Sādhanā (2019) 44:68
https://doi.org/10.1007/s12046-018-1044-2

Unsteady hydromagnetic flow and heat transfer of a viscous fluid near a suddenly accelerated flat surface

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MS received 11 January 2018; revised 12 August 2018; accepted 11 October 2018; published online 23 February 2019

Abstract. The unsteady boundary layer flow and heat transfer near a suddenly accelerated flat surface in an unbounded mass of hydromagnetic viscous fluid with the combined influence of the magnetic field, viscous dissipation, internal heat generation/absorption and thermal radiation have been investigated. A new similarity transformation is recommended, which transforms the hydromagnetic boundary layer equations into a set of non-linear ordinary differential equations. These equations are then solved numerically using the finite-difference method for some values of the governing parameters that involve unsteadiness parameter β, heat source/sink parameter λ, Eckert number E, magnetic interaction parameter M, radiation parameter N and Prandtl number Pr. The influence of these parameters on the velocity as well as the temperature field is investigated in detail. In particular, the impact of viscous dissipation (measured through E), which is a strong function of constant reference temperature T_{ref}, on the temperature field has been investigated in different situations. This entails considering the value of T_{ref} as positive or negative depending on whether the surface temperature is higher or lower than the fluid temperature. The analysis reveals that there exists a critical value of E depending upon the values of the other physical parameters for which the surface heat flux vanishes. Below this critical value, heat flows either from the surface to the fluid or from the fluid to the surface depending on whether $T_{ref} >$ or <0.

Keywords. Magnetohydrodynamic flow; similarity solution; accelerated plate; viscous dissipation.

1. Introduction

The study of the unsteady boundary layer flow is important due to the fact that most of the flow problems in the practical fluid dynamics are (in the sense) unsteady. Unsteady boundary layer flow is more complicated than the steady boundary layer flow owing to the additional time-dependent terms, which have a significant effect on the boundary layer flow characteristics such as separation and reattachment of the flow. Excellent reviews of the unsteady boundary layer flows under various conditions are found in the papers published by Stewartson [1], Stuart [2], Riley [3], Wang [4] and Ma and Hui [5], among others. Very recently, Dholey and Gupta [6] and Dholey [7–9] provided some basic ideas of the unsteady boundary layer flows in their articles. In fact, unsteady boundary layer flows are investigated actively today as a part of an effort to calculate the friction drag and the heat transfer rate as well as to comprehend the correct behaviour of the flow dynamics inside the layer. It is noteworthy to mention here that some types of machines are reliant on the time-dependent motion for performing their basic functions. However, the unsteadiness in all the afore-mentioned studies arises due to the time-dependent free stream velocity, which is inversely proportional to the time.

A similar kind of unsteady boundary layer flow occurs in the practical fluid mechanics when an infinite flat surface is suddenly set in motion in its own plane through an unbounded mass of fluid that is at rest. Indeed, the study of this unsteady boundary layer flow has attracted much attention to the research community due to its practical applications in various physical problems, including start-up flows, shut-down flows, periodic fluid motion, flows over the aerodynamic surfaces of vehicles, submarines and many others. Actually, the flow problem that originates solely from the impulsive motion of an infinite flat surface in its own plane in an infinite mass of viscous fluid was first studied by Stokes [10] in his famous pendulum problem. This special type of flow problem is generally known as the 'First Stokes problem' in the literature. As Rayleigh [11] also considered this problem, it is often referred to as the 'Rayleigh problem' (see Schlichting and Gersten [12], Currie [13], Rosenhead [14], Telionis [15], Panton [16] and the references therein).

Magnetohydrodynamics (MHD) deals with the investigation of the flow dynamics of an electrically conducting fluid in the presence of a magnetic field. The interaction between the fluid flow and the external magnetic field generates mechanical forces, which significantly modify the fluid motion inside the boundary layer. A wide class of natural phenomena as well as many engineering problems are highly influenced by the hydromagnetic (MHD) flows such as MHD power generators and pumps, MHD accelerators, plasma devices, nuclear reactors, etc. Recently, MHD flow has attracted the attention of the researchers, not only for its inherent interest but also for its applications in many problems having geophysical and astrophysical significance. Rossow [17] studied the MHD flow induced by an impulsive start of an infinite flat surface in an unbounded mass of viscous fluid. It is important to mention here that the effects of radiation as well as the heat generated (or absorbed) in the system are very useful in the framework of space technology and the processes involve high temperature. In the presence of a magnetic field, the radiative heat flows of an electrically conducting fluid with high temperature are encountered in MHD power generation, solar power technology, astrophysical flows, nuclear engineering applications, etc.

All of these deliberations have motivated us for the present study, where we analyse the unsteady boundary layer flow and heat transfer of an electrically conducting viscous fluid near a suddenly accelerated flat surface in the presence of a transverse magnetic field. Viscous dissipation, thermal radiation and internal heat generation/absorption terms are considered in the energy equation. Time-dependent plate velocity, magnetic field and temperature distribution along the surface are taken into consideration, for which a similarity solution exists. Here our main concern is on the impact of viscous as well as Ohmic dissipation on the behaviour of the thermal characteristics of this flow field as well. The present analysis reveals that the surface temperature due to frictional heating (or cooling) through viscous as well as Ohmic dissipation can be kept under control up to a reasonable value (in a practical situation) by introducing a suitable magnitude of heat absorption (or generation) parameter into the system. Another interesting result arising from this analysis is the dual nature of the magnetic field, which opposes or assists the heat flow depending on whether the surface temperature is greater or less than the ambient fluid temperature.

2. Physical model and mathematical formulation

Consider a long flat plate kept in an unbounded mass of viscous incompressible and electrically conducting fluid in the presence of a transverse magnetic field fixed to the fluid. Initially, we assume that both the plate and the fluid are at rest with the same temperature T_∞. The plate is assumed to move suddenly in its own plane with a velocity $U(t)$ and enter a system so that its temperature instantaneously rises or falls to $T_w(t)$. We introduce a Cartesian (x, y)-coordinates system in which the x-axis is taken along the plate surface, and the y-axis is perpendicular to it, as depicted in figure 1. Obviously, the fluid flow is originated solely by the translatory motion of the plate surface. Hence the pressure gradient is hydrostatic and it does not, therefore, affect the velocity parallel to the plate surface. This flow is two-dimensional and the only nonzero component of velocity is u, along x-direction, which will be a function of y-coordinate and time t. A time-dependent magnetic field $\mathbf{B}(t)$ is applied in the direction of positive y-axis, which induces an electric field (see Shercliff [18]). This electric field produces an additional Lorentz force term in the flow system. Here we assume that the magnetic Reynolds number R_m is very small (i.e., $R_m \ll 1$). As a consequence, the applied magnetic field contributes only to the x-component of Lorentz force $(-\sigma \mathbf{B}^2 u/\rho)$ in the momentum equation (see Dholey [19]), where σ and ρ are electrical conductivity and density of the fluid, respectively. Finally, we assume that the entire flow field is exposed to thermal radiation.

Under these assumptions, the equations of momentum and energy governing this unsteady flow and heat transfer are given by

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mathbf{B}^2}{\rho} u, \tag{1}$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma \mathbf{B}^2}{\rho c_p} u^2 - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{Q_s}{\rho c_p}(T - T_\infty), \tag{2}$$

where ν, T, α, c_p, q_r and Q_s denote kinematic coefficient of viscosity, temperature, thermal diffusivity, specific heat, radiative heat flux and volumetric rate of heat generation/absorption, respectively. It is important to note that the afore-mentioned radiative heat transfer is applicable only when the thermal boundary layer is very thick or the medium is greatly absorbing (see Chung [20]). Hence, using the Roseland approximation of radiation, one can obtain q_r as (see Mahapatra *et al* [21])

$$q_r = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y}, \tag{3}$$

where σ^* and k^* are the Stefan–Boltzman constant and the mean absorption coefficient, respectively.

The boundary conditions compatible for this flow model are

$$\begin{aligned} t \geq 0: \quad & u(0,t) = U(t), \quad T(0,t) = T_w(t) \quad \text{and} \\ & u(y,t) \to 0, \quad T(y,t) \to T_\infty \quad \text{as} \quad y \to \infty. \end{aligned} \tag{4}$$

Besides these, an initial condition on $u(0, t)$ must be included for a well-posed problem; for this, let the plate be at rest (i.e., $u(0,t) = 0$) at time $t < 0$.

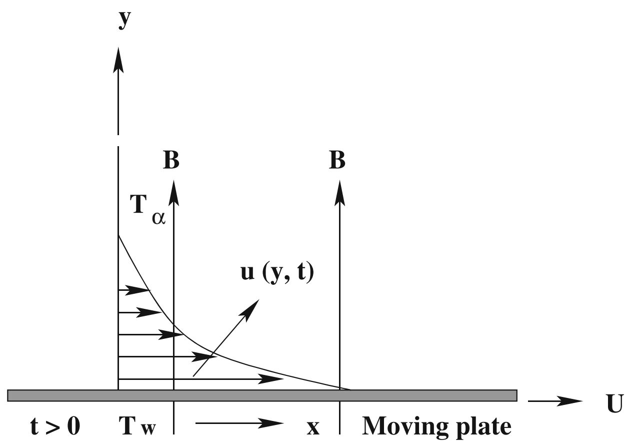

Figure 1. Sketch of the flow and the coordinate system.

3. Similarity transformation and solution procedure

Here, we propose a new similarity transformation to investigate the impacts of the magnetic field, viscous dissipation, internal heat generation/absorption and thermal radiation in the present unsteady flow dynamics. First, we define a stream function $\psi(y,t)$ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{5}$$

which automatically satisfies the continuity equation. The similarity variable η and the new dependent variables f and θ are defined as

$$\eta = \sqrt{A(\bar{t})}\,\bar{y}, \quad \bar{\psi} = \sqrt{A(\bar{t})}\int_0^\eta f(\eta)\,d\eta \quad \text{and} \quad \theta(\eta) = \frac{T(\eta,\bar{t}) - T_\infty}{T_w - T_\infty}, \tag{6}$$

$$\text{where} \quad \bar{y} = \frac{yU_0}{\nu},\ \bar{t} = \frac{tU_0^2}{\nu},\ \bar{\psi} = \frac{\psi}{\nu},\ \bar{u} = \frac{u}{U_0} \quad \text{and} \quad \bar{U} = \frac{U}{U_0}, \tag{7}$$

U_0 being the representative of u as a whole in the flow field and $A(\bar{t})(>0)$ is an as yet unknown function of time $\bar{t}$ to be determined during the course of the solution process. Here, the surface temperature depends on time $\bar{t}$ and we assume that it varies in accordance with $T_w(\bar{t}) = T_\infty + T_{ref}A^2(\bar{t})$ where T_{ref} is a constant reference value of temperature. Thus, the wall temperature T_w represents a situation in which the surface temperature falls or rises depending on whether $T_{ref} <$ or > 0. It should be emphasized that the last transformation of (6) is valid only when $T_w \neq T_\infty$. In particular, when $T_w = T_\infty$, the trivial solution $T(\eta,t) = T_\infty$ solves the energy equation (2) subject to the thermal boundary conditions given in (4) and there is no need for a similarity transformation. It is noticeable that the fluid outside the thermal boundary layer remains isothermal with temperature T_∞ since this portion of the fluid is unaffected by the heat transfer process. Finally, we consider the functional forms of $\mathbf{B}(\bar{t})$, $Q_s(\bar{t})$ and $\bar{U}(\bar{t})$ as follows:

$$\mathbf{B}(\bar{t}) = B_0\sqrt{A(\bar{t})}, \quad Q_s(\bar{t}) = Q_0 \quad A(\bar{t}) \quad \text{and} \quad \bar{U}(\bar{t}) = C\,A(\bar{t}), \tag{8}$$

where B_0 is the magnetic field strength, Q_0 is the heat generation/absorption coefficient and $C(>0)$ is a dimensionless constant.

Using (1)–(4) and (6)–(8), one can now obtain the following boundary value problem (BVP):

$$f'' - \left(\frac{\dot{A}}{A^2}\frac{\eta}{2}\right)f' - \left(\frac{\dot{A}}{A^2} + M^2\right)f = 0, \tag{9}$$

$$\frac{\theta''}{P_{eff}} - \left(\frac{\dot{A}}{A^2}\frac{\eta}{2}\right)\theta' + \left(\lambda - 2\frac{\dot{A}}{A^2}\right)\theta = -E\left(f'^2 + M^2 f^2\right), \tag{10}$$

$$f(0) = C, \quad \theta(0) = 1 \quad \text{and} \quad f(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty, \tag{11}$$

where a prime and a dot denote derivatives with respect to η and $\bar{t}$, respectively. Here $E = \left(U_0^2/c_pT_{ref}\right)$ is the Eckert number that measures the effect of viscous dissipation and $M = (\sigma B_0^2\nu/\rho U_0^2)^{1/2}$ is the Hartmann number, which is also familiar as 'interaction parameter' in the literature. The impact of the magnetic field on the present flow field is characterized by this parameter. Also, $\lambda = (Q_0\nu/\rho c_p U_0^2)$ is the heat sink or source parameter accordingly as $\lambda <$ or > 0. Furthermore, $P_{eff} = Pr/(1+N)$ is the effective Prandtl number where $Pr(=\nu/\alpha)$ is the Prandtl number and $N(=16\sigma^* T_\infty^3/3\alpha\rho c_p k^*)$ is the radiation parameter. It is noticeable that the parameter P_{eff} appears in Eq. (10) with the combination of two usual parameters Pr and N. Indeed, this two-parameter approach will be confusing and it is not necessary. Therefore, we will show only the effect of P_{eff}, from which one can easily appreciate the effects of the others.

It is well known that for the existence of similarity solution, all the coefficients in (9) and (10) should either be constants or functions of η only. As the velocity $\bar{u}$ and temperature T are uniquely related to the nondimensionl velocity $f(\eta)$ and temperature $\theta(\eta)$ according to Eq. (6), the requirements for similarity are satisfied. Hence we consider

$$\beta = \frac{\dot{A}}{A^2} \tag{12}$$

as a dimensionless constant that measures the unsteadiness of the flow. Moreover, Eqs. (9) and (10) require that β be a nonzero constant in time since $A(\bar{t})$ is the time evolution of the flow strength. However, integrating (12), once one can easily obtain $A(\bar{t})$ as

$$A(\bar{t}) = \frac{1}{\beta\,(t_{ref} - \bar{t})} \tag{13}$$

with $A(0) = (\beta t_{ref})^{-1} > 0$ and t_{ref} is the constant reference value of time $\bar{t}$. Interestingly enough for $\beta = 0$, Eq. (12) provides us $A(\bar{t})$ as constant $(= 1$, for example) and consequently the whole field will be independent of time $\bar{t}$. In this case, the governing boundary layer equations (9) and (10) yield the analytic solutions $f(\eta) = Ce^{-M\eta}$ and $\theta(\eta)$ will be of the form either $\theta(\eta) = (1 + D)e^{-\sqrt{-\lambda P_{eff}}\,\eta} - De^{-2M\eta}$ or $\theta(\eta) = (1 - D_1)e^{-\sqrt{-\lambda P_{eff}}\,\eta} + D_1 e^{-2M\eta}$ accordingly as $(4M^2 + \lambda P_{eff}) \neq 0$ or 0. Here $D = 2C^2 EM^2 P_{eff}/(4M^2 + \lambda P_{eff})$; $D_1 = C^2 EMP_{eff}/(2\sqrt{-\lambda P_{eff}})$ and the value of λ is always negative. Again, for $\beta = -2M^2 < 0$, Eq. (9) subject to (11) conceives an analytic solution $f(\eta) = Ce^{-(M^2/2)\eta^2}$, which gives us $f'(0) = 0$. Since the flow is originated by the impulsive motion of the plate surface, it exerts a dragging force on the fluid body, for which $f'(0)$ is always negative for this flow model. Moreover, this flow problem is unsteady and the assumption $A(\bar{t}) > 0$ give us two distinct possibilities with the initial condition specified at $\bar{t} = 0$: (*a*) $\beta > 0$ and $0 \leq \bar{t} < t_{ref}$ and (*b*) $\beta < 0$ and $t_{ref} < 0 \leq \bar{t} < \infty$. Figure 2a and b explicitly follows these two cases. Figure 2a demonstrates that for a given value of $\beta(> 0)$, the flow strength $A(\bar{t})$ is an increasing function of $\bar{t}$. This suggests that the case (*a*) is appropriate for this flow problem since the considered flow is accelerating. On the other hand, case (*b*) follows the decelerating flow in time $\bar{t}$. Henceforth we will consider only the positive value of β for the analysis of the present flow problem.

Using a finite-difference method (Thomas algorithm [22]), the non-linear two-point BVP (9)–(11) is solved numerically for various values of the governing parameters with the step length $d\eta = 0.01$. Obviously, the fluid velocity is enhanced with the increase of C for the given values of the other physical parameters. Hence for conciseness of this paper we will consider only the fixed value of $C(= 1)$ throughout the analysis unless stated otherwise. We are however not aware of any theoretical and numerical as well as any experimental data with which our results may be compared.

4. Physical quantities of interest

In practical purposes, the significant physical quantities are skin-friction coefficient C_f, shear layer thickness (or the penetration depth of the velocity into the fluid body) $\bar{\delta}$ and Nusselt number Nu, which are given as follows.

Skin-friction coefficient: The skin-friction coefficient C_f $(= \tau_w/\rho U_0^2)$ is obtained as

$$C_f = \frac{1}{[\beta(t_{ref} - \bar{t})]^{3/2}} f'(0). \tag{14}$$

Equation (14) ensures that the skin-friction coefficient C_f tends to zero with increase in the value of $\beta(> 0)$ in the proportion $1/\beta^{3/2}$, and it becomes infinity with the increase of time $\bar{t}$ in $0 \leq \bar{t} < t_{ref}$. Furthermore, C_f is directly connected with the wall gradient $f'(0)$, the main physical quantity of interest for the present study. However, our main aim is to illustrate the change of $-f'(0)$, since $f'(0) < 0$ in the present flow problem, with the parameters M and β.

Shear layer thickness: The fluid flow originates solely due to the impulsive motion of the plate surface, which is an arbitrary function of time $\bar{t}$. As time increases, however, the effect of the plate motion propagates farther and farther out into the body of the fluid since the momentum is transferred normal to the plate surface by molecular diffusion and finally a series of velocity profiles is achieved over the plate surface. However, the velocity $f(\eta)$ decreases continuously and ultimately tends to its limiting value zero

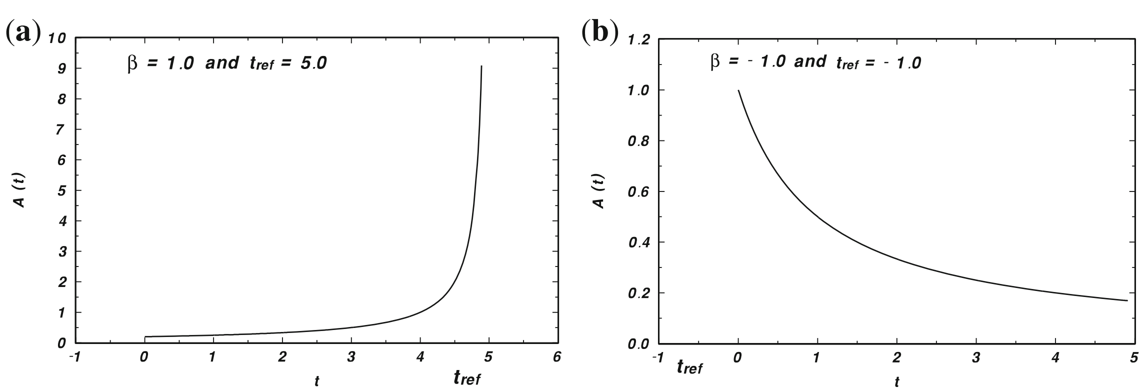

Figure 2. Time evolution of the flow strength $A(\bar{t})$ for the cases: (**a**) $\beta > 0$ and $0 \leq \bar{t} < t_{ref}$ and (**b**) $\beta < 0$ and $t_{ref} < 0 \leq \bar{t} < \infty$.

at a certain distance from the plate surface (i.e., for $\eta \to \infty$). We define the shear layer thickness δ, corresponding to the value of y, as such distance η_s of η where the wall effect on the fluid has dropped to one per-cent. Hence from (6) and (13) the dimensionless shear layer thickness $\bar{\delta}(= \delta U_0/v)$ is obtained as

$$\bar{\delta} = \eta_s \sqrt{\beta(t_{ref} - \bar{t})}\,. \tag{15}$$

The distance $\bar{\delta}$ measures the degree of penetration of the momentum (velocity) into the fluid body and it is proportional to the square root of the product of $\beta(>0)$ and $\bar{t}(0<\bar{t}<t_{ref})$. It is interesting to note that when $(\beta(t_{ref} - \bar{t}))$ is large, $\bar{\delta}$ becomes infinite, and consequently the whole field over the plate surface receives the effect of the velocity of the plate surface. On the other hand, for small values of $(\beta(t_{ref} - \bar{t}))$, $\bar{\delta}$ will be small and once again we will have a boundary layer flow.

Nusselt number: The local dimensionless heat transfer coefficient, generally known as Nusselt number $Nu(x, \bar{t})$, is defined as follows:

$$Nu(x, \bar{t}) = \frac{x q_w(\bar{t})}{\alpha \rho c_p (T_w - T_\infty)}, \tag{16}$$

where the surface heat flux $q_w(\bar{t})$ at a given location x on the plate surface is given by

$$q_w(\bar{t}) = -\left(\alpha \rho c_p + \frac{16\sigma T_\infty^3}{3k^*}\right)\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{17}$$

Hence the explicit expression of Nusselt number, which is related to various thermophysical quantities and governs the heat transfer rate at the wall, is obtained using (6), (7) and (13) as

$$Nu(x, \bar{t}) = -\frac{(1+N)}{[\beta(t_{ref} - \bar{t})]^{1/2}} \theta'(0) Re_x, \tag{18}$$

where $Re_x = (U_0 x/v)$ is the local Reynolds number. Equation (18) proves that Nusselt number is enhanced by the radiation parameter as well as the local Reynolds number, whereas it is reduced by the unsteadiness parameter.

5. Results and discussion

From Eqs. (9)–(11), it is obvious that the present flow problem completely depends on the following five dimensionless quantities: (*a*) magnetic parameter M, (*b*) unsteadiness parameter β, (*c*) effective Prandtl number P_{eff}, (*d*) heat source/sink parameter λ and (*e*) Eckert number E. Each and every parameter has a significant influence on this flow dynamics. Here, we have highlighted only the main numerical results in the form of figures and have discussed these results with corresponding physics.

5.1 *Effect of magnetic parameter on the variation of velocity*

Figure 3 shows the similarity profiles $f(\eta)$ against η for different values of M when $\beta = 1$. This figure clearly shows that the largest velocity occurs in the absence of a magnetic field. Furthermore, the velocity at a given location decreases continuously with the increase of the Hartmann number M. This is due of the fact that any movement of an electrically conducting fluid in the presence of a transverse magnetic field generates a drag force (Lorentz resistive force) acting opposite to the direction of the fluid motion. Obviously, this drag force will be increased with an increase of the magnetic field (see Eq. (1)). As a result, the fluid velocity near the plate surface decreases consistently with increasing value of M and simultaneously the magnitude of the velocity gradient near the surface is enhanced. This result indicates the reduction of the shear layer thickness, which is reflected in this figure as well. Thus we see that the magnetic field has an inhibiting, resisting influence on the velocity profiles as well as on the shear layer thickness.

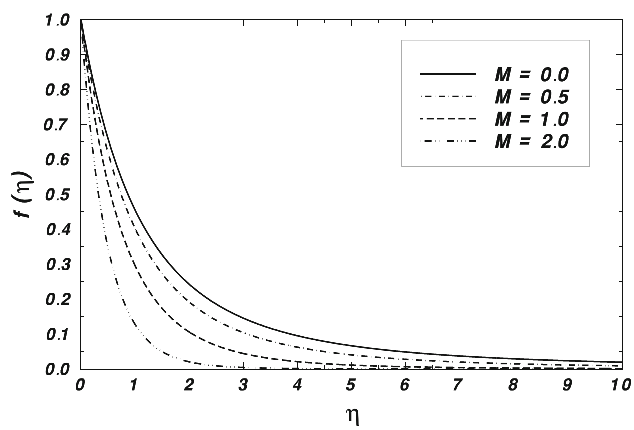

Figure 3. Velocity profiles for different values of the interaction parameter M when $\beta = 1$.

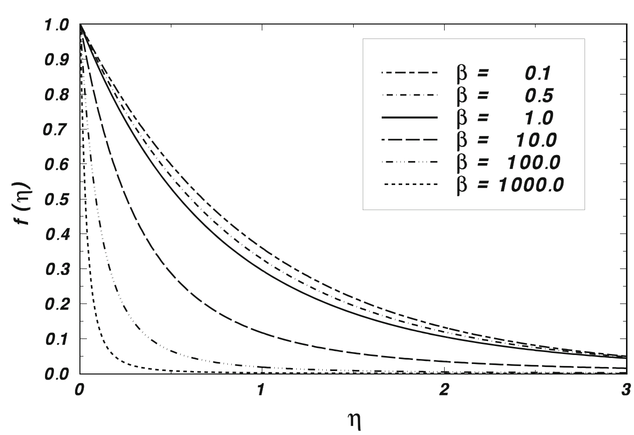

Figure 4. Velocity profiles for different values of the unsteadiness parameter β when $M = 1$.

The influence of the unsteadiness parameter β on the similarity profiles $f(\eta)$ has been shown in figure 4. It is readily seen that the velocity at a given location decreases with the increase of β and finally it tends to zero with the limiting values of β (i.e., for $\beta \to \infty$). A physical clarification of this result is as follows. It is noticeable that the present flow problem has originated solely from the time-dependent translating motion of an infinite flat plate. Therefore, the fluid that is dragged along the plate surface has a velocity that is also a function of time and hence dependent on the flow strength function $A(\bar{t})$. This flow strength function is inversely proportional to the unsteadiness parameter β for a given value of $\bar{t}$ in $0 \leq \bar{t} < t_{ref}$ (see Eq. (13)). Hence, for a fixed value of $\bar{t}$, the strength of the flow decreases appreciably with the increase of β and results in the reduction of velocity inside the boundary layer. Finally, we see that the velocity at a given location increases or decreases depending on whether $\beta <$ or > 1 (in contrast with the corresponding velocity for $\beta = 1$) in proportion to $(1/\beta)$. Hence we come to the conclusion that the magnetic field can be used to control the motion of an electrically conducting fluid over a moving flat surface as well, and the addition of the unsteadiness parameter highly promotes the effect of the magnetic field.

5.2 *Effect of magnetic parameter on the skin-friction coefficient*

We have pointed out earlier that the velocity gradient at the wall is always negative for this flow problem as the flow is originated by the impulsive motion of the plate. Physically, negative velocity gradient means the plate surface exerts a dragging force on the fluid body. The variation of the skin-friction coefficient $-f'(0)$ against the magnetic interaction parameter M for three different values of the unsteadiness parameter $\beta(= 0.5,\ 1.0$ and $1.5)$ is depicted in figure 5a. From this figure it is clear that for any given value of β the magnitude of the skin-friction coefficient $|f'(0)|$ increases continuously with increasing value of M. We have shown previously that the velocity inside the layer is reduced by the Lorentz resistive force and therefore the magnitude of the skin-friction coefficient is increased (see figure 3). It is also found that for a given value of M, $|f'(0)|$ increases with β. From a mathematical point of view, increasing β is an indication of the decrease in flow strength $A(\bar{t})$, which in turn leads to the decrease in velocity and thereby increases the magnitude of the skin-friction coefficient. In addition to this, the increasing rate of $|f'(0)|$ with β is more pronounced for higher values of M. In fact, the unsteadiness parameter β opposes the flow and the addition of the magnetic interaction parameter M enhances the opposing effect of β. The variation of $-f'(0)$ as a function of β for three different values of $M(= 0.0$, 0.5 and 1.0) is shown in figure 5b.

5.3 *Effects of magnetic and unsteadiness parameters as well as effective Prandtl and Eckert numbers on the variation of temperature*

In this section, the effects of the physical parameters, viz., magnetic parameter M, unsteadiness parameter β, effective Prandtl number P_{eff} and Eckert number E, on the temperature field will be discussed in detail. Before we continue further to give details of the numerical results for different values of these physical parameters, we shall focus our attention on the definitions of the effective Prandtl number P_{eff} and the Eckert number E. It is noticeable that the influences of the Prandtl number Pr as well as the radiation parameter N on the temperature field can easily be obtained from the relation $P_{eff} = Pr/(1+N)$. This relation clearly indicates that increasing P_{eff} is equivalent to increasing Pr or decreasing N. Physically, increasing Pr implies the decrease of thermal diffusivity and decreasing N implies the decrease of radiation of heat from the system. Furthermore, from the definition of Eckert number it is clear that the values of E can be either positive or negative accordingly as the temperature of the plate surface rises or falls, i.e., when T_{ref} is positive or negative.

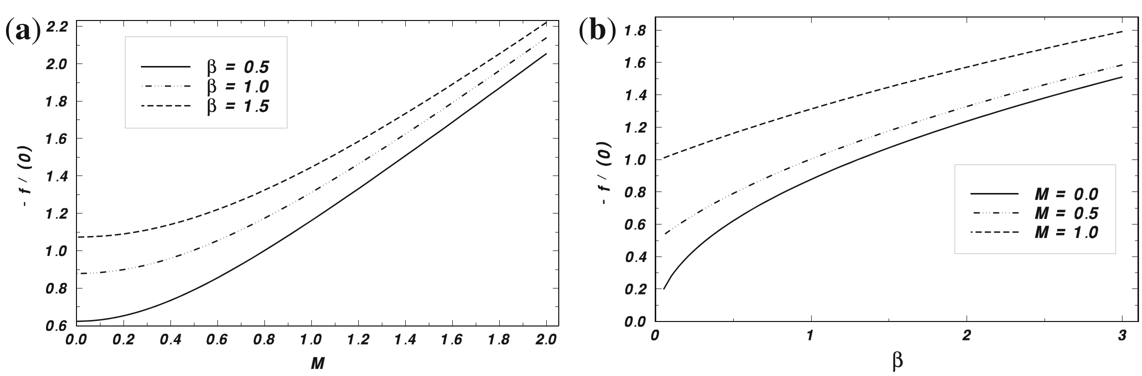

Figure 5. Reduced skin-friction coefficient $-f'(0)$ as a function of (**a**) M for various values of β and (**b**) β for various values of M.

The influence of the magnetic field on the dimensionless temperature profiles $\theta(\eta)$ is displayed in figure 6. It is found that the temperature at a given location increases with an increase in the magnetic interaction parameter M. This is due of the fact that the fluid velocity within the boundary layer is reduced by the Lorentz resistive force, which enhances the corresponding temperature profiles within the flow. Actually, in the presence of a transverse magnetic field, the total amount of heat transferred from the surface is accumulated in a small region over the plate surface in the view of an isothermal free surface temperature. This is the reason for which the temperature within the flow is high in the magnetic case. Indeed, when a metallic plate moves with high speed in the presence of a magnetic field, a motional electromagnetic field (EMF) is developed due to the change of magnetic flux. As a result, an eddy current circulates within the metallic plate, which in turn develops heat and consequently the temperature of the plate surface rises, which increases the thickness of the thermal boundary layer. This result ensures the enhancement of the wall temperature gradient with the increase in the magnetic parameter (see figure 10b).

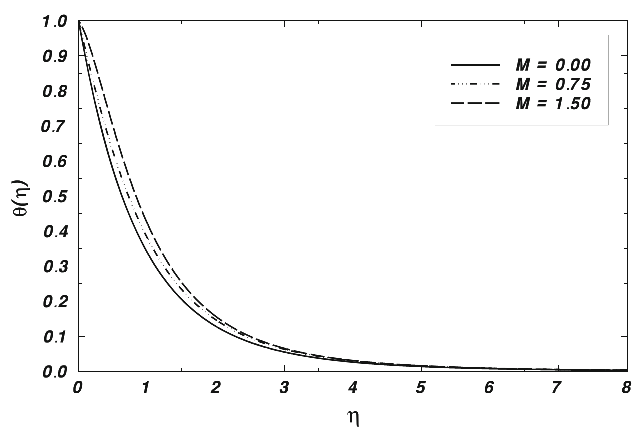

Figure 6. Variation of $\theta(\eta)$ with η for various values of M when $\beta = P_{eff} = E = 1$ and $\lambda = 0$.

Figure 7 shows the variation of $\theta(\eta)$ with η for various values of the unsteadiness parameter β with the fixed values of $M = P_{eff} = E = 1$ and $\lambda = 0$. From this figure it is observed that the thickness of the thermal boundary layer along with the temperature at a given location decreases steadily with increase of the unsteadiness parameter β. It is noticeable that the impact of the unsteadiness parameter β on the temperature distribution is the same as that on the velocity field. The underlying physics behind such behaviour of the temperature field is closely related to the distribution of temperature as given in Eq. (6). In fact, the fluid velocity and temperature are highly dependent on the time evolution function $A(\bar{t})$, which controls (via unsteadiness parameter) both the distributions of velocity and temperature within the boundary layer.

Figure 8 shows the variation of $\theta(\eta)$ with η for different values of P_{eff}. This figure clearly exhibits that the temperature at a given location decreases with increase in the value of P_{eff}. This suggests that the effect of Prandtl number Pr is to reduce the temperature within the flow and consequently rate of heat transfer enhancement at the plate surface is observed. For this reason, higher Prandtl number fluids are used for cooling the system. Obviously, the impact of the radiation parameter N is to increase the temperature within the flow. Moreover, the heat generation parameter $(\lambda > 0)$ augments the thermal diffusion layer, whereas the absorption parameter $(\lambda < 0)$ curtails it in comparison with the zero heat generation or absorption $(\lambda = 0)$ in the system.

We have considered a fixed positive value of E $(= 1)$ for delineating figures 6–8 in order to investigate the effects of the other physical parameters on the temperature field. However, the qualitative behaviours of $\theta(\eta)$ for any given value of $E(< 0)$ follow similar patterns while quantitatively they get reduced as shown in figures 7 and 8 since the unsteadiness parameter has an opposing effect inherently and the Prandtl number is a material property of the fluid. However, the effect of M on $\theta(\eta)$ for a given value of E (< 0) is completely opposite to that of a positive value of E

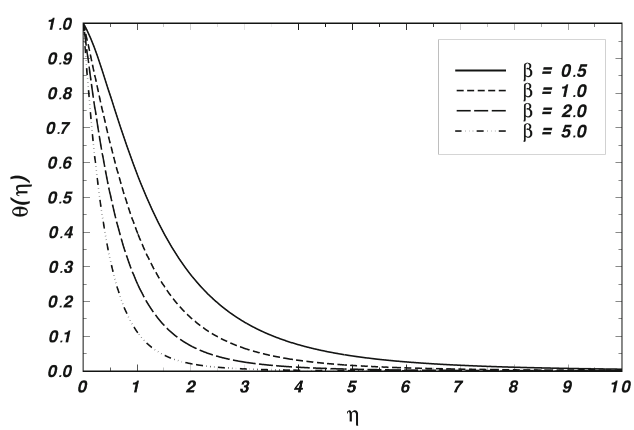

Figure 7. Variation of $\theta(\eta)$ with η for various values of β when $M = P_{eff} = E = 1$ and $\lambda = 0$.

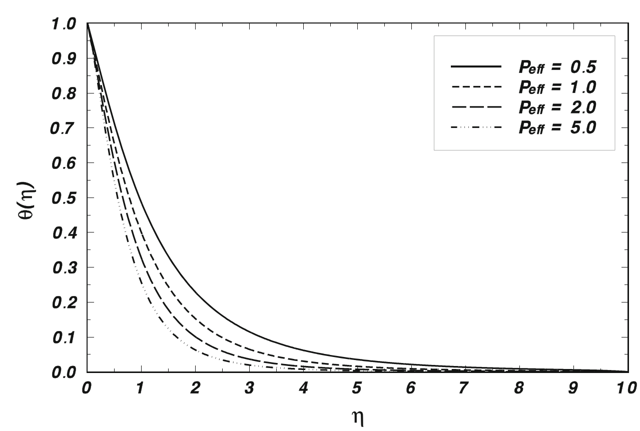

Figure 8. Variation of $\theta(\eta)$ with η for various values of P_{eff} with $M = \beta = E = 1$ and $\lambda = 0$.

(see figure 6). In fact, a negative value of E (i.e., when $T_{ref} < 0$) implies higher fluid temperature than the surface temperature even when the sign of the dependent variable $\theta(\eta)$ remains unchanged (see Eq. (6)). Actually, in this case the direction of heat flow will be changed whereas the direction of the applied magnetic field remains the same as before. As a result, the temperature of the fluid within the flow will be decreased with an increase of the magnetic field.

The temperature boundary layer's response to the variation of the Eckert number $E(\geq 0)$ is illustrated in figure 9. Increase in the temperature profile with E ensures that more heat is dissipated within the flow owing to the effect of viscous dissipation. Again, the temperature overshoot in figure 9 can be described as follows. For small enough values of the plate velocity, viscous dissipation in the system is very small and therefore considerable heat is not produced in the system. Hence, for a small value of E, heat is transferred from the plate surface to the fluid $(\theta'(0) < 0)$ as $T_w > T_\infty$. However, for a large value of E (i.e., when the plate velocity is high), significant heat is produced inside the layer owing to the effect of viscous dissipation in combination with the magnetic field (Ohmic dissipation). In this case the temperature of the fluid very close to the plate surface is higher than the surface temperature T_w, and consequently heat transfer occurs from the fluid to the plate surface $(\theta'(0) > 0)$ even when the condition $T_w > T_\infty$ is maintained. As a result, the heated fluid particles near the wall are convected downstream to a position where the surface temperature is lesser. Hence we come to this conclusion that there is a critical value E_{cr} (say) of E, dependent on the values of the other physical parameters, for which the surface heat flux vanishes. Below this critical value, heat flows from the surface to the fluid, essentially making the thermal boundary layer thinner, and above this critical value, an opposite trend is observed in this variation. On the contrary, one must find the undershoot behaviour of the temperature after a certain negative value of E depending upon the values of the other parameters.

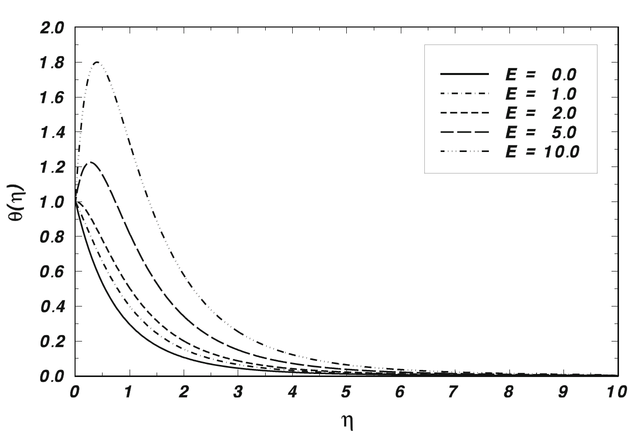

Figure 9. Variation of $\theta(\eta)$ with η for several values of $E(\geq 0)$ with $M = P_{eff} = \beta = 1$ and $\lambda = 0$.

5.4 *Effects of viscous dissipation and heat generation/absorption parameter on the heat transfer coefficient*

The temperature gradient at the wall $|\theta'(0)|$ is significant for evaluating the rate of heat transfer in the flow system. Physically, a negative value of $\theta'(0)$ implies heat transfer (from the plate surface to the fluid) and a positive value implies heat absorption. We have already mentioned that the main objective of this study is to assess the impacts of viscous as well as Ohmic dissipation in conjunction with the internal heat source/sink parameter on the thermal characteristics of this flow problem. For this purpose, we have delineated figure 10a and b for three dissimilar values of λ as indicated in each figure.

Figure 10a displays the variation of $\theta'(0)$ as a function of Eckert number E for three dissimilar values of λ ($=-1$, 0 and 1) when $\beta = P_{eff} = M = 1$. This figure clearly shows that the dimensionless wall temperature gradient increases with E for all values of λ considered in the present study. This is consistent with the fact that an increase in E implies increase of viscous dissipation in the system, which leads to the increase in temperature of the fluid within the layer. It is readily seen that the effect of viscous dissipation is enhanced/reduced with heat source/sink parameter relative to zero heat source $(\lambda = 0)$ in the system. A closer scrutiny of this figure discloses that there is a critical value E_{cr} of E depending upon the values of λ (as well as β, M, Pr and N) for which no heat flows from one side to other. Below this critical value, heat is transferred from the surface to the fluid, and beyond this the plate surface absorbs heat from the fluid body. Finally, we see that the critical value E_{cr} continuously decreases/increases with increase in heat generation/absorption in the system.

On the other hand, figure 10b exhibits the dual nature of the magnetic field, which opposes the heat flow for a positive value of E and assists it for negative values of E. The physical clarification of this fact has already been given in section 5.3. Here we have found the critical value M_{cr} of M (like E_{cr}) only for a positive value of E. However, no such critical value exists in this flow problem for a negative value of E. It is important to mention in this context here that in the absence of viscous dissipation (i.e., for $E = 0$), the temperature field does not depend on the magnetic field since the thermal boundary layer equation is decoupled from the momentum boundary layer equation (see Eq. (10)). As a result, the heat transfer remains constant with the magnetic field. In this case (i.e., for $E = 0$), the values of $-\theta'(0)$ found are 1.64674, 1.31938 and 0.88124 for $\lambda = -1$, 0 and 1, respectively.

From the ongoing analysis, it is clear that the plate velocity and the magnetic field cause the generation of viscous and Ohmic dissipation in the system, which increases the fluid temperature within the flow. In fact, heat energy is accumulated in the fluid elements owing to the frictional (viscous) heating and Eckert number E measures

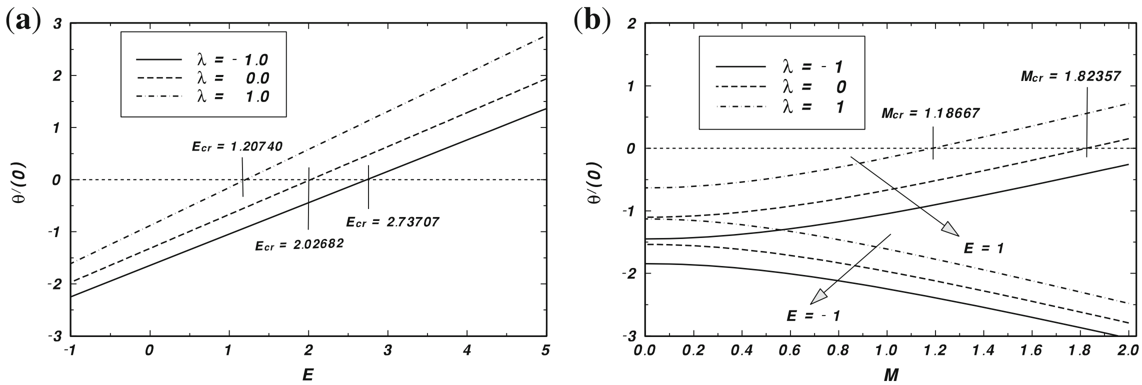

Figure 10. (**a**) Variation of $\theta'(0)$ as a function of E for three dissimilar values of λ $(= -1, 0$ and $1)$ when $\beta = P_{eff} = M = 1$. (**b**) Variation of $\theta'(0)$ as a function of M for two different values of E $(= -1$ and $1)$. The other physical parameters have the same values as mentioned in case (**a**).

the produced heat. Positive value of E means higher surface temperature than the fluid temperature since T_{ref} is positive. One more important thing to note here is that the surface heat flux decreases or increases with the magnetic interaction parameter M accordingly as $E >$ or < 0. This implies that the surface temperature increases with the magnetic parameter for a positive value of E, while it decreases for a negative value. Hence we can conclude that for maintaining a constant temperature of the system as well as to continue the stable performance without changing the performance parameters of the system, the system must be connected to a heat source at constant high temperature (generator) of infinite heat capacity to supply heat energy to the system so that after supply of heat energy to the system its temperature does not fall. Again, to reject the unwanted heat due to afore-mentioned causes, the system must be connected to a sink (absorber) at constant low temperature of infinite heat capacity so that after absorbing the extra unwanted heat energy from the system its temperature does not rise.

Figure 11 shows the regions of heat flows in (E, λ)-plane for magnetic $(M = 1)$ as well as nonmagnetic $(M = 0)$ cases when $\beta = P_{eff} = 1$. It is important to note that any pair of values of (E, λ) on the M-curves are the critical values, i.e., the M-curves demarcate the regions of heat transfers (or absorptions) from one side to the other. Actually, for any pair of values of (E, λ) on this curve, no heat flows from one side to the other, which means that the surface heat flux vanishes on the M-curves. We note that for any pair of values (E, λ) that is below this curve, heat flows from the surface to fluid; above this, the surface absorbs heat from the fluid body. As expected, the region of heat transfer from surface to fluid decreases with the increase of the magnetic field. This result is consistent with our earlier observation made in figure 6. One more important result that can also be found from this figure is that increasing

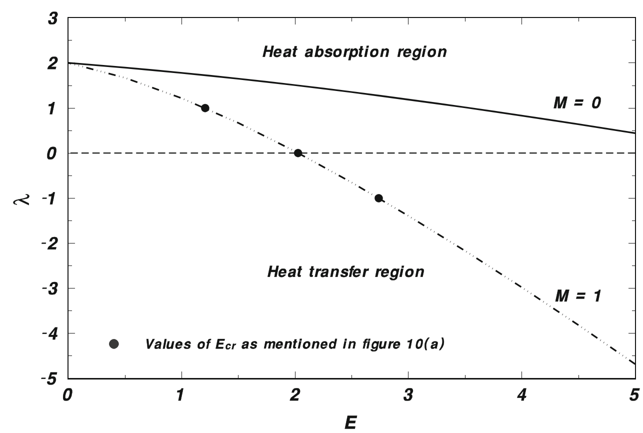

Figure 11. Heat flow regions in (E, λ)-plane for both magnetic M $(= 1)$ and nonmagnetic M $(= 0)$ cases. Other physical parameters have the following values: $\beta = 1$ and $P_{eff} = 1$. Any point on the M-curves has the corresponding critical values of (E, λ) for which no heat flows from one side to other. Heat transfer region decreases with the increase of the magnetic parameter M.

value of $E(> 0)$ enhances the effect of the magnetic interaction parameter M in the thermal boundary layer.

We conclude our discussion by making a few remarks about the penetration depth of velocity into the fluid body by the molecular diffusion of momentum. Numerical computations indirectly reveal the decreasing influence of both parameters M and β on the shear layer thickness $\bar{\delta}$ (see figures 3 and 4), whereas Eq. (15) ensures only the increasing influence of β on $\bar{\delta}$. In spite of the decreasing effect of β on the dimensionless distance η_s of η, the shear layer thickness $\bar{\delta}$ is enhanced rapidly with increase of β owing to the presence of the term $\sqrt{\beta(t_{ref} - \bar{t})}$ in Eq. (15). As a result, the velocity penetration into the fluid body becomes larger and larger with increasing values of β. On the other hand, the penetration depth of the velocity becomes shorter and shorter with increase of M. Hence we come to the conclusion that for a given value of β there is a large value M_0 of M beyond which the present unsteady boundary layer flow will cease. However, the solutions of the unsteady boundary layer equations, when they exist (i.e., when $M < M_0$), will be the exact solutions of the unsteady Navier–Stokes equations and have the boundary layer character.

6. Conclusion

In this paper, the flow behaviour and heat transfer characteristics of an incompressible viscous and electrically conducting fluid near a suddenly accelerated flat surface in the presence of a transverse magnetic field are investigated numerically. The present analysis is based on the solutions of the governing hydromagnetic equations with the assumption that the magnetic Reynolds number is very small. A new kind of similarity transformation is employed to investigate the impacts of the governing parameters on this flow model. A novel result of this analysis is that the present flow problem is possible up to a certain value of the magnetic interaction parameter M depending upon the values of the unsteadiness parameter β. We have established that the solutions of the governing hydromagnetic equations, when they exist, are the exact solutions of the full Navier–Stokes equations and have the correct asymptotic behaviour. It is found that the thermal boundary layer is coupled with the momentum boundary layer via viscous dissipation. Hence, the absence of viscous dissipation decouples the temperature field from the velocity field, for which the temperature field does not depend on the magnetic field. Another important result of this analysis is that the wall temperature gradient is enhanced/reduced with increase of the positive/negative values of Eckert number and this effect is successfully controlled by introducing adjustable amount of heat absorption/generation parameter into the system. Finally, we can conclude that the results obtained from the present analysis upgrade the understanding of the heat transfer characteristics, which is useful for controlling the rate of heat transferred/absorbed into the system.

Acknowledgements

The author expresses his very sincere gratitude to the editors and reviewers for their time and interest as well as valuable comments and constructive suggestions for improving this paper. The author would also like to thank Shreya Dholey and Anita Dholey for their kind cooperation during the work. The author acknowledges the financial assistance of Science and Engineering Research Board (SERB) of India through Grant No. EMR/2016/005533.

Nomenclature

$\mathbf{B}$ applied magnetic field
B_0 strength of the magnetic field
c_p specific heat
C dimensionless positive constant
C_f skin-friction coefficient
E Eckert number
E_{cr} critical value of E
f similarity function
k^* mean absorption coefficient
M interaction parameter (Hartmann number)
N radiation parameter
Nu Nusselt number
Pr Prandtl number
P_{eff} effective Prandtl number
q_r radiative heat flux
q_w surface heat flux
Q_0 heat generation/absorption coefficient
Q_s volumetric rate of heat generation/absorption
R_m magnetic Reynolds number
t time
t_{ref} constant reference value of t
T temperature of the fluid within the viscous layer
T_{ref} constant reference value of T
T_w wall temperature
T_∞ ambient fluid temperature
u, v components of velocity in x- and y-directions, respectively
U velocity of the plate
U_0 velocity scale
x, y coordinates along and transverse to the plate surface, respectively

Greek symbols

α thermal diffusivity of the fluid
β unsteadiness parameter
λ heat source/sink parameter
δ penetration depth of the velocity into the fluid body
η dimensionless distance normal to the plate surface
η_s corresponding value of η where the wall effect on the fluid has dropped to one per-cent
ν kinematic coefficient of viscosity
ψ stream function of the fluid
σ electrical conductivity of the fluid
σ^* Stefan–Boltzmann constant
ρ density of the fluid
θ dimensionless temperature of the fluid

Subscripts

cr critical
ref reference
w wall

Superscripts

$'$ differentiation with respect to η

$\dot{}$ differentiation with respect to $\bar{t}$
$\bar{}$ dimensionless quantities

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