Multi-objective solid transportation problem under stochastic environment

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Abstract. In real life, three-dimensional (solid) transportation problem is an uncertain multi-objective decision-making (MODM) problem. In particular, it involves searching for the best transportation set-up that meets the decision maker's preferences by considering the conflicting objectives/criteria such as transportation cost, transportation time, environmental and social issues. To tackle such complex situations, this paper proposes a general formulation of the multi-objective solid transportation problem (STP) with some random parameters. The paper makes the following contributions: (i) proposes a solution methodology based on chance-constraint programming technique to solve an STP with the uncertainty characterized by gamma distribution, (ii) proposes the initial feasibility conditions for the problem and (iii) extends fuzzy programming approach for solving the multi-objective stochastic problems. A numerical example is presented to illustrate the model and methodology.

Keywords. Solid transportation problem; multi-objective decision making; stochastic programming; chance-constraint programming; gamma distribution.

1. Introduction

In 1941, Hitchcock [1] formulated the classical form of the transportation problem (TP), which is a special type of linear programming problem. In a classical TP, homogeneous products are transported from several sources to numerous destinations with minimum transportation cost. A more general form of TP is the transshipment problem in which the transportation of goods from one source to another source is permissible. Three-dimensional TP or solid transportation problem (STP) is an extended form of the well-known TP, first modelled by Schell [2] and developed by Haley [3]. The aim of STP is to transport homogenous products from sources to destinations by different types of conveyances such that the total transportation cost is minimized [4]. The availability of the products at the source points, the requirements of the products at the destination points and capacity of different modes of conveyances (such as trucks, cargo flights, goods trains, ships, etc.) used to transport the product from sources to destinations are the parameters of a three-dimensional TP. In real life, due to the presence of several factors such as machine breakdown and labour problem for production, market mode, road condition and weather condition for transportation, the parameters of the problem are not deterministic, rather uncertain. Sometimes, random

variables are used to characterize these uncertainties (especially stochastic). During the formulation of a real-world STP, we need to consider the optimization of several objectives such as minimization of transportation cost, minimization of loss during the transportation, minimization of transportation time, etc. This information leads us to consider a stochastic multi-objective STP.

STP is one of the important research topics from both theoretical and practical aspects. Several researchers have contributed significantly in this area of research. In 1993, Bit *et al* [5] used fuzzy linear programming for solving a linear multi-objective STP. Gen *et al* [6] studied bi-criterion STP where all the parameters are fuzzy numbers. Jménez and Verdegay [7] studied two types of uncertain STP in which they considered the supplies, demands and conveyance capacities as interval numbers and fuzzy numbers, respectively. Yang and Liu [8] investigated fixed charge STP in fuzzy environment. They developed expected value models, chance-constraint programming model and dependent-chance programming model for the problem. Liu [9] presented fuzzy STP where all the parameters such as cost coefficients, the supply and demand quantities and conveyance capacities are fuzzy numbers. Rani *et al* [10] studied multi-objective TP under intuitionistic fuzzy environment. Ebrahimnejad [11] studied the TP which involves interval-valued trapezoidal fuzzy numbers for the parameters. Rani *et al* [10] presented fuzzy multi-objective multi-item STP under uncertain environment. Yang and

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Feng [12] studied a bi-criterion STP with fixed charge under stochastic environment and proposed three models for the problem. Roy [13] studied multi-choice stochastic TP where the parameters related to supplies and demands follow Weibull distribution. Roy *et al* [14] established an equivalent deterministic model of a multi-choice stochastic TP where supplies and demands follow exponential distribution. Recently, Cui and Sheng [15] studied STP with normal random parameters. Das and Bera [16] presented a bi-objective STP with fuzzy transportation cost and fuzzy transportation time. Zhang *et al* [17] dealt with a fixed charge STP where the supplies, demands, conveyance capacities, the direct costs, and the fixed charges are uncertain variables. Chen *et al* [18] studied an uncertain solid transportation model that involves entropy function as a new objective. Chen *et al* [4] presented goal programming technique for solving a bi-criterion STP under uncertain environment. They presented two models depending on expected value goal programming and chance-constrained goal programming to transform the problem into equivalent deterministic form. Also, they presented an expanded literature review on STP under uncertain environment. Dalman and Sivri [19] studied multi-objective STP with interval uncertainty. Rani *et al* [20] studied unbalanced transportation problem in which all the parameters are considered to be fuzzy numbers.

It is observed in the literature of multi-objective STP that most of the studies have been done by considering the supply, demand and conveyance capacity as fuzzy variables and random variables (Weibull, normal and log normal). However, if we consider the situation mentioned later, then there does not exist any model or methodology to tackle the situation.

A sugar production company has n number of sugar mills for sugar production. For the purpose, the company buys sugarcane from the farmers directly and it is the company's responsibility to transport the sugarcane from fields to the mills. Let there be m number of sources from where sugarcane is supplied. For a smooth production process, the manager of each mill divides the transported sugarcane into number of stocks and each stock consists of α quintals of sugarcane. Due to several factors (e.g., machine breakdown, manpower, etc.), the time taken to process one stock is not fixed. Let the time taken to process one stock be approximately β days. Also, the production process of a stock will not start until unless the process is finished for the previous stock. Hence, the demands of sugarcane at the mills become random and can be approximated as gamma random variable with α, β as its parameters. Due to the random demands, the supplies and the conveyance capacities become random also. Under these circumstances, the transportation manager of the company wants to design a transportation schedule such that the transportation cost and time are minimized.

Thus, this paper presents a mathematical model for multi-objective STP where the supplies, demands and conveyance capacities follow gamma distribution or Erlang

distribution. Furthermore, an efficient methodology to solve the proposed problem is developed. The remainder of the paper is organized as follows. The mathematical formulation of the three-dimensional multi-objective TP with gamma or Erlang distribution is presented in section 2. An equivalent deterministic model of the considered problem is established in section 3. Then the fuzzy programming technique to solve the multi-objective model and a numerical example are presented in sections 4 and 5, respectively. Some conclusions are drawn in section 6.

2. Formulation of mathematical model for multi-objective stochastic STP

STP is an extension of two-dimensional TP where homogeneous products are transported from a number of production houses to several destinations. In addition, the mode of transportation is considered in STP. Hence, there are three types of constraints corresponding to supplies from the sources, demands at the destinations and mode of conveyances, which are of different capacities. In STP, we need to model an efficient transportation plan that mainly minimizes the transportation cost by satisfying all the constraints. In many real-life situations, more than one objectives (instead of a single objective) are considered and optimized at the same time. A decision maker (DM) may consider the following objectives:

- (i) minimization of the total transportation cost,
- (ii) minimization of total transportation time,
- (iii) minimization of total loss during transportation,
- (iv) minimization of total deterioration of product, etc.

In order to determine an effective transportation planning, the past records for the parameters of the problem need to be studied. However, these parameters (supply, demands and conveyances) may not be always precise and certain. Therefore, the aforesaid parameters are considered to be uncertain. These uncertainties come from linguistic information, fluctuating financial market, insufficient information, imperfect statistical analysis, etc. Random set theory is used and successfully applied to deal with these kinds of uncertainties. Thus, the mathematical model of a multi-objective STP with random supply, demand and conveyance capacity is formulated.

Let there be m number of origins, say $O_i (i = 1, 2, \dots, m)$ from which the product should be transported to n number of destinations, say $D_j (j = 1, 2, \dots, n)$ using ℓ number of transportation modes, say $E_k (k = 1, 2, \dots, \ell)$. Under this situation, the DM wants to find the optimal transportation strategy by considering P number of objective functions, say $Z_p (p = 1, 2, \dots, P)$. In this context, to formulate the mathematical model of the problem, let us define the following parameters and variables:

x_{ijk} : amount of product transported from i -th origin to j -th destination using k -th transportation mode ($i = 1, 2, \dots, m; j = 1, 2, \dots, n; k = 1, 2, \dots, \ell$)

c_{ijk}^p : cost coefficient of the p -th objective function ($p = 1, 2, \dots, P$)

a_i : amount of supply from i -th origin O_i ($i = 1, 2, \dots, m$)

b_j : amount of demand at j -th destination D_j ($j = 1, 2, \dots, n$)

e_k : conveyance capacity of k -th transportation mode E_k ($k = 1, 2, \dots, \ell$)

According to the aforementioned definitions, the problem can be expressed as follows:

$$\min Z_p = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^{\ell} c_{ijk}^p x_{ijk}, \quad p = 1, 2, \dots, P, \quad (1)$$

subject to

$$\sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk} \leq a_i, \quad i = 1, 2, \dots, m, \quad (2)$$

$$\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk} \geq b_j, \quad j = 1, 2, \dots, n, \quad (3)$$

$$\sum_{i=1}^m \sum_{j=1}^n x_{ijk} \leq e_k, \quad k = 1, 2, \dots, \ell, \quad (4)$$

$$x_{ijk} \geq 0 \quad \forall \quad i, j \text{ and } k. \quad (5)$$

Note that, all the parameters except cost coefficients of this model are random variables.

Conventionally, in a random experiment, how long some event occurs follows exponential distribution but how long some different events occur follow gamma distribution, i.e., the sum of independent exponential distributions follows gamma distribution. Gamma distribution is one of the popular continuous distributions that is commonly used for elapsed times and some financial variables such as wireless communication for the multipath fading of signal power, modelling the size of insurance claims and rainfalls. It is also used to model the errors in multi-level Poisson regression models, etc. Roy *et al* [14] studied classical TP with exponential distribution. Gamma distribution can be used in TP where the probability of demand of certain product is very high at initial stage of a certain period and gradually decreases with the time. Depending on demands, the supplies and conveyance capacities also vary. For example, the probability of demand of rice in a market is very high at the starting of a month and it decreases gradually till the end of the month. Depending on this demand, suppliers have to supply conveniently. Hence, in this study, we consider that the random variables of the problem (1–5) follow independent gamma distribution or Erlang distribution

with known means and variances. Also, the independent gamma distributions have different shape parameters and the same scale parameter. In the next section, the methodology to formulate the deterministic model of the problem is presented.

Due to the presence of uncertainty in the parameters, the TP is not balanced. For an unbalanced STP, there are two feasibility conditions: (i) total supply should be more than total demand and (ii) total capacity of the conveyances should be higher than total demand. Mathematically, these conditions are expressed as

$$\sum_{i=1}^m a_i \geq \sum_{j=1}^n b_j, \quad \sum_{k=1}^{\ell} e_k \geq \sum_{j=1}^n b_j. \quad (6)$$

The inequality relation is well defined if the parameters are deterministic or known in advance. However, in our problem, all these parameters are random in nature. Therefore, these ordered relations are not applicable here. To overcome these difficulties, some new feasibility conditions for the problem are proposed in the next section.

3. Equivalent deterministic model formulation

In this section, an equivalent deterministic model for the stochastic problem is established. To do so, firstly the initial feasibility conditions for the problem is presented. For an unbalanced STP, if the parameters are deterministic, then, the initial feasibility conditions are given by Eq. (6). If any one of those parameters becomes stochastic and follows gamma distribution then the feasibility conditions for the problem are presented by the following theorems.

Theorem 3.1 *If the supply parameters a_i of an STP follow $\text{gamma}(\alpha_i, \beta)$ ($i = 1, 2, \dots, m$), then the feasibility conditions of the problem are*

$$\exp\left(-\frac{\sum_{j=1}^n b_j}{\beta}\right) \left(\sum_{r=0}^{\sum_{i=1}^m \alpha_i - 1} \left(\frac{\sum_{j=1}^n b_j}{\beta} \right)^r \frac{1}{r!} \right) \geq 1 - \gamma', \quad (7)$$

and

$$\sum_{k=1}^{\ell} e_k \geq \sum_{j=1}^n b_j, \quad (8)$$

where γ' is the level of significance and other symbols are defined earlier.

Proof For any unbalanced STP the total supply as well as total conveyance capacity should always be greater than the total demand. Hence, the feasibility conditions for deterministic STP are

$$\sum_{i=1}^m a_i \geq \sum_{j=1}^n b_j, \quad \sum_{k=1}^{\ell} e_k \geq \sum_{j=1}^n b_j.$$

In our case, only a_i ($i = 1, 2, \dots, m$) are not deterministic; they follow independent gamma distributions with known means and variances, and the same scale parameter, i.e., $a_i \sim \text{gamma}(\alpha_i, \beta)$. Chance-constraint approach is employed to find the deterministic form of the first feasibility condition. Hence, the first condition is

$$\Pr\left(\sum_{i=1}^m a_i \geq \sum_{j=1}^n b_j\right) \geq 1 - \gamma', \quad (9)$$

where γ' is the level of significance; e.g., if $\gamma' = 0.01$, then total supply will be greater than total demand with 99% surety.

It is known that the sum of independent gamma distributions with the same scale parameter follows a gamma distribution. Hence, $A = \sum_{i=1}^m a_i$ follows gamma distribution with shape parameter $\alpha = \sum_{i=1}^m \alpha_i$ and scale parameter β . Then the feasibility condition (9) leads to

$$\left(\int_{\sum_{j=1}^n b_j}^{\infty} \frac{1}{\Gamma(\alpha)\beta^\alpha} A^{\alpha-1} \exp\left(\frac{-A}{\beta}\right) dA\right) \geq 1 - \gamma_i, \quad (10)$$

$$\Rightarrow \exp\left(\frac{-\sum_{j=1}^n b_j}{\beta}\right) \left(\sum_{r=0}^{\alpha-1} \left(\frac{\sum_{j=1}^n b_j}{\beta}\right)^r \frac{1}{r!}\right) \geq 1 - \gamma'. \quad (11)$$

Hence, the deterministic form of Eq. (9) is

$$\exp\left(\frac{-\sum_{j=1}^n b_j}{\beta}\right) \left(\sum_{r=0}^{\sum_{i=1}^m \alpha_i - 1} \left(\frac{\sum_{j=1}^n b_j}{\beta}\right)^r \frac{1}{r!}\right) \geq 1 - \gamma'.$$

□

Theorem 3.2 If the demand parameters b_j of an STP follow gamma distributions $\text{gamma}(\alpha'_j, \beta')$, then the feasibility conditions of the problem are

$$\exp\left(\frac{-\sum_{i=1}^m a_i}{\beta'}\right) \left(\sum_{r=0}^{\sum_{j=1}^n \alpha'_j - 1} \left(\frac{\sum_{i=1}^m a_i}{\beta'}\right)^r \frac{1}{r!}\right) \leq \eta'_1, \quad (12)$$

and

$$\exp\left(\frac{-\sum_{k=1}^{\ell} e_k}{\beta'}\right) \left(\sum_{r=0}^{\sum_{j=1}^n \alpha'_j - 1} \left(\frac{\sum_{k=1}^{\ell} e_k}{\beta'}\right)^r \frac{1}{r!}\right) \leq \eta'_2, \quad (13)$$

where η'_1, η'_2 are the levels of significance.

Proof Proof is similar to that of theorem (3.1).

Theorem 3.3 If the parameters corresponding to the conveyance capacity e_k of an STP follow $\text{gamma}(\alpha''_k, \beta'')$, then the feasibility conditions of the problem are

$$\exp\left(\frac{-\sum_{j=1}^n b_j}{\beta''}\right) \left(\sum_{r=0}^{\sum_{k=1}^{\ell} \alpha''_k - 1} \left(\frac{\sum_{j=1}^n b_j}{\beta''}\right)^r \frac{1}{r!}\right) \geq 1 - \zeta', \quad (14)$$

and

$$\sum_{i=1}^m a_i \geq \sum_{j=1}^n b_j, \quad (15)$$

where ζ' is the level of significance.

Proof Proof is similar to that of theorem (3.1).

The mathematical model (1–5) is a multi-objective stochastic programming problem, as the right hand side parameters of the constraints are random variables. To solve this model, the deterministic form of the problem is established by removing the randomness of the parameters. The chance-constraint programming technique [21] is used to tackle the stochastic parameters. In this technique, the constraints are allowed to violate up to a given probability level. Considering the chance constraints for all the uncertain constraints, we have the following model:

$$\min Z_p = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^{\ell} c_{ijk}^p x_{ijk}, \quad p = 1, 2, \dots, P, \quad (16)$$

subject to

$$\Pr\left(\sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk} \leq a_i\right) \geq 1 - \gamma_i, \quad i = 1, 2, \dots, m \quad (17)$$

$$\Pr\left(\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk} \geq b_j\right) \geq 1 - \eta_j, \quad j = 1, 2, \dots, n \quad (18)$$

$$\Pr\left(\sum_{i=1}^m \sum_{j=1}^n x_{ijk} \leq e_k\right) \geq 1 - \zeta_k, \quad k = 1, 2, \dots, \ell \quad (19)$$

$$x_{ijk} \geq 0 \quad \forall \quad i, j \text{ and } k \quad (20)$$

where $(1 - \gamma_i)$, $(1 - \eta_j)$ and $(1 - \zeta_k)$ are the probability levels provided as an appropriate safety margin by the DM. Any vector $\mathbf{x} \in \mathbb{R}^n (\geq 0)$ is a feasible solution of the problem if it satisfies all the constraints (17)–(19). Also, it will be a Pareto optimal solution if there does not exist a feasible $\mathbf{x}'$ such that, $Z_p(\mathbf{x}) \geq Z_p(\mathbf{x}'), p = 1, 2, \dots, P$ and there exists at least one objective for which the inequality holds strictly.

Uncertainty is based on different circumstances that are already discussed. It is not necessary that every time all the

parameters of the model will be random variables. Therefore, the problem is classified into four models depending on different situations. They are presented as follows:

- (i) only supply parameters $a_i (i = 1, 2, \dots, m)$ follow gamma distribution;
- (ii) only demand parameters $b_j (j = 1, 2, \dots, n)$ follow gamma distribution;
- (iii) only conveyance capacities $c_k (k = 1, 2, \dots, \ell)$ follow gamma distribution and
- (iv) supplies $a_i (i = 1, 2, \dots, m)$, demands $b_j (j = 1, 2, \dots, n)$ and conveyance capacities $e_k (k = 1, 2, \dots, \ell)$ follow gamma distribution.

Depending on the choice of deterministic and random nature of the parameters, the equivalent deterministic model becomes linear or nonlinear. We assume that the parameters follow a gamma distribution with shape parameter (> 1), so the deterministic form will be nonlinear. From the classification, it is also observed that the first three models are sub-models of the fourth model. All these models can be derived from the last model by considering some parameters as deterministic. Note that if all the parameters of a chance constraint are deterministic, then the probability of the chance-constraint will be 1. Hence, the formulation of the deterministic model for the last case is discussed here. We know that, to use traditional solution methodology for solving the problem, we need the deterministic equivalents of the chance constraints. However, this process is usually hard to execute and successful only for some special cases. In the following theorems, the deterministic equivalent of the chance constraint having gamma random variables is presented.

Theorem 3.4 *If the supply parameters $a_i (i = 1, 2, \dots, m)$ are assumed to be independent gamma random variables,*

then $Pr\left(\sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk} \leq a_i\right) \geq 1 - \gamma_i$ if and only if

$$\exp\left(\frac{-\sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk}}{\beta}\right) \left(\sum_{r=0}^{\alpha_i-1} \frac{1}{r!} \left(\frac{\sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk}}{\beta}\right)^r\right) \geq (1 - \gamma_i), \quad (21)$$

where α_i and β represent the shape and scale parameters of the gamma distribution corresponding to $a_i (i = 1, 2, \dots, m)$.

Proof The probability density function (pdf) of gamma distribution corresponding to a_i having α_i and β as the shape and scale parameters, respectively, is

$$f(a_i) = \frac{1}{\Gamma(\alpha_i)\beta^{\alpha_i}} a_i^{\alpha_i-1} \exp\left(\frac{-a_i}{\beta}\right), \quad 0 < a_i < \infty, \\ \alpha_i, \beta \in \mathbb{R}^+, i = 1, 2, \dots, m.$$

Let us take $h_i = \sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk}$. Then the constraint (17)

becomes

$$P(a_i \geq h_i) \geq 1 - \gamma_i, \quad (22)$$

$$\left(\int_{h_i}^{\infty} \frac{1}{\Gamma(\alpha_i)\beta^{\alpha_i}} a_i^{\alpha_i-1} \exp\left(\frac{-a_i}{\beta}\right) da_i\right) \geq 1 - \gamma_i, \quad (23)$$

$$\exp\left(\frac{-\sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk}}{\beta}\right) \left(\sum_{r=0}^{\alpha_i-1} \frac{1}{r!} \left(\frac{\sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk}}{\beta}\right)^r\right) \geq (1 - \gamma_i). \quad (24)$$

That is, the deterministic equivalent of the chance constraint is (21). The theorem is proved. $\square$

Theorem 3.5 *If the demand parameters $b_j (j = 1, 2, \dots, n)$ are assumed to be independent gamma random variables,*

then $Pr\left(\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk} \geq b_j\right) \geq 1 - \eta_j$ if and only if

$$\exp\left(\frac{-\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk}}{\beta'}\right) \left(\sum_{r=0}^{\alpha'_j-1} \left(\frac{\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk}}{\beta'}\right)^r \frac{1}{r!}\right) \leq \eta_j, \quad (25)$$

where α'_j and β' represent, respectively, the shape and scale parameters of the gamma distribution corresponding to $b_j (j = 1, 2, \dots, n)$.

Proof The pdf of gamma distribution corresponding to b_j is

$$f(b_j) = \frac{1}{\Gamma(\alpha'_j)(\beta')^{\alpha'_j}} b_j^{\alpha'_j-1} \exp\left(\frac{-b_j}{\beta'}\right), \\ 0 < b_j < \infty, j = 1, 2, \dots, n.$$

Let us take $g_j = \sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk}$. Then the constraint

$Pr\left(\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk} \geq b_j\right) \geq 1 - \eta_j$ can be written as

$$Pr(b_j \geq g_j) \leq \eta_j.$$

After integration, the equivalent deterministic form of the constraint is

$$\exp\left(\frac{-\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk}}{\beta'}\right) \left(\sum_{r=0}^{\alpha'_j-1} \left(\frac{\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk}}{\beta'}\right)^r \frac{1}{r!}\right) \leq \eta_j. \quad (26)$$

That is, the deterministic equivalent of the chance constraint is (27). The theorem is proved. $\square$

Using the same logic, we can prove the following theorem also:

Theorem 3.6 *If the conveyance capacities e_k ($k = 1, 2, \dots, \ell$) are assumed to be independent gamma random variables, then $Pr\left(\sum_{i=1}^m \sum_{j=1}^n x_{ijk} \leq e_k\right) \geq 1 - \zeta_k$ if and only if*

$$\exp\left(\frac{-\sum_{i=1}^m \sum_{j=1}^n x_{ijk}}{\beta''}\right) \left(\sum_{r=0}^{\alpha_i''-1} \frac{1}{r!} \left(\frac{\sum_{i=1}^m \sum_{j=1}^n x_{ijk}}{\beta''}\right)^r\right) \geq (1 - \zeta_k), \quad (27)$$

where α_i'' and β'' represent, respectively, the shape and scale parameters of the gamma distribution corresponding to e_k ($k = 1, 2, \dots, \ell$).

With the help of theorems (3.4)–(3.6), we establish the equivalent deterministic model of the problem where all the resource parameters follow gamma distributions with known means and variances. The deterministic model is

$$\min \quad Z_p = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^{\ell} c_{ijk}^p x_{ijk}, \quad p = 1, 2, \dots, P, \quad (28)$$

subject to

$$\exp\left(\frac{-\sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk}}{\beta_i}\right) \left(\sum_{r=0}^{\alpha_i-1} \frac{1}{r!} \left(\frac{\sum_{j=1}^n \sum_{k=1}^{\ell} x_{ijk}}{\beta_i}\right)^r\right) \geq (1 - \gamma_i), \quad (29)$$

$i = 1 \dots m,$

$$\exp\left(\frac{-\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk}}{\beta'_j}\right) \left(\sum_{r=0}^{\alpha'_j-1} \left(\frac{\sum_{i=1}^m \sum_{k=1}^{\ell} x_{ijk}}{\beta'_j}\right)^r \frac{1}{r!}\right) \leq \eta_j, \quad (30)$$

$j = 1, \dots, n,$

$$\exp\left(\frac{-\sum_{i=1}^m \sum_{j=1}^n x_{ijk}}{\beta''_k}\right) \left(\sum_{r=0}^{\alpha''_k-1} \frac{1}{r!} \left(\frac{\sum_{i=1}^m \sum_{j=1}^n x_{ijk}}{\beta''_k}\right)^r\right) \geq (1 - \zeta_k) \quad (31)$$

$k = 1 \dots \ell,$

$$x_{ijk} \geq 0 \quad \forall \quad i, j \text{ and } k. \quad (32)$$

The obtained equivalent deterministic model is a nonlinear multi-objective programming problem. It is assumed that the model is feasible and has an optimal compromised solution. To tackle the conflicting nature of the multiple objectives and to find a compromised solution, we use the fuzzy programming technique. In the following section, the fuzzy programming technique for solving multi-objective model is presented. Also, note that if the shape and scale parameters are integers, then the distribution becomes Erlang distribution. Hence, if the parameters follow Erlang distribution, then the model is a special case of this model.

4. Fuzzy programming approach for multi-objective model

In a multi-objective programming problem, the DM needs to optimize a number of conflicting objective functions at a time. Since the objective functions are conflicting in nature, it is impossible to find a single optimal point where all the objectives attain their optimal values. Hence, we need to find a compromised solution or a Pareto optimal solution. In the literature, there exist several methodologies like goal programming approach [22, 23], weighted sum method [24, 25], ϵ -constraint method [26, 27], fuzzy programming technique [28, 29] and fuzzy goal programming approach [30–32], for solving a multi-objective problem. All the aforementioned techniques except fuzzy programming approach need prior information on objectives (goals and weights) from the DM for solving the problem. Fuzzy programming technique uses the concept of the shortest distance from the positive ideal solution (PIS) to find a compromise solution for the problem; the method does not need any prior information on objective function from the DM. Hence, to find a compromise solution for our problem, we apply the fuzzy programming technique. The steps of fuzzy programming technique for solving the problem are given here.

4.1 Steps of the fuzzy programming technique

Step 1 Solve the model by considering one objective at a time and the obtained optimal solution is the ideal solution for the corresponding objective. All the optimal values for different objectives together will construct the PIS point for the multi-objective deterministic model.

Step 2 The *pay-off* matrix for the objectives is formulated in the following way:

$$\begin{matrix} & Z_1 & Z_2 & \dots & Z_P \\ \begin{matrix} X_1 \\ X_2 \\ \dots \\ X_P \end{matrix} & \begin{pmatrix} Z_{11} & Z_{12} & \dots & Z_{1P} \\ Z_{21} & Z_{22} & \dots & Z_{2P} \\ \dots & \dots & \dots & \dots \\ Z_{P1} & Z_{P2} & \dots & Z_{PP} \end{pmatrix} \end{matrix}$$

where X_p is the optimal point for the single objective deterministic problem with the p -th objective function.

$Z_{ij} = Z_j(X_i)$ is the i -th row and j -th column of the *pay-off* matrix ($i = 1, 2, \dots, P$ and $j = 1, 2, \dots, P$).

Step 3 Find the lower bound L_p and upper bound U_p for each objective function from the *pay-off* matrix.

Step 4 Define linear membership function for each objective as

$$\mu_{Z_p}(X) = \begin{cases} 1 & \text{if } Z_p \leq L_p, \\ \frac{U_p - Z_p(X)}{U_p - L_p} & \text{if } L_p \leq Z_p \leq U_p, \\ 0 & \text{if } Z_p \geq U_p. \end{cases} \quad (33)$$

Step 5 Construct the equivalent crisp model as follows:

$$\max \quad \lambda \quad (34)$$

subject to

$$\lambda \leq \mu_{Z_p}(X), \quad p = 1, 2, \dots, P, \quad (35)$$

$$\text{with the original constraint set of the problem.} \quad (36)$$

Using the steps described here, we obtain a compromise solution for the equivalent multi-objective model and hence find a compromise solution for the original problem.

5. Numerical example

To illustrate the proposed study, we consider the following example of sugar TP where supplies, demands and conveyance capacities are considered as gamma random variables. Suppose there are three sugar factories O_1, O_2, O_3 from where the sugar is supplied to three cities D_1, D_2, D_3 . Conveyances with three different capacities E_1, E_2, E_3 are available to be selected for transporting sugar. The transportation cost and the transportation time are given in tables 1 and 2, respectively.

Here

- t_{ijk} is the number of days taken to transport one thousand tonnes of sugar from i -th origin to j -th destination using k -th type of transportation mode; e.g., it will take 8 days to transport one thousand tonnes of

Table 2. Transportation cost in rupees (ten thousands per day).

c_{ijk}	E_1			E_2			E_3		
	E_1			E_2			E_3		
	E_1			E_2			E_3		
	D_1			D_2			D_3		
O_1	6	7	8	10	6	5	11	3	7
O_2	13	8	11	12	2	9	20	15	13
O_3	5	6	7	11	9	7	10	5	2

sugar from the first sugar factory to the first market using the first type of transportation mode.

- c_{ijk} is the transport cost (in ten thousand rupees per day) to transport one thousand tonnes of sugar from i -th origin to j -th destination using k -th type of transportation mode; e.g., it will take 60,000 rupees per day to transport one thousand tonnes of sugar from the first sugar factory to the first market using the first type of transportation mode.
- a_i is total availability of sugar at i -th sugar factory (in thousand tonnes).
- b_j is total demand at j -th destination or market (in thousand tonnes).
- e_k is conveyance capacity of k -th transportation mode (in thousand tonnes).

The decision variables are defined as follows:

x_{ijk} is amount of sugar transported from i -th origin to j -th destination using k -th type of transportation mode (in thousand tonnes).

Depending on the nature of different parameters of the problem, we formulate four different models for the example. These models are described here.

Case 1

First, we consider the market mode where the demand and conveyance capacities are fixed but the supplies follow gamma distribution. The pdf of the supplies a_i follow $G(\alpha_i, 15)$, where $\alpha_1 = 3$, $\alpha_2 = 2$ and $\alpha_3 = 4$. Let the probability levels corresponding to the supply constraints be $\gamma_1 = 96\%$, $\gamma_2 = 91\%$ and $\gamma_3 = 90\%$. Demands and conveyance capacities (in thousand tonnes) are $b_1 = 7$, $b_2 = 15$, $b_3 = 12$ and $e_1 = 10$, $e_2 = 14$, $e_3 = 13$, respectively.

Using the given data and with the help of the model (28)–(32), the deterministic model for the problem is established. Also, we observe that

$$\exp\left(\frac{-\sum_{j=1}^3 b_j}{\beta}\right) \left(\sum_{r=0}^{\sum_{i=1}^3 \alpha_i - 1} \left(\frac{\sum_{j=1}^3 b_j}{\beta} \right)^r \frac{1}{r!} \right) = 0.9994207 \geq 0.99.$$

Hence, the initial feasibility condition is satisfied.

The optimal solutions of the problem with individual objective function are

Table 1. Transportation time in days.

t_{ijk}	E_1			E_2			E_3		
	E_1			E_2			E_3		
	E_1			E_2			E_3		
	D_1			D_2			D_3		
O_1	8	2	3	7	3	9	5	6	4
O_2	4	3	8	4	6	2	5	8	4
O_3	9	4	5	8	4	7	2	3	3

$Z_1 = 81.33753$ and $x_{112} = 6.489906$, $x_{113} = 0.5100941$,
 $x_{122} = 4.19275$, $x_{223} = 7.489906$, $x_{322} = 3.317344$,
 $x_{331} = 10$, $x_{333} = 2$.
 $Z_2 = 118.04026$ and $x_{122} = 2.82738$, $x_{123} = 4.682685$,
 $x_{132} = 3.682685$, $x_{222} = 7.489935$, $x_{311} = 7$,
 $x_{333} = 8.317315$.

Hence the *pay-off* matrix corresponding to the problem is

$$\begin{array}{c} Z_1 \quad Z_2 \\ X_1 \begin{pmatrix} 81.33753 & 275.9318 \end{pmatrix} \\ X_2 \begin{pmatrix} 172.46972 & 118.04038 \end{pmatrix} \end{array}$$

From the *payoff* matrix, the lower bound L_p and upper bound U_p for the objective functions Z_p ($p = 1, 2$) are set as $L_1 = 81.33753 \leq Z_1 \leq 172.46972 = U_1$ and $L_2 = 118.04038 \leq Z_2 \leq 275.9318 = U_2$. Using these bounds for the objective functions, we construct the membership function corresponding to each objective. Hence, the fuzzy programming model is

$$\max \quad \lambda,$$

subject to

$$\begin{aligned} & \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 t_{ijk}^p x_{ijk} + (U_1 - L_1)\lambda \leq U_1, \\ & \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 c_{ijk}^p x_{ijk} + (U_2 - L_2)\lambda \leq U_2, \\ & \exp\left(\frac{-\sum_{j=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta_i}\right) \left(\sum_{r=0}^{\alpha_i-1} \left(\frac{\sum_{j=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta_i}\right)^r \frac{1}{r!}\right) \\ & \geq (1 - \gamma_i), \\ & \quad i = 1, 2, 3, \\ & \sum_{i=1}^3 \sum_{k=1}^3 x_{ijk} \geq b_j, \quad j = 1, 2, 3, \\ & \sum_{i=1}^3 \sum_{j=1}^3 x_{ijk} \leq e_k, \quad k = 1, 2, 3, \\ & \lambda, x_{ijk} \geq 0 \quad \forall \quad i, j \text{ and } k. \end{aligned}$$

Using Lingo 11.0, we obtain a compromised solution as $x_{112} = 3.682644$, $x_{122} = 7.510070$, $x_{221} = 5.020898$, $x_{222} = 1.464032$, $x_{223} = 1$, $x_{311} = 1.979102$, $x_{312} = 1.338254$, $x_{333} = 12$. The corresponding objective's values are computed as 119.95821 and 184.9528.

Case 2

In this case, supply and conveyance capacity are deterministic, but all the parameters corresponding to demands follow the gamma distribution. The transportation time (t_{ijk}) and transportation costs (c_{ijk}) are given in tables 1 and 2, respectively. Supplies and conveyance capacities are $a_1 = 11$, $a_2 = 13$, $a_3 = 14$ and $e_1 = 11$, $e_2 = 10$, $e_3 = 17$, respectively. The demands are $b_1 \sim \text{gamma}(4, 1.7)$, $b_2 \sim$

$\text{gamma}(3, 1.7)$ and $b_3 \sim \text{gamma}(2, 1.7)$; probability levels corresponding to the demand constraints are $\eta_1 = 95\%$, $\eta_2 = 93\%$, $\eta_3 = 98\%$, respectively. Using the data given by the tables and with the help of the model (28)–(32), we establish the deterministic model for the problem. The optimal solutions of the problem with individual objective function are

$$\begin{aligned} & Z_1 = 71.38207 \text{ and } x_{112} = 8.782628, x_{113} = 2.217372, \\ & x_{211} = 0.963844, x_{212} = 1.217372, x_{223} = 9.910936, \\ & x_{331} = 9.917667. \\ & Z_2 = 129.05406 \text{ and } x_{111} = 10.917667, x_{113} = 0.082333, \\ & x_{212} = 0.089063, x_{213} = 2.00982, x_{222} = 9.910937, \\ & x_{311} = 0.082333, x_{333} = 9.917667. \end{aligned}$$

Hence the *pay-off* matrix corresponding to the problem is

$$\begin{array}{c} Z_1 \quad Z_2 \\ X_1 \begin{pmatrix} 71.38207 & 290.33538 \end{pmatrix} \\ X_2 \begin{pmatrix} 193.89368 & 129.05406 \end{pmatrix} \end{array}$$

Using the information from the *payoff* matrix, we set the lower bound L_p and upper bound U_p of the objective functions Z_p where $p = 1, 2$ as $L_1 = 71.38207 \leq Z_1 \leq 193.89368 = U_1$ and $L_2 = 129.0541 \leq Z_2 \leq 290.33538 = U_2$. Using these bounds for the objective functions, we construct the membership functions corresponding to each objective. Hence, the fuzzy programming model is

$$\max \quad \lambda,$$

subject to

$$\begin{aligned} & \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 t_{ijk}^p x_{ijk} + (U_1 - L_1)\lambda \leq U_1, \\ & \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 c_{ijk}^p x_{ijk} + (U_2 - L_2)\lambda \leq U_2, \\ & \sum_{j=1}^3 \sum_{k=1}^3 x_{ijk} \leq a_i, \quad i = 1, 2, 3, \\ & \exp\left(\frac{-\sum_{i=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta'_j}\right) \left(\sum_{r=0}^{\alpha'_j-1} \left(\frac{\sum_{i=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta'_j}\right)^r \frac{1}{r!}\right) \\ & \leq (\eta_j), \\ & \quad j = 1, 2, 3, \\ & \sum_{i=1}^3 \sum_{j=1}^3 x_{ijk} \leq e_k, \quad k = 1, 2, 3, \\ & \lambda, x_{ijk} \geq 0 \quad \forall \quad i, j \text{ and } k, \end{aligned}$$

where $\alpha'_1 = 4$, $\alpha'_2 = 3$, $\alpha'_3 = 2$, $\beta'_1 = 1.7$.

The obtained compromise solution is

$$\begin{aligned}x_{111} &= 3.828601, x_{112} = 6.882876, x_{113} = 0.288523, \\x_{211} &= 2.098883, x_{222} = 3.117124, x_{223} = 6.79381, \\x_{311} &= 0.0823331, x_{333} = 9.917667.\end{aligned}$$

The corresponding objective's values are computed as $Z_1 = 116.44002$ and $Z_2 = 188.37094$.

Case 3

In this case, supplies and demands are deterministic but conveyance capacities follow the gamma distribution. The transportation time (t_{ijk}) and transportation cost (c_{ijk}) are given in tables 1 and 2, respectively. Supplies and demands are $a_1 = 11, a_2 = 17, a_3 = 19$ and $b_1 = 15, b_2 = 16, b_3 = 14$, respectively. The conveyance capacities are given by the following gamma random variables: $e_1 \sim \text{gamma}(2, 14)$, $e_2 \sim \text{gamma}(3, 14)$ and $e_3 \sim \text{gamma}(4, 14)$; the significant levels of probability corresponding to the conveyance constraints are $\eta_1 = 96\%$, $\eta_2 = 97\%$, $\eta_3 = 94\%$, respectively.

Using the data given in the tables and with the help of the model (28)–(32), we establish the deterministic model for the problem. The optimal solutions of the problem with individual objective function are

$$\begin{aligned}Z_1 &= 97.85936 \quad \text{and} \quad x_{112} = 10.57032, \\x_{212} &= 1, x_{223} = 16, x_{312} = 3.429678, x_{331} = 14; \\Z_2 &= 145 \quad \text{and} \quad x_{111} = 5.501044, x_{132} = 4.498956, \\x_{222} &= 16, x_{311} = 9.498956, x_{333} = 9.501044.\end{aligned}$$

Hence the *pay-off* matrix corresponding to the problem is

$$\begin{array}{cc} & \begin{matrix} Z_1 & Z_2 \end{matrix} \\ \begin{matrix} X_1 \\ X_2 \end{matrix} & \begin{pmatrix} 97.85936 & 386.57032 \\ 280.99583 & 145 \end{pmatrix} \end{array}$$

Using the information from *pay-off* matrix, we set the lower bound L_p and upper bound U_p of the objective functions Z_p where $p = 1, 2$ as $L_1 = 97.85936 \leq Z_1 \leq 280.99583 = U_1$ and $L_2 = 145 \leq Z_2 \leq 386.57032 = U_2$. Using these bounds for the objective functions, we construct the membership function corresponding to each objective. Hence, the fuzzy programming model is

$$\max \quad \lambda,$$

subject to

$$\begin{aligned}&\sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 t_{ijk}^p x_{ijk} + (U_1 - L_1)\lambda \leq U_1, \\&\sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 c_{ijk}^p x_{ijk} + (U_2 - L_2)\lambda \leq U_2, \\&\sum_{j=1}^3 \sum_{k=1}^3 x_{ijk} \leq a_i, \quad i = 1, 2, 3, \quad \sum_{i=1}^3 \sum_{k=1}^3 x_{ijk} \geq b_j, \quad j = 1, 2, 3, \\&\exp\left(\frac{-\sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta_k''}\right) \left(\sum_{r=0}^{\alpha_k''-1} \left(\frac{\sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta_k''}\right)^r \frac{1}{r!}\right) \\&\geq (1 - \xi_k), \quad k = 1, 2, 3, \quad \lambda, x_{ijk} \geq 0 \quad \forall \quad i, j \text{ and } k,\end{aligned}$$

where $\alpha_1'' = 2, \alpha_2'' = 3, \alpha_3'' = 4, \beta'' = 14$.

Using Lingo 11.0, we solve this model and obtain a compromise solution as

$$\begin{aligned}x_{112} &= 9, x_{211} = 1, x_{221} = 3.464947, \\x_{222} &= 12.535052, x_{311} = 1.04129, x_{312} = 3.95871, \\x_{331} &= 5.522861, x_{333} = 8.477139.\end{aligned}$$

The corresponding objective's values are computed as $Z_1 = 172.753693$ and $Z_2 = 243.791071$.

Case 4

Finally, we consider the case where all the parameters of the problem (except cost coefficient) follow gamma distribution. We use the same data used for the previous cases. Using the data given in the tables and with the help of the model (28)–(32), we establish the deterministic model for the problem. The optimal solutions of the problem with individual objective functions are

$$\begin{aligned}Z_1 &= 77.38333 \text{ and } x_{112} = 8.3641, x_{223} = 7.489906, \\x_{312} &= 4.477751, x_{313} = 0.339365, x_{322} = 1.558039, \\x_{331} &= 9.917667; \\Z_2 &= 108.51142 \text{ and } x_{123} = 1.558039, x_{222} = 7.489906, \\x_{311} &= 13.181216, x_{333} = 9.917667.\end{aligned}$$

Hence the *pay-off* matrix corresponding to the problem is

$$\begin{array}{cc} & \begin{matrix} Z_1 & Z_2 \end{matrix} \\ \begin{matrix} X_1 \\ X_2 \end{matrix} & \begin{pmatrix} 77.38333 & 268.39893 \\ 207.34568 & 108.51142 \end{pmatrix} \end{array}$$

Using the information from *pay-off* matrix, we set the lower bound L_p and upper bound U_p for the objective functions Z_p where $p = 1, 2$ as $L_1 = 77.38333 \leq Z_1 \leq U_1 = 207.34568$ and $L_2 = 108.5114 \leq Z_2 \leq U_2 = 268.39893$. Using these bounds for the objective functions, we construct the membership function corresponding to each objective. Hence, the fuzzy programming model is

$$\max \quad \lambda,$$

subject to

$$\begin{aligned}&\sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 t_{ijk}^p x_{ijk} + (U_1 - L_1)\lambda \leq U_1, \\&\sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 c_{ijk}^p x_{ijk} + (U_2 - L_2)\lambda \leq U_2,\end{aligned}$$

$$\begin{aligned}
& \exp\left(\frac{-\sum_{j=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta_i}\right) \left(\sum_{r=0}^{\alpha_i-1} \left(\frac{\sum_{j=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta_i}\right)^r \frac{1}{r!}\right) \\
& \geq (1 - \gamma_i), \\
& \quad i = 1, 2, 3, \\
& \exp\left(\frac{-\sum_{i=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta'_j}\right) \left(\sum_{r=0}^{\alpha'_j-1} \left(\frac{\sum_{i=1}^3 \sum_{k=1}^3 x_{ijk}}{\beta'_j}\right)^r \frac{1}{r!}\right) \\
& \leq (\eta_j), \\
& \quad j = 1, 2, 3, \\
& \exp\left(\frac{-\sum_{i=1}^3 \sum_{j=1}^3 x_{ijk}}{\beta''_k}\right) \left(\sum_{r=0}^{\alpha''_k-1} \left(\frac{\sum_{i=1}^3 \sum_{j=1}^3 x_{ijk}}{\beta''_k}\right)^r \frac{1}{r!}\right) \\
& \geq (1 - \xi_k), \\
& \quad k = 1, 2, 3, \\
& \lambda, x_{ijk} \geq 0 \quad \forall \quad i, j \text{ and } k,
\end{aligned}$$

where $\alpha_1 = 3, \alpha_2 = 2, \alpha_3 = 4, \beta = 15, \alpha'_1 = 4, \alpha'_2 = 3, \alpha'_3 = 2, \beta' = 1.7, \alpha''_1 = 2, \alpha''_2 = 3, \alpha''_3 = 4, \beta'' = 14$.

Using Lingo 11.0, we solve this model and obtain a compromise solution as

$$\begin{aligned}
x_{122} &= 0.866256, x_{222} = 3.183096, x_{223} = 4.30681, \\
x_{312} &= 13.181216, x_{322} = 0.691783, x_{333} = 9.917667.
\end{aligned}$$

The corresponding objective's values are computed as $Z_1 = 115.55596$ and $Z_2 = 155.47369$.

5.1 Result analysis

In the previous subsection, the mathematical model of the example for different cases is formulated and our methodology to establish the deterministic model for each case is used. The equivalent deterministic forms of the chance constraints are non-linear constraints. Hence, the deterministic form of the multi-objective STP with gamma random variables becomes multi-objective non-linear programming problem. Fuzzy programming approach is used to tackle the multiple conflicting objectives. All the models are solved on a personal computer with a 1.9 GHz CPU and a 4.00 GB memory space. The optimization engine used to solve those non-linear programming models is Lingo 11.0.

For the first model, we have obtained the optimal values for the objectives as $z_1 = 119.95821$ and $z_2 = 184.95281$, i.e. it takes approximately 120 days to transport 34 thousand tonnes of sugar from three different sugar factories to three different markets. The minimum transportation cost is Rs. 1,849,528.1. Achievement rates of objectives in fuzzy programming approach are 0.57621 and 0.57623 for first and second objective, respectively. Also, the multi-objective problem is solved using the weighted sum method, ϵ -constraint method and Topsis method [33]. For all the four cases, the Pareto front obtained by those methods and the

compromised solution obtained from fuzzy programming approach and Topsis method are shown in figures 1–4. These plots indicate that the Pareto optimal fronts obtained

Figure 1. Comparison of solutions obtained using different methods when only availabilities are gamma random variables.

Figure 2. Comparison of solutions obtained using different methods when only demands are gamma random variables.

Figure 3. Comparison of solutions obtained using different methods when only conveyance capacities are gamma random variables.

Figure 4. Comparison of solutions obtained using different methods when all the parameters are gamma random variables.

by different methods are the same and the obtained compromise solutions are also Pareto optimal solutions.

For the first case, the optimal value for supply parameters are $a_1 = 11.19275$, $a_2 = 7.48991$, $a_3 = 15.31794$. We use ϵ -constraint method to check the sensitivity of the solution for all the cases. From the solution, it is observed that if we decrease the transportation time by one day then it will cost an additional Rs. 15,057.1.

From the solution of second model, it is observed that it will take approximately 117 days to transport 33.009817 thousand tonnes of sugar, which is the total demand for the case. In this case, demands at destination points are $b_1 = 13.181216$, $b_2 = 9.910934$ and $b_3 = 9.917667$. The transportation cost is Rs. 1,883,709.4. Achievement rates of objectives in fuzzy programming approach are 0.632213 and 0.632215 for the first and second objective, respectively.

In the third case, the compromised solution obtained using fuzzy programming approach shows that the minimum time taken to transport 45 thousand tonnes of sugar is approximately 173 days and the minimum transportation cost is Rs. 2,437,910.7. The achievement rate is 0.591046 for both objectives. Also, the amount of sugar transported using the first type of conveyance is 11.029098 thousand tonnes. Using second and third types of conveyances, 25.493763 and 8.477139 thousand tonnes of sugar are transported, respectively.

Finally, in the last case, the fuzzy compromised solution shows that it takes approximately 116 number of days and Rs. 1,554,736.9 to transport 32.146828 thousand tonnes of sugar. The achievement rates of the objectives are the same and it is 0.70628. The amounts of sugar transported from factories are 0.866256, 7.489906 and 23.790666 thousand tonnes. Demands from the markets are 13.181216, 9.047945 and 9.917667 thousand tonnes. The solution indicates that the first type of conveyance is not used for the transportation and the amount of sugar transported using second and third types of conveyances are 17.922351 and 14.224477 thousand tonnes, respectively.

6. Conclusion

In this paper, the solution procedure for multi-objective stochastic STP is presented where the parameters like supplies, demands and conveyance capacities are considered as gamma or Erlang distribution. Chance-constraint programming technique is used to establish the equivalent deterministic model of the problem. In this problem, the deterministic model becomes nonlinear, whereas the deterministic model remains linear for distributions like uniform or exponential or normal. Since random variables are present in the problem, it is not possible to apply the initial feasible condition. For the considered problem the feasibility conditions are obtained using chance-constraint technique. Due to conflicting nature of the objective functions, fuzzy programming technique has been applied for finding compromised optimal solution. It is clear from the figure where the solution obtained by different methods is presented that the transportation cost increases rapidly as the transportation time decreases. Here, less transportation time will increase the goodwill of the customer. The problem can be extended to a model where the coefficients of the objective related to time follow gamma distribution. Also, it will be interesting to study the stochastic STP where several DMs with different objectives are present in a hierarchy.

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