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# Longitudinal oscillations of a circular cylinder in a micro-polar fluid: case of resonance

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**Abstract.** The problem of the longitudinal oscillations of a circular cylinder along its axis of symmetry in an incompressible micro-polar fluid and the flow generated due to these oscillations in the fluid is considered. The Stokes flow is considered by neglecting nonlinear convective terms in the equations of motion on the assumption that the flow is so slow that oscillations' Reynolds number is less than unity. Here we get a rare, but distinct special case referred to as *resonance* in which material constants are interrelated in a particular way. In *non-resonance* case, all material constants are independent and are not related. The solution in this case cannot be obtained as limiting case of a non-resonance problem. The velocity and micro-rotation components of the flow for the case of *resonance* and *non-resonance* are obtained. The skin friction acting on the cylinder is evaluated and the effect of physical parameters like micro-polarity and couple stress parameter on the skin friction due to oscillations is shown through graphs.

**Keywords.** Micro-polar fluids; longitudinal oscillations; resonance flow; skin friction.

## 1. Introduction

There is a vast literature available on Stokesian flows generated in micro-polar fluids over the past half a century ever since the classical theory of micro-polar fluids was formulated by Eringen [1]. With the aim of obtaining drag or couple, Lakshmana Rao *et al* in [2–5] studied the oscillatory flows in the case of a circular cylinder, sphere, spheroid and elliptic cylinder in incompressible micro-polar fluids. However, in all these above problems, a special case, named as "Resonance" type flow that arises when the material parameters of the fluids are related in a special form (will be defined later) have not been investigated until recently. The rare but distinct possibility of this type of resonance flows has been noticed in [4, 5] and the study of the solution in this particular case is mathematically tougher than in the usual case of non-resonance type flows. These resonance flows usually arise in non-Newtonian fluids whenever flows are generated due to oscillations of the bodies. For example, this case arises in the papers of [6–8], but these cases were not attempted by the authors. Recently Ramana Murthy *et al* [9, 10] investigated the resonance case of rectilinear oscillations of circular cylinder and sphere. In [11], the resonance case of rotary oscillations of the sphere is investigated. In the present paper, we propose to investigate this case of resonance type flow, in micro-polar fluids, due to longitudinal oscillations of a circular cylinder along its axis of symmetry.

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## 2. Mathematical formulation

### 2.1 *Basic equations*

The basic equations of an incompressible micro-polar fluid as introduced by Eringen [1] are given by:

2.1a *Continuity equation*

$$\nabla_1.\bar{Q} = 0 \tag{1}$$

2.1b *Linear Momentum equation*:

$$\rho\left(\frac{\partial \bar{Q}}{\partial \tau} + \bar{Q}.\nabla_1\bar{Q}\right) = -\nabla_1 P + k\nabla_1 \times \bar{l} - (\mu + k)\nabla_1 \times \nabla_1 \times \bar{Q} \tag{2}$$

2.1c *Angular momentum equation*

$$\rho J\left(\frac{\partial \bar{l}}{\partial \tau} + \bar{Q}.\nabla_1\bar{l}\right) = -2k\bar{l} + k\nabla_1 \times \bar{Q} - \gamma\nabla_1 \times \nabla_1 \times \bar{l} + (\alpha + \beta + \gamma)\nabla_1(\nabla_1.\bar{l}) \tag{3}$$

where $\bar{Q}$, $\bar{l}$ are fluid velocity and micro-rotation vectors, $\rho$ is density, $\tau$ is time, $J$ is gyration coefficient, $\mu$ is viscosity coefficient, $k$ is micro-viscosity coefficient and $\alpha$, $\beta$, $\gamma$ are couple stress viscosity coefficients. For micro-polar fluids, the stress components $T_{ij}$ and couple stress components $M_{ij}$ satisfy the following constitutive equations.

$$T_{ij} = -P\delta_{ij} + \frac{1}{2}(2\mu + k)\left(Q_{i,j} + Q_{j,i}\right) + ke_{ijr}(w_r - l_r) \quad (4)$$

$$M_{ij} = \alpha l_{i,i}\delta_{i,j} + \beta l_{i,j} + \gamma l_{j,i} \quad (5)$$

where $w_r$ = r th component of ½ (curl $\bar{Q}$) and $e_{ijr}$ is permutation tensor = 0 if any two indices are equal and = 1 if i, j, r are cyclic and = −1 if i, j, r are acyclic.

Neglecting the nonlinear convective terms in Eqs. (2) and (3), the linearised version of the equations are given by,

$$div\bar{Q} = 0 \quad (6)$$

$$\rho\frac{\partial\bar{Q}}{\partial\tau} = -\nabla_1 P + k\nabla_1 \times \bar{l} - (\mu + k)\nabla_1 \times \nabla_1 \times \bar{Q} \quad (7)$$

$$\rho J\frac{\partial\bar{l}}{\partial\tau} = -2k\bar{l} + k\nabla_1 \times \bar{Q} - \gamma\nabla_1 \times \nabla_1 \times \bar{l} + (\alpha + \beta + \gamma)\nabla_1(\nabla_1.\bar{l}) \quad (8)$$

## 2.2 *Statement of the problem*

A circular cylinder of radius $a$ and, of infinite length is performing longitudinal oscillations with velocity $W_0e^{i\sigma\tau}$along its axis of symmetry in an infinite vat containing incompressible micro-polar fluid (see figure 1). A cylindrical coordinate system $(R, \theta, Z)$ with base vectors $(\boldsymbol{e_R}, \boldsymbol{e_\theta}, \boldsymbol{e_Z})$ with origin on the axis of the cylinder is considered. Since the flow is generated by these oscillations, the fluid velocity will be in the crosssectional plane of the cylinder containing the base vectors $(\boldsymbol{e_R}, \boldsymbol{e_Z})$. We assume the flow is axially symmetric and hence the velocity and micro-rotation are assumed in the following form:

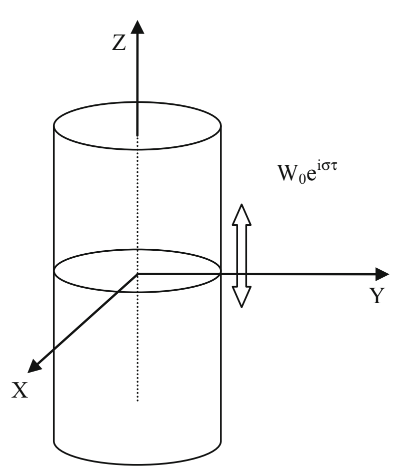


**Figure 1.** Geometry of the problem.

$$\boldsymbol{Q} = e^{i\sigma\tau}W(R)\boldsymbol{e_z} \quad \text{and} \quad \boldsymbol{l} = e^{i\sigma\tau}\mathcal{B}(R)\boldsymbol{e_\theta} \quad (9)$$

The following non-dimensional scheme is introduced. Capitals and LHS terms indicate physical quantities and small letters and RHS terms indicate the corresponding non-dimensional quantities.

$$\mathrm{R} = ar, \mathrm{W} = w\mathrm{w}_0, \mathbf{Q} = \boldsymbol{q}\mathrm{w}_0, \mathcal{B} = \mathrm{B}\sigma, \boldsymbol{l} = \boldsymbol{\upsilon}\frac{\boldsymbol{w_0}}{\boldsymbol{a}}, \quad \mathrm{P} = p\rho\mathrm{w}_0^2, \mathrm{Z} = az \text{ and } \tau = \frac{at}{\mathrm{w}_0} \quad (10)$$

The following non-dimensional parameters are introduced viz, $j$ gyration parameter, $\varpi$ frequency parameter, $s$ couple stress parameter, $c$ cross viscosity or micro-polarity parameter and $R_0$ oscillations Reynolds number for micro-polar fluids.

$$j = \frac{\mathrm{J}\rho\mathrm{w}_0a}{\gamma}, \varpi = \frac{\mathrm{a}\sigma}{\mathrm{w}_0}, s = \frac{\mathrm{k}a^2}{\gamma}, c = \frac{\mathrm{k}}{\mu + \mathrm{k}} \quad \text{and} \quad R_0 = \frac{\rho w_0 a}{\mu + k} \quad (11)$$

Substituting this non-dimensional scheme Eq. (10) and non-dimensional parameters Eq. (11), the equations of motion are reduced to

$$R_0\frac{\partial\bar{q}}{\partial t} = -R_0\nabla p + c\nabla \times \bar{\upsilon} - \nabla \times \nabla \times \bar{q} \quad (12)$$

$$\frac{j}{\varpi}\frac{\partial\bar{\upsilon}}{\partial t} = -2s\bar{\upsilon} + s\nabla \times \bar{q} - \nabla \times \nabla \times \bar{\upsilon} \quad (13)$$

Further, by the choice of the velocity field in Eq. (9), the equations of motion are reduced to as

$$iR_0\varpi w = -R_0p_0 + \frac{c}{r}\frac{d}{dr}(rB) + \frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) \quad (14)$$

$$ijB = -2sB - s\frac{dw}{dr} + \frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rB)\right) \quad (15)$$

where $p_0$ constant pressure gradient along z-direction.

From Eq. (14)

$$-\frac{c}{r}\frac{d}{dr}(rB) = -R_0p_0 + w'' + \frac{1}{r}w' - i\varpi R_0w \quad (16)$$

From Eq. (15)

$$(2s + ij)B = -sw' + D^2B \quad (17)$$

where $D^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}$

Differentiating Eq. (16) to eliminate $p_0$, we get

$$-cD^2B = \frac{d}{dr}\left(w'' + \frac{1}{r}w'\right) - i\varpi R_0w' \quad (18)$$

Substituting Eq. (18) in Eq. (17), we get

$$c(2s+ij)B = -\frac{d}{dr}\left(w'' + \frac{1}{r}w'\right) + (i\varpi R_0 - cs)w' \quad (19)$$

Eliminating B from Eqs. (18) and (19) we get

$$\left(D^2-\lambda_1^2\right)\left(D^2-\lambda_2^2\right)w' = 0 \quad (20)$$

where

$$\begin{aligned}\lambda_1^2+\lambda_2^2 &= (2-c)s + i(j+\varpi R_0) \quad \text{and} \\ \lambda_1^2\lambda_2^2 &= i\varpi R_0(2s+ij)\end{aligned} \quad (21)$$

The solution for $w'$ if $\lambda_1 \neq \lambda_2$ in Eq. (20) is given in [8]. The solution for w′ for the case, $\lambda_1 = \lambda_2$ cannot be obtained as a limiting case of $\lambda_1 \rightarrow \lambda_2$. This case is referred to as "*Resonance*". This resonance occurs if the material coefficients follow the following relation:

$$\frac{\gamma}{J} = \frac{(2\mu+k)(\mu+k)}{2\mu+3k} \quad \text{and} \quad \rho\sigma = \frac{(2\mu+k)k+\gamma\rho\sigma}{J(\mu+k)}$$

In non-dimensional form, these conditions are given by

$$(2-c)s = j - R_0\varpi \quad \text{and} \quad (2-c)j = (2+c)\varpi R_0 \quad (22)$$

If $\lambda_1 = \lambda_2$, then the two solutions $w_1$ and $w_2$ will be equal and cannot be independent. Hence we need special attention for the case of resonance

In this paper, we are interested in the solution for *w* and B for the case of *resonance*. In this case, the equations for *w* and B are given by

$$\left(D^2-\lambda^2\right)^2 w' \quad \text{and} \quad cB = -\frac{i\varpi R_0}{\lambda^4}(D^2-2\lambda^2)w' - w' \quad (23.1)$$

and for the case of non-resonance

$$\begin{aligned}&\left(D^2-\lambda_1^2\right)\left(D^2-\lambda_2^2\right)w' = 0 \quad \text{and} \\ &cB = -\frac{i\varpi R_0}{\lambda_1^2\lambda_1^2}(D^2-\lambda_1^2-\lambda_2^2)w' - w'\end{aligned} \quad (23.2)$$

Substituting this again in Eq. (14), we get

$$iR_0\varpi w = -R_0 p_0 - \frac{i\varpi R_0}{\lambda^4 r}\frac{d}{dr}\left\{r\left(D^2-2\lambda^2\right)w'\right\} \quad (24.1)$$

and for the case of non-resonance

$$iR_0\varpi w = -R_0 p_0 - \frac{i\varpi R_0}{\lambda_1^2\lambda_2^2 r}\frac{d}{dr}\left\{r\left(D^2-\lambda_1^2-\lambda_2^2\right)w'\right\} \quad (24.2)$$

## 2.3 *Boundary conditions*

By no-slip condition, (the non-dimensional velocity of a fluid particle on the circular cylinder Γ is same as the velocity of cylinder i.e., w=1) and by hyper-stick condition, (the micro-rotation vector of a particle on the cylinder is ½ of angular velocity of the particle on the cylinder i.e., B = ½ (Curl $Q_\Gamma)_\theta$ (where the suffix represents the component along that direction θ) i.e., B=0 and hence we have;

$$\text{On} \quad r=1, \quad w=1 \quad \text{and} \quad B=0 \quad (25)$$

# 3. Solution of the problem

## 3.1 *Case of non-resonance*

For a solution of Eq. (20), $w'$ is assumed in the form

$$w' = c_1 w_1' + c_2 w_2' \quad (26)$$

where

$$\left(D^2-\lambda_1^2\right)w_1' = 0 \quad \text{and} \quad \left(D^2-\lambda_2^2\right)w_2' = 0 \quad (27.1)$$

This gives the solution as below

$$w_1' = K_1(\lambda_1 r) \quad \text{and} \quad w_2' = K_1(\lambda_2 r) \quad (28.1)$$

The following results are useful to note for the purpose of simplification.

$$D^2 w_1' = \lambda_1^2 w_1' \quad \text{and} \quad D^2 w_2' = \lambda_2^2 w_2' \quad (29.1)$$

Substituting these solutions of Eq. (28.1) in Eq. (23.1) we get,

$$i\varpi R_0 w = -\frac{p_0}{i\varpi} + \frac{c_1}{\lambda_1}K_0(\lambda_1 r) + \frac{c_2}{\lambda_2}.K_0(\lambda_2 r)] \quad (30.1)$$

$$cB = \left(\frac{i\varpi R_0}{\lambda_1^2} - 1\right)c_1 K_1(\lambda_1 r) + \left(\frac{i\varpi R_0}{\lambda_2^2} - 1\right)c_2 K_1(\lambda_2 r) \quad (30.2)$$

From Eqs. (28.1) and (23.1), the conditions for $c_1$, $c_2$ are given by

$$\begin{bmatrix}\lambda_2^2 K_0(\lambda_1) & \lambda_1^2 K_0(\lambda_2) \\ a_1 K_1(\lambda_1) & a_2 K_1(\lambda_2)\end{bmatrix}\begin{bmatrix}c_1 \\ c_2\end{bmatrix} = \begin{bmatrix}i\varpi + p_0 \\ 0\end{bmatrix} \quad (31.1)$$

## 3.2 *Case of resonance*

For a solution of Eq. (20), $w'$ is assumed in the form

$$w' = c_1 w_1' + c_2 w_2' \quad (26)$$

where

$$\left(D^2-\lambda^2\right)w_1' = 0 \quad \text{and} \quad \left(D^2-\lambda^2\right)^2 w_2' = 0 \quad (27.2)$$

This will yield the solutions as below:

$$w'_1 = K_1(\lambda r) \quad \text{and} \quad w'_2 = \lambda r K'_1(\lambda r) \tag{28.2}$$

The following results are useful to note for the purpose of simplifications.

$$D^2 w'_1 = \lambda^2 w'_1 \quad \text{and} \quad D^2 w'_2 = \left(2\lambda^2 w'_1 + \lambda^2 w'_2\right) \tag{29.2}$$

Substituting these solutions of Eq. (28.1) in Eq. (23.1) we get, for the case of resonance as

$$i\varpi R_0 w = -R_0 p_0 + \frac{i\varpi R_0}{\lambda^2}\{(c_1 - 2c_2)\lambda K_0(\lambda r) + c_2.\lambda[K_0(\lambda r) + \lambda r K_1(\lambda r)]\} \tag{30.1}$$

$$cB = \left(\frac{i\varpi R_0}{\lambda^2} - 1\right)\left(c_1 K_1(\lambda r) + c_2 \lambda r K'_1(\lambda r)\right) - \frac{2c_2 i\varpi R_0}{\lambda^2}.K_1(\lambda r) \tag{30.2}$$

The constants $c_1$, $c_2$ are obtained from the boundary conditions Eq. (25) as follows:

$$\begin{bmatrix} \frac{K_0(\lambda)}{\lambda} & \frac{\lambda K_1(\lambda) - K_0(\lambda)}{\lambda} \\ K_1(\lambda) & \lambda K'_1(\lambda) + \frac{2i\varpi R_0}{\lambda^2 - i\varpi R_0} K_1(\lambda) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 - \frac{ip_0}{\varpi} \\ 0 \end{bmatrix} \tag{31.2}$$

where $a_1 = \frac{i\varpi R_0}{\lambda_1^2} - 1$ and $a_2 = \frac{i\varpi R_0}{\lambda_2^2} - 1$

From Eq. (31) we can calculate $c_1$ and $c_2$. Hence w′ is known.

## 4. Skin friction acting on the cylinder per length L

Skin friction acting on the circular cylinder

$$c_f = \frac{2T_{rz}}{\rho w_0^2} \tag{32}$$

From Eq. (4), we get

$$T_{rz} = \left\{(2\mu + k)\frac{1}{2}\frac{\partial w}{\partial r} + k\left(\frac{1}{2}\frac{\partial w}{\partial r} + B\right)\right\}e^{i\varpi t} = \frac{(\mu + k)W_0}{a}\left\{\frac{dw}{dr} + cB\right\}e^{i\varpi t} \tag{33}$$

Substituting Eq. (33) in Eq. (32), the skin friction acting on the circular cylinder (after deleting the factor $e^{i\varpi t}$) is obtained as:

$$c_f = \frac{2}{R_o}\left(\frac{dw}{dr}\right)_{r=1}, \quad c_f = \frac{2}{R_0}\left[c_1 K_1(\lambda) + c_2 \lambda K'_1(\lambda)\right] \tag{34.1}$$

In the non-resonance case, the skin friction is given by

$$c_f = \frac{2}{R_0}[c_1 \mathrm{K}_1(\lambda_1) + c_2 K_1(\lambda_2)] \tag{34.2}$$

## 5. Results and discussions

For resonance case, the value of λ cannot be taken randomly. In the case of resonance, the values of λ are obtained from Eq. (16) by solving the following equation for x.

$$x^2 - [(2 - c)s + i(j + \varpi R_0)]x + i\varpi R_0(ij + 2s) = 0$$

Then in the resonance case, the values of λ are given by

$$\lambda = \sqrt{x} = \sqrt{(2 - c)s + \frac{i(j + \varpi.R_0)}{2}} \tag{35}$$

This equation involves 5 parameters which are related by two equations in Eq. (15). Hence we choose three parameters as an independent. Here $\varpi$, $R_0$ and c are selected independently, with $0 \leq c \leq 1$, $R_0 \ll 1$ and $\varpi \gg 1$ such that $\varpi.R_0$ is not negligibly small (say>1). For this range of values of $R_0$, the nonlinear convective terms can be neglected but local derivative is retained. After selecting $c$, $R_0$ and $\varpi$, the values of $s$ and $j$ are obtained from Eq. (15) and then λ is obtained from Eq. (35). In the case of non-resonance, all 5 parameters are independent. The values of λ are complex. These values for λ are substituted in Eq. (31) and the constants $c_1$ and $c_2$ are obtained.

### 5.1 *Velocity*

It can be observed from figure 2 that as Reynolds number increases, the velocity w decreases near to the cylinder and then increases slightly and tends to zero at a distance four times the radius of the cylinder. In the case of resonance, the velocity w becomes zero at a longer distance than in the case of non-resonance. *Hence observe that Reynolds number, in the case of resonance, decreases the velocity w near to the cylinder and velocity vanishes at a longer distance than in the case of non-resonance*.

Similarly, from figure 3, as micro-polarity parameter $c$ increases, in the case of resonance, velocity w increases in the range of distance from 1.5 to 2.5. Whereas the effect of micro-polarity $c$ is negligible in the case of non-resonance. *Hence, the conclusion is that the micropolarity parameter increases the velocity in the case of resonance and has no effect on velocity in the non-resonance case.*

Velocity w in the direction Z axis is computed by using Eq. (30.1).

### 5.2 *Microratation*

It is observed that micro-rotation component B is positive, in the case of resonance, and becomes zero at

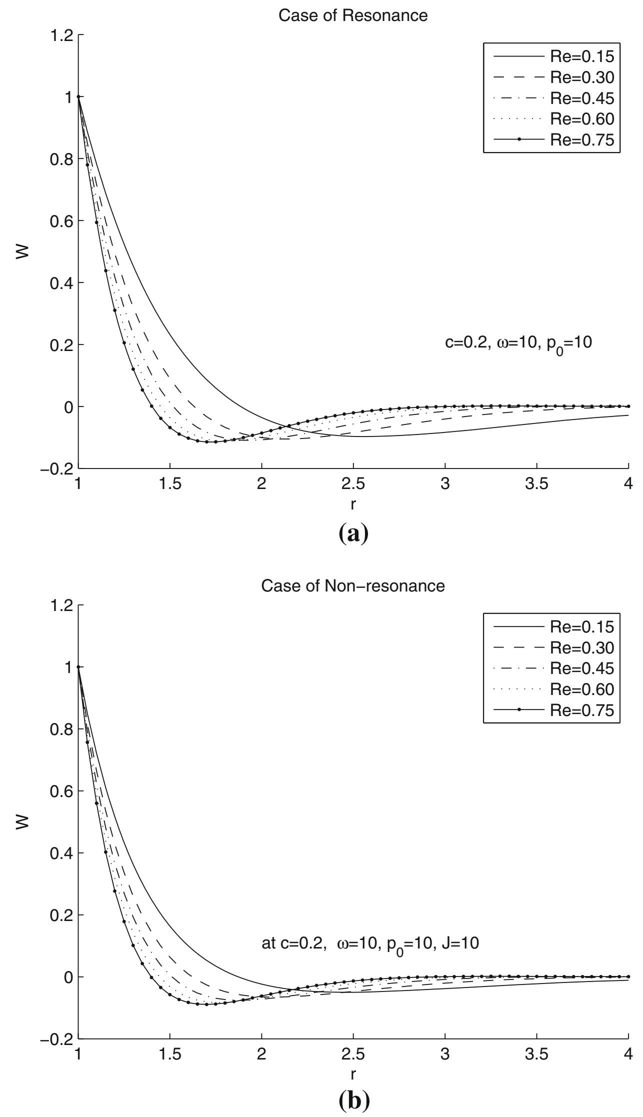


**Figure 2.** Velocity at different values of Reynolds numbers for the case of (**a**) resonance and (**b**) non-resonance.

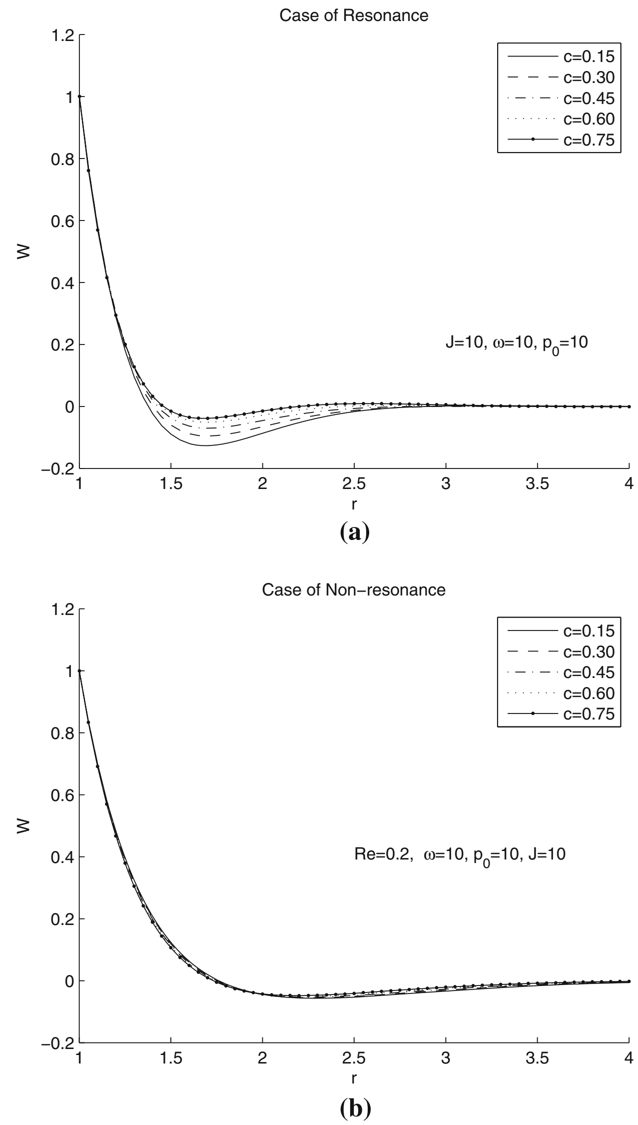


**Figure 3.** Velocity at different values of micro-polarity parameter $c$ for the case of (**a**) resonance and (**b**) non-resonance.

a long distance from the cylinder. We notice from figure 4 that as Reynolds number increases, in the case of resonance, micro-rotation increases near to the cylinder. But in the case of non-resonance, as Reynolds number increases, micro-rotation increases from negative values to positive values and then soon becomes zero. *It can be concluded that in the case of resonance, micro-rotation vanishes at a long distance from the cylinder and in the case of non-resonace, it vanishes relatively near to the cylinder.*

It is observed from figure 5 that in the case of resonance, as micro-polarity increases, micro-rotation increases and is always positive. But in the case of non-resonance, micro-rotation decreases and increases from negative values to positive values and the effect of micro-polarity on micro-rotation is almost negligible.

*The difference between resonance and non-resonance is not very much clear for velocity, but for micro-rotation vector, the difference is dominating.*

## 5.3 *Skin friction*

We can observe from figure 6 that resonance flow occurs only a particular range of gyration parameter J and Re.

Skin friction is much smaller in the case of resonance and as Gyration parameter increases, skin friction decreases. But in the case of non-resonant, as gyration parameter increases, skin friction also increases. Resonance decreases the skin friction drastically to a low value (from 500 non-resonance range to 30 s resonance). *It is noticed that as gyration parameter j increases, in the case of resonance,*

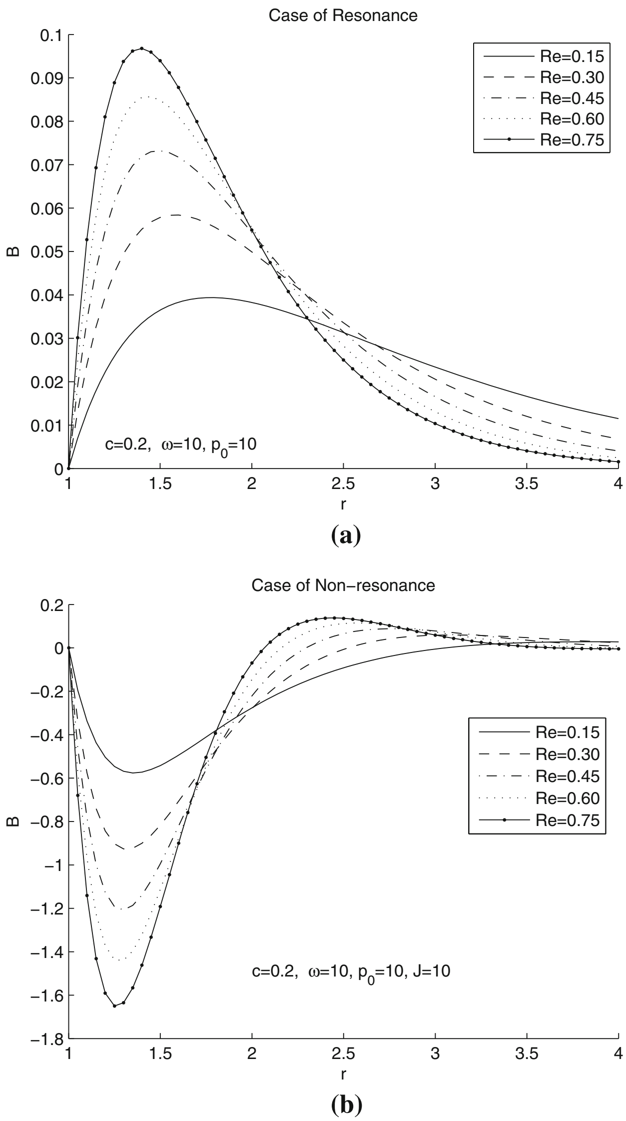


**Figure 4.** Micro-rotation at different values of Reynolds number for the case of (**a**) resonance and (**b**) non-resonance.

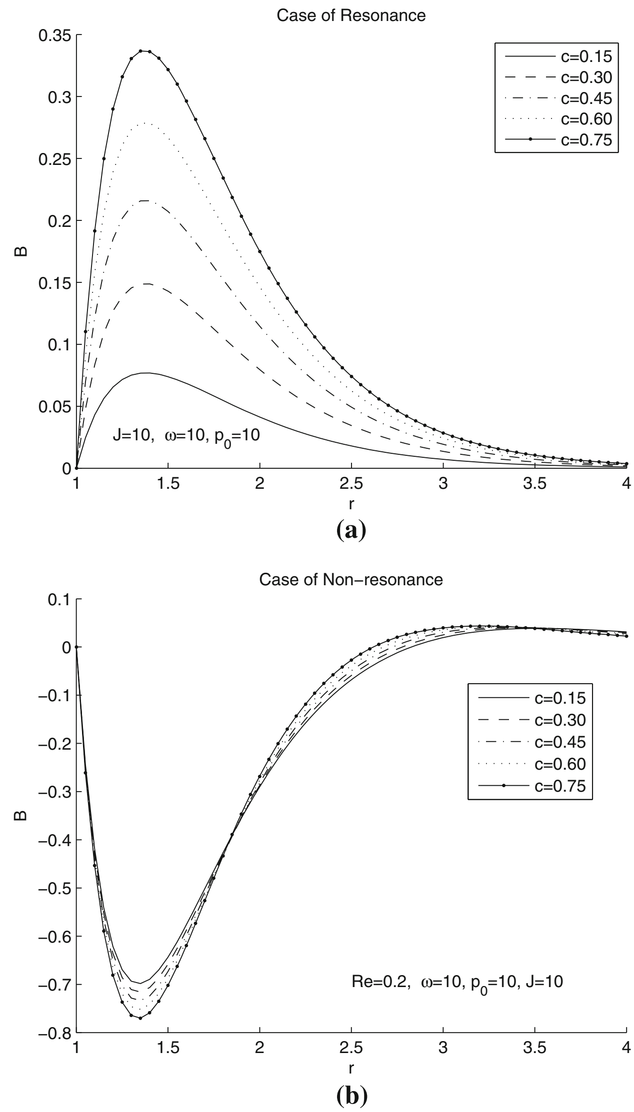


**Figure 5.** Micro-rotation at different values of micro-polarity parameter $c$ for the case of (**a**) resonance and (**b**) non-resonance.

*skin friction decreases and in the case of resonance, skin friction increases.*

*Physically we assume that since in the case of resonance, micro-rotation vector takes only positive values, much energy is saved. while in the case of micro-rotation changing its direction of flow, much energy is spent. This affects skin friction to reduce much for the case of resonance.*

It is observed from figure 7 that skin friction is not affected much by variation in micro-polarity in the case of resonance. But opposite to this in the case of non-resonance, as micro-polarity increases, skin friction decreases drastically. In figure 8, as Reynolds number increases, skin friction decreases. This is expected, since in the formula Eq. (34), Reynolds number is in the denominator. It is very interesting to note that the skin friction in the case of resonance is much smaller than in the case of non-resonance.

Hence the conclusion is that as Reynolds number or micro-polarity increases, skin friction decreases but the case of resonance offers less resistance for the flow and hence skin friction is very much lesser than in the case of non-resonance.

## 6. Conclusions

(1) The case of resonance makes the micro-rotation as unidirectional (i.e., positive only). In non-resonance, micro-rotation raises from negative values to positive values and then vanishes.

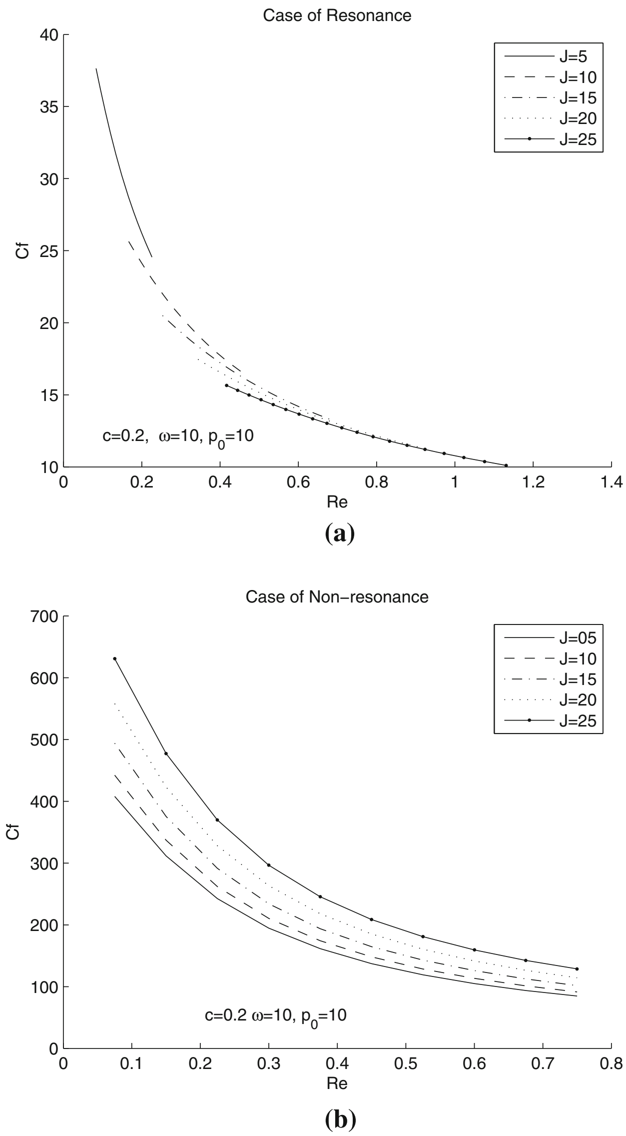


**Figure 6.** Skin friction at different values of gyration parameter J for the case of (**a**) resonance and (**b**) non-resonance.

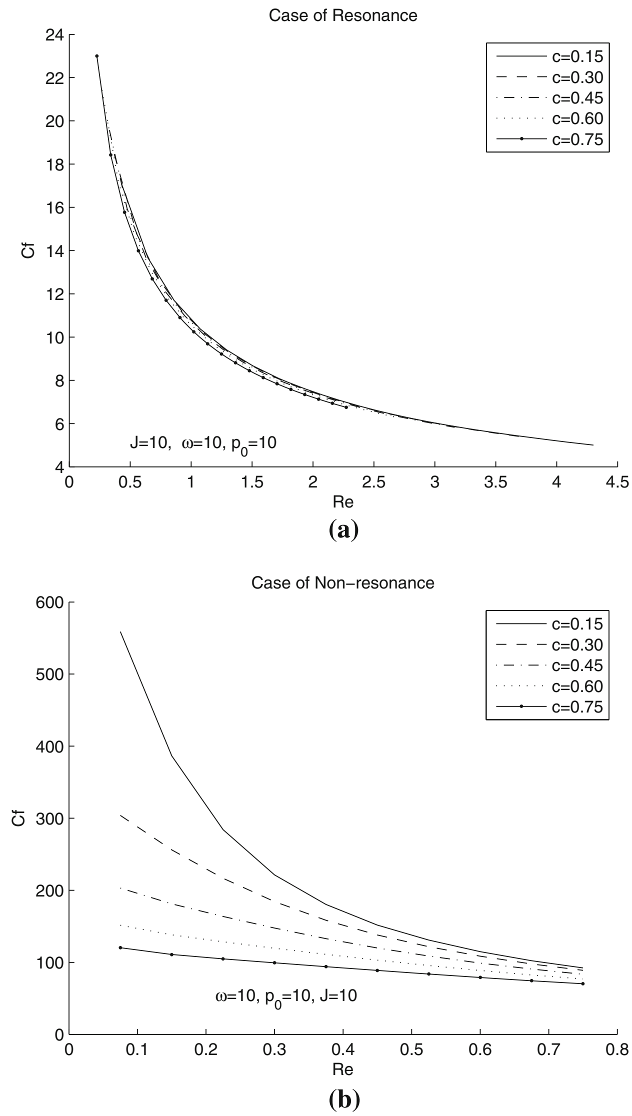


**Figure 7.** Skin friction at different values of micro-polarity $c$ for the case of (**a**) resonance and (**b**) non-resonance.

(2) The case of resonance offers less resistance to the flow and hence decreases skin friction.
(3) These above two observations are very important to focus our attention on the case of "resonance". They may have an Industrial application, for producing a suitable micro-polar fluid to get minimum skin friction.

## Nomenclature

| | |
|---|---|
| $\bar{Q}$ | Fluid velocity vector ($ms^{-1}$) |
| $\bar{l}$ | Micro-rotation vector |
| $\rho$ | Density of the fluid (kg $m^{-3}$) |
| $\tau$ | Time (s) |
| $P$ | Fluid pressure at any point (kg $m^{-1}$ $s^{-2}$) |
| $W$ | Velocity component (m $s^{-1}$) |
| $\mathcal{B}$ | Micro-rotation component |
| $\sigma$ | Frequency parameter ($s^{-1}$) |
| $J$ | Gyration coefficient (kg m $s^{-1}$) |
| $\bar{q}$ | Non dimensional Fluid velocity vector |
| $\bar{\upsilon}$ | Non dimensional Micro-rotation vector |
| $t$ | Non dimensional time |
| $p$ | Non dimensional Fluid pressure at any point |
| $w$ | Non dimensional Velocity component |
| $B$ | Non dimensional Micro-rotation components |
| $\varpi$ | Non dimensional Frequency parameter |
| $j$ | Non dimensional Gyration coefficient |
| $\mu$ | Viscosity coefficient (kg $m^{-1}$ $s^{-1}$) |
| $k$ | Micro-viscosity coefficient (kg $m^{-1}$ $s^{-1}$) |
| $\alpha, \beta, \gamma$ | Couple-stress viscosity coefficients (kg $m^{-1}$ $s^{-1}$) |
| $T_{ij}$ | Stress components |

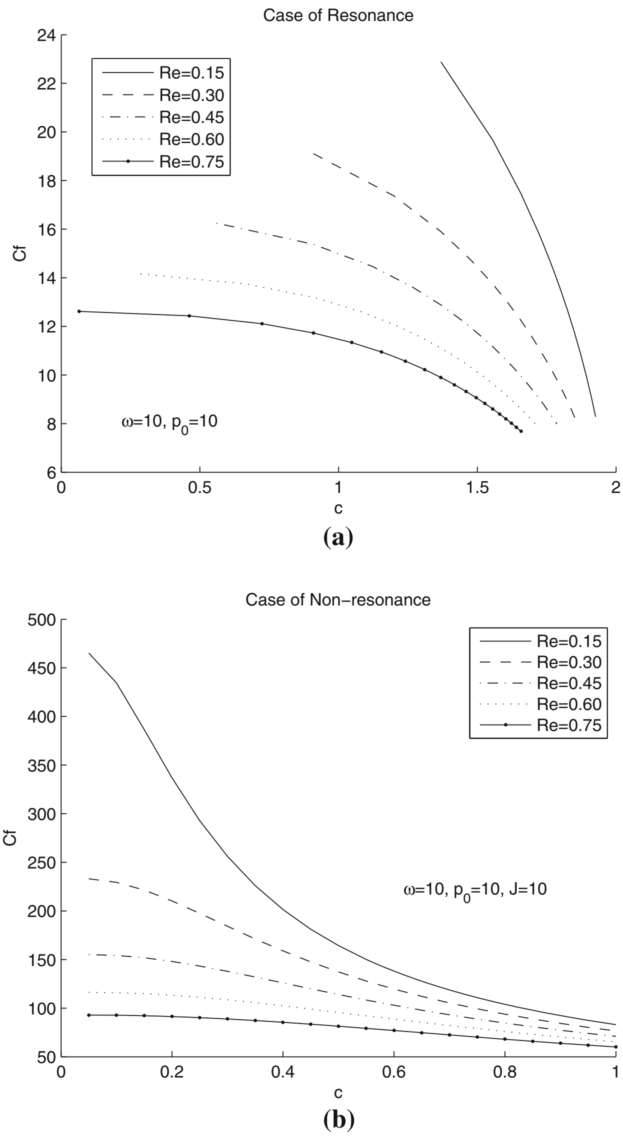


**Figure 8.** Skin friction at different values of Reynolds number Re for the case of (**a**) resonance and (**b**) non-resonance.

| | |
|---|---|
| $M_{ij}$ | Couple-stress components |
| $s$ | Couple-stress parameter for micro-polar fluid |
| $c$ | Cross viscosity coefficient or micro-polarity parameter |
| $R_0$ | Oscillations Reynolds number |

## References

[1] Eringen A C 1966 Theory of micropolar fluids. *J. Math. Mech.* 16: 1–18

[2] Lakshmana Rao S K and Bhuganga Rao P 1971 The oscillations of a sphere in a micro-polar fluid. *I.J.E.S.* 9: 651–672

[3] Lakshmana Rao S K and Bhuganga Rao P 1972 Circular cylinder oscillating about a mean position in an incompressible micro-polar fluid. *I.J.E.S.* 10: 185–191

[4] Lakshmana Rao S K and Iyengar T K V 1983 Rotary oscillations of a spheroid in an incompressible micro-polar fluid. *I.J.E.S.* 21: 973–987

[5] Lakshmana Rao S K, Iyengar T K V and Venkatapathi Raju K 1987 The rectilinear oscillations of an elliptic cylinder in an incompressible micro-polar fluid. *I.J.E.S* 25: 531–548

[6] Ramana Murthy J V, Sai K S and Bahali N K 2011 Steady flow of micropolar fluid in a rectangular channel under transverse magnetic field with suction. In: *AIP ADVANCES*1, 032123, pp. 1–10

[7] Aparna P and Ramana Murthy J V 2012 Rotary oscillations of a permeable sphere in an incompressible micropolar fluid. *Int. J. Appl. Math. Mech.* 8: 79–91

[8] Nagaraju G and Ramana Murthy J V 2014 Unsteady flow of a micropolar fluid generated by a circular cylinder subject to longitudinal and torsional oscillations. *Theor. Appl. Mech. TEOPM7* 41: 71–91

[9] Ramana Murthy J V, Bhaskara Rao G S and Govinda Rao T 2014 Resonance flow due to rectilinear oscillations of a circular cylinder in a micropolar fluid. In: *59th Proceedings of ISTAM*, pp. 1–7

[10] Ramana Murthy J V, Bhaskara Rao G S and Govinda Rao T 2015 Resonance flow due to rectilinear oscillations of a sphere in a micropolar fluid. *J. Phys. Conf. Ser.* 662: 1–8

[11] Ramana Murthy J V, Bhaskara Rao G S and Govinda Rao T 2015 Resonance flow due to rotary oscillations of a sphere in a micropolar fluid. *Proc. Eng.* 127: 1323–1329