A data envelopment analysis approach for resource allocation with undesirable outputs: an application to home appliance production companies

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Abstract. Traditional data envelopment analysis (DEA) models use all multiple inputs and outputs to estimate efficiency scores of decision-making units (DMUs). Each unit may consist of several subunits in cases such as manufacturing systems, and each subunit may produce both desirable and undesirable outputs. Providing information about the proportion of resources for each subunit could assist managers in making better decisions for increasing the efficiency of production systems. The current study proposes a new approach for resource allocation and efficiency estimation of production units by considering partial impacts among inputs and outputs in the DEA framework. A weak disposable technology is used in these evaluations, and an empirical application of the proposed approach for obtaining performance of home appliances production companies is provided for illustration purposes.

Keywords. Data envelopment analysis (DEA); partial impacts; undesirable outputs; weak disposability; resource allocation.

1. Introduction

Data envelopment analysis (DEA) was introduced by Charnes *et al* [1]. DEA is a practical tool for evaluating the performance of peer decision-making units (DMUs) that use multiple inputs to produce multiple outputs. In general expression, the aim is to minimize input and maximize output in classical production theory methods and in particular the expression in DEA. As discussed in Wu *et al* [2], due to resource constraints and unlimited needs of the community, the issue of optimal allocation of resources is raised. Optimal allocation of resources is the most important tool for implementing a long-term strategy and plan in any organization. In other words, the policies and goals of each organization's program are reflected in the allocation of resources in an optimal manner. The amount of achievement depends on how resources are allocated and controlled. One of the important issues that should be considered in the production process is the optimal allocation of resources with undesirable factors. In the real world applications, there may be both desirable and undesirable outputs in production processes such as factories, hospitals, greenhouses or powerhouses where several departments and organizations work in conjunction. Desirable outputs may include health services, crops,

power, etc. and undesirable outputs could include aerosols, waste and pollution. In production theory literature, many authors considered undesirable outputs in the production processes (DMUs) and they presented many approaches for measuring the efficiency of DMUs by consideration the optimal allocation of resources. The concept of optimal allocation of resource can be studied in different dimensions.

In most of DEA approaches, the resources allocation is in such a way that all inputs are used completely to produce good outputs and bad outputs. For example, Hailu and Veeman [3] extended the Chavas–Cox approaches [4] to the non-parametric analysis by incorporating undesirable outputs and applied their model to measuring the efficiency of Canadian pulp and paper industry. Kuosmanen [5] presented a production technology based on weak disposability assumptions to calculate the environmental performance of units with undesirable outputs. Zhou *et al* [6] suggested a non-radial DEA approach to measure the environmental performance of OECD countries. Bain and Yang [7] developed several DEA models for measuring the aggregated efficiency of resource and environment based on environment DEA technology, which can be provided with an index to analyse DMU energy efficiencies and environmental efficiencies simultaneously. A centralized resource allocation (CRA) DEA method for accidental data, namely SCRA model, was suggested by Hosseinzadeh Lotfi *et al*

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[8]. Their approach creates a stochastic DEA structure for CRA. Using their method, accidental propositions are transformed into decisive and analysable propositions. Then by using the decisive CRA models, the accidental propositions can be converted into efficient ones. Mandal [9] coined the term “energy efficiency” to calculate the performance of Indian cement industry. Wu *et al* [2] proposed a DEA approach for allocation of the resource by considering bad outputs. In their approach, two objective functions are modelled, i.e., maximization of all good outputs and minimization all bad outputs. Dakpo *et al* [10] explained the recent developments on the inclusion of undesirable outputs in production technology modelling, namely material balance principles and weak G-disposability, by-product modelling and cost disposability assumptions. For further reference see Yan *et al* [11], Färe and Grosskopf [12], Basso and Pecati [13], Hadi-Vencheh *et al* [14], Cecilio and Diego [15], Gomes and Lins [16], Lin *et al* [17], Korhonen and Syrjänen [18] and Yang *et al* [19].

In the recent DEA literature, some other approaches such as network DEA methods and some other methods were proposed for measuring the efficiency of the production process by considering the optimal allocation of resource in the presence of undesirable outputs. Some of them are as follows. Amirteimoori *et al* [20] offered a two-stage network structure with undrainable outputs and applied their model to evaluate the performance of Green Hen poultry chains in the Guilan province, Iran. Ming-Miin [21] proposed a centralized network DEA model that merges the centralized DEA model and network DEA for the allocation of resources between subunits. His approach considers that in the circumstances in which bad outputs are commonly generated by good outputs, the decrease of bad outputs is related to the decrease of inputs, and some inputs are appropriated to the special subunit while some inputs are allocated between subunits. Fukuyama and Weber [22] offered a two-stage network structure for estimating the performance of DMUs by considering that inputs are consumed to create only final outputs and they applied their model for calculating the inefficiency of Japanese banks. Wu *et al* [23] suggested two-stage network processes. In their approach, some of the resources are shared and resources are recovered by bad outputs. Also, the non-cooperative performance values are presented for the system. In addition, a heuristic algorithm is proposed for converting the nonlinear structure into a linear form. Mogale *et al* [24] proposed a two-stage model for transportation and allocation of food grain in the supply chain network structure in India.

The concept of resource allocation was also studied in supply chain management. Mogale *et al* [25] proposed a Mixed Integer Non-Linear Programming (MINLP) model for resource allocation. In their method, some decisions are taken for collecting the food grain in supply chain structure in India. Also, different operations like purchasing, collecting, transporting and distributing are considered in food grain supply chain of India. Their approach presents an

MINLP model for scheming in collecting food grain from surplus states to deficit states considering the seasonal procurement, silo capacity, demand satisfaction and vehicle capacity constraints. Mogale *et al* [26] presented a multi-period inventory transporting method for tactical scheming of food grain supply chain. In their approach, an MINLP model is modelled, which asks for minimizing the overall cost comprising those of the total of food grain goods, collection and execution

In assessing the performance of industrial units and supply chain, knowledge of environmental factors such as undesirable outputs play an important role. By studying previous methods for performance evaluation of DMUs in the presence of undesirable outputs, it is found that resources are shared equally among both desirable and undesirable outputs in all models. However, it is possible that there is some partial impact between inputs and good outputs and bad outputs.

The current work attempts to estimate the percentage of each resource that is used by each production line without considering any network structure. Gauging the partial impacts of inputs on desirable and undesirable outputs has been attempted with regard to the recent work by Imanirad *et al* [27]. Weak disposability assumption is applied as proposed by Kuosmanen [5] to suggest a novel DEA approach for estimating the aggregate efficiency of DMUs by taking undesirable outputs into account. The model of Kuosmanen [5] is an important development in calculating the efficiency of DMUs with undesirable outputs and his method was also studied in the following papers: Yang and Pollitt [28], Wu *et al* [2], Kuosmanen and Podinovski [29].

Our proposed approach makes it possible to estimate efficiencies and to analyse performances of production systems with partial impacts among inputs and outputs when both desirable and undesirable outputs coexist. An empirical application of the proposed approach for performance evaluation of 60 home appliance production companies is also provided, which demonstrates the applicability of the new approach in practical applications. Some of the novelty points that distinguish the proposed approach in this article from those of previous models are as follows:

- for the first time, a new approach is suggested in this paper to calculate the aggregate efficiency of production units by considering bad outputs;
- in the new proposed model, the percent of resources (inputs) that lead to good output and bad output in each subunit is estimated;
- the current model can provide estimation of subunit efficiency in each unit;
- the overall efficiency is estimated by combining subunit efficiency.

The rest of the paper is organized as follows. Section 2 presents a brief review of weakly disposable technology and an example of resource allocation in traditional DEA

models with undesirable outputs. A new DEA method is suggested for considering partial input–output impact in the presence of undesirable outputs in section 3, and section 4 is devoted to presenting an application of the new method to home appliance production companies. Further discussion is presented in section 5, and section 6 presents general conclusions and suggestions for future studies.

2. DEA resource allocation with undesirable output

A production process may include undesirable (bad) outputs as well as desirable (good) outputs. Several DEA models have been suggested for evaluating the performance of industrial units in the presence of undesirable outputs in the past decades. Kuosmanen [5] introduced a production technology based on the weakly disposable axiom of outputs for modelling undesirable outputs in DEA framework.

For a formal presentation, assume a set of J DMUs to be evaluated in terms of a set of I inputs ($x_i, i = 1, \dots, I$), M good outputs ($v_m, m = 1, \dots, M$) and S bad outputs ($w_s, s = 1, \dots, S$). Inputs and good outputs are assumed to be freely disposable under this setting. Desirable and undesirable outputs are weakly disposable and output set $\hat{P}(X)$ is assumed to be convex under a constant return to scale (CRS) assumption. The linear programming form of this weakly disposable technology for evaluating the performance of DMU_o is formulated as follows; for further reading see Kuosmanen and Kazemi Matin [30]:

$$\begin{aligned} \max \quad & \sum_{m=1}^M u_m v_{mj_o} - \sum_{s=1}^S l_s w_{sj_o} \\ \text{subject to} \quad & \sum_{m=1}^M u_m v_{mj} - \sum_{s=1}^S l_s w_{sj} - \sum_{i=1}^I d_i x_{ij} \leq 1 \quad j = 1, \dots, J \\ & \forall m, i : u_m, d_i \geq 0 \quad \forall s : l_s \text{ free in sign.} \end{aligned} \quad (1)$$

Figure 1 shows the default of how inputs affect desirable and undesirable outputs when using Model (1) and all traditional DEA models including undesirable outputs. It is assumed that there are three inputs: two good outputs and one bad output, in each DMU.

Figure 1. Impact of input on good and bad output in Model (1).

Note that all good and bad outputs are affected by all inputs in Model (1) as shown in figure 1. In other words, all inputs are shared equally among all good and bad outputs. It is possible to have situations where a unit has multiple subunits that use existing shared resources according to their needs in DEA applications. A simple example is used to illustrate this issue better.

2.1 Motivating example

Traditional models like Model (1) are useful for evaluating efficiency scores of DMUs where the entire output spectrum including both good and bad outputs is influenced by all inputs. Consider an industrial unit with two production lines as outputs. Undesirable outputs such as wasted products also coexist in the second production line. Therefore, managers should consider a recycling cost as input for waste products. Therefore, two inputs are considered in this evaluation: raw materials and recycling cost. In applying Model (1) for efficiency evaluation of units, the default interrelation between inputs and good and bad outputs is shown in table 1 and depicted in figure 2.

The following features of traditional DEA models such as Model (1) may be pointed out:

- as shown in figure 2, all good and bad outputs are assumed to be affected by all inputs, whereas recycling cost should have effects on only production line 2;
- provided efficiency score by Model (1) may underestimate the true efficiency score by considering the effect of recycling cost for both production lines.

As a result of these observations, it may be concluded that traditional DEA models are not able to calculate the correct efficiency of DMUs when partial impact between inputs and good and bad outputs exists.

Business units may have several production lines in practical applications where undesirable outputs affect only

Table 1. Interrelations between inputs and good and bad outputs in industrial units.

DMU	Production line 1 Good output	Production line 2	
		Good output	Bad output
Raw materials	✓	✓	✓
Recycling cost	✓	✓	✓

Figure 2. Default interrelations between inputs and good and bad outputs in industrial units.

some inputs but not outputs. Standard DEA models need to be modified before application to performance assessment in such cases.

An innovative DEA model is presented in the next section to estimate the efficiency score that takes into account both issues of undesirable outputs and the existence of shared resource impacts.

3. New proposed method

3.1 Structure of model

A practical application is considered in this section with bad outputs to present the idea of partial input–output impact. In this evaluation, there are 60 home appliance firms, which include three production lines. These production lines include both good outputs and bad outputs:

- Audio and video devices such as TVs, home theaters, camcorders, digital cameras, video projectors, etc.
- Heating and cooling devices such as air conditioners, fans, heaters, etc.
- Home appliance devices such as refrigerators, juicers, meat grinders, vacuum cleaners, cookers, washing machines, dishwashers, etc.

Furthermore, bad outputs may be considered as the number of waste byproducts that are present in each production line. For example, suppose there are 10 defective TVs or cameras out of each 100 assemblies in production line 1. Resource inputs are considered as (1) firm labour, (2) pressing machines, (3) pressing and (4) dyeing.

Table 2 shows the interrelationship between resources and production lines. As shown, not all production lines including

good and bad outputs are influenced by all resource inputs. In other words, each production line uses resources according to its needs. As an example, the usage of pressing may not be needed for producing heating and cooling devices, while pressing is required in the production of audio and video devices. The interrelation between resource inputs and production lines that include good and bad outputs could be classified as a bundle.

For formal presentation of the proposed model, suppose each firm contains k subunits so that each subunit k is represented by its own resource/production line bundle $(I_k, R_k(O_G, O_B))$ $k = 1, \dots, K$. Resource inputs are numbered as 1, 2, 3 and 4 for firm labour, pressing, packing machines and dyeing, respectively. Additionally, production lines are numbered as 1, 2 and 3, for audio and video devices, heating and cooling devices and home appliance devices, respectively. Home appliances firms have 2 subunits or bundles $(I_1, R_1(O_G, O_B))$, $(I_2, R_2(O_G, O_B))$, where $I_1 = (1, 2, 3, 4)$, $I_2 = (1, 3)$ and $R_1((1_G, \emptyset), (3_G, 3_B))$, $R_2(2_G, 2_B)$. It is assumed that production line 1 does not have any bad output in bundle $R_1((1_G, \emptyset), (3_G, 3_B))$.

Notations “ G ” and “ B ” for definitions of “good output” and “bad output” in bundle $R_k(O_G, O_B)$ are used from this point on.

It is further assumed that inputs are divisible and can be allocated between subunits. Split variables β_{ik} are defined for the amount of good and bad output usage from inputs, in the value $\beta_{ik}x_i$ where $\sum_k \beta_{ik} = 1$. Figure 3 shows the types

of resource allocation between good and bad outputs in production lines. An algorithm for creating maximal bundles in a general case is presented in the appendix.

A novel DEA approach is proposed for calculating the efficiency of DMUs in such situations and presented in the next section.

3.2 A new proposed method

A new model is suggested in this section for performance evaluation of home appliance companies including several production lines, subunits in the presence of undesirable outputs. The performances of DMUs are calculated in three steps: 1—aggregate efficiency, 2—subunit efficiency and 3—overall efficiency. Following three steps are proposed for achieving this goal:

Step 1. Split variables β_{ik} are defined for obtaining values for good and bad output usage from the input i in each bundle k . The aggregate efficiency of DMUs is then evaluated.

Step 2. Efficiencies for each production line subunit are estimated using split variables β_{ik} .

Step 3. Overall efficiency of the business unit under evaluation is assessed.

- Step 1: Calculating aggregate efficiency

It is supposed that a DMU is the sum of several sections, and there are no economies or diseconomies of scope. Aggregate efficiency is a weighted average of k subunit performances.

Table 2. Interrelation between production lines and resources.

Resources	Production lines		
	Production line 1 Audio and video devices (Including Good outputs)	Production line 2 Heating and cooling devices (Including Good and Bad outputs)	Production line 3 Home appliance devices (Including Good and Bad outputs)
Firm labour (input 1)	✓	✓	✓
Pressing (input 2)	✓	–	✓
Packing machines (input 3)	✓	✓	✓
Dyeing (input 4)	✓	–	✓

Figure 3. Resource allocation among subunits in the new model.

For this purpose, the CRS radial Model (1) is modified for deriving the β -split of inputs with the objective of maximizing this aggregate efficiency with undesirable outputs:

$$e_{agg} = \max \sum_{k=1}^K H_{kj} \left[\frac{\sum_{m \in R_k(O_G, O_B)} u_m v_{mj} - \sum_{s \in R_k(O_G, O_B)} l_s w_{sj}}{\sum_{i \in I_k} d_i \beta_{ik} x_{ij}} \right] \quad (2)$$

subject to

$$\sum_{k=1}^K H_{kj} \left[\frac{\sum_{m \in R_k(O_G, O_B)} u_m v_{mj} - \sum_{s \in R_k(O_G, O_B)} l_s w_{sj}}{\sum_{i \in I_k} d_i \beta_{ik} x_{ij}} \right] \leq 1, \quad \forall j \quad (2.1)$$

$$\frac{\sum_{m \in R_k(O_G, O_B)} u_m v_{mj} - \sum_{s \in R_k(O_G, O_B)} l_s w_{sj}}{\sum_{i \in I_k} d_i \beta_{ik} x_{ij}} \leq 1, \quad \forall k, j \quad (2.2)$$

$$\sum_{k \in T_i} \beta_{ik} = 1, \quad \forall i \quad (2.3)$$

$$a_{ik} \leq \beta_{ik} \leq b_{ik}, \quad \forall i, k \quad (2.4)$$

$$u_m, d_i, \beta_{ik} \geq \varepsilon \quad \forall m, i, k \quad (2.5)$$

$$l_s \text{ free in sign} \quad \forall s. \quad (2.6)$$

The notation e_{agg} is used and it indicates the mean “aggregate” efficiency score for the units under study. Further, constraints (2.2) defined for the variable β_{ik} should be selected in such a way that the efficiency score of each subunit k of DMU j does not intrude on the unity of corresponding values of weights u_m , d_i and l_s . Constraints (2.3) impose the convexity restriction on β_{ik} values among subunit k for each input i that is exerted on those subunits. To

insert restriction on the size of β_{ik} variables, constraints (2.4) are imposed. Since there are weak disposability assumptions on undesirable outputs in constraints (2.6), variables l_s are defined free of the sign.

Based on the modified Model (2), the weights H_{kj} are defined as follows to indicate the significance of each subunit for DMU j under consideration:

$$H_{kj} = \sum_{i \in I_k} d_i \beta_{ik} x_{ij} / \sum_{k=1}^K \left[\sum_{i \in I_k} d_i \beta_{ik} x_{ij} \right]. \quad (3)$$

Therefore, aggregate efficiency is represented as a convex combination of k subunit efficiencies.

3.2a Linearization: Based on the definition of H_{kj} as offered in Model (3), the objective function (2) may be rewritten as follows:

$$e_0 = \max \sum_{k=1}^K \sum_{m \in R_k(O_G, O_B)} u_m v_{mj} - \sum_{s \in R_k(O_G, O_B)} l_s w_{sj} / \sum_i d_i x_{ij}. \quad (2')$$

In order to convert the present non-linear structure of Model (2) to a linear form, the variable substitution $z_{ik} = d_i \beta_{ik}$ is used, and as a result $\sum_{k \in T_i} d_i \beta_{ik} = d_i$ is obtained, which may be rewritten as $\sum_{k \in T_i} z_{ik} = d_i$. Utilizing the standard Charnes and Cooper [31] transformation technique $t = 1 / \sum_i d_i x_{ij}$ and by defining $\mu_m = t v_m$, $\rho_i = t d_i$, $\delta_{ik} = t z_{ik}$, $\theta_s = t l_s$, Model (2) is converted to the following linear programming form:

$$e_{agg} = \max \sum_{k=1}^K \left[\sum_{m \in R_k(O_G, O_B)} \mu_m v_{mj_o} - \sum_{s \in R_k(O_G, O_B)} l_s w_{sj_o} \right] \quad (4)$$

subject to

$$\sum_i \rho_i x_{ij_o} = 1, \quad (4.1)$$

$$\sum_{m \in R_k(O_G, O_B)} \mu_m v_{mj} - \sum_{s \in R_k(O_G, O_B)} \theta_s w_{sj} - \sum_{i \in I_k} \delta_{ik} x_{ij} \leq 0, \quad \forall j, k \quad (4.2)$$

$$\sum_{k \in T_i} \delta_{ik} = \rho_i, \quad \forall i \quad (4.3)$$

$$\rho_i a_{ik} \leq \delta_{ik} \leq \rho_i b_{ik}, \quad \forall i, k \quad (4.4)$$

$$\mu_m, \rho_i, \delta_{ik} \geq \varepsilon \quad \forall m, i, k \quad (4.5)$$

$$\theta_s \text{ free in sign} \quad \forall s. \quad (4.6)$$

Note that the set T_i in restrictions (4.3) is defined as the set of all subunits k that have i as a member. For example, as shown in figure 2, input X_2 is allocated only to subunit 1; therefore, the weights $\delta_{2,1}$ and d_2 are equivalent. Since $\delta_{2,1}$ is limited as $d_2 a_{2,1} \leq \delta_{2,1} \leq d_2 b_{2,1}$ in restriction (4.4), this restriction becomes redundant.

- Step 2: Calculating subunit efficiency score

One of the key results from the solution of Model (4) is the resource split-variable β_{ik} . Note that $\beta_{ik} = \delta_{ik} / \rho_i$. Some of the production lines follow bad outputs. Therefore, the efficiency score may be derived by considering that each bundle output uses an appropriate portion of inputs ($\beta_{ik} x_{ij}$) according to its respective production line subunits. This model makes a valuable contribution to theoretical DEA literature by demonstrating how resources are allocated when the partial impact of input to good output and bad output exists. Finally, Model (2) is applied with the knowledge that the evaluated “DMU” is the k^{th} subunit where its good outputs and bad outputs are the members of R_k in the amounts v_{mj} and w_{sj} , and its inputs are the members of I_k in amounts $\beta_{ik} x_{ij}$.

- Step 3: Obtaining the overall efficiency score

To calculate the overall efficiency scores e_{ove} of the DMUs, the weighted average of the k subunit scores that were obtained in stage 2 is needed while using H_{kj} defined in (3). In order to compute H_{kj} , a suitable set of input multipliers d_i must be chosen. These multipliers should be calculated under the conditions where all subunits are being compared similarly. The aggregate Model (4) offers such a condition. If DMU_{j_o} is to be evaluated, the input portion of constraint (4.2), i.e., $\sum_{i \in I_k} \delta_{ik} x_{ij}$, represents the amount of

DMU resources that is attributed to subunit k . The sum of all consumed resources by DMU_{j_o} is shown by $\sum_i \rho_i x_{ij_o}$, which is naturally scaled to unity by constraint (4.1). Therefore, the weights H_{kj} can be written as $H_{kj} = \sum_{i \in I_k} \delta_{ik} x_{ij}$.

Weights H_{kj} are used to obtain the overall efficiency scores for the DMUs.

4. Illustrative application and discussions

4.1 Illustrative application

In this section, the proposed approach is applied to performance measurement of 60 home appliance production companies in Iran. Collected data¹ are reported in table 3.

- Calculating aggregate efficiency

Aggregate efficiency is calculated in the interval $[a_{ik}, b_{ik}] = [0.3, 0.9]$ by Model (4) and results are presented in table 4. For example, to calculate the aggregate efficiency of DMU 60 we need to solve the following linear programming optimization model:

$$\max \quad 365\mu_1 + 18731\mu_3 - 1151\theta_2 + 2088\mu_2 - 0$$

subject to

$$452\rho_1 + 18.97\rho_2 + 13.17\rho_3 + 1.114\rho_4 = 1$$

$$1783\mu_1 + 12854\mu_3 - 1155\theta_2 - 270\delta_{1,1}$$

$$- 10.1\delta_{2,1} - 10\delta_{3,1} - 1.2\delta_{4,1}$$

$$\leq 0 \quad (\text{subunit 1 DMU 1})$$

$$\text{subunit 1 DMU 2, } \dots, \text{ subunit 1 DMU 59}$$

$$2086\mu_1 + 18731\mu_3 - 1151\theta_2 - 452\delta_{1,1} - 18.97\delta_{2,1}$$

$$- 13.17\delta_{3,1} - 1.114\delta_{4,1} \leq 0$$

$$(\text{subunit 1 DMU 60})$$

$$412\mu_1 - 30\theta_1 - 270\delta_{1,2} - 10\delta_{3,2} \leq 0 \quad (\text{subunit 2 DMU 1})$$

$$\text{subunit 2 DMU 2, } \dots, \text{ subunit 2 DMU 59}$$

$$356\mu_1 - 0\theta_1 - 452\delta_{1,2} - 13.17\delta_{3,2} \leq 0$$

$$(\text{subunit 2 DMU 60})$$

$$\delta_{1,1} + \delta_{1,2} = \rho_1 \quad \delta_{2,1} = \rho_2 \quad \delta_{3,1} + \delta_{3,2} = \rho_3$$

$$\delta_{4,1} = \rho_4 \quad \delta_{1,1} \geq 0.3\rho_1$$

$$\delta_{1,2} \geq 0.3\rho_1 \quad \delta_{3,1} \geq 0.3\rho_3 \quad \delta_{3,2} \geq 0.3\rho_3 \quad \delta_{1,1} \leq 0.9\rho_1$$

$$\delta_{1,2} \leq 0.9\rho_1 \quad \delta_{3,2} \leq 0.9\rho_3 \quad \delta_{3,1} \leq 0.9\rho_3$$

$$\rho_1 \dots \rho_4, \delta_{1,1}, \delta_{1,2}, \delta_{3,1}, \delta_{3,2}, \mu_1, \mu_2, \mu_3 \geq \varepsilon = 0.00001,$$

$$\text{free in sign } \theta_1, \theta_2.$$

¹www.igs.nigc.ir/NAFT/SOURCING/ANJOMAN-LIST.HTM.

Table 3. Input–output data for 60 firms.

DMU	Inputs				Production lines				
	Labour X1	Pressing X2	Packing X3	Dyeing X4	Good outputs			Bad outputs	
					V1	V2	V3	W1	W2
1	270	10.1	10	1.2	1783	412	12854	30	1155
2	260	14.3	19	0.7	2266	765	24673	40	2164
3	350	21.4	10.5	0.8	6872	298	43525	35	2342
4	380	18.4	11	0.9	2372	176	93184	38	5346
5	156	12.3	16.1	0.5	2763	287	21286	28	1155
6	140	9.3	12.2	1.6	6234	276	31125	37	2338
7	170	18	20.2	2.1	3454	762	42986	31	3441
8	285	19	18.6	1.6	3324	871	63437	1	6315
9	285	16	12.5	1.45	6369	655	42872	25	4248
10	160	11	17.8	1.15	7662	297	64476	1	6132
11	280	11.7	12.2	1.15	5632	546	42235	25	4414
12	332	13.5	18.9	0.64	4577	231	12245	1	1233
13	228	14.7	15.7	1.09	3545	264	27287	1	2126
14	248	16.8	14.6	0.99	4852	463	64674	32	6134
15	179	12.3	22.7	1.22	8762	582	54740	26	5343
16	299	11.4	18.9	0.49	8983	398	32492	1	3142
17	257	10.1	18.88	0.55	5786	365	21481	25	2523
18	355	25.5	13.2	1.16	2634	873	43265	32	4528
19	220	19.7	14.2	1.68	7281	547	34764	33	3439
20	197	15.8	20.9	1.22	6491	325	11461	27	1343
21	125	12.3	14.2	0.93	1254	287	35296	10	3123
22	167	11.5	10.4	1.21	3421	542	27548	1	2658
23	145	13.6	11.3	1.7	8723	358	87354	12	8456
24	165	15.5	12.8	0.34	1254	673	23461	32	2428
25	172	16.8	16.8	0.13	1367	982	87354	39	8763
26	187	21.4	21.5	0.983	1366	283	45268	25	4254
27	129	18.4	17.9	0.543	7623	126	43286	98	4643
28	231	23.6	18.3	0.429	2876	284	54628	4	5528
29	248	10.4	14.9	0.284	3523	339	32671	32	3428
30	321	14.6	23.6	0.183	3790	397	67298	22	5762
31	362	15.6	11.8	0.329	6732	653	37519	45	3163
32	183	13.5	17.6	0.678	5432	473	28745	1	2983
33	193	16.7	18.4	0.382	8715	459	27546	63	2572
34	159	18.9	15.8	0.983	5629	256	87456	12	4467
35	219	14.6	21.4	0.632	3891	334	58563	13	4863
36	264	23	19	0.829	3764	654	46839	16	3537
37	251	17	15	1.5	8743	562	28303	1	4729
38	182	16.4	16.3	1.59	3454	238	35520	32	2452
39	143	12.6	19.3	1.63	4982	783	38760	25	5562
40	128	10.7	12.5	2.3	7652	712	46598	22	3427
41	342	13.6	13.9	2.1	4328	831	65790	12	4271
42	295	12.4	18.2	1.73	6712	432	87220	1	2528
43	234	16.4	13.8	1.234	5639	901	64830	74	1173
44	260	14.4	14.2	1.821	3519	619	23489	33	3352
45	237	13.3	18.2	1.438	7645	563	56382	27	4278
46	138	18	12	1.292	8632	198	36581	0	3753
47	276	14	10.3	1.573	4529	231	47891	15	3563
48	249	19	15	1.629	3562	100	32980	10	3825
49	176	19.5	17.3	1.392	2382	173	76830	1	2476
50	147	20	13.9	1.733	1853	351	77652	23	2387
51	432	17.2	12.01	1.324	6822	406	75376	14	1633
52	342	10.85	10.69	1.945	1460	819	69546	67	2687
53	612	19.24	13.73	0.536	2789	412	40991	34	2496
54	213	14.67	10.33	0.443	1540	963	82577	52	3623

Table 3 continued

DMU	Inputs				Production lines				
	Labour X1	Pressing X2	Packing X3	Dyeing X4	Good outputs			Bad outputs	
					V1	V2	V3	W1	W2
55	763	14.54	12.96	0.8	7602	747	63896	49	3131
56	692	10.18	17.82	1.315	9866	328	81342	10	2526
57	319	12.84	12.95	1.499	4829	760	14562	29	1120
58	476	12.57	16.94	1.347	6450	265	83896	60	5461
59	936	16.07	12.46	1.419	3395	382	48031	30	5589
60	452	18.97	13.17	1.114	2086	356	18731	1	1151

Table 4. Efficiency scores resulting from Model (1) and Model (4).

DMU	Efficiency scores Model (1)	Aggregate efficiency scores Model (4)	DMU	Efficiency scores Model (1)	Aggregate efficiency scores Model (4)
1	1.0000	0.4770	31	1.0000	0.9925
2	1.0000	0.4904	32	0.8230	0.7366
3	1.0000	0.9095	33	1.0000	0.9996
4	1.0000	0.9070	34	1.0000	0.9640
5	1.0000	0.5303	35	0.7761	0.7424
6	1.0000	0.9563	36	1.0000	0.5550
7	0.8767	0.6831	37	1.0000	0.8218
8	0.7604	0.8407	38	0.4824	0.4241
9	0.9348	0.7048	39	1.0000	0.8676
10	1.0000	0.9998	40	1.0000	1.0000
11	0.9411	0.7161	41	1.0000	0.9615
12	0.6743	0.6375	42	1.0000	0.9995
13	0.4608	0.4084	43	1.0000	1.0000
14	0.7650	0.7107	44	1.0000	0.4454
15	1.0000	0.9826	45	0.9156	0.8056
16	1.0000	0.9994	46	1.0000	0.9168
17	0.8519	0.7459	47	1.0000	0.5508
18	1.0000	0.8316	48	0.4202	0.3546
19	0.8498	0.6702	49	0.9954	0.8781
20	1.0000	0.9993	50	1.0000	0.9399
21	0.6589	0.4886	51	1.0000	0.9994
22	0.8052	0.9390	52	1.0000	1.0000
23	1.0000	0.9997	53	0.8690	0.5084
24	1.0000	0.6609	54	1.0000	1.0000
25	1.0000	1.0000	55	1.0000	0.9697
26	0.6409	0.4158	56	1.0000	0.9989
27	1.0000	1.0000	57	1.0000	0.7453
28	0.6855	0.5549	58	0.9915	0.9671
29	0.8803	0.7975	59	0.7725	0.5780
30	1.0000	0.9991	60	0.4835	0.4879

- Calculating production line (subunit) efficiency

Values of split variables β_{ik} are obtained from Model (4) for calculating production line efficiencies. These values are then multiplied in inputs according to each good and bad output usage from inputs. Model (2) is finally applied, and results are collected in table 7. Split variables β_{ik}

provide appropriate inputs, i.e., $(\beta_{ik}x_{ij})$ to their respective production line. The values of split variables β_{ik} are reported in table 5.

- Calculating overall efficiency scores

The H_{kj} values are computed from Model (4) for two subunits $k = 1, 2$, where $H_{kj} = \sum_{i \in I_k} \delta_{ik}x_{ij}$. Results are

Table 5. β_{ik} values resulting from Model (4).

DMU	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{2,1}$	$\beta_{3,1}$	$\beta_{3,2}$	$\beta_{4,1}$	DMU	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{2,1}$	$\beta_{3,1}$	$\beta_{3,2}$	$\beta_{4,1}$
1	0.3	0.7	1	0.1	0.9	1	31	0.5	0.5	1	0.7	0.3	1
2	0.3	0.7	1	0.1	0.9	1	32	0.3	0.7	1	0.5	0.5	1
3	0.7	0.3	1	0.7	0.3	1	33	0.7	0.3	1	0.1	0.9	1
4	0.5	0.5	1	0.7	0.3	1	34	0.7	0.3	1	0.7	0.3	1
5	0.7	0.3	1	0.5	0.5	1	35	0.7	0.3	1	0.5	0.5	1
6	0.7	0.3	1	0.7	0.3	1	36	0.3	0.7	1	0.3	0.7	1
7	0.3	0.7	1	0.5	0.5	1	37	0.7	0.3	1	0.7	0.3	1
8	0.3	0.7	1	0.1	0.9	1	38	0.7	0.3	1	0.3	0.7	1
9	0.5	0.5	1	0.7	0.3	1	39	0.3	0.7	1	0.5	0.5	1
10	0.7	0.3	1	0.7	0.3	1	40	0.7	0.3	1	0.7	0.3	1
11	0.5	0.5	1	0.7	0.3	1	41	0.3	0.7	1	0.1	0.9	1
12	0.5	0.5	1	0.1	0.9	1	42	0.7	0.3	1	0.1	0.9	1
13	0.7	0.3	1	0.7	0.3	1	43	0.7	0.3	1	0.1	0.9	1
14	0.5	0.5	1	0.7	0.3	1	44	0.3	0.7	1	0.1	0.9	1
15	0.7	0.3	1	0.5	0.5	1	45	0.7	0.3	1	0.7	0.3	1
16	0.7	0.3	1	0.5	0.5	1	46	0.7	0.3	1	0.7	0.3	1
17	0.4	0.6	1	0.1	0.9	1	47	0.5	0.5	1	0.7	0.3	1
18	0.5	0.5	1	0.1	0.9	1	48	0.5	0.5	1	0.7	0.3	1
19	0.7	0.3	1	0.7	0.3	1	49	0.7	0.3	1	0.5	0.5	1
20	0.7	0.3	1	0.5	0.5	1	50	0.7	0.3	1	0.7	0.3	1
21	0.7	0.3	1	0.5	0.5	1	51	0.5	0.5	1	0.1	0.9	1
22	0.3	0.7	1	0.1	0.9	1	52	0.5	0.5	1	0.7	0.3	1
23	0.7	0.3	1	0.1	0.9	1	53	0.5	0.5	1	0.7	0.3	1
24	0.3	0.7	1	0.1	0.9	1	54	0.5	0.5	1	0.1	0.9	1
25	0.3	0.7	1	0.7	0.3	1	55	0.5	0.5	1	0.7	0.3	1
26	0.7	0.3	1	0.5	0.5	1	56	0.7	0.3	1	0.7	0.3	1
27	0.3	0.7	1	0.5	0.5	1	57	0.7	0.3	1	0.7	0.3	1
28	0.7	0.3	1	0.7	0.3	1	58	0.5	0.5	1	0.1	0.9	1
29	0.4	0.6	1	0.1	0.9	1	59	0.5	0.5	1	0.7	0.3	1
30	0.4	0.6	1	0.1	0.9	1	60	0.5	0.5	1	0.1	0.9	1

shown in table 6. Variables are used for obtaining the overall efficiency score of DMUs. To compute the overall efficiency scores e_{ove} of the DMUs, weighted average of the k subunit scores that were obtained in stage 2 is needed, using H_{kj} as defined in relation (3). Overall efficiency scores are listed in table 7. For example, for DMU 60 we have

$$\begin{aligned}
 H_{1,60} &= \delta_{1,1}x_{1,60} + \delta_{2,1}x_{2,60} + \delta_{3,1}x_{3,60} + \delta_{4,1}x_{4,60} \\
 &= 0.00001 \times 452 + 0.00001 \times 18.97 + 0.00756 \\
 &\quad \times 13.17 + 0.00001 \times 1.114 \\
 &= 0.1004.
 \end{aligned}$$

$$\begin{aligned}
 H_{2,60} &= \delta_{1,2}x_{1,60} + \delta_{3,2}x_{3,60} = 0.00001 \times 452 + 0.06827 \\
 &\quad \times 13.17 = 0.8997.
 \end{aligned}$$

Further, in order to calculate the overall efficiency of DMU 60

$$\begin{aligned}
 e_{ove60} &= H_{1,60} \times \text{eff subunit } 1_{60} + H_{2,60} \times \text{eff subunit } 2_{60} \\
 &= 0.1004 \times 0.2818 + 0.8997 \times 0.2405 = 0.2447.
 \end{aligned}$$

4.2 Discussion

Computed results are used for directing resource transfers within units. The proposed approach provides information that demonstrates how much of primary materials each production line uses, how this material is consumed and how much of it is involved in the production of pollution. Therefore, the proposed method may be used by managers in utilizing efficiency scores and related resource usage to both efficiency improvement and controlling the production of pollutants and waste produced in production lines.

As discussed in previous sections, each DMU viewed as a business unit consists of several subunits. In the suggested approach for sharing the resource in subunits, a β_{ik} split variable is defined. This variable provides an appropriate portioning of inputs ($\beta_{ik}x_{ij}$) to their respective subunits. It is known that sharing input is not similar to this approach in conventional DEA models affected by undesirable output.

DMUs 43 and 44 are analysed for usage of inputs and their performance as regards bad outputs. There are 4 inputs, 3 good outputs and 2 bad outputs in each DMU, and each DMU has 2 subunits as shown in table 2. For DMUs

Table 6. H_{kj} values arising from Model (4); this variable is used to obtain overall efficiency.

DMU_j	H_{1j}	H_{2j}	DMU_j	H_{1j}	H_{2j}	DMU_j	H_{1j}	H_{2j}
1	0.1002	0.8999	21	0.8513	0.1488	41	0.1019	0.8982
2	0.3064	0.6937	22	0.1002	0.8999	42	0.9991	0.001
3	0.807	0.1931	23	0.9995	0.0006	43	0.9976	0.0025
4	0.7936	0.2065	24	0.2354	0.7647	44	0.1016	0.8985
5	0.9041	0.096	25	0.7057	0.2944	45	0.8847	0.1154
6	0.9009	0.0992	26	0.7193	0.2808	46	0.8555	0.1446
7	0.3001	0.7	27	0.7074	0.2927	47	0.7975	0.2026
8	0.1209	0.8792	28	0.7499	0.2502	48	0.9766	0.0235
9	0.7961	0.204	29	0.9993	0.0008	49	0.8616	0.1385
10	0.9995	0.0006	30	0.9989	0.0012	50	0.8825	0.1176
11	0.9718	0.0283	31	0.978	0.0221	51	0.9992	0.0009
12	0.999	0.0011	32	0.4732	0.5269	52	0.8173	0.1828
13	0.8479	0.1522	33	0.9985	0.0016	53	0.8208	0.1793
14	0.8938	0.1063	34	0.9489	0.0512	54	0.9995	0.0006
15	0.9578	0.0423	35	0.9374	0.0627	55	0.9079	0.0922
16	0.9991	0.001	36	0.3924	0.6077	56	0.9987	0.0014
17	0.9991	0.001	37	0.8191	0.181	57	0.7584	0.2417
18	0.1004	0.8997	38	0.8138	0.1863	58	0.9987	0.0014
19	0.8346	0.1655	39	0.3001	0.7	59	0.9968	0.0033
20	0.9994	0.0007	40	0.8238	0.1763	60	0.1004	0.8997

Table 7. Subunit efficiency scores arisen from Model (1) and overall efficiency scores.

DMU	Subunit 1	Subunit 2	Overall	DMU	Subunit 1	Subunit 2	Overall
1	0.3114	0.2951	0.2968	31	0.9716	0.7854	0.9676
2	0.4576	0.3114	0.3562	32	0.8401	0.4947	0.6582
3	0.7075	0.6819	0.7026	33	1.0000	1.0000	1.0001
4	0.8589	0.5512	0.7954	34	1.0000	0.3638	0.9675
5	0.5305	0.5641	0.5338	35	0.7419	0.2933	0.7138
6	1.0000	1.0000	1.0000	36	0.6537	0.287	0.4309
7	0.8009	0.4362	0.5457	37	0.7653	1.0000	0.8079
8	0.8436	0.585	0.6163	38	0.4282	0.5444	0.4499
9	0.6108	0.8862	0.6670	39	1.0000	0.4324	0.6028
10	1.0000	0.829	1.0000	40	1.0000	1.0000	1.0000
11	0.7111	0.7268	0.7116	41	0.8843	0.4468	0.4914
12	0.7139	0.1865	0.7134	42	1.0000	0.654	0.9998
13	0.4071	0.5171	0.4239	43	1.0000	1.0000	1.0000
14	0.7038	0.5583	0.6884	44	0.5075	0.3047	0.3253
15	1.0000	0.6544	0.9855	45	0.8385	0.6307	0.8146
16	1.0000	0.5945	0.9997	46	0.9838	0.6408	0.9343
17	0.9945	0.1983	0.9938	47	0.5566	0.3262	0.5100
18	0.6506	0.3758	0.4034	48	0.3672	0.1552	0.3623
19	0.6194	0.8429	0.6565	49	0.9453	0.439	0.8753
20	1.0000	0.4332	0.9997	50	1.0000	0.6834	0.9629
21	0.4800	0.4269	0.4721	51	1.0000	0.1943	0.9994
22	0.8191	0.6212	0.6411	52	0.9215	1.0000	0.9359
23	1.0000	0.454	0.9998	53	0.4763	0.3951	0.4618
24	0.7619	0.3979	0.4836	54	1.0000	0.7148	0.9999
25	1.0000	0.9806	0.9944	55	0.9448	0.835	0.9348
26	0.3739	0.4207	0.3871	56	1.0000	0.3254	0.9992
27	1.0000	1.0000	1.0000	57	0.7018	0.9926	0.7722
28	0.5084	0.3995	0.4812	58	1.0000	0.3269	0.9992
29	0.9197	0.2595	0.9193	59	0.5891	0.409	0.5886
30	1.0000	0.1496	0.9991	60	0.2818	0.2405	0.2447

43 and 44, using conventional DEA Model (2), by undesirable output the efficiency scores are obtained as 1 and 1, respectively, as shown in table 4. Aggregate efficiencies for selected intervals $[a_{ik}, b_{ik}] = [0.3, 0.9]$ are obtained as 0.4454 and 1, respectively, by employing Model (4) as shown in table 5. It should be noted that the arbitrarily chosen intervals a_{ik} and b_{ik} may be any positive numbers between 0 and 1.

For DMU 44 in table 5 and in accordance with values obtained for variable β_{ik} , subunit 1 uses 30% of input 1, 100% of input 2, 10% of input 3 and 100% of input 4; using Model (2), efficiency score for subunit 1 is calculated to be 0.5075 as indicated in table 7. Additionally, subunit 2 uses 70% of input 1 and 90% of input 3; utilizing Model (2), efficiency score in subunit 2 is calculated to be 0.3047. In DMU 43, subunit 1 uses 70% of input 1, 100% of input 2, 10% of input 3 and 100% of input 4; employing Model (2), efficiency score in subunit 1 is calculated to be 1. Similarly, subunit 2 uses 30% of input 1 and 90% of input 3; using Model (2), efficiency score in subunit 2 is computed to be 1 as shown in table 7. The overall efficiencies of DMUs 43 and 44 are 1 and 0.3253, respectively, as shown in table 7.

Two significant items are noticed:

- i One of the important features of Model (4) is that each output in this model uses input according to its requirement. As an example, in DMUs 43 and 44 and before evaluation in Model (4), efficiency of both DMUs is 1 considering bad output. DMU 43 was efficient and DMU 44 was inefficient in the case of Model (4).
- ii Intervals $[a_{ik}, b_{ik}]$ that were defined for β_{ik} may help a manager impose different sharing limits on shared input resources.

5. Conclusions

Recent work by Imanirad *et al* [27] considers a DMU as a business unit consisting of several subunits. A traditional DEA model CCR was extended in their approach to permit efficiency measurement in situations where partial input–output interaction exists. The term “aggregate efficiency” is suggested for measuring performance of several subunits. An important issue that should be considered in evaluating the performance of a business unit is undesirable outputs.

This work extends the current approach to take into account a business unit consisting of several subunits that produce undesirable outputs as a by-product of desirable outputs. For this purpose, the weak disposable production technology proposed by Kuosmanen [5] is used for modelling undesirable outputs. This approach may be similar to network structure DEA in resource allocation, but it is clear that these approaches are not the same. The effort presented here may be further complemented in future studies. The following are suggested efforts:

Suggestion 1: As discussed in section 3, each input x_i is separable, but there are situations where each input x_i may be non-separable. Model (4) may be modified to address non-separable inputs.

Suggestion 2: As described in section 3, all DMUs are assumed to have the same resource-sharing profile. However, it is possible to have a different profile for each DMU.

Suggestion 3: The proposed model may be further developed to consider non-discretionary input or output into consideration.

List of Symbols

$j \in J, j = 1, \dots, n$	collection of DMUs
$m = 1, \dots, M$	set of good outputs
$s = 1, \dots, S$	set of bad outputs
$i = 1, \dots, I$	set of inputs
$k = 1, \dots, K$	number of bundles
DMU_o	DMU under evaluation
v_{mj}	m th good output of j th DMU
w_{sj}	s th bad output of j th DMU
x_{ij}	i th input of j th DMU
v_{mo}	m th good output of DMU_o
w_{so}	s th bad output of DMU_o
x_{io}	i th input of DMU_o
μ_m	weight for the m th good output
θ_s	weight for the s th bad output
ρ_i	weight for the i th input
β_{ik}	split variable for x_{ij} in k th bundle
I_k	set of inputs in k th bundle
T_i	set of all k bundles that have i as a member
$R_k(O_G, O_B)$	k th bundle that has good or bad outputs
ε	non-Archimedean and very small number
H_{kj}	convex combination of k bundles' efficiencies
e_{agg}	aggregate efficiency
e_{ove}	overall efficiency

Appendix A

Creating the input, good output and bad output bundle

Following the notations used in Imanirad *et al* [27], input, good output and bad output bundles may be generated for each $k=1, \dots, K$, where I_k and $R_k(O_G, O_B)$ represent a set of inputs, and good output and bad output, respectively. These bundles are generated so that R_k is from a mutually exclusive set and for each k , $(I_k, R_k(O_G, O_B))$ $k = 1, \dots, K$ is maximal based on the following definition.

Definition 1 An input, good output and bad output bundle $(I_k, R_k(O_G, O_B))$ is considered to be maximal if it possesses the following two properties:

1. every good output v and every bad output w in $R_k(O_G, O_B)$ are influenced by every input i in I_k and no other input outside of I_k may influence any good output v or any bad output w in $R_k(O_G, O_B)$;
2. there exists no output outside of $R_k(O_G, O_B)$ whose input bundle is identical to that of $R_k(O_G, O_B)$.

Algorithm 1 (creating maximal bundles)

Step 1: Define S to be an empty set.

Step 2: For each good output and bad output (m, s) , derive $I(m, s)$, the set of all inputs i that influence (m, s) ; transfer $I(m, s)$ to S and set the bundle counter $ask = 1$.

Step 3: For each $I(m, s)$ in S , compare it with every other $I(m', s')$ in S , and identify all $I(m', s')$ that have the same input elements as in $I(m, s)$. If no such (m', s') is identified, create bundle $(I_k, R_k(O_G, O_B))$ using $I(m, s)$ and (m, s) so that $(I_k, R_k(O_G, O_B)) = (I(m, s), (m, s))$. Remove $I(m, s)$ from S . Go to step 4. Otherwise, group good output and bad output (m, s) and all identified (m', s') (having the same input sets) together to derive $R_k(O_G, O_B)$ and create bundle $(I_k, R_k(O_G, O_B))$ using $I(m, s)$ and $R_k(O_G, O_B)$ so that $(I_k, R_k(O_G, O_B)) = (I(m, s), (m, s))$. Remove $I(m, s)$ and all identified $I(m', s')$ from S . Go to step 4.

Step 4: If S is nonempty, set $k = k + 1$ and go to step 3. Otherwise, terminate having formed the set of all bundles.

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