Design and architecture of new 11:2 decimal compressors

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MS received 5 January 2018; revised 11 March 2019; accepted 19 March 2019; published online 25 April 2019

Abstract. Design and architectures of new 11:2 decimal compressors have been reported in this paper. Two design methodologies viz. delay and area optimized compressors have been introduced and implemented through tree structure of decimal number system. The architectures have been realized through vertical carry save addition algorithm, wherein to build up such addition of unconventional (4221 and 5211) binary coded decimal technique has been incorporated. Configurations of such compressors have been prototyped and transistor level implementation have been carried out to evaluate performance parameters like speed (propagation delay), power dissipation, area and area delay product. The architecture has been validated by Cadence virtuoso platform using 90 nm, 65 nm and 45 nm CMOS technology which provides useful statistics to determine the compressor's performance parameters. The propagation delay of the delay optimized compressor equals to ~ 0.094 ns while the propagation delay of the area optimized compressor is ~ 0.124 ns using 90 nm CMOS technology at 1 V supply voltage. The reported architectures are $\sim 24\%$ and $\sim 41\%$ faster from its counterpart.

Keywords. Cadence; decimal compressor; layout area; power consumption; propagation delay.

1. Introduction

Currently, general purpose computers are performing decimal computations through binary arithmetic [1]. However, decimal computer arithmetic is gaining interest for decimal data processing like scientific, commercial, financial and internet-based applications [2]. Though, many decimal numbers like 0.1, 0.2, etc. cannot be converted in exact binary format due to finite word-length effect, thereby error-free computation is impracticable [3]. Recently, decimal arithmetic is becoming commercialized for general purpose computer [4], where, Binary Coded Decimal (BCD) encoding techniques maintaining the relationship between binary and decimal number system. Decimal codes are employed in such a way that the sum of the weights of each 4-bit decimal, i.e., $\sum_{i=0}^3 r_i = 9$ [5, 6]. The family of such type of codes includes 4221, 5211, 3321, etc., where all the combinations are suitable for decimal arithmetic operations [5–7].

Efficient hardware implementation of multi-operand decimal addition [8–10] plays a vital role during the partial product reduction during decimal multiplication. Decimal numbers in binary format can be coded directly in digit by digit format, whereas, the addition of such BCD numbers summation exceed the decimal value 9 which creates the overflow [10]. To overcome the problem, higher order decimal compressors are helpful in reducing number of steps for partial product reduction of decimal parallel

multiplication; hence it reduces the overall delay of the multiplication. Moreover, almost all the parallel decimal multipliers suffer from high area requirement [11, 12], thereby the reported area optimized compressors are helpful to reduce the overall area.

In this paper, two different architecture viz. delay and area optimized 11:2 decimal compressor has been introduced using unconventional (4221 and 5211) coding techniques. This paper is an extension of a previous work of the same authors [13] and others where concept of vertical bit array compressor has been introduced which has become the building block of higher order decimal compressors. This vertical bit array compressor has made the higher order digit compressor more simple and concise. Implementation of the delay and area optimized versions of the compressors have been carried out through vertical carry save addition algorithm. The functionality of the reported architectures were checked and transistor level implementations have been carried out for practical VLSI applications. Performance parameters like propagation delay, power consumption, area and area delay product have been computed in Cadence virtuoso platform using 90 nm CMOS technology which provides useful statistics to evaluate the compressor's performance parameters. The results offered the pros and cons of the reported approaches.

In this work, two different 11:2 decimal compressor architectures (delay and area optimized) have been implemented and their functionalities have been examined. The basic compressors design idea along with the proposed

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implementation model has been described in section 2. The circuit modules to build up the compressors have been discussed in section 3, followed by the results and discussions in section 4. Finally, the conclusions have been provided in section 5.

2. Decimal compressor architecture

4-bit BCD-8421 is the most common representation of decimal values. The binary representation of 0 to 9 is valid in BCD; hence, additional logic is required for the conversion (decimal value 10 to 15), moreover, BCD-8421 is not self-complementing. These limitations can be overcome by a family of weighted self complementing codes (4221, 5211) where sum of weights ($r_3 \ r_2 \ r_1 \ r_0$) of each 4-bit decimal is $\sum r_j = 9$, where j varies from 0 to 3 unlike conventional BCD. The reduction by columns is done by compressor that adds a column of m digits of the same weight and produce n digits of adjacent weights [7, 14–16]. Mathematically the idea can be represented as:

$$\sum_{i=0}^{m-1} x_i = \sum_{j=0}^{n-1} y_j 10^j \quad (1)$$

The relation between m and n can be formulated as $10^n - 1 \geq m$. A $(m:n)$ compressor module consists m inputs and n output shown in figure 1. In figure 1, the numerical examples also considered by taking the highest decimal digit, i.e., 9, thereby highest number of rows which can be compressed into two digits should be 11. To fully understand the concept of the $(11:2)$ digit compressor the following mathematical equation can be formulated. The inputs are eleven digits variable $X = (x_{10}, x_9, \dots, x_1, x_0)$ and the output Y which is of 2-bits can be formulated as:

$$Y = \sum_{i=0}^{m-1} x_i = \sum_{j=0}^{n-1} y_j 10^j \quad (2)$$

where $\forall(x_i, y_j) \in (0..9)$.

2.1 Decimal 3:2 compressor

Taking a brief look at decimal carry save addition [5–7, 11], which have stabilized a known position in related designs and provided the basis for this work, indicates that a typical realization has been implemented by 4 conventional binary adder (3:2 binary compressor). Mathematical representation for the addition technique can be represented as:

$$\begin{aligned} A_i + B_i + Cin_i &= \sum_{i=0}^3 (a_{i,j} + b_{i,j} + cin_{i,j}) r_i \\ &= \sum_{j=0}^3 s_{i,j} r_j + 2 \times \sum_{j=0}^3 c_{i,j} r_j = S_i + 2Cout_i \end{aligned} \quad (3)$$

Figure 1. Compressor (a) $(m:n)$ compressor, (b) decimal 11:2 compressor.

where A_i , B_i and Cin_i are defining the decimal digits, $s_{i,j}$ and $c_{i,j}$ are the sum and carry bits of full adders respectively. $Cout_i, S_i \in [0, 9]$, are the decimal carry and sum digits respectively, and r in the digit sets based on 4211 and 5211 style. Decimal correction is not required for this 4-bit vector expressions of $Cout_i$, S_i because of coding (4221 and 5211) techniques. Moreover, $\times 2$ (decimal) is required for further usage of carry digit $Cout_i$. The implementation procedure (algorithmic level) for the usage of the carry bit has been shown in figure 2 [5]. In figure 2(a), a numeric example has been considered, where the implementation procedure has been described. In figure 2(b) the calculation of the carry bit has been described for the further usage of the carry bit based on the context of 5211 coding style.

The gate level implementation for the further usage of the carry bit based on 5211 coding has been described in figure 3. This architecture (figure 3) has been implemented through 4 binary full adders, combinational left shifter and digit recoder. From Eq. (3), it has been observed that decimal multiplication by 2 is required for carry digit which has been implemented through digit recoder followed by combinational left shifter [6]. Using this 3:2 digit

Figure 2. (a) Flow diagram for the calculation of the carry digit for the further usage. (b) One numerical example for the carry calculation.

compressor higher order decimal compressors can be fostered. In order to build up higher order compressor, the following two rules must be adhered to:

- All the decimal inputs of a digit compressor should follow same number system.
- All the decimal inputs of a digit compressor should have same digit weightage.

2.2 9:4 vertical bit array compressor

A family of fast decimal digit adders has been designed by using vertical bit array compressors. Vertical bit array compressor is used to compress a larger array of bits with equal binary weights into a smaller array of bits having different binary weights. A bit array compressor sums a binary bit column of same weight and produces 3, 4 or 5 weighted bits of (421), (4221) or (42221), respectively.

If we consider the same thing based on the conventional BCD coding techniques, then after every stage decimal correction stage is required. 9:4 bit array compressor has been shown in figure 4. The architecture has also been implemented through two stages of full adders. Here $A_i(-j)$ are the inputs of 9:4 bit array compressor; where 'j' varies from 0 to 8 and 'i' represents the ith column of vertical bit array compressors in 9:4 digit compressor. The mathematical expressions for the implementation of the 9:4 bit array compressor are given by the following Eq. (4).

$$\sum_{j=0}^8 A_i(j) = 2 \times (q_{1,i} + q_{3,i} + q_{5,i}) + (q_{0,i} + q_{2,i} + q_{4,i}) = Q_i(4221)$$

where

$$\begin{aligned} q_{0,i} &= a_i(0) \oplus a_i(1) \oplus a_i(2) \\ q_{1,i} &= a_i(0)a_i(1) + a_i(1)a_i(2) + a_i(2)a_i(0) \\ q_{2,i} &= a_i(3) \oplus a_i(4) \oplus a_i(5) \\ q_{3,i} &= a_i(3)a_i(4) + a_i(4)a_i(5) + a_i(5)a_i(3) \\ q_{4,i} &= a_i(6) \oplus a_i(7) \oplus a_i(8) \\ q_{4,i} &= a_i(6)a_i(7) + a_i(7)a_i(8) + a_i(8)a_i(6) \end{aligned}$$

(4)

Thus the equation can be reformulated as:

$$\begin{aligned} Q_i(4221) &= \\ q_i(0) &= q_{0,i} \oplus q_{2,i} \oplus q_{4,i} \\ q_i(1) &= q_{0,i}q_{2,i} + q_{2,i}q_{4,i} + q_{4,i}q_{0,i} \\ q_i(2) &= q_{1,i} \oplus q_{3,i} \oplus q_{5,i} \\ q_i(3) &= q_{1,i}q_{3,i} + q_{3,i}q_{5,i} + q_{5,i}q_{1,i} \end{aligned}$$

(5)

It has been observed that nine binary bits of same weight are arranged in a vertical stack and compressed into four bits ($q_i(3), q_i(2), q_i(1), q_i(0)$) with respective binary weights as $\times 4, \times 2, \times 2$ and $\times 1$. For a binary full adder the weight of the carry bit is $\times 2$, where the same for sum bit is $\times 1$. For nine bits, three binary full adders are used at the first stage;

Figure 3. Architecture of basic digit compressor.

Figure 4. Architecture of 9:4 vertical bit array compressor.

each binary full adder creates one carry bit ($\times 2$) and one sum bit ($\times 1$). So after first stage there are three bits which are having binary weightage $\times 2$ and same for weight bits $\times 1$. Thereby, three binary bits of binary weight $\times 2$ are given to another binary full adder at the second stage which results in two bits (carry and sum) of binary weights $\times 4$ and $\times 2$, respectively. Alongside three bits of binary weight $\times 1$ are also given to another 3:2 binary compressor which gives two binary bits, carry ($\times 2$) and sum ($\times 1$) as result. So, at the end, as a result we get a digit (Q_i) represented in 4221 coding as in Eq. (5).

2.3 9:4 decimal compressor

Dot representation of 9:4 digit counters is shown in figure 5. 9:4 decimal compressor compresses nine decimal digits into four decimal digits using vertical bit array compressor. Four 9:4 vertical bit array compressors are used to design one 9:4 decimal compressor. Now, on the other hand, if we accommodate nine decimal digits (5211) in that row of four 9:4 bit array compressors in such a way

Figure 5. 9:4 decimal compressor based on 5211 coding.

that the weightage of the columns turn out to be $\times 5$, $\times 2$, $\times 1$ and $\times 1$ from left to right, the resultant bits will have different binary weights if we look at these from left to right and top to down. If we take resultant digits from left to right as indicated by an arrow in figure 5, these four digits will be in 5211 coding with binary weightage $\times 4$, $\times 2$, $\times 2$ and $\times 1$.

2.4 Proposed 11:2 delay optimized compressor

Delay optimized 11:2 digit compressor has been shown in figure 6. The design has been implemented through 9:4 digit compressor, basic 3:2 compressors, digit recoders and left shifters. Input digits (in_i ; where i varies from 1 to 10 and each digit is of 4 bits) of the 9:4 decimal compressor (explained in section 2.2) are in 5211 ($\times 5$, $\times 2$, $\times 1$ and $\times 1$) format and the output digits (Q_3 , Q_2 , Q_1 , Q_0) are in 4221 format coding with different binary weights ($\times 4$, $\times 2$, $\times 2$ and $\times 1$ respectively) shown in the magnified view in figure 6. In magnified view $x_k = in_i(k)$; where k varies from 0 to 3 and for each value of k , i varies from 1 to 10. Another input of the compressor is c_{in} . Three inputs (c_{in} , in_1 and in_2) in 5211 format are given as inputs of one 3:2 decimal compressor whose two outputs are also in 5211 number system and has digit weight of $\times 2$ and $\times 1$. Following the compression rules dictated in section 2.2, all the outputs with same number system and same digit weight are put together to other digit compressors for further compression.

If outputs available are not of same number system or not of same digit weight then those outputs are to be converted into some other number system or digit weight in order to compress them in the next stage and this process goes on until we end up with two digits after compression. Like $\times 5$ operation for 4221 coding can be done by left shifting of the bits of 3 times and the resulting digit with weight $\times 1$ would be in 5211 coding as shown in figure 6. $\times 2$ operation can be done by using the $\times 2$ multiplier block for 4221 coding (section 3.2a). Total five 3:2 decimal compressors have been used to design this delay optimized 11:2 decimal compressor. The weights of the sum and carry digit of digit compressors have been decided according to the weight of the input digit.

2.5 Proposed 11:2 area optimized compressor

Area optimized 11:2 digit compressor has been shown in figure 7. To implement this compressor only 3:2 decimal compressor (basic compressor), $\times 5$ multiplier block, $\times 2$ multiplier block, 5211-4221 recoder and 4221-5211 recoder have been used. In this architecture (figure 7) the input digits have been taken in BCD 5211 coding format. These inputs are given to a row of four 3:2 decimal compressor, as an example C_{in} , In_0 and In_1 are the input of the first (taken from right) compressor similarly In_2 , In_3 and In_4 are the input of the second compressor. In figure 7, basic compressors have been used to implement the higher order

Figure 6. Proposed 11:2 delay optimized compressor.

compressors in parallel manner. $\times 5$ multiplication and $\times 2$ multiplication are the intermediate stage of the BCD 5211 and 4221 coding style. These inputs are divided in the group of three and given to a row of four 3:2 decimal compressors. Each of these compressors produces two digits; sum and carry of weights $\times 1$ and $\times 2$ respectively (as explained in section 2.1). Carry digits from the 1st stage (digit weight $\times 2$) are taken together and given as inputs to a 3:2 decimal compressor in 2nd stage. The carry output of this decimal compressor has digit weight of $\times 2(\times 2)$ i.e., $\times 4$ and at the same time digit weight for the sum digit is $\times 1(\times 2)$ i.e., $\times 2$. Similarly, three digits with digit weight $\times 1$ are given to another 3:2 decimal compressor of 2nd stage which results one sum and carry digit with weight-age of $\times 1$ and $\times 2$, respectively.

It has been observed that at the end of two 3:2 decimal compressors of second stage we get four digits (5211) with binary weight $\times 4$, $\times 2$, $\times 2$ and $\times 1$, respectively and each weighted digit consists of 4-bits, represented vertically in dot format as in figure 7. If we align the bits as shown in figure 7 and take 4-bits horizontally as a digit as

represented with rectangle in the figure, then we get 4-digits which follow number system 4221 and have digit weights $\times 5$, $\times 2$, $\times 1$ and $\times 1$. Now all the signals will follow the algorithm for compression (as discussed in section 2.1) for rest of the tree; same as delay optimized compressor (figure 6). At the end, two output digits are converted into 5211 style representation to get the outputs in 5211 coding same as inputs.

3. Circuit modules

In this section, different circuit modules like digit recoder $\times 2$ multiplier and $\times 5$ multiplier have been described to build up the delay and area optimized 11:2 decimal compressors.

From table 1, it has been observed that number representation based on BCD 5211, and 4221 is not unique, thereby different digital circuits implementations are possible. To make it uniform digit recoder is required. Changing the intermediate results from one coding style to

Figure 7. Area optimized decimal compressor.

Table 1. Decimal digit represented in 4221 and 5211.

Decimal	4221	5211
0	0000	0000
1	0001	0001 0010
2	0100 0010	0011 0100
3	0011 0101	0101 0110
4	0110 1000	0111
5	0111 1001	1000
6	1010 1100	1010 1001
7	1101 1011	1100 1011
8	1110	1110 1101
9	1111	1111

another like 5211 to 4221 or vice versa this digit recoder is required. Furthermore, $\times 2$ multiplier and $\times 5$ multiplier are the intermediate stage of the basic compressor as well as the higher order (11:2) compressor. The individual modules and their corresponding description including gate level implementation of the above mentioned modules have been described in this section.

3.1 Digit recoder

Table 1 represents the coding style of the decimal numbers based on 4221 and 5211 coding style. For each of the coding style of some numbers there are multiple representations are possible. The improved recorder is compatible for all the representations of the decimal digits based on 4221 coding.

3.1a 4221 to 5211 digit recoder: Hardware implementation of digit recoder (4221 to 5211) have been re-designed for the proposed decimal compressors, and shown in figure 8. From table 1, it has been observed that shaded digits of (4221) coding schemes are different from (5211) coding schemes and the rest of the digit combinations are same for both cases. Thereby, shaded combinations have been treated separately and modification has been carried out for the conversion. The modification has been carried out for the following cases:

- S1: This case appears for decimal digits 4 and 5.

When the combinations appear '0110' and '0111' then value of S1 has been considered as high else low.

- S2: This case appears for decimal digits 4 and 5.

When the combinations appear '1000' and '1001' then value of S2 has been considered as high or low.

- S3: This case appears for decimal digits 6 and 7.

Either of the shaded combination appears then S3 become high or low.

Figure 8. 4221 to 5211 recoder.

- S4: This case appears for decimal digits 2 and 3.

Either of the shaded combination appears then S4 become high else low.

For non-shaded combinations (table 1) no modification has been carried out because they are same for both the coding techniques. After identification of the digit combinations, the value has been promoted towards multiplexers, for bit modification facilitation 4221 to 5211 conversions. This proposed digit recoder gives us flexibility to use all the sixteen combinations of each coding techniques unlike [5, 6] where number of combinations of decimal digit representation considered, was restricted. The same digit recoder can be used when the input digits of decimal compressors are in 4221 code and also when inputs are coded in 5211. The expressions of converting 4221 ($a_3 a_2 a_1 a_0$) to 5211 ($b_3 b_2 b_1 b_0$) is given in Eq. (6).

$$\begin{aligned}
 b(0) &= \overline{a(3)a(2)} + a(2)(a(1) \oplus a(3)) \oplus a(0) \\
 b(1) &= \overline{a(3)}\overline{a(0)a(1)a(2)} + a(0)a(1)a(2) \oplus a(1) \\
 b(2) &= \overline{a(0)}\overline{a(1)}(a(2) \oplus a(3)) + a(0)\overline{a(3)}a(1) \oplus a(2) \\
 b(3) &= \overline{a(3)}\overline{a(0)a(1)a(2)} + a(0)a(1)a(2) \oplus a(3)
 \end{aligned} \quad (6)$$

3.1b 5211 to 4221 digit recoder: Figure 9 shows a digit recoder of 5211 to 4221 coding. The output weighted bit $\times 4$ is same as input weighted bit $\times 5$ and one of the weighted bit $\times 2$ of the output digit is same as the $\times 2$ weighted bit of the input digit. Another $\times 2$ weighted bit of the output digit comes from the output of one 2:1 MUX as shown in figure 10. This bit is basically the carry output of the full adder whose corresponding inputs are $\times 5$, $\times 1$ and $\times 1$ weighted bits of the input digit (5211) of the digit recoder. The output $\times 1$ bit is the sum of the above mentioned full adder as shown in figure 9. The mathematical expression of 5221 to 4211 recoding scheme has been shown in Eq. (7).

Figure 9. 5211 to 4221 digit recoder.

Figure 10. $\times 2$ multiplication block.

$$\begin{aligned}
 b(0) &= a(0) \oplus a(1) \oplus a(3) \\
 b(1) &= a(1) \bullet \overline{a(0) \oplus a(1)} + a(3) \bullet (a(0) \oplus a(1)) \\
 b(2) &= a(2) \\
 b(3) &= a(3)
 \end{aligned} \quad (7)$$

3.2 Multiplier

3.2a $\times 2$ multiplication: From Eq. (3), it has been observed that one decimal $\times 2$ operation is needed for decimal basic compressor. Multiplication of $\times 2$ block diagram is shown in figure 10. If the input to the multiplier block is in 4221 coding then the multiplier needs one 4221 to 5211 digit recoder first and then one bit left shift block. In case of 5211 coding, one left shift block is required first and then one 4221-5211 digit recoder to perform $\times 2$ decimal multiplication.

3.2b $\times 5$ multiplication: Figure 11(a) shows an example where H has been represented through 4221 coding scheme and the multiplication of the same digit by 5 has been represented by W (5211 coding scheme). If H is left shifted by 3 bits, we can get 5 times of H (5211 coding) i.e., 5W. Thereby, one $\times 5$ multiplication block consists of three cascaded 1-bit left shift blocks shown in figure 11(b). This multiplication block takes input in 4221 coding and gives output in 5211 coding. If the output need to be converted into 4221 coding, one 5211 to 4221 digit recoder (figure 9) is required.

4. Comparison

In this paper, transistor level implementation of the reported architecture has been carried out and their functionality has been checked. Implementation has been carried out for the further usage of the compressors in different circuits. One of the applications of this

Figure 11. (a) Architecture of the $\times 5$ multiplication block. (b) A numerical example, in this example input has been considered in decimal 4221 coding and output have been observed in decimal 5211 coding.

compressor circuit is multi-operand decimal adders. The details of the multi-operand decimal adders are shown in section 2.1. The references have been taken from the different sources and have been implemented using same technological environment. Cadence Virtuoso platform has been used for the simulation of the above mentioned compressors. The simulation results are compared with [5, 7] architectures and shown in tables 2 and 3. From tables 2 and 3, it has been observed that [5] has the superior results than [7], thereby, comparisons have been made with respect to [5].

4.1 Simulation environment

The decimal 11:2 compressors have been synthesized using Cadence Virtuoso platform with the library file of gpdn of 90 nm, 65 nm and 45 nm CMOS technology in room temperature. We have also implemented the reference architecture under the same environmental condition for the comparison. Input data was taken in a regular fashion for experimental purpose. For each transition, the delay is measured from 50% of the input voltage swing to 50% of the output voltage swing and the average value has been reported.

Table 2. Performance parameter like as propagation delay and power consumption analysis of the 11: 2 delay optimized compressor.

Supply voltage/ architecture	1 V		1.5 V		2.0 V		2.5 V		3.0 V	
	Delay (ns)	Power (mW)	Delay (ns)	Power (mW)	Delay (ns)	Power (mW)	Delay (ns)	Power (mW)	Delay (ns)	Power (mW)
(a) Delay and power analysis based on 90 nm CMOS technology file										
Ref. [5]	0.124	10.079	0.182	11.011	0.267	16.850	0.298	19.455	0.301	22.810
Ref. [7]	0.137	11.023	0.196	12.053	0.293	17.082	0.310	21.054	0.323	24.055
Proposed	0.094	10.077	0.176	11.179	0.260	16.923	0.282	20.367	0.285	23.068
(b) Delay and power analysis based on 65 nm CMOS technology file										
Ref. [5]	0.113	11.52	0.163	11.89	0.241	17.69	0.278	21.57	0.298	23.13
Ref. [7]	0.117	11.93	0.183	12.39	0.253	18.79	0.299	23.01	0.321	25.32
Proposed	0.089	11.50	0.159	11.87	0.229	18.09	0.253	22.36	0.296	24.67
(c) Delay and power analysis based on 45 nm CMOS technology file										
Ref. [5]	0.107	12.12	0.149	12.99	0.23	18.59	0.259	23.73	0.285	25.32
Ref. [7]	0.113	12.93	0.157	13.38	0.27	20.52	0.31	25.36	0.331	28.97
Proposed	0.085	12.57	0.148	13.17	0.22	19.72	0.268	24.79	0.283	26.98

Table 3. Performance parameter like as propagation delay and power consumption analysis of the 11: 2 area optimized compressor.

Supply voltage/ architecture	1 V		1.5 V		2.0 V		2.5 V		3.0 V	
	Delay (ns)	Power (mW)	Delay (ns)	Power (mW)	Delay (ns)	Power (mW)	Delay (ns)	Power (mW)	Delay (ns)	Power (mW)
(a) Delay and power analysis based on 90 nm CMOS technology file										
Ref. [5]	0.219	10.082	0.277	11.088	0.345	17.090	0.372	20.071	0.413	23.231
Ref. [7]	0.225	10.21	0.281	11.520	0.351	17.910	0.387	20.815	0.429	23.00
Proposed	0.198	10.09	0.232	10.986	0.269	16.265	0.304	19.391	0.301	22.709
(b) Delay and power analysis based on 65 nm CMOS technology file										
Ref. [5]	0.20	11.12	0.253	11.97	0.301	19.01	0.345	21.71	0.39	24.23
Ref. [7]	0.21	11.52	0.263	12.04	0.312	20.01	0.353	21.99	0.41	24.93
Proposed	0.18	11.1	0.229	11.13	0.267	18.1	0.315	20.39	0.35	23.7
(c) Delay and power analysis based on 45 nm CMOS technology file										
Ref. [5]	0.19	11.92	0.23	12.53	0.283	19.0	0.345	22.89	0.361	25.23
Ref. [7]	0.20	12.10	0.24	12.93	0.291	19.97	0.351	23.13	0.384	26.23
Proposed	0.17	11.23	0.18	11.98	0.237	18.1	0.29	21.79	0.31	24.79

4.2 Critical path delay

The critical path delay of the proposed delay optimized 11:2 compressor (figure 6) consists of one 9:4 decimal compressor, 2 numbers of 1-bit left shifter, three numbers of 3:2 digit compressors and three numbers of $\times(4221-5211)$ digit recoder). 3:2 digit compressor (figure 3) has been implemented through full adder, digit recoder and left shifter. Critical path delay of full adder approximated as ~ 2 XOR gate. Digit recoder (figure 8) has the critical path delay of ~ 3 XOR gate. And 1-bit left shifter has 1 XOR gate delay. Thus, overall 3:2 basic compressor has 6 XOR gate delay. It has been (figure 4) observed that vertical bit has been implemented in 2 stage of full adder, thus critical path delay is equals to 4 XOR gate. 9:4 digit compressors (figure 5) consists of 9:4 vertical bit array compressor and 4221-5211 decoder. 4221-5211 (figure 8) has a critical path delay ~ 8 XOR gate. Thereby critical path delay for the 9:4 digit compressor is ~ 13 XOR gate. Thereby total critical path delay of the delay optimized 11:2 compressor is equal to $(13 + 2 + 3 \times 6 + 3 \times 8)$ 57 XOR gate delay. Similarly, critical path delay of the area optimized 11:2 compressor (figure 7) have been implemented by two 1-bit left shifter, six number 3:2 digit compressor and four number 4221-5211 digit recoder. Thereby total critical path delay $(2 \times 1 + 6 \times 6 + 4 \times 8) \sim 70$ XOR gate delay.

5. Results

Performance parameters as a function of propagation delay and dynamic power consumption [17] analysis of the proposed delay optimized 11:2 compressor architecture have been shown in table 2. Table 2a is based on 90 nm CMOS technology [18–22], table 2b is based on 65 nm technology file [22] and figure 2c is based on 45 nm technology file [22]. Supply voltage has been varied linearly from 1 V to

3 V with the linear increment of 0.5 V step. From table 2a it has been observed that propagation delay of the proposed compressor is 0.094 ns, when the supply voltage is 1 V, at 90 nm CMOS technology. Moreover, the power consumption of the same at 1 V supply is 10.077 mw. The power is comparable with the reference paper. But the compressor is $\sim 24\%$ faster (less propagation delay) from the reference voltage. When the supply voltage is increasing then also maintains the same ratio $\sim 25\%$ is lesser than the reference compressor. When the performance parameters are measuring at 65 nm technology file at 1 V supply, propagation delay is only 0.089 ns and this compressor is $\sim 22\%$ faster than the reference compressor, and the power is almost equal. Same thing is happening when results are compared at 45 nm technology. The details chart of the implementation is shown in table 2.

Table 3 is also describing the propagation delay and power consumption analysis of the area optimized compressor. Table 3a is based on 90 nm CMOS technology [18–22], table 3b is the based on 65 nm technology file [22] and figure 3c is based on 45 nm technology file [22]. Dynamic switching power can be calculated theoretically as $P = \frac{1}{T} \int_0^T I_d(t) V_{DD}(t) dt$, where T is the switching time interval of the circuit [17]. In this area optimized compressor, at 1 V power supply, the propagation delay is 0.198 ns, still the compressor is $\sim 9\%$ faster than the reference architecture and the power consumption is almost equal with respect to their counterpart an 90 nm CMOS technology. Increasing the supply voltage the propagation delay is reducing $\sim 7\%$ to $\sim 10\%$. The power is comparable with the reference architecture. Using the 65 nm and 45 nm technology file, the propagation delay is reducing by $\sim 7\%$ to 10% , and the power is comparable with the reference architecture.

Table 4 describes the layout area of the proposed and the reference compressor. Area delay product measurement of the compressors has been shown in table 4.

Table 4. Area comparison.

		Measurement
Delay optimized compressor		
Ref. [5]		No. of NAND gates: 658 Layout area: $151.8\lambda \times 317.5\lambda$
Proposed		No. of NAND gates: 629 Layout area: $121.8\lambda \times 315.2\lambda$
Area optimized compressor		
Ref. [5]		No. of NAND gates: 626 Layout area: $119.4\lambda \times 315.2\lambda$
Proposed		No. of NAND gates: 604 $238.4\lambda \times 134.7\lambda$

6. Discussions

From table 2, it has been observed that dynamic switching power consumption is slightly higher for this proposed architecture as compared to the reference architecture as a function of the supply voltage. The propagation delay of the delay optimized compressor equals to 0.094 ns (table 2), while the propagation delay of the area optimized compressor is 0.128 ns (table 3) at 1 V supply voltage. The proposed compressor is $\sim 24\%$ faster than the reference compressor in case of delay optimized (table 2) design. While, for the area optimized case the proposed methodology is $\sim 41\%$ faster from its counterpart (table 3) at 1 V supply voltage. The propagation delay comparison among two different proposed methodology (i.e., delay and area optimized compressors) delay optimized compressor is $\sim 26\%$ faster from the area optimized compressor. To take the view point of area delay product (table 4) it has been observed that delay optimized compressor is more beneficial with respect to delay and area measurement (table 5).

Table 5. Performance parameter analysis as a function of area delay product.

Supply voltage/ architecture	Area delay product (10^{-2})				
	1 V	1.5 V	2.0 V	2.5 V	3.0 V
Delay optimized					
Ref. [5]	0.582	0.855	1.254	1.400	1.414
Proposed	0.357	0.668	0.988	1.071	1.083
Area optimized					
Ref. [5]	0.700	0.886	1.104	1.190	1.321
Proposed	0.516	0.720	1.108	1.125	1.152

7. Conclusions

In this paper, design and architectures of new 11:2 decimal compressors have been reported. Transistor level implementations of such compressors have been carried out and functionalities have been verified in Cadence Virtuoso platform. Performance parameters as a function of delay,

power and area have been calculated through Cadence with the library file of gpdk of 90 nm, 65 nm and 45 nm CMOS technology. Simulation results offered that propagation delay of the delay optimized compressor is ~ 0.094 ns (at 90 nm CMOS technology with a supply voltage 1 V) and $\sim 24\%$ faster from its counterpart. To take the view point of area delay product, it has been observed that delay optimized compressor is more economical from its counterpart and can be a prominent solution for the decimal computing system.

References

- [1] James R K, Shahana T K, Jacob K P and Sasi S 2008 Fixed point decimal multiplication using RPS algorithm. In: *Proceedings of the International Symposium on Parallel and Distributed Processing with Applications*, ISPA '08, pp. 343–350
- [2] Cowlshaw M F 2003 Decimal floating-point: algorithm for computers. In: *Proceedings of the 16th IEEE Symposium on Computer Arithmetic*, Santiago de Compostela, pp. 104–111
- [3] Véstias M P and Neto H C 2010 Parallel decimal multipliers using binary multipliers. In: *Proceedings of the Southern Programmable Logic Conference (SPL)*, pp. 73–78
- [4] Jaberipur G and Kaivani A 2007 Binary-coded decimal digit multipliers. *IET J. Comput. Digit. Tech.* 1(4): 377–381
- [5] Vazquez A, Antelo E and Montuschi P 2010 Improved design of high-performance parallel decimal multipliers. *IEEE Trans. Comput.* 59(5): 679–693
- [6] Vazquez A, Antelo E and Montuschi P 2007 A new family of high performance parallel decimal multipliers. In: *18th IEEE Symposium on Computer Arithmetic*, pp. 195–204
- [7] Castellanos I D and Stine J E 2008 Compressor trees for decimal partial product reduction. In: *Proceedings of the ACM 18th ACM Great Lakes Symposium on VLSI (GLSVLSI '08)*, pp. 107–110
- [8] Kenney R D and Schulte M J 2004 Multioperand decimal addition. In: *Proceedings of the IEEE Computer Society Annual Symposium on VLSI*, pp. 251–253
- [9] Kenney R D and Schulte M J 2004 High-speed multioperand decimal adders. *IEEE Trans. Comput.* 54(8): 953–963
- [10] Dadda L and Nannarelli A 2009 *Fast Multi Operand Decimal Adders Using Digit Compressors with Decimal Carry Generation*. Technical University of Denmark, 2009 (IMM Technical Report-2009-05)
- [11] Veeramachaneni S, Krishna M K, Avinash L, Reddy S and Srinivas M B 2007 Novel, high-speed 16-digit BCD adders conforming to IEEE 754r format. In: *IEEE Computer Society Annual Symposium on VLSI (ISVLSI '07)*, pp. 43–50
- [12] Gonzalez-Navarro S, Tsen C and Schulte M J 2013 Binary integer decimal-based floating-point multiplication. *IEEE Trans. Comput.* 62(7): 1460–1463
- [13] Saha P, Samanta P and Kumar D 2016 4:2 and 5:2 Decimal compressors. In: *Proc. 7th Int. Conf. on Intelligent Systems, Modelling and Simulation*, pp. 424–429
- [14] Erle M A, Hickmann B J and Schulte M J 2009 Decimal floating-point multiplication. *IEEE Trans. Comput.* 58(7): 902–916
- [15] Richards R K 1955 *Arithmetic operations in digital computers*. Princeton, NJ: Van Nostrand Company
- [16] Ercegovic M D and Lang T 2004 *Digital Arithmetic*. San Francisco: Morgan Kaufman
- [17] Uyemura J P 2001 *CMOS Logic Circuit Design*. Kluwer, New York
- [18] Zhao W and Cao Y 2006 New generation of Predictive Technology Model for sub-45 nm early design exploration. In: *International Symposium on Quality Electronic Design (ISQED'06)*, pp. 590–596
- [19] Cao Y, Sato T, Sylvester D, Orshansky M and Hu C 2000 New paradigm of predictive MOSFET and interconnect modeling for early circuit design. In: *Proceedings of the Custom Integrated Circuits Conference*, pp. 201–204
- [20] Ueno K 2011 Material and process challenges for interconnects in nanoelectronics era. In: *Int. Symposium on VLSI Technology, Systems and Applications (VLSI-TSA)*, pp. 64–65
- [21] Kosonocky S, Burd T, Kasprak K, Schultz R and Stephany R 2012 Designing in scaled technologies: 32 nm and beyond. In: *Symposium on VLSI Technology (VLSIT)*, pp. 147–148
- [22] <http://ptm.asu.edu/latest.html>