A comprehensive study of an economic order quantity model under fuzzy monsoon demand

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Abstract. This article deals with a classical economic order quantity (EOQ) model under monsoon type fuzzy demand rate. It is nothing but the generalization of cloudy fuzzy model. We split the EOQ model into three parts according to the real-time fuzzy components of the demand rate. To defuzzify the model we develop an algorithm and the solution is obtained with the help of a nonlinear optimization technique that requires maximum aspiration level of the fuzzy membership of the objective function. Moreover, for comparative study we take numerical results of the crisp, general fuzzy and cloudy fuzzy model also. By this study we have shown that the decision maker might have to choose the monsoon type fuzzy environment all the time to control the proposed inventory. Moreover, sensitivity analysis and graphical illustrations are made to justify the new fuzzy approach.

Keywords. Inventory; fuzzy cloud; fuzzy monsoon; new solution algorithm; optimization.

1. Introduction

From the development of classical economic order quantity (EOQ) model, a lot of research works have been made on the inventory control system. Harris [1] studied the foundation in management problems in the field of Operations Research (OR). Nevertheless, owing to today's turbulent business environment, it is a common occurrence in the inventory planning that much relevant input data are either not known precisely, or cannot be found due to the lack of sufficient information [2]. For instance, due to various uncertainties of the market, the demand rate usually fluctuates from one cycle time to another. Moreover, for developing inventory models, a major difficulty faced by a decision maker (DM) is to forecast the annual demand. Through its process the characteristic of the EOQ model with backorders is studied by Yao and Su [3], Wu and Yao [4] and Björk [5], the model for imperfect quality is developed by Chang [6], Mahata and Goswami [7] and Sharifi *et al* [8], and that for deteriorating items is investigated by De *et al* [9, 10] and Mahata and Goswami [11].

In fuzzy decision-making problems, Bellman and Zadeh [12] first used the fuzzy programming technique to get the final decision. After this, numerous research techniques have been developed [13–16]. Authors like Chen and Chang [17] and Shekarian *et al* [18] discussed problems on defective products, and that for deteriorating items was

discussed by Pal *et al* [19]. Báez-Sánchez *et al* [20] contributed genuine knowledge on polygonal fuzzy sets. Wang and Tang [21] investigated an economic production quantity (EPQ) model characterizing the set-up cost, the holding cost and the elapsed time as fuzzy variables. Zhang *et al* [22] assumed that the inventory set-up cost, holding cost and the other parameters are fuzzy variables. Hu *et al* [23] studied a model of fuzzy random variable where the elapsed time when the machine shifts from one state to another has been considered. Chakraborty *et al* [24] developed a deteriorating multi-item inventory model with price discount and variable demands via fuzzy logic under resource constraints. De [25] developed an EOQ model with natural idle time and wrongly measured demand rate. De and Sana [26] studied fuzzy order quantity inventory model with fuzzy shortage quantity and fuzzy promotional index. Kumar *et al* [27] developed a fuzzy model for ramp type demand and partial backlogging. De and Sana [28] developed a backlogging EOQ model considering a pentagonal fuzzy number and they used the centre of gravity method to defuzzify the model followed by phi coefficient test. Das *et al* [29] presented an integrated production inventory model in interactive fuzzy credit period for deteriorating items. Kumar and Goswami [30] proposed a fuzzy random EPQ model for imperfect quality items with possibility and necessity constraints. Recently, Mahata [31] investigated the learning effect on inventory modelling. Currently, Shekarian *et al* [32] wrote a comprehensive review of fuzzy inventory models.

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Alternatively, De and Beg [33, 34] introduced dense fuzzy approach to capture the degree of learning experiences in recent times. De and Mahata [35] extended the dense fuzzy set to cloudy fuzzy set and gave a new formula for defuzzification. Several attempts on the application of dense fuzzy rule were studied by De [36], Karmakar *et al* [37, 38] and Maity *et al* [39] extensively.

However, in the literature, several defuzzification methods have been found [40–44]. Cheng [45] discussed a new approach for ranking fuzzy numbers by distance method. Buckley and Chanas [46] discussed a fast method of ranking alternatives using fuzzy numbers. Ezatti and Saneifard [47] developed a method of continuous weighted quasi-arithmetic means. The extensive works on L–R fuzzy numbers analysed by Wang *et al* [48], Kumar *et al* [49] and Hajjari and Abbasbandy [50] left a mark in the subject based on deviation degree. Recently, Kumar [51] developed a defuzzification method of type-2 fuzzy variable pertaining to interval approximation. However, after the invention of cloudy fuzzy number [35] there are no such defuzzification methods available in the literature yet.

In this article, we consider a classical EOQ model where the demand rate follows monsoon type fuzzy uncertainty. A solution algorithm has been developed for its numerical illustration. Comparing the numerical result with that of the other related existing model we have shown the actual justification for the new approach. Sensitivity analysis and graphical illustration are made for its global acceptance.

2. Preliminaries

Here we shall discuss a few essential definitions and formulas, which have been used frequently for solving the proposed model.

2.1 Normalized general triangular fuzzy number (NGTFN)

Let $\tilde{D}$ be an NGTFN (shown in figure 1) having the form $\tilde{D} = (D_1, D_2, D_3)$. Then its membership function is defined by

$$\mu(\tilde{D}) = \begin{cases} 0 & \text{if } D < D_1 \text{ and } D > D_3, \\ \frac{D - D_1}{D_2 - D_1} & \text{if } D_1 \leq D \leq D_2, \\ \frac{D_3 - D}{D_3 - D_2} & \text{if } D_2 \leq D \leq D_3. \end{cases} \quad (1)$$

Now, the left and right α -cuts of $\mu(\tilde{D})$ are given by

$$L(\alpha) = D_1 + \alpha(D_2 - D_1) \quad \text{and} \\ R(\alpha) = D_3 - \alpha(D_3 - D_2).$$

Utilizing Yager's [52] index method, the index value of $\tilde{D}$ is given by

$$I(\tilde{D}) = \frac{D_1 + 2D_2 + D_3}{4}. \quad (2)$$

2.2 Cloudy normalized triangular fuzzy number [33, 35]

Definition 1 Let $\tilde{A}$ be the fuzzy set whose components are the elements of $\mathcal{R} \times \mathcal{R}$, $\mathcal{R}$ being the set of real numbers with the membership grade satisfying the functional relation $\mu : \mathcal{R} \times \mathcal{R} \rightarrow [0, 1]$. In particular, let one of the set $\mathcal{R}$ be taken from the time domain T . Thus we redefine the membership value μ such that $\mu : \mathcal{R} \times T \rightarrow [0, 1]$. Now, as $t \rightarrow \infty$, if $\mu(x, t) \rightarrow 1$ for some $x \in \mathcal{R}$ and $t \in T$ then we call the set $\tilde{A}$ as a cloudy fuzzy number. If we consider the fuzzy number $\tilde{A}$ of the form $\tilde{A} = a_1, a_2, a_3$ then we call it “cloudy triangular fuzzy number”. Now, if for all t in T , $\mu(x, t)$ attains the highest membership degree 1 then we can express this fuzzy number as “cloudy normalized triangular fuzzy number” or CNTFN. A graphical representation of CNTFN is shown in figure 2.

2.3 Distinction between cloudy fuzzy model and monsoon type fuzzy model

Let us assume a triangular cloudy fuzzy number of the form

$$\tilde{A} = \left\langle a \left(1 - \frac{\rho}{1+t} \right), \quad a, \quad a \left(1 + \frac{\sigma}{1+t} \right) \right\rangle, \quad \text{for} \\ 0 < \rho, \quad \sigma < 1$$

where t denotes the time variable and a is a fixed number; ρ and σ are the left and right deviations (or fuzzy system parameters), respectively, of the fuzzy number $\tilde{A}$. If we take the time variable t as the cycle time of the process, then the model associated with this process is called cloudy fuzzy model. If we consider the time variable t as an ordinary time variable throughout any process, then the model associated with this process is called monsoon type fuzzy model. Basically, in cloudy fuzzy set the time variable t is the ultimate goal (represents optimum cycle time) but it is not true for monsoon type fuzzy set. In cloud model, the fuzzification is done after the formation of crisp objective function; however, in monsoon model, fuzzy classification is done during the formulation of the fuzzy objective function. However, the defuzzification methods (they are based on a single objective for cloud model but multiple objectives for monsoon model) are different for the two fuzzy optimization problems.

Note that, by virtue, the cloud index of monsoon type fuzzy set is always higher than that of cloudy fuzzy set. Let T_1 and T_2 be the optimum cycle time in monsoon model and cloudy fuzzy model, respectively; then the cloud indexes (degrees of uncertainty) are

Figure 1. Membership function of NGTFN.

Figure 2. Membership function of CNTFN.

$$\begin{cases} \delta_{monsoon} = \frac{\log(1+T_1)}{T_1}, \\ \delta_{cloud} = \frac{\log(1+T_2)}{T_2}. \end{cases} \quad (3)$$

Now, by definition, $\delta_{monsoon} > \delta_{cloud}$. Alternatively, since $T_1 < T_2$, $\frac{\log(1+T_1)}{T_1} > \frac{\log(1+T_2)}{T_2}$ implies $\delta_{monsoon} > \delta_{cloud}$. In addition, we observe that the monsoon will converge to cloud if and only if $\delta_{monsoon} \rightarrow \delta_{cloud}$. In other words

$$\delta_{monsoon} - \delta_{cloud} = \frac{\log(1+T_1)}{T_1} - \frac{\log(1+T_2)}{T_2} = \varepsilon \rightarrow 0.$$

Also, we see that $\lim_{t \rightarrow \infty} a \left(1 - \frac{\rho}{1+t}\right) \rightarrow a$ and $\lim_{t \rightarrow \infty} a \left(1 + \frac{\sigma}{1+t}\right) \rightarrow a$, hence $\tilde{A} \rightarrow \{a\}$.

2.4 Bundle type/tunnel type curve

Definition 2 Let a_i be the fuzzy component associated with the i -tuple fuzzy number $\tilde{A}$. Then at $a_i, i \in N$ if the objective functions z_i follow independent (non-intersecting but can take curvature all the time) curves $z_1, z_2, z_3, \dots, z_n$ then we call such curves as bundle type curves. Moreover, if any one of them coincides with another or we are interested to study the situation between two independent curves then the curves along with their geometrical shape are known as tunnel type curves. Consider the case of triangular fuzzy number $\tilde{A} = a_1, a_2, a_3$; then the following situations may arise. Figure 3a–c indicates the converging, diverging and natural tunnel type curves of the fuzzy objective functions with respect to the fuzzy set $\tilde{A}$.

2.5 The conventional left fuzzy (L-fuzzy) number

Let a fuzzy number $\tilde{b}$ have U and L as upper and lower bounds. Then we call this fuzzy number as the left fuzzy number if its membership function can be expressed in the following way (shown in figure 4):

$$\mu_{\tilde{b}}(x) = \begin{cases} 0 & \text{if } x > U, \\ \frac{U-x}{U-L} & \text{if } L \leq x \leq U, \\ 1 & \text{if } x \leq L. \end{cases} \quad (4)$$

3. Assumptions and notations

The following notations and assumptions are used to develop the model.

Assumptions

1. Replenishments are instantaneous.
2. The time horizon is infinite (weeks).
3. Shortages are not allowed.

Notations

- q the order quantity per cycle (a decision variable)
- d demand rate per week
- k set-up cost per cycle (\$)
- h inventory holding cost per unit quantity per cycle (\$)
- c purchasing price of unit item (\$)
- t cycle time (weeks) (a decision variable)
- z average total cost of the inventory (\$) (a decision variable)

3.1 Crisp mathematical model

Let the inventory start with order quantity q and be depleted at the constant demand rate D . Then after the cycle time T it reaches zero. The costs associated with it are unit purchasing

(a) Fuzzy element

(b) Fuzzy element

(c) Fuzzy element

Figure 3. (a) Converging tunnel over time t . (b) Diverging tunnel over time t . (c) Natural tunnel over time t .

price c , unit holding cost h and set-up cost k only. Therefore, the inventory problem for average total cost is given as

$$\begin{cases} \text{minimize } z = cD + \frac{k}{T} + \frac{hDT}{2} \\ \text{subject to } q = DT. \end{cases} \quad (5)$$

3.2 Formulation of fuzzy monsoon mathematical model

Let us assume that the demand rate of the proposed inventory model varies with the real-time fuzzy number

Figure 4. L-fuzzy membership function.

(fuzzy monsoon). We consider the fuzzy monsoon over the demand rate as

$$\tilde{D} = d_1, d_2, d_3 = \left\langle d \left(1 - \frac{\rho}{1+t} \right), d, d \left(1 + \frac{\sigma}{1+t} \right) \right\rangle, \quad \text{for} \quad 0 < \rho, \quad \sigma < 1 \quad (6)$$

where ρ and σ are left and right deviations, respectively, of the fuzzy set $\tilde{D}$. The left component of $\tilde{D}$ will generate the left cloud model, the middle component will create the crisp model (or central cloud model) and the right component will generate the right cloud model.

3.2a Left cloud model: We develop the objective function at the left fuzzy element (increasing function of t) of (6). Let $I_1(t)$ be the inventory level at time t ; then the governing differential equation of the problem is given by

$$\begin{cases} \frac{dI_1(t)}{dt} = -d \left(1 - \frac{\rho}{1+t} \right), & 0 \leq t \leq T_1 \\ \text{subject to } I_1(0) = q & \text{and } I_1(T_1) = 0. \end{cases} \quad (7)$$

Solving (7), we get

$$I_1(t) = d(T_1 - t) - d\rho \log \left(\frac{1+T_1}{1+t} \right), \quad 0 \leq t \leq T_1. \quad (8)$$

$$\text{Order quantity } q = I_1(0) = dT_1 - d\rho \log(1+T_1). \quad (9)$$

$$\text{Purchasing cost} = cq = cd[T_1 - \rho \log(1+T_1)]. \quad (10)$$

$$\begin{aligned} \text{Holding cost} &= h \int_0^{T_1} I_1(t) dt \\ &= hd \left[\frac{T_1^2}{2} + \rho \{ \log(1+T_1) - T_1 \} \right]. \end{aligned} \quad (11)$$

Thus, total average inventory cost is given by

$$\begin{aligned}
z_1(T_1) &= \frac{\text{purchasing cost} + \text{holding cost} + \text{set-up cost}}{\text{cycle time}} \\
&= \frac{cd}{T_1} [T_1 - \rho \log(1 + T_1)] + \frac{k}{T_1} + \frac{hd}{T_1} \\
&\quad \left[\frac{T_1^2}{2} + \rho \{ \log(1 + T_1) - T_1 \} \right] \\
&= cd + \frac{hdT_1}{2} + \frac{k}{T_1} - \rho d \left[h + (c - h) \frac{\log(1 + T_1)}{T_1} \right].
\end{aligned} \tag{12}$$

Therefore, our problem is given as follows:

$$\begin{cases} \text{minimize } z_1(T_1) = cd + \frac{hdT_1}{2} + \frac{k}{T_1} - \rho d \left[h + (c - h) \frac{\log(1 + T_1)}{T_1} \right] \\ \text{subject to } q = dT_1 - d\rho \log(1 + T_1). \end{cases} \tag{13}$$

3.2b Right cloud model: We develop the objective function at the right fuzzy element (decreasing function of t) stated in (6). Let $I_3(t)$ be the inventory level at time t ; then the governing differential equation of the problem is

$$\begin{cases} \frac{dI_3(t)}{dt} = -d \left(1 + \frac{\sigma}{1+t} \right), & 0 \leq t \leq T_3 \\ \text{subject to } I_3(0) = q & \text{and } I_3(T_3) = 0. \end{cases} \tag{14}$$

Solving (14) and proceeding as earlier, we have

Now, for better understanding, the crisp problem can be redefined as follows.

$$\begin{cases} \text{minimize } z_3(T_3) = cd + \frac{hdT_3}{2} + \frac{k}{T_3} + \sigma d \left[h + (c - h) \frac{\log(1 + T_3)}{T_3} \right] \\ \text{subject to } q = d[T_3 + \sigma \log(1 + T_3)]. \end{cases} \tag{15}$$

3.2c Central cloud model: Here, we shall redefine the crisp problem stated as follows:

$$\begin{cases} \text{minimize } z_2 = cd + \frac{k}{T_2} + \frac{hdT_2}{2} \\ \text{subject to } q = dT_2. \end{cases} \tag{16}$$

Thus, the common problem of the fuzzy monsoon model is to find the decision variables T and q such that they would be able to minimize the objective functions stated in (13), (15) and (16) at a time.

3.3 Solution procedure of the proposed model

Here, we shall develop a solution algorithm to find a common solution of the problems stated in (13), (15) and (16).

3.3a. Solution algorithm:

Step 1. Solve (13), (15) and (16) separately, get individual minimum of each objective and find their corresponding optimum decision variables.

Step 2. Find the minimum and maximum values among all the optimum objectives obtained in Step1. Find the minimum and maximum values of the decision variables also.

Step 3. Take a new objective from the average of all the objectives. To construct the appropriate membership functions of that objective function, order quantity and cycle time, assume $z^{Min} = \text{Min}\{ \text{Min}(z_1), \text{Min}(z_2),$

$\text{Min}(z_3) \}$ and $z^{Max} = \text{Max}\{ \text{Min}(z_1), \text{Min}(z_2), \text{Min}(z_3) \}$; $q^{Min} = \text{Min}\{ q_1^*, q_2^*, q_3^* \}$, $q^{Max} = \text{Max}\{ q_1^*, q_2^*, q_3^* \}$; $T^{Min} = \text{Min}\{ T_1^*, T_2^*, T_3^* \}$, $T^{Max} = \text{Max}\{ T_1^*, T_2^*, T_3^* \}$, respectively, as their corresponding upper and lower bounds.

Step 4. Construct the fuzzy membership functions of the objective function, order quantity and the cycle time as follows:

$$\mu(z_m) = \begin{cases} 0 & \text{if } z_m > z^{Max} \\ \frac{z^{Max} - z_m}{z^{Max} - z^{Min}} & \text{if } z^{Min} \leq z_m \leq z^{Max} \\ 1 & \text{if } z_m \leq z^{Min} \end{cases} \tag{17}$$

and

$$\begin{cases} \gamma(T) = \begin{cases} 0 & \text{if } T > T^{Max} \\ \frac{T^{Min} - T}{T^{Max} - T^{Min}} & \text{if } T^{Min} \leq T \leq T^{Max} \\ 1 & \text{if } T \leq T^{Min} \end{cases} \\ \vartheta(q) = \begin{cases} 0 & \text{if } q > q^{Max} \\ \frac{q^{Min} - q}{q^{Max} - q^{Min}} & \text{if } q^{Min} \leq q \leq q^{Max} \\ 1 & \text{if } q \leq q^{Min} \end{cases} \end{cases} \tag{18}$$

Step 5. Using (17) and (18), formulate a nonlinear programming problem taking the aspiration level α with appropriate constraints. Maximize the aspiration level and get the result. In this proposed model our aim is to optimize the objective function $z_m = \frac{z_1 + z_2 + z_3}{3}$ where z_1, z_2 and z_3 are obtained from (13), (15) and (16), respectively. Using (13), (15) and (16)–(18), the nonlinear programming problem becomes

$$\begin{cases}
\text{maximize } \alpha \\
\text{subject to } z_m < z^{\text{Min}} + (1 - \alpha)(z^{\text{Max}} - z^{\text{Min}}) \\
T < T^{\text{Min}} + (1 - \alpha)(T^{\text{Max}} - T^{\text{Min}}) \\
q < q^{\text{Min}} + (1 - \alpha)(q^{\text{Max}} - q^{\text{Min}}) \\
\text{where } z_m = \frac{z_1(T) + z_2(T) + z_3(T)}{3} \\
z_1(T) = z_2(T) - \rho d \left[h + (c - h) \frac{\log(1 + T)}{T} \right] \\
z_2(T) = cd + \frac{k}{T} + \frac{hdT}{2} \\
z_3(T) = z_2(T) + \sigma d \left[h + (c - h) \frac{\log(1 + T)}{T} \right].
\end{cases} \quad (19)$$

Thus, the membership functions of the cloudy fuzzy objective and its corresponding order quantity are given as

3.4 Cloudy fuzzy model [33, 35]

We rewrite the crisp problem (5) as $D = \frac{z - \frac{k}{T}}{c + \frac{hT}{2}} = \frac{q}{T}$. Now, the membership function of the demand rate is given as

$$\mu(D, T) = \begin{cases} 0 & \text{if } D < d \left(1 - \frac{\rho}{1 + T} \right) \text{ and } D > d \left(1 + \frac{\sigma}{1 + T} \right) \\ \left\{ \frac{D - d \left(1 - \frac{\rho}{1 + T} \right)}{\frac{\rho d}{1 + T}} \right\} & \text{if } d \left(1 - \frac{\rho}{1 + T} \right) \leq D \leq d \\ \left\{ \frac{d \left(1 + \frac{\sigma}{1 + T} \right) - D}{\frac{\sigma d}{1 + T}} \right\} & \text{if } d \leq D \leq d \left(1 + \frac{\sigma}{1 + T} \right). \end{cases} \quad (20)$$

and

$$\pi(z, T) = \begin{cases} 0 & \text{if } z < \frac{k}{T} + d \left(1 - \frac{\rho}{1 + T} \right) \left(c + \frac{hT}{2} \right) \text{ and } z > \frac{k}{T} + d \left(1 + \frac{\sigma}{1 + T} \right) \left(c + \frac{hT}{2} \right) \\ \left\{ \frac{z - \frac{k}{T} - d \left(1 - \frac{\rho}{1 + T} \right) \left(c + \frac{hT}{2} \right)}{\frac{\rho d}{1 + T} \left(c + \frac{hT}{2} \right)} \right\} & \text{if } \frac{k}{T} + \left(c + \frac{hT}{2} \right) d \left(1 - \frac{\rho}{1 + T} \right) \leq z \leq \left(c + \frac{hT}{2} \right) d + \frac{k}{T} \\ \left\{ \frac{\frac{k}{T} + d \left(c + \frac{hT}{2} \right) \left(1 + \frac{\sigma}{1 + T} \right) - z}{\frac{\sigma d}{1 + T} \left(c + \frac{hT}{2} \right)} \right\} & \text{if } \frac{k}{T} + d \left(c + \frac{hT}{2} \right) \leq z \leq \frac{k}{T} + d \left(1 + \frac{\sigma}{1 + T} \right) \left(c + \frac{hT}{2} \right) \end{cases} \quad (21)$$

$$w(q, T) = \begin{cases} 0 & \text{if } q \left\langle dT \left(1 - \frac{\rho}{1+T} \right) \text{ and } q \right\rangle dT \left(1 + \frac{\sigma}{1+T} \right) \\ \left\{ \frac{q - dT \left(1 - \frac{\rho}{1+T} \right)}{\frac{\rho dT}{1+T}} \right\} & \text{if } dT \left(1 - \frac{\rho}{1+T} \right) \leq q \leq dT \\ \left\{ \frac{dT \left(1 + \frac{\sigma}{1+T} \right) - q}{\frac{\sigma dT}{1+T}} \right\} & \text{if } dT \leq q \leq dT \left(1 + \frac{\sigma}{1+T} \right) \end{cases} \quad (22)$$

respectively, where $0 < \rho, \sigma < 1$ and T is the inventory cycle time. Now, the α cuts of the membership function of the cloudy fuzzy objective and that of order quantity obtained from (21)–(22) are given by $\left[\frac{k}{T} + \left(c + \frac{hT}{2} \right) \left\{ d \left(1 - \frac{\rho}{1+T} \right) + \frac{\rho d\alpha}{1+T} \right\}, \frac{k}{T} + \left(c + \frac{hT}{2} \right) \left\{ d \left(1 + \frac{\sigma}{1+T} \right) - \frac{\sigma d\alpha}{1+T} \right\} \right]$ and $\left[dT \left(1 - \frac{\rho}{1+T} \right) + \frac{\rho dT\alpha}{1+T}, dT \left(1 + \frac{\sigma}{1+T} \right) - \frac{\sigma dT\alpha}{1+T} \right]$, respectively; the index value of cloudy fuzzy objective is written as

$$\begin{aligned} I(z) &= \frac{1}{2\tau} \int_{\epsilon}^{\tau} \int_0^1 \left[\frac{k}{T} + \left(c + \frac{hT}{2} \right) \left\{ d \left(1 - \frac{\rho}{1+T} \right) + \frac{\rho d\alpha}{1+T} \right\} + \frac{k}{T} \right. \\ &\quad \left. + \left(c + \frac{hT}{2} \right) \left\{ d \left(1 + \frac{\sigma}{1+T} \right) - \frac{\sigma d\alpha}{1+T} \right\} \right] d\alpha dT \\ &= \frac{1}{2\tau} \int_{\epsilon}^{\tau} \left[\frac{2k}{T} + \left(c + \frac{hT}{2} \right) \left\{ d \left(2 - \frac{\rho - \sigma}{1+T} \right) + \frac{(\rho - \sigma)d}{2(1+T)} \right\} \right] dT \\ &= \frac{k \log(\tau/\epsilon)}{\tau} + cd + \frac{hd\tau}{4} - \frac{3cd(\rho - \sigma) \log(1 + \tau)}{4\tau} - \frac{hd(\rho - \sigma)}{8} \\ &\quad \left\{ 1 - \frac{\log(1 + \tau)}{\tau} \right\} \end{aligned} \quad (23)$$

and that for order quantity as

$$\begin{aligned} I(q) &= \frac{1}{2\tau} \int_{\epsilon}^{\tau} \int_0^1 \left[dT \left(1 - \frac{\rho}{1+T} \right) + \frac{\rho dT\alpha}{1+T} \right. \\ &\quad \left. + dT \left(1 + \frac{\sigma}{1+T} \right) - \frac{\sigma dT\alpha}{1+T} \right] d\alpha dT \\ &= \frac{d\tau}{2} - \frac{d(\rho - \sigma)}{4} \left\{ 1 - \frac{\log(1 + \tau)}{\tau} \right\}. \end{aligned} \quad (24)$$

3.5 Implication of cloudy fuzzy and fuzzy monsoon in inventory process

In this uncertain world, the concepts of various parameters involved in managerial decisions are changing in a rapid way. A few of them are fixed over time and the others are time

dependent. We know that in any inventory process, for a particular region, the customers' needs and choices are almost the same and predictable. In fact, at the beginning of a newly opened shop the number of customers is totally unknown. As time progresses, customers' motives and needs become clear and visible over the managerial framework. On the basis of this, DM can decide whether the system requires a strategic change or not. This way, the DM would have observed how much customers accept and adopt this process. In cloudy fuzzy environment the DM gains experiences from one cycle to another. However, in fuzzy monsoon approach the DM gains experiences continuously from the very beginning of the opening of the inventory process and has the options to change his/her strategy (through some path of thinking) any time without compromising on the normal cycle time of the process itself. The learning process over cycle time is shown in figure 5.

3.6 Conceptual convergence of physical monsoon and fuzzy monsoon

We know that, at the monsoon season, the sky is always covered by clouds and the clouds accumulate in more and more denser and deeper manner over time. Whenever the clouds get dense, the roads/paths of that particular place

Figure 5. Learning process of cloud and monsoon type fuzzy model.

become darkish and consequently the people begin to feel uneasy to walk across the road. However, as soon as the clouds get thinner, the path becomes visible and therefore people begin to walk freely. Similarly, in fuzzy monsoon, the solutions of the objective function are made by the DM with higher uncertainties than those of cloudy fuzzy model. A practical example is shown in appendix section to justify the proposed approach. The graphical illustrations are stated in figure 6a–c.

4. Numerical illustrations

Here we consider numerical solutions of all the objective functions with constraints stated in (13), (15) and (16) that are associated to monsoon type fuzzy model.

4.1 Numerical example 1

Let us consider $c = 10$, $h = 5$, $k = 200$, $d = 10$, $\sigma = 0.2$, $\rho = 0.8$ in Eqs. (13), (15) and (16). Then, we get the solution presented in table 1.

Figure 6. (a) Facts on physical monsoon. (b) Facts on fuzzy monsoon. (c) Facts on cloudy fuzzy environment.

Table 1. Optimal solution of the classical EOQ model.

Decision variables	Optimize z_1	Optimize z_2	Optimize z_3
T_1^*	2.662		
T_2^*		2.828	
T_3^*			2.871
q_1^*	16.236		
q_2^*		28.084	
q_3^*			31.421
z_1^*	182.18		
z_2^*		241.42	
z_3^*			256.15

Table 2. Optimal solution for the EOQ model.

Model	Optimal cycle time T^* (weeks)	Order quantity q^* (units)	Average inventory cost z^* (\$)	Cloud index
Crisp	2.828	28.084	241.42	–
General fuzzy	2.794	28.635	245.68	–
Cloudy fuzzy	7.689	36.649	238.95	0.281
Fuzzy monsoon	2.780	17.170	226.66	0.477

Table 3. Solution of cloudy fuzzy and fuzzy monsoon EOQ model.

Cycle time T (weeks)	Cloudy fuzzy		Crisp model		Fuzzy monsoon	
	q^*	z^*	q^*	z^*	q^*	z^*
1	4.233	537.20	10.00	325.00	4.455	308.07
2	8.873	372.64	20.00	250.00	11.211	234.51
3	13.655	305.43	30.00	241.67	18.909	227.05
4	18.506	271.55	40.00	250.00	27.125	235.98
5	23.396	253.33	50.00	265.00	35.670	251.42
6	28.311	243.81	60.00	283.33	44.433	270.09
7	33.242	239.66	70.00	303.57	53.364	290.60
8	38.187	239.08	80.00	325.00	62.422	312.25
9	43.140	241.00	90.00	347.22	71.579	334.66
10	48.099	244.75	100.00	370.00	80.817	357.60

In table 1, we see that the bounds of the desired objective function are (182.18, 256.15), bounds of the optimum cycle time are (2.662, 2.871) and those of the order quantity are (16.236, 31.421). Therefore, utilizing these results in (19) we can construct the problem of fuzzy monsoon model. However, we obtain the solution of crisp problem (16), solution of general fuzzy problem of (16) [utilizing (2) and taking fuzzy demand rate as $\tilde{D} = \langle 6, 10, 15 \rangle$] and that for cloudy fuzzy problem (23)–(24) taking $\epsilon = 0.09$ and the outputs are shown in table 2 also. Table 3 shows 10 weeks comprehensive studies of the average inventory cost and their respective decision variable in cloudy fuzzy model, crisp model and monsoon type fuzzy model.

Table 4. Sensitivity analysis of the fuzzy monsoon model.

Parameters	Change (%)	Cycle time T^* (weeks)	Order quantity q^*	Average total cost z^*	$\frac{(z^* - z_*)}{z_*} \times 100\%$	Aspiration level α^*	Cloud index
c	+50	2.744	30.082	271.86	12.61	0.717	0.481
	+30	2.761	29.852	253.79	5.12	0.774	0.480
	-30	2.811	17.475	199.52	-17.36	0.945	0.476
	-50	2.584	15.629	182.00	-24.61	1.00	0.494
h	+50	2.295	25.237	255.61	5.88	0.768	0.520
	+30	2.458	27.057	244.72	1.37	0.803	0.505
	-30	3.304	35.956	205.61	-14.83	0.926	0.442
	-50	3.880	38.419	188.92	-21.75	0.978	0.409
d	+50	2.267	37.550	300.40	24.44	0.628	0.522
	+30	2.438	34.580	271.68	12.53	0.718	0.507
	-30	4.117	31.102	180.85	-25.09	1.00	0.397
	-50	4.175	14.298	143.12	-40.72	1.00	0.394
k	+50	3.423	37.172	258.87	7.23	0.758	0.434
	+30	3.183	34.653	246.763	2.21	0.796	0.449
	-30	2.324	13.669	203.17	-15.84	0.933	0.517
	-50	1.957	10.899	184.48	-23.44	0.992	0.554
ρ	+50	2.758	30.228	216.80	-10.2	0.890	0.480
	+30	2.769	30.346	220.74	-8.56	0.878	0.479
	-30	2.803	30.701	232.57	-3.67	0.841	0.477
	-50	2.814	22.848	236.50	-2.03	0.829	0.476
σ	+50	2.793	17.265	229.12	-5.09	0.852	0.477
	+30	2.790	17.582	228.14	-5.50	0.855	0.478
	-30	2.782	29.602	225.18	-6.72	0.864	0.478
	-50	2.780	17.425	224.20	-7.13	0.867	0.478

Figure 7. Total average cost vs. cycle time.

4.2 Discussion on tables 2 and 3

From table 2, we see that for the cases of crisp, general fuzzy and monsoon type fuzzy model the optimal solution exists near 3 weeks of cycle time and among them monsoon type model gets the lowest value than others. However, for the case of cloud model, the optimality reaches near 8

Figure 8. Optimum cost for fuzzy monsoon inventory.

weeks cycle time, giving the average inventory cost of \$ 238.95, which is \$ 12.29 more than that of monsoon model. Also, we see that the cloud index for cloud model is 0.281 but that for monsoon model is 0.477 exclusively. Moreover, table 3 reveals that for the case of 10 weeks extensive studies the cloud model gives optimum average inventory cost of \$ 239.08 at 8 weeks cycle time with respect to the order quantity of 38.187 units. However, at only 3 weeks cycle time, crisp, as well as monsoon, model gives optimum average inventory cost function in which the monsoon model assumes the lowest inventory cost value of the whole table values. It is also seen that since cloud index is directly related to cycle time, the solution in higher uncertainty

Figure 9. Cloudy fuzzy vs. fuzzy monsoon solution.

Figure 10. Measures of cloud index.

system assumes lesser inventory cost than lesser uncertainty systems.

4.3 Sensitivity analysis

Let us consider the sensitivity of each of the parameters c , h , d , k , ρ , σ (-50% to $+50\%$) to study the effective variation of the monsoon model. The observations are listed in table 4. From table 4 we see that the demand parameter d is moderately sensitive and the corresponding inventory cost assumes minimum value with respect to the cycle time of 4.18 weeks (approximately) whenever a -50% change is made keeping the cloud index as 0.394 only. The parameters c , k and h are also moderately sensitive for changes of -30% to -50% alone. The value of the cost function has average sensitivity for $+30\%$ to $+50\%$ changes also. Here, in almost all the cases, the cycle time assumes values within the range of 1.96–3.88 weeks and the range of average inventory cost is \$182.00–\$271.86 with respect to the range of order quantity of 15.629–37.172 units only. Again, the left and right fuzzy deviation parameters ρ , σ are less sensitive for -50% to $+50\%$ changes. We also notice that the average cost function assumes lesser values throughout the changes keeping the cost value at \$216.80–\$236.50 with respect to order quantity within 17.265–30.701 units

exclusively. However, if we go across the whole table, we see that the cloud index assumes maximum value of 0.554 with maximum aspiration level and it came from -50% change of set-up cost k only.

5. Graphical illustrations of the model

Here we discuss figures 7–10 of the proposed model. Figure 7 shows the nature of cost function in monsoon type fuzzy model indicating the model optimum near 2.5–3.5 weeks cycle time. Figure 8 shows the values of the average inventory cost in crisp, general fuzzy, cloudy fuzzy and monsoon type fuzzy environment, where the monsoon type fuzzy model gets lower values than that of others. Figure 9 reveals that near 3 weeks cycle time the monsoon model gives finer result than others. It also reveals that, near 5 weeks cycle time, all curves assume closure values around a neighbourhood point. However, all the curves begin to diverge as the cycle time increases. Figure 10 shows the measures of cloud index in the monsoon type fuzzy environment.

6. Conclusion

Here we have studied the classical EOQ model in cloudy fuzzy environment. The model is studied in three different cases: (i) left cloud model, (ii) crisp model, (iii) right cloud model and (iv) monsoon model, which includes the models (i)–(iii). For the DM, the monsoon model is quite suitable as per the numerical study conducted in developing the model. Moreover, our findings support the basic code of business ethics “no risk no gain” or “high risk high gain”. Alternatively, “high risk (uncertainty) low cost (high profit)” attitudes have been established because of the superiority of the monsoon (having higher cloud index) type fuzzy model with respect to the others (having less cloud index). Finally, the basic characteristics of this model are as follows:

- (i) The components of each fuzzy parameter correspond to several increasing and decreasing functions of the time variable and their combinations also.
- (ii) To get better defuzzified value of the fuzzy objective function it is essential to have closure of the different fuzzy objective functions at least once even if they diverge at all other times separately.
- (iii) Fuzzification exclusively depends on real-time variable; beyond this, everything is crisp deterministic.

Incorporating some other algorithms for multi-objective functions in monsoon type fuzzy model, several new research works can be developed.

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Appendix A

A practical example of the proposed model

Suppose a salesman (DM) purchases (orders) goods after six days in a week. The amount of stock is decided according to weekly demand rate by the seller. Learning and gaining demand status of the first week(s) are used to order the items accordingly for the second week. Similarly, gaining the experience from the second week(s), he may change the amounts of order quantity accordingly for the next week and so on.

On the other hand, if the salesman wants to apply his/her learning experiences day-wise for the selling process, the he/she may shorten the length of cycle time accordingly. In this case the demand rate might vary in any two or three consecutive days, and hence the order quantity also. Thus the problem is as follows:

- (i) Is it profitable if the DM replenishes his/her order quantities once after each and every cycle time, where the length of each cycle time does not vary in a non-random uncertain system?
- (ii) Is it profitable if the DM replenishes his/her order quantities once after each and every cycle time, where the length of each cycle time varies from one cycle to another in a non-random uncertain system?

The answer is too much popular, because the basic aim of inventory management is to minimize system cost (or maximizing profit); hence, obviously the second case will give the minimum system cost and hence it is a part of monsoon type fuzzy model.

Since the basic aim of learning is to minimize inventory cost, obviously for the second case the average inventory cost will be minimum.

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