{\text{in}}^{(l)}(n+1) - \mathbf{x}'_{\text{in}}^{(l)}(n+1)) \right) \right\|^2 \\ &\leq \sum_{l=1}^L \left\| \left((1 - a^{(l)}) \mathbf{E}^{(l)} + \sum_{i=1}^{N_i^{(l)}} \lambda_i^{(l)} \mathbf{W}^{(l)} \right) (\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)) \right. \\ & \quad \left. + \sum_{i=1}^{N_i^{(l)}} \left(\lambda_i^{(l)} \mathbf{W}_{\text{in}}^{(l)} (\mathbf{i}^{(l)}(n+1) - \mathbf{i}'^{(l)}(n+1)) \right) \right\|^2 \\ &\leq \sum_{l=1}^L \left\| \left((1 - a^{(l)}) \mathbf{E}^{(l)} + \sum_{i=1}^{N_i^{(l)}} \lambda_i^{(l)} \mathbf{W}^{(l)} \right) (\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)) \right\|^2 \\ & \quad + \sum_{l=1}^L \left\| \sum_{i=1}^{N_i^{(l)}} \left(\lambda_i^{(l)} \mathbf{W}_{\text{in}}^{(l)} (\mathbf{i}^{(l)}(n+1) - \mathbf{i}'^{(l)}(n+1)) \right) \right\|^2 \\ &\leq \sum_{l=1}^L \left(1 - a^{(l)} + \sum_{i=1}^{N_i^{(l)}} |\lambda_i^{(l)}| A^{(l)} \right)^2 \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2 \\ & \quad + \sum_{l=1}^L \left(\sum_{i=1}^{N_i^{(l)}} |\lambda_i^{(l)}| A_{\text{in}}^{(l)} \right)^2 \|\mathbf{i}^{(l)}(n+1) - \mathbf{i}'^{(l)}(n+1)\|^2. \end{aligned} \quad (\text{B.1})$$

According to Eq. (A.7)

$$\begin{aligned} & \|\mathbf{x}(n+1) - \mathbf{x}'(n+1)\|^2 \\ &= \sum_{l=1}^L \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\ &\leq \sum_{l=1}^L \left(1 - a^{(l)} + \sum_{i=1}^{N_i^{(l)}} |\lambda_i^{(l)}| A^{(l)} \right)^2 \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2 \\ & \quad + \sum_{l=2}^L \left(\sum_{i=1}^{N_i^{(l)}} |\lambda_i^{(l)}| A_{\text{in}}^{(l)} \right)^2 \|\mathbf{x}^{(l-1)}(n+1) - \mathbf{x}'^{(l-1)}(n+1)\|^2 \\ &\leq \sum_{l=1}^L \left(1 - a^{(l)} + \sum_{i=1}^{N_i^{(l)}} |\lambda_i^{(l)}| A^{(l)} \right)^2 \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2 \\ & \quad + \sum_{l=1}^{L-1} \left(\sum_{i=1}^{N_i^{(l+1)}} |\lambda_i^{(l+1)}| A_{\text{in}}^{(l+1)} \right)^2 \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \end{aligned} \quad (\text{B.2})$$

Transform Inequality (B.2) as follows:

$$\begin{aligned} & \sum_{l=1}^L \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\ & \quad - \sum_{l=1}^{L-1} \left(\sum_{i=1}^{N_i^{(l+1)}} |\lambda_i^{(l+1)}| A_{\text{in}}^{(l+1)} \right)^2 \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\ & \leq \sum_{l=1}^L \left(1 - a^{(l)} + \sum_{i=1}^{N_i^{(l)}} |\lambda_i^{(l)}| A^{(l)} \right)^2 \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2 \end{aligned} \quad (\text{B.3})$$

Transform Inequality (B.3) as follows:

$$\begin{aligned} & \sum_{l=1}^{L-1} \left(1 - \left(\sum_{i=1}^{N_i^{(l+1)}} |\lambda_i^{(l+1)}| A_{\text{in}}^{(l+1)} \right)^2 \right) \\ & \quad \|\mathbf{x}^{(l)}(n+1) - \mathbf{x}'^{(l)}(n+1)\|^2 \\ & \quad + \|\mathbf{x}^{(L)}(n+1) - \mathbf{x}'^{(L)}(n+1)\|^2 \\ & \leq \sum_{l=1}^L \left(1 - a^{(l)} + \sum_{i=1}^{N_i^{(l)}} |\lambda_i^{(l)}| A^{(l)} \right)^2 \\ & \quad \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2 \end{aligned} \quad (\text{B.4})$$

Similar to Appendix A, when the following inequations

$$\begin{aligned} & \frac{\left(1 - a^{(l)} + \sum_{i=1}^{N_i^{(l)}} |\lambda_i^{(l)}| A^{(l)} \right)^2}{1 - \left(\sum_{i=1}^{N_i^{(l+1)}} |\lambda_i^{(l+1)}| A_{\text{in}}^{(l+1)} \right)^2} \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2 \\ & \leq \|\mathbf{x}^{(L)}(n) - \mathbf{x}'^{(L)}(n)\|^2, l = 1, 2, \dots, L-1 \\ & \left(1 - a^{(l)} + \sum_{i=1}^{N_i^{(l)}} |\lambda_i^{(l)}| A^{(l)} \right)^2 \|\mathbf{x}^{(l)}(n) - \mathbf{x}'^{(l)}(n)\|^2 \\ & \leq \|\mathbf{x}^{(L)}(n+1) - \mathbf{x}'^{(L)}(n+1)\|^2, l = L \end{aligned} \quad (\text{B.5})$$

are satisfied, the sufficient condition is obtained. According to Inequalities (B.3)–(B.6), one can obtain following inequations:

$$\left\{ \begin{array}{l} \left(1 - a^{(l)} + \sum_{i=1}^{N_l^l} |\lambda_i^{(l)}| A^{(l)} \right)^2 \leq 1 - \left(\sum_{i=1}^{N_l^{l+1}} |\lambda_i^{(l+1)}| A_{in}^{(l+1)} \right)^2, l=1, 2, \dots, L-1 \\ \left(1 - a^{(l)} + \sum_{i=1}^{N_l^l} |\lambda_i^{(l)}| A^{(l)} \right)^2 \leq 1, l=L \end{array} \right. \quad (B.7)$$

Inequation (B.7) develops to the following:

$$\left\{ \begin{array}{l} 0 < A^{(l)} < 1, l=1, 2, \dots, L-1 \\ 0 < A^{(l)} < a^{(l)} / \sum_{i=1}^{N_l^l} |\lambda_i^{(l)}| A^{(l)}, l=L \\ A_{in}^{(l+1)} \leq \sqrt{1 - \left(1 - a^{(l)} + \sum_{i=1}^{N_l^l} |\lambda_i^{(l)}| A^{(l)} \right)^2} / a^{(l+1)}, l=1, 2, \dots, L-1 \end{array} \right. \quad (B.8)$$

Then, the proof process is completed.

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