Predicting the mechanical properties of ordinary concrete and nano-silica concrete using micromechanical methods

M LEZGY-NAZARGAH^{*}, S A EMAMIAN, E AGHASIZADEH and M KHANI

Faculty of Civil Engineering, Hakim Sabzevari University, Sabzevar, P.O. Box 9617976487, Iran
e-mail: m.lezgy@hsu.ac.ir

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Abstract. By combining several materials with specific mechanical properties, new materials with unknown mechanical properties are obtained. Various experiments are required to determine the mechanical properties of the produced composite materials. Since conducting experiment processes is costly and time-consuming, comprehensive studies have been conducted in recent years to solve the problem. Fortunately, it is possible to easily predict the mechanical properties of composite materials without the need to construct them, by inspecting their constituent's properties using micromechanical methods. Although various micromechanical methods have been presented so far, few of them yielded precise predictions of the properties of composite materials. Therefore, selecting a method suitable to predict the properties of composite materials is of much importance. In this study, some micromechanical approaches, including Hirsch, Hansen, Bache, Cavento, Mori–Tanaka, Eshelby, self-consistent, effective interface and double-inclusion models, were employed for the estimation of elasticity modulus and Poisson's ratio of ordinary and nanomaterial concretes. The results obtained from the micromechanical methods were compared to those obtained from experimental tests. The obtained numerical results showed that Bache's model is the most accurate micromechanics model for predicting the elastic mechanical properties of ordinary and nanomaterial concretes.

Keywords. Micromechanics; concrete; nano-silica; homogenization.

1. Introduction

Composite materials, though homogenous in large or macro-scale, have heterogeneous structure in small or micro-scale. They are actually composed of a combination of several materials with specific mechanical properties. For example, in manufacturing concrete, different materials (phases) such as aggregate and cement paste are combined so that the final material will have the expected properties like compressive strength and ductility. Accurate experiments are needed to determine the properties of multi-phase materials. Properties of composite materials depend on the volume percent and features of primary constituents. Due to the fact that the volume percent of phases of composite materials varies, determining the properties of composite materials and selecting the best choice to use involve trial and error. Conducting multiple experiments and investigating the obtained results are very costly and time-consuming. As a result, attempt has been made by researchers to find theoretical solutions to predict the properties of composite materials. The theoretical solutions used to predict the properties of composite materials are now known as micromechanical methods and are given attention by many researchers.

Studies on micromechanics emerged about a century ago in 1907 by Volterra [1]. He started his research by studying the elastic stress and strain fields surrounding concavities and defects of materials. Based on the principles of continuum mechanics of materials, Volterra developed theories that relate the reaction of materials to parameters inside the materials. By studying the non-local behaviour, Volterra found that stress at one spot does not merely depend on strain at that spot but it also depends on strain at surrounding spots. Mindlin [2] developed a micromechanical linear elasticity theory in 1964, which allows stresses to be dependent on both strains and a small transformation tensor. Related research such as Eringen [3] led to the development of micro-polar and couple-stress theories in 1968. These theories consider material transformation to include a series of translational and rotational degrees of freedom. Integrating the theories of continuum mechanics with micro-polar and couple-stress theories led to more developments in micromechanical methods. Among these studies, the studies of Bardenhagen and Triantafyllidis [4] and Chang and Gao [5] can be referred to.

Regarding the conducted studies, Cowin and Nunziato [6] developed a theory of composite material elasticity by taking cavities into consideration. Another prominent theory is doublet mechanics suggested by Ferrari *et al* [7]. This

^{*}For correspondence

theory considers composite materials as an arrangement of points or particles interacting according to micromechanical rules. Other studies by Budianski and O'Connell [8] and Kachanov [9] have also investigated composite materials with scattered cracks. In some studies, statistical and probabilistic methods were used to develop models with random changes in mechanical properties, among which the study by Starzewski-Ostoja and Wang [10] can be referred to. Mura [11] collected the results of all studies on micromechanics and invented the eigen-strain technique. Elastic modelling of defects, cracks and other heterogeneities became feasible using the eigen-strain technique of Mura. Shokriyeh and Parsayi [12] studied micro-buckling of composite materials under compressive loading using three types of volume elements: hexagon, square and circle. The results of this study showed that when fibre-reinforced composite materials are compressively loaded in line with fibres, they are destroyed before fibre buckling, due to the effect of fibre buckling or the effect of transverse fracture in resin. In compressive loading, there are two buckling states for fibres: the first one is shear buckling and the second one is lateral buckling. In this study, the type of volume element suitable for micro-modelling was investigated. The results of the study of Shokriyeh and Parsayi [12] showed that in fibre buckling in micro-scale, the resin that surrounds the fibres does not undergo buckling. Norris [13] studied the mechanical properties of two-phase composites using a discrete element method. Dai *et al* [14] were able to predict the effective elasticity modulus of multi-phase hybrid composites by combining Mori–Tanaka and self-consistent models. Shokriyeh and Elahi [15] presented a new model for estimating the elasticity modulus of polymer concrete using micromechanical models. The model of these researchers is based on Mori–Tanaka's model. Since the Mori–Tanaka model is weak in the estimation of elasticity modulus of composite materials with high impurity volume percent, an attempt was made by Shokriyeh and Elahi [15] to eliminate this drawback by taking into consideration the effect of impurity in the matrix phases. Mean-field homogenization techniques are commonly based on the results of Eshelby [16]. Eshelby surveyed issues such as determining impurity strain in infinite space of matrix and calculating stress in elliptical impurities of matrix. Eshelby's computations, which account for the interior and exterior of impurities, are now the basis of many micromechanical theories. Based on iso-field assumptions, Lezgy-Nazargah [17] derived explicit formulations for computing the effective coupled thermo-electro-elastic material properties of Macro Fibre Composites in the linear regime. Lezgy-Nazargah and Eskandari-Naddaf [18] developed a novel extended iso-field micromechanical model for estimating the effective coupled thermo-electro-elastic material coefficients of three-phase piezoelectric structural fibre composites.

Concrete is one of the most applicable materials in civil engineering. Accurate prediction of the mechanical

properties of concrete materials is of great interest for civil engineers. Beycioğlu *et al* [19] introduced a rule based on Fuzzy Logic model for predicting the flexural strengths and compressive strengths of blended cements at elevated temperatures. For prediction of the concrete properties, Chen *et al* [20] collected sets of mixtures that include fly ash, slag and silica. These researchers developed a new database platform, with the stepwise multiple linear regression and artificial neural networks programs for identifying correlations between the concrete properties and the dosage and/or quality of materials. Najjar *et al* [21] used a database to develop Fuzzy Logic models as predictive tools for the grout flow ability and mechanical properties of two-stage concrete mixtures. They evaluated the performance of the developed models using statistical parameters and error analyses. By combining two different techniques, Neural Networks and Principal Component Analysis, Boukhatem *et al* [22] successfully predicted the slump and compressive strength of concrete with mineral additives such as blast furnace slag, fly ash and silica fume. Chu *et al* [23] proposed prediction models for estimating the hydration properties of concrete, such as the compressive strength, the splitting tensile strength, the elastic modulus and the autogenous shrinkage.

Although studies on micromechanics methods emerged about a century ago, applicable research works on these methods for predicting the mechanical properties of concrete materials are limited. Indeed, finding a micromechanics method suitable to predict the properties of concrete materials is of much importance for civil engineers. The purpose of this research was to predict the mechanical properties of ordinary concrete and nano-silica concrete using different micromechanical methods. Hirsch, Hansen, Cavento, Batch, self-consistent, Eshelby, Mori–Tanaka, effective interface and double-inclusion models were investigated in this research. The elasticity modulus and Poisson's ratio of different samples of ordinary concrete and nano-silica concrete made up of various mixtures were calculated using the mentioned micromechanical methods. The estimated results obtained from the micromechanical methods were compared to experimental results. Finally, in order to adopt a solution suitable for precise prediction of mechanical properties of concrete samples, a discussion is carried out on the results estimated by the different micromechanical methods. The effect of different parameters on the accuracy of various micromechanical methods was also investigated.

2. Micromechanical methods

Different micromechanical equations were employed for the prediction of the mechanical properties of the concrete material as follows.

Hirsch model [24]:

$$\frac{1}{E} = (1-x) \left[\frac{C_2}{E_2} + \frac{C_1}{E_1} \right] + x \left[\frac{1}{C_2 E_2 + C_1 E_1} \right] \quad (1)$$

where E is the effective elasticity modulus of concrete. E_1 and E_2 are the elasticity modulus of matrix (cement paste) and impurity (aggregate), respectively. Volume percent of matrix and aggregate are represented by C_1 and C_2 , respectively; x is a number between 0 and 1 that determines the relative share of parallel (equivalent-constant-strain state) and series (equivalent-constant-stress state) models in the calculation of effective elasticity modulus.

Hansen model [25]:

$$E = \left(\frac{C_1 E_1 + (1 + C_2) E_2}{(1 + C_2) E_1 + C_1 E_2} \right) E_1. \quad (2)$$

Cavento model [26]:

$$\frac{1}{E} = \frac{1 - \sqrt{C_2}}{E_1} + \frac{1}{\left(\frac{1 - \sqrt{C_2}}{\sqrt{C_2}} \right) E_1 + E_2} \quad (3)$$

Batch model [27]:

$$E = E_1^{C_1} E_2^{C_2} \quad (4)$$

Mori–Tanaka method [28]:

$$\mathbf{D} = (C_1 \mathbf{D}_1 + C_2 \mathbf{D}_2 \mathbf{T}^p) (C_1 \mathbf{I} + C_2 \mathbf{T}^p)^{-1} \quad (5)$$

where $\mathbf{D}$ is the tensor of effective elasticity constants of the composite material (concrete); $\mathbf{D}_1$ and $\mathbf{D}_2$ are the tensors of matrix (cement paste) and impurity (aggregate) elasticity constants; $\mathbf{I}$ is the unit tensor (that is, a tensor whose entries or elements of its main diagonal are 1 and other entries are 0) and $\mathbf{T}^p$ is the tensor of centralized strain of impurities calculated as follows:

$$\mathbf{T}^p = \left[\mathbf{I} + \mathbf{S}^p (\mathbf{D}_1)^{-1} (\mathbf{D}_2 - \mathbf{D}_1) \right]^{-1} \quad (6)$$

where $\mathbf{S}^p$ is Eshelby's tensor, which depends on the form of impurities. For instance, for a composite material with isotropic matrix and spherical impurities, entries of Eshelby's tensor are calculated using the following equations:

$$\begin{aligned} S_{1111} &= S_{2222} = S_{3333} = \frac{7-5\nu}{15(1-\nu)} \\ S_{1122} &= S_{2233} = S_{3311} = S_{1133} = S_{2211} = S_{3322} = \frac{5\nu-1}{15(1-\nu)} \\ S_{1212} &= S_{2323} = S_{3131} = \frac{4-5\nu}{15(1-\nu)} \end{aligned} \quad (7)$$

where ν is Poisson's ratio of the matrix phase.

Eshelby's method [16, 29]:

$$K = K_1 + \frac{C_2(K_2 - K_1)(3K_1 + 4G_1)}{3K_2 + 4G_1} \quad (8)$$

$$G = G_1 + \frac{5C_2 G_1 (G_2 - G_1)(3K_1 + 4G_1)}{3K_1(3G_1 + 2G_2) + 4G_1(2G_1 + 3G_2)} \quad (9)$$

where K and G , respectively, are the effective bulk and shear modulus of the composite material. Subscripts 1 and 2 denote the matrix (cement paste) and impurity (aggregate), respectively.

Self-consistent method [30]:

$$K = K_1 + \frac{C_2(K_2 - K_1)(3K + 4G)}{3K_2 + 4G} \quad (10)$$

$$G = G_1 + \frac{5C_2 G (G_2 - G_1)(3K + 4G)}{3K(3G + 2G_2) + 4G(2G + 3G_2)} \quad (11)$$

Effective interface method [31]:

$$\begin{aligned} \mathbf{D} &= \mathbf{D}_1 + [(C_2 + C_i)(\mathbf{D}_i - \mathbf{D}_1) \mathbf{T}^{pi} \\ &\quad + C_2(\mathbf{D}_2 - \mathbf{D}_i) \mathbf{T}^p] \\ &\quad \times [C_1 \mathbf{I} + (C_2 + C_i) \mathbf{T}^{pi}]^{-1} \end{aligned} \quad (12)$$

where

$$\mathbf{T}^p = \mathbf{I} - \mathbf{S}^p [\mathbf{S}^p + (\mathbf{D}_2 - \mathbf{D}_1)^{-1} \mathbf{D}_1]^{-1} \quad (13)$$

$$\begin{aligned} \mathbf{T}^{pi} &= \mathbf{I} - \mathbf{S}^p \times \left\{ \frac{C_2}{C_i + C_2} [\mathbf{S}^p + (\mathbf{D}_2 - \mathbf{D}_1)^{-1} \mathbf{D}_1]^{-1} \right. \\ &\quad \left. + \frac{C_i}{C_i + C_2} [\mathbf{S}^p + (\mathbf{D}_i - \mathbf{D}_1)^{-1} \mathbf{D}_1]^{-1} \right\} \end{aligned} \quad (14)$$

and C_i is the volume percent of the interface between two phases of matrix and impurity. $\mathbf{D}_i$ denotes the tensor of elasticity constants of the interface of the two phases.

Double-inclusion method [32]:

$$\mathbf{D} = \mathbf{D}_1 + C_2(\mathbf{D}_2 - \mathbf{D}_1) \mathbf{A} \quad (15)$$

where

$$\mathbf{A} = \left\{ (1 - \lambda) [\mathbf{A}^1]^{-1} + \lambda [\mathbf{A}^u]^{-1} \right\}^{-1} \quad (16)$$

$$\mathbf{A}^1 = \left\{ \mathbf{I} + \mathbf{S}^p [(\mathbf{D}_1)^{-1} \mathbf{D}_2 - \mathbf{I}]^{-1} \right\}^{-1} \quad (17)$$

$$\mathbf{A}^u = \mathbf{I} + \mathbf{S}^p [(\mathbf{D}_2)^{-1} \mathbf{D}_1 - \mathbf{I}]^{-1} \quad (18)$$

$$\lambda = \frac{1}{2} C_2 (1 + C_2). \quad (19)$$

3. Numerical results

Here, elasticity modulus and Poisson's ratio of ordinary concrete and nano-silica concrete materials were determined using the micromechanical methods presented

previously. Based on the mathematical formulation introduced in the previous section, a computer code is developed in MATLAB programming software for each micromechanics model. Results obtained from these different micromechanical methods are compared to experimental results and the accuracy of each method is evaluated.

3.1 Estimating the mechanical properties of ordinary concrete

Concrete is one of the most common materials used in civil engineering applications. In this section, the mechanical properties of some ordinary concrete samples with different mix designs have been estimated using different micromechanics models. Details of the mix design of these different concrete samples are shown in table 1. As shown in this table, four types of aggregate, including steel, glass, sand and rubble, and three types of cement paste with water to cement ratios (W/C) 0.286, 0.294 and 0.315 were used. In preparing concrete samples, the volume percent of aggregate was considered as 10%, 20% and 30%. Although the volume fraction of aggregate in the concrete is generally higher than 30%, for the purpose of comparison with data available in literature such values are assumed. In

table 1, Poisson's ratios as well as modulus of elasticity of cement paste and aggregate are also given for each concrete sample. These values are obtained by Huang *et al* [33] via experimental tests.

It is well known that concrete has several phases, including aggregate, cement paste, air bubbles and micro-cracks. Although it is necessary to account for all these mentioned phases in the determination of the effective elasticity modulus of the concrete, due to complexities, only two phases, that is, aggregate and cement paste, are being used in calculations. Indeed, the cement paste is assumed to be the matrix phase while the aggregates are assumed to be the impurity phase. The results obtained from estimation of elasticity modulus of ordinary concrete using Hirsch, Hansen, Cavento and Batch micromechanical methods are shown in tables 2 and 3. It is worth noting here that these estimated results are obtained from a MATLAB code whose algorithm is based on the mathematical formulations given in section 2. In tables 2 and 3, the estimated micromechanics results have been compared with experimental results of Huang *et al* [33]. It can be deduced from table 2 that the results of Hirsch model are more accurate for mixtures with aggregate volume percent less than 20%. Table 2 also reveals the fact that the accuracy of Hirsch and Hansen models in the prediction of the elasticity

Table 1. Mechanical properties and specifications of mixtures of the ordinary concrete samples [33].

	W/C	Type of aggregate	Volume percent of cement paste	Volume percent of aggregate	Poisson's ratio of aggregate	Elasticity modulus of aggregate (GPa)	Poisson's ratio of cement paste	Elasticity modulus of cement paste (GPa)
1	0.286	Steel	90	10	0.33	205.8	0.228	32.11
2	0.286	Steel	80	20	0.33	205.8	0.228	32.11
3	0.286	Steel	70	30	0.33	205.8	0.228	32.11
4	0.286	Glass	90	10	0.28	72.2	0.228	32.11
5	0.286	Glass	80	20	0.28	72.2	0.228	32.11
6	0.286	Rubble	90	10	0.23	38.0	0.228	32.11
7	0.286	Rubble	80	20	0.23	38.0	0.228	32.11
8	0.286	Rubble	70	30	0.23	38.0	0.228	32.11
9	0.294	Steel	90	10	0.33	205.8	0.217	28.46
10	0.294	Steel	80	20	0.33	205.8	0.217	28.46
11	0.294	Glass	90	10	0.28	72.2	0.217	28.46
12	0.294	Glass	80	20	0.28	72.2	0.217	28.46
13	0.294	Glass	70	30	0.28	72.2	0.217	28.46
14	0.294	Rubble	90	10	0.23	38.0	0.217	28.46
15	0.294	Rubble	80	20	0.23	38.0	0.217	28.46
16	0.294	Rubble	70	30	0.23	38.0	0.217	28.46
17	0.315	Steel	90	10	0.33	205.8	0.208	23.83
18	0.315	Steel	80	20	0.33	205.8	0.208	23.83
19	0.315	Steel	70	30	0.33	205.8	0.208	23.83
20	0.315	Glass	90	10	0.28	72.2	0.208	23.83
21	0.315	Glass	80	20	0.28	72.2	0.208	23.83
22	0.315	Glass	70	30	0.28	72.2	0.208	23.83
23	0.315	Sand	90	10	0.23	54.0	0.208	23.83
24	0.315	Sand	80	20	0.23	54.0	0.208	23.83
25	0.315	Sand	70	30	0.23	54.0	0.208	23.83
26	0.315	Rubble	90	10	0.23	38.0	0.208	23.83

Table 2. Estimation of elasticity modulus of ordinary concrete using Hirsch, Hansen, Cavento and Batch models (results are stated in GPa).

Sample no.	Hirsch model ($x = 0.3$)	Error (%)	Hirsch model ($x = 0.5$)	Error (%)	Hansen model	Error (%)	Cavento model	Error (%)	Batch model	Error (%)	Experimental results [33]
1	38.426	0.78	41.046	7.65	37.167	2.52	40.115	5.21	38.665	1.40	38.13
2	44.231	1.40	48.965	9.15	43.090	3.95	46.972	4.71	46.558	3.79	44.86
3	50.396	8.12	56.928	3.79	50.119	8.62	54.378	0.86	56.063	2.21	54.85
4	34.607	0.52	35.026	0.68	34.676	0.33	35.266	1.37	34.820	0.09	34.79
5	37.236	1.74	38.019	3.88	37.457	2.34	38.236	4.47	37.758	3.17	36.60
6	32.640	1.57	32.657	1.52	32.654	1.53	32.676	1.46	32.655	1.52	33.16
7	33.182	1.15	33.212	1.07	33.207	1.08	33.236	0.99	33.210	1.07	33.57
8	33.736	8.20	33.776	8.09	33.770	8.11	33.800	8.03	33.774	8.10	36.75
9	34.517	0.72	37.204	8.56	33.121	3.35	36.014	5.09	34.686	1.21	34.27
10	39.920	4.97	44.718	6.45	38.615	8.08	42.421	0.98	42.274	0.63	42.01
11	31.014	3.65	31.513	2.10	31.045	3.55	31.742	1.39	31.236	2.96	32.19
12	33.694	6.12	34.628	3.51	33.877	5.61	34.799	3.04	34.284	4.47	35.89
13	36.575	1.78	37.878	1.72	36.992	0.67	37.963	1.94	37.629	1.05	37.24
14	29.258	1.09	29.303	0.94	29.288	0.98	29.349	0.78	29.294	0.96	29.58
15	30.084	5.34	30.164	5.08	30.142	5.15	30.221	4.90	30.153	5.12	31.78
16	30.939	8.65	31.047	8.33	31.021	8.41	31.102	8.17	31.038	8.36	33.87
17	29.484	0.98	32.233	10.39	27.931	4.34	30.693	5.12	29.563	1.24	29.20
18	34.291	0.86	39.102	13.05	32.806	5.16	36.432	5.33	36.676	6.03	34.59
19	39.357	0.81	45.888	15.65	38.694	2.48	42.708	7.63	45.500	14.67	39.59
20	26.404	3.42	27.013	1.19	26.357	3.59	27.191	0.54	26.623	2.62	27.34
21	29.075	8.05	30.216	4.44	29.169	7.75	30.271	4.26	29.744	5.93	31.62
22	31.953	7.89	33.55	3.28	32.313	6.85	33.485	3.47	33.231	4.20	34.69
23	25.701	0.99	26.018	2.23	25.571	1.19	26.198	2.94	25.861	1.62	25.45
24	27.671	5.34	28.264	7.59	27.835	5.96	28.425	8.21	28.065	6.84	26.27
25	29.784	5.13	30.607	8.04	30.101	6.25	30.720	8.44	30.458	7.51	28.33

modulus of ordinary concrete is almost equal. According to the results of Batch model in table 2, it can be stated that the elasticity modulus of mixtures having less aggregate percent is more in agreement with experimental results. In the present study, the concrete material is modelled as a composite formed by a matrix (cement paste) with embedded inclusions (aggregate). Since the size of inclusions (i.e., aggregate) in comparison with other impurities is relatively high, the mechanical properties of interface between the cement paste and aggregates affects the mechanical properties of concrete. It is well known that the structure of cement paste in the vicinity of the aggregates differs from that of bulk cement paste. The phase in which the presence of the aggregates affects the properties of cement paste is taken as the Interfacial Transition Zone (ITZ). Therefore, concrete is a three-phase material consisting of cement paste matrix, discrete inclusions of aggregate and an ITZ between the matrix and inclusions [34, 35]. With increasing percentage aggregate volume, the effects of elastic properties of ITZ phase on the mechanical properties of concrete increase. Since the considered micromechanics models used in the present study are not able to consider the effects of ITZ phase in the formulation, the discrepancy between the present and experimental results increases with increasing percentage aggregate

volume. It appears that more refined micromechanics models are needed for the accurate prediction of mechanical properties of concrete materials with high percentage aggregate volume. This issue will be addressed by the authors in another paper.

The modulus of elasticity of the concrete samples estimated by Mori–Tanaka, self-consistent, effective interface and double-inclusion micromechanics models is presented in table 3. In estimating the elasticity modulus of concrete using Mori–Tanaka model, three different forms were considered for the impurity shape (here aggregate): spherical, elliptical with ratio 1:2 and elliptical with ratio 1:3. As can be observed, the estimated elasticity moduli by assuming spherical shape for aggregates are smaller than the estimated elasticity moduli by assuming elliptical shape for aggregates. This difference is more obvious in mixtures in which the difference between the elasticity modulus of aggregate and matrix is high. As shown in table 3, it can be concluded that the Mori–Tanaka method by assuming elliptical shapes with ratio 1:3 for the impurity (aggregates) leads to more accurate results in predicting the elasticity modulus of concrete. Since the shape of aggregates in reality is closer to an ellipse than a sphere, the results of Mori–Tanaka method by assuming elliptical shape for aggregates are closer to the experimental results. In

Table 3. Estimation of elasticity modulus of the ordinary concrete using Mori–Tanaka, self-consistent, effective interface and double-inclusion models (results are stated in GPa).

No. sample	Mori–Tanaka assuming spherical impurity	Mori–Tanaka assuming elliptical impurity (1:2)	Error (%)	Mori–Tanaka assuming elliptical impurity (1:3)	Error (%)	Self- consistent model	Error (%)	Effective interface model	Error (%)	Double- inclusion model	Error (%)	Experimental results [33]
1	37.16	38.30	2.54	39.29	1.74	37.44	1.81	36.60	4.01	36.93	3.15	38.13
2	43.076	45.46	3.98	47.47	3.54	44.41	1.01	42.45	5.37	42.11	6.30	44.86
3	50.098	53.81	8.66	56.89	0.91	53.61	2.27	49.41	9.92	47.78	12.89	54.85
4	34.67	34.95	0.34	35.14	0.73	34.70	0.27	34.02	2.21	34.59	0.57	34.79
5	37.45	37.98	2.32	38.34	4.26	37.59	2.71	36.76	0.44	37.13	1.45	36.60
6	32.65	32.66	1.54	32.67	1.49	32.65	1.55	32.02	3.44	32.64	1.57	33.16
7	33.207	33.22	1.08	33.23	1.03	33.19	1.13	32.56	3.01	33.18	1.16	33.57
8	77.33	33.79	8.11	33.81	8.03	33.76	8.13	33.11	9.90	33.73	8.22	36.75
9	33.11	34.23	3.38	35.20	1.30	33.39	2.56	32.58	4.93	32.90	4.00	34.27
10	38.60	40.94	8.12	42.93	0.18	39.97	4.85	38.02	9.50	37.70	10.26	42.01
11	31.04	31.36	3.57	31.59	2.22	31.09	3.42	30.45	5.41	30.96	3.82	32.19
12	33.88	34.49	5.60	34.93	3.29	34.05	5.12	33.24	7.38	33.52	6.60	35.89
13	39.99	37.88	0.67	38.50	2.55	37.42	0.48	36.30	2.52	36.15	2.93	37.24
14	29.28	29.31	1.01	29.33	0.88	29.28	1.00	28.71	2.94	29.27	1.05	29.58
15	30.14	30.19	5.16	30.23	4.94	30.14	5.14	29.55	7.02	30.09	5.32	31.78
16	31.02	31.09	8.41	31.14	8.13	31.03	8.38	30.41	10.22	30.92	8.71	33.87
17	27.93	28.97	4.35	29.89	0.79	28.21	3.40	27.46	5.96	27.74	5.00	29.20
18	32.80	35.00	5.17	36.93	3.98	34.19	1.15	32.29	6.65	31.99	7.52	34.59
19	38.69	42.19	2.49	45.20	10.12	42.49	7.08	38.13	3.91	36.73	43/7	39.68
20	26.36	26.73	3.58	27.00	1.74	26.41	3.41	25.85	5.45	26.26	3.95	27.34
21	29.17	29.90	7.75	30.43	4.60	29.42	6.95	28.61	9.52	28.78	9.98	31.62
22	32.32	33.38	6.83	34.14	2.68	32.91	5.12	31.71	8.59	31.40	9.48	34.69
23	25.75	25.95	1.18	26.09	2.24	25.77	1.25	25.24	0.83	25.69	0.94	25.45
24	27.83	28.22	5.94	28.49	7.94	27.94	6.36	27.29	3.88	27.59	02/5	26.27
25	30.10	30.65	6.25	31.03	8.86	30.32	7.03	29.51	4.17	29.53	4.24	28.33
26	24.94	25.01	7.97	25.05	7.64	24.95	7.93	24.45	9.78	24.92	8.04	27.10

Table 4. Results obtained from estimation of Poisson's ratio of ordinary concrete using different micromechanical methods.

Sample no.	Self-consistent	Mori–Tanaka assuming spherical impurity	Mori–Tanaka assuming elliptical impurity (1:2)	Mori–Tanaka assuming elliptical impurity (1:3)	Effective interface	Double inclusion
1	0.23	0.23	0.23	0.24	0.23	0.23
2	0.24	0.23	0.24	0.24	0.23	0.23
3	0.24	0.24	0.25	0.25	0.24	24/0
4	0.23	0.23	0.23	0.23	0.23	0.23
5	0.24	0.24	0.24	0.24	0.24	0.24
6	0.23	0.23	0.23	0.23	0.23	0.23
7	0.23	0.23	0.23	0.23	0.23	0.23
8	0.23	0.23	0.23	0.23	0.23	0.23
9	0.22	0.22	0.22	0.23	0.22	0.22
10	0.23	0.23	0.23	0.24	0.22	0.22
11	0.22	0.22	0.22	0.22	0.22	0.22
12	0.23	0.23	0.23	0.23	0.23	0.23
13	0.23	0.23	0.24	0.24	0.23	0.23
14	0.22	0.22	0.22	0.22	0.22	0.22
15	0.22	0.22	0.22	0.22	0.22	0.22
16	0.22	0.22	0.22	0.22	0.22	0.22
17	0.21	0.21	0.22	0.22	0.21	0.21
18	0.22	0.22	0.22	0.23	0.22	0.21
19	0.23	0.22	0.23	0.24	0.22	0.22
20	0.21	0.21	0.22	0.22	0.21	0.21
21	0.22	0.22	0.22	0.22	0.22	0.22
22	0.23	0.22	0.23	0.23	0.22	0.22
23	0.21	0.21	0.21	0.21	0.21	0.21
24	0.21	0.21	0.21	0.21	0.21	0.21
25	0.21	0.21	0.21	0.22	0.21	0.21
26	0.21	0.21	0.21	0.22	0.21	0.21

predicting the elasticity modulus of concrete using the effective interface micromechanics model, the volume percent of the interface of the two phases, that is, aggregate and cement paste, is considered as 1%. In other words, 1% air bubble is considered in the concrete. Comparison of the results of tables 2 and 3 shows that the Batch model with mean error of 2.67% has the least error in estimating the elasticity modulus of the concrete samples.

Poisson's ratio determination is another application of the micromechanical methods. In table 4, Poisson coefficients of concrete samples with different mix designs are calculated using some of the micromechanical models (Mori–Tanaka, self-consistent, effective interface and double inclusion). As shown, different micromechanical models lead to almost equal results in predicting Poisson's ratio of the ordinary concrete material.

3.2 Estimating the mechanical properties of nano-silica concrete

In recent years, nano-silica particles have been widely used for increasing the compressive strength of the ordinary concrete. Therefore, studying the effect of nano-silica particles on strength and elasticity modulus of the concrete material is of much importance. In this example, the

attempt was to calculate the elasticity modulus and Poisson's ratio of nano-silica concretes using the micromechanical models. With this aim, some nano-silica concrete samples with different types of mixtures are considered. Mechanical characteristics and specifications of four different types of mixtures used in the preparation of nano-silica concrete samples are shown in table 5. In the micromechanics calculations, the concrete material is assumed as the matrix while the added nano-silica is considered as the inclusion phase. For each micromechanics model described in section 2, a computer code is developed in MATLAB. The results obtained from different micromechanical models have been compared to experimental results of Saloma *et al* [36].

The effective elasticity modulus of nano-silica concrete estimated using different micromechanical methods is presented in table 6. Poisson's ratio of nano-silica concrete samples is also calculated using micromechanical models and the obtained results are shown in table 7. In the estimation of elasticity modulus of nano-silica concrete samples using the Hirsch equation, it is assumed that $x = 0.3$. The uptrend of elasticity modulus of the concrete on increasing nano-silica particles can be explicitly observed in table 6. As shown, the results of the micromechanical methods have an excellent agreement to experimental results. Batch and Mori–Tanaka (assuming elliptical

Table 5. Mechanical properties and mix design of the nano-silica concrete samples [36].

Sample no.	Elasticity modulus of phase 1 (ordinary concrete)	Poisson's ratio of phase 1 (ordinary concrete)	Elasticity modulus of phase 2 (nano-silica particles)	Poisson's ratio of phase 2 (nano-silica particles)	Volume percent of nano-silica particles	Volume percent of ordinary concrete
1	39.7	0.3	88.7	0.23	2.5	97.5
2	39.7	0.3	88.7	0.23	5.0	95.0
3	39.7	0.3	88.7	0.23	7.5	92.5
4	39.7	0.3	88.7	0.23	10.0	90.0

Table 6. Estimation of elasticity modulus of nano-silica concrete using different micromechanical methods (results are stated in GPa).

Micromechanical model	Sample 1	Sample 2	Sample 3	Sample 4	Error (%) of Sample 1	Error (%) of Sample 2	Error (%) of Sample 3	Error (%) of Sample 4
Hirsch	40.45	41.22	41.98	42.76	0.92	0.23	0.89	0.80
Hansen	40.46	41.24	42.04	42.85	0.89	0.16	0.76	0.60
Cavento	40.75	41.72	42.65	43.57	0.19	0.99	0.69	1.06
Batch	40.50	41.33	42.17	43.02	0.79	0.04	0.45	0.20
Self-consistent	40.46	41.27	42.10	42.95	0.90	0.11	0.62	0.38
Eshelby	39.80	39.91	40.02	40.12	2.50	3.38	5.52	6.92
Mori–Tanaka assuming spherical impurity	40.47	41.27	42.08	42.90	0.88	0.10	0.66	0.46
Mori–Tanaka assuming elliptical impurity (1:2)	40.56	41.43	42.32	43.22	0.59	0.44	0.11	0.52
Mori–Tanaka assuming elliptical impurity (1:3)	40.62	41.55	42.49	43.45	0.59	0.44	0.11	0.52
Effective interface	40.47	41.27	42.08	42.90	0.88	0.10	0.66	0.49
Double inclusion	40.47	41.24	42.02	42.80	0.88	0.17	0.80	0.72
Experimental results [36]	40.83	41.31	42.36	43.11	–	–	–	–

Table 7. Results obtained from estimation of Poisson's ratio of nano-silica concrete using different micromechanical methods.

Sample no.	Micromechanics model						
	Self-consistent model	Eshelby model	Mori–Tanaka model assuming spherical impurity	Mori–Tanaka model assuming elliptical impurity (1:2)	Mori–Tanaka model assuming elliptical impurity (1:3)	Effective interface model	Double-inclusion model
1	0.3	0.3	0.3	0.3	0.3	0.3	0.3
2	0.3	0.3	0.3	0.3	0.3	0.3	0.3
3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
4	0.3	0.31	0.3	0.3	0.3	0.3	0.3

impurity) methods having, respectively, 0.37% and 0.41% mean discrepancy have the least error in estimating the elasticity modulus of nano-silica concrete. In the present case study, the concrete material is assumed to be the matrix and the added nano-silica is considered to be the inclusion phase. Since the size of nano-silica particles is very small, the mechanical properties of ITZ do not affect the mechanical properties of resulting nano-silica concrete. In case of ordinary concrete, cement paste is assumed to be the matrix phase while the aggregates are considered to be the inclusion phase. As stated in the previous example, the effects of elastic properties of ITZ phase should be considered in micromechanical formulations of ordinary

concrete. As the effects of ITZ phase are neglected in the present study, the accuracy of employed two-phase micromechanical models in the prediction of mechanical properties of nano-silica concrete is more than that of ordinary concrete.

4. Conclusion

In this research, the elasticity modulus and Poisson's ratio of ordinary concrete and nano-silica concrete was calculated using micromechanical methods. Various methods such as Hansen, Cavento, Batch, Hirsch, Mori–Tanaka,

effective interface, Eshelby and double inclusion were used for the prediction of the mechanical properties of the concrete material. The results obtained from the micromechanical models were validated by comparing them to experimental results available in open literature.

Investigation of the results of the estimated elasticity modulus of concrete using micromechanical methods showed that the accuracy of these methods has an inverse relationship with the volume percent of aggregates. On increasing the volume percent of aggregates in the concrete mixture, the accuracy of all micromechanical methods is decreased. Numerical results obtained from the Batch model have the highest agreement with the experimental results.

Numerical results obtained from micromechanical formulations show that addition of nano-silica particles into concrete paste has significant effect on elasticity modulus of concrete, such that on increasing the volume percent of nano-silica particles, the elasticity modulus of concrete is significantly increased. Batch and Mori–Tanaka (assuming elliptical impurity) methods have the highest accuracy in the estimation of the elasticity modulus of nano-silica concrete. Unlike elasticity modulus, the effective Poisson's ratio of the nano-silica concrete is independent of the volume percent of nano particles and it is almost equal to Poisson's ratio of the ordinary concrete material.

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