Recognition of online unconstrained handwritten Gurmukhi characters based on Finite State Automata

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Abstract. This paper presents a language-based efficient post-processing algorithm for the recognition of online unconstrained handwritten Gurmukhi characters. A total of 93 stroke classes have been identified to recognize the Gurmukhi character set in this work. Support Vector Machine (SVM) classifier has been employed for stroke classification. The main objective of this paper is to improve the character level recognition accuracy using an efficient Finite State Automata (FSA)-based formation of Gurmukhi characters algorithm. A database of 21,945 online handwritten Gurmukhi words is primarily collected in this experiment. After analysing the collected database, we have observed that a character can be written using one or more strokes. Therefore, a total of 65,946 strokes have been annotated using the 93 identified stroke classes. Among these strokes, 15,069 stroke samples are considered for training the classifier. The proposed system achieved promising recognition accuracy of 97.3% for Gurmukhi characters, when tested with a new database of 8,200 characters, written by 20 different writers.

Keywords. Online handwriting recognition; Gurmukhi characters; post-processing; strokes association; Finite State Automata; SVM classifier.

1. Introduction

Study in the field of online handwriting recognition for Indian scripts has become popular nowadays. More attention is given towards online handwriting recognition because of these reasons: (i) handwriting is easier than typing, (ii) typing is not possible everywhere, (iii) non-availability of all languages keyboard and (iv) typing is difficult for varied size of characters. Moreover, due to rapid revolution in devices related to Information Technology, including mobiles based on touch screens, digital tablets, notepads, *etc.*, the demand of applications based on online handwriting recognition is increasing day by day. These devices help us in capturing information with the help of a digital-pen/stylus. This captured information is stored as x - y traces with progressive time. A set of such traces is referred to as a stroke in an online handwriting recognition system. A stroke is thus the set of coordinates captured between the two events, namely, pen-down and pen-up with the help of a writing device. A stroke is the building block in online handwriting recognition systems.

Online handwritten character recognition of any scripting language is a difficult task due to the problems of different handwriting styles of different individuals, complex

structure of a language and different touch-based handwriting capturing devices used by the individuals. A good amount of research work has been carried out on online handwriting recognition in the recent past for different scripts such as Chinese, Japanese, Korean and Arabic [1–4]. On the other hand, works on isolated character recognition for many Indic languages like Bangla, Hindi, Tamil and Telugu have been reported by many researchers in the past few years [5–8]. The present study deals with recognition of Gurmukhi script, the most renowned script of north India used to write the Punjabi language. A stroke is considered as the smallest unit in online Gurmukhi script recognition. Further, a Gurmukhi character can be formed using a single recognized stroke or with a combination of more than one recognized stroke. The most common difficulties that occur in the online handwritten Gurmukhi script recognition are

- complex structure of the script,
- similar shape of characters,
- huge variation in handwriting styles,
- formation of a character by combining the strokes, *etc.*

The post-processing phase of online handwriting recognition system deals with character formation, which plays a crucial role in order to improve the accuracy of the recognition system. In the literature, the researchers have

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mostly used the rule-based approach to form a character from the recognized strokes [9–12]. Rule-based methods have certain limitations such as the following: (i) not giving the desired output in case the input data are insufficient and (ii) difficult to write the generic rules to deal with all the situations. However, in the present work, a Gurmukhi character is formed using a different approach, based on Finite State Automata (FSA) theory, which covers all the possible variations used to write a Gurmukhi character and overcome the limitations of rule-based approach. We here present two algorithms, one is an efficient algorithm that reorders the lists of recognized strokes in such a way so as to render formed character(s) in a valid sequence. Another one is FSA-based Gurmukhi character formation algorithm, which forms a character from the sequence of strokes available in the lists.

The rest of the paper is organized as follows. In section 2, we present a brief review of the related works. A brief discussion about the Gurmukhi script is given in section 3. In section 4, various challenges to Gurmukhi character are discussed. Section 5 describes about the online handwritten data collection. The pre-processing steps and features used in the present study are discussed in section 6. The classification tool used for stroke classification has been discussed in section 7. Section 8 elaborates the detailed process of Gurmukhi character formation. Experimental results are presented in section 9 and concluding remarks are given in section 10.

2. Related work

This section briefly reviews the literature on online handwriting recognition systems. Tappert *et al* [13] have described the state of art in online handwritten character and word recognition systems. Shape recognition algorithms, pre-processing techniques and post-processing techniques have extensively been surveyed in their paper. Gao *et al* [14] presented an online handwritten Japanese text recognition system. Their system includes an offline character recognizer and other components for linguistic and geometric contexts. A linear Markov Random Field (MRF) chain model has been used in their work. They have reported an accuracy varying from 91.1% to 95.7% by combined the recognizer. Aparna *et al* [15] presented a recognition system for online handwritten Tamil characters. In their work, a set of 18 shape features is used for stroke recognition and they implemented the character formation scheme using FSA theory. They employed an accuracy of 82.80% for Tamil characters. Swethalakshmi *et al* [16] developed an online handwritten character recognition system for Devanagari and Telugu scripts. The Support Vector Machine (SVM) classifier has been used for stroke recognition. In post-processing, rule-based strokes database is used for character formation in their work. Rampalli and

Ramakrishnan [17] have proposed an online handwritten character recognition system working in blend with an offline recognition system for Kannada. In their work, online written strokes are converted into an offline image and concurrently recognized by both online and offline strategies. They have reported an improvement of 11.0% in accuracy by combining the online recognition system with offline one. Roy *et al* [18] have discussed online handwritten Bangla characters formation. In their study, they have computed a total of 15 structural features and 90 point-based features. The Multilayer Perceptron Neural Network scheme has been used for stroke recognition. They have followed rule-based strategy for Bangla character formation. Later on, Samanta *et al* [19] proposed a novel approach for online unconstrained handwritten Bangla words recognition using the hidden Markov (HMM) model. In their work, a whole word sample is considered as a basic unit for recognition instead of recognizing individual sub-strokes. Circular feature, linear feature and combination of circular and linear features have been computed at sub-strokes level after segmenting the word into sub-strokes. They have implemented fully connected non-homogeneous HMMs in their experiments. The applied smoothing of the probability estimates at two levels shows the promising results in online handwritten Bangla word recognition.

A very limited amount of research work has been carried out on online handwritten character recognition for Gurmukhi script in recent past. Initially, the work in online handwritten Gurmukhi script was started with basic characters recognition using elastic matching technique [20]. In this study, online Gurmukhi character is formed using a single or more than one stroke and the strokes are pre-processed through five steps (i.e., normalization and centering, interpolating missing points, smoothing, slant correction and re-sampling of points) in sequential order. Later, Sharma *et al* [21] extended this work by introducing a new approach of rearrangement of recognized strokes on the basis of dependent and major dependent strokes before recognition of a particular Gurmukhi word. Afterwards, in [22], the 41 Gurmukhi characters recognition was studied using a HMM-based classifier. In 2013, Kumar and Sharma [10] presented an efficient algorithm for post-processing of online handwritten Gurmukhi character recognition where the characters were formed by a rule-based method. Later on, Kumar *et al* [11] discussed the recognition of multi-stroke-based online handwritten Gurmukhi Aksharas, where SVM classifier has been used for stroke recognition and rule-based approach was used for character and Aksharas formation. In their work, they have tested 4,310 Gurmukhi Aksharas, written by 10 different writers. They have also shown the results of 35 Gurmukhi characters with a combination of *Vowel* and *Nasal*. In [23], online handwriting recognition procedure for Gurmukhi script was discussed. Various challenges, like zone identification of a stroke, identification of location of headline and dot detection, were discussed in pre-processing phase.

Moreover, in classification phase, rule-based methods and statistical classification methods like TDNN, HMM and SVM were also discussed. In a recent study [24] of online handwritten Gurmukhi character recognition, a voting-based classification model is implemented based on HMM and SVM classifiers. In their work a total of 74 stroke classes have been identified in order to implement Gurmukhi character recognition. Stroke classification has been carried-out based on five different features, namely, normalized x - y traces (N_{xy}), region-based features (R_{xy}), curvature features (C_{xy}), curvature-feature-based classes (C_{xy}^N) and directional features (D_{xy}). In post-processing, they have used the rule-based method proposed by Kumar and Sharma [10] for character formation from the recognized strokes. In experimentation, they have considered a test data set of 35 Gurmukhi *consonants*, which was written by 10 different writers, five times each. Additionally, the major issues of confusing characters formation have also been studied by them.

3. Gurmukhi script

Gurmukhi is the script used for writing Punjabi language. Punjabi is an Indo-Aryan language spoken by 102 million speakers worldwide. It is the 10th most widely spoken language in the world. In the 16th century, Shri Guru Angad Dev Ji, second Sikh Guru, standardized the Gurmukhi

Consonants	ੳ	ਅ	ੲ	ਸ	ਹ	ਕ	ਖ	ਗ	ਘ	ਙ	
	ਚ	ਛ	ਜ	ਝ	ਞ	ਟ	ਠ	ਡ	ਢ	ਣ	
	ਤ	ਥ	ਦ	ਧ	ਨ	ਪ	ਫ	ਬ	ਭ	ਮ	
	ਯ	ਰ	ਲ	ਵ	ੜ	ਸ਼	ਖ਼	ਗ਼	ਜ਼	ਫ਼	ਲ਼
Vowels	ਾ	ਿ	ੀ	ੁ	ੂ	ੇ	ੈ	ੋ	ੌ		
Nasal Symbols	ੰ	ੰ	ੰ								

Figure 1. Gurmukhi character set.

Figure 2. Online handwritten Gurmukhi script representation in three zones. Different colours indicate different strokes within a character sample.

script. The name of Gurmukhi is derived from the old Punjabi term *Guramukhi*, which means from the mouth of Guru. The following are some of the basic characteristics of Gurmukhi script: (i) the writing structure of Gurmukhi script is cursive and (ii) it is written from left to right direction. Figure 1 illustrates various characters of the Gurmukhi script. It has a basic set of 53 characters: 41 *consonants* (35 basic *consonants* + 6 additional modified *consonants* (i.e., ਞ, ਝ, ਞ, ਜ਼, ਢ, and ਲ਼), 9 *vowels* and 3 nasal symbols. Among these 9 *vowels*, 4 *vowels* (i.e., ੇ, ੈ, ੌ and ੐), considered as upper matras, reside above the *consonants*; 2 *vowels* (i.e., ੱ and ੲ), considered as lower matras, reside below the *consonants* and 3 *vowels* (i.e., ਾ, ਿ and ੀ) appear adjacent to a *consonant*. The nasal symbols (i.e., ੰ, ੱ and ੲ) are also considered as upper matras and they also reside above the *consonants* [25].

3.1 Set of strokes considered in this work

Online handwritten strokes for Gurmukhi script can be drawn in one of the three horizontal zones, namely, Upper Zone, Middle Zone and Lower Zone as shown in figure 2. In Gurmukhi script the *consonants* are formed along with a horizontal line, called *head-line*, drawn above the *consonant*. Upper matras strokes can be separated on the basis of this *head-line*. To detect this *head-line* is the major problem in Gurmukhi character recognition. Samanta *et al* [19] proposed an algorithm to detect the *headline* and *baseline* to classify the strokes into three zones. In the present work, the same scheme is used to detect the *headline*, which helps classify the strokes into two zones (i.e., Upper Zone and MiddleLower Zone).

To identify the stroke classes for online handwritten Gurmukhi character recognition system, we analysed the collected handwritten data. During analysis, we identified the shapes of strokes, used by different writers, to write a Gurmukhi character. In this way, all the possible variations for writing the Gurmukhi characters have been observed. This resulted in a total of 93 stroke classes. Among these identified stroke classes, there exist some strokes that are similar in shape and participate in the formation of *consonants* and *vowels*, leading to a confusion to the recognizer, if classified using a single classifier. In order to reduce this confusion between the stroke classes and increase stroke-level classification accuracy, these stroke classes are further decomposed into two subsets: (i) Upper Zone stroke set and (ii) MiddleLower Zone stroke set, shown in figure 3. The Upper Zone stroke set contains 12 stroke classes to classify the upper matras and the MiddleLower Zone stroke set contains 81 stroke classes to classify all the *consonants* and lower matras. Therefore, two zone-wise classifiers, namely, *UpperZoneClassifier* and *MiddleLowerZoneClassifier*, are trained to classify the input strokes in the respective

zones. In the present study, the reasons to keep the stroke classes of lower matras and *consonants* in a single set are the following: (i) the count of lower matras is less in comparison with *consonants* (i.e., lower matras = 2, and *consonants* = 41) and (ii) the stroke classes lower matras

are confused with the stroke classes of *consonants*. Therefore, the stroke classes to classify the lower matras (ॐ, ॐ, and ॐ) are separated from MiddleLower Zone on the basis of their position on y-axis in post-processing phase.

Upper Zone stroke set	121	—	MiddleLower Zone stroke set	141	ॐ	156	ॐ	172	ॐ	188	ॐ	204	ॐ	223	ॐ
	122	ॐ		142	ॐ	157	ॐ	173	ॐ	189	ॐ	205	ॐ	224	ॐ
	123	ॐ		143	ॐ	158	ॐ	174	ॐ	190	ॐ	206	ॐ	225	ॐ
	124	ॐ		144	ॐ	159	ॐ	175	ॐ	191	ॐ	207	ॐ	121	—
	125	ॐ		145	ॐ	160	ॐ	176	ॐ	192	ॐ	208	ॐ	351	ॐ
	126	ॐ		146	ॐ	161	ॐ	177	ॐ	193	ॐ	209	ॐ	102	.
	127	ॐ		147	ॐ	162	ॐ	179	ॐ	194	ॐ	210	ॐ		
	128	ॐ		148	ॐ	164	ॐ	180	ॐ	195	ॐ	211	ॐ		
	131	ॐ		149	ॐ	165	ॐ	181	ॐ	196	ॐ	212	ॐ		
	132	ॐ		150	ॐ	166	ॐ	182	ॐ	197	ॐ	214	ॐ		
	133	ॐ		151	ॐ	167	ॐ	183	ॐ	198	ॐ	215	ॐ		
	134	ॐ		152	ॐ	168	ॐ	184	ॐ	200	ॐ	216	ॐ		
				153	ॐ	169	ॐ	185	ॐ	201	ॐ	217	ॐ		
				154	ॐ	170	ॐ	186	ॐ	202	ॐ	218	ॐ		
				155	ॐ	171	ॐ	187	ॐ	203	ॐ	222	ॐ		

Figure 3. Shapes of strokes considered for Gurmukhi character recognition system. The three digits numerical values represent stroke labels assigned to the respective stroke shapes.

Figure 4. Variations in writing Gurmukhi character ‘ਗ’: (a) the character ‘ਗ’ is written in a single stroke, (b) written in two strokes, (c) written in two strokes, with different shapes, (d) written in three strokes and with different shapes and (e) written in two strokes and again with different shapes. (For interpretation of the references to colour in this figure, the reader is referred to the web version of this paper).

Figure 5. Similar features of strokes in online handwritten Gurmukhi word “ਗੁਰੂ”. In (a), the stroke label of the lower matras strokes s_4 and s_5 is “121”, which is replaced with the new stroke label “101” after extracting from MiddleLower Zone.

4. Challenges in online handwritten characters formation

Due to variation in handwriting, writers have their own way of writing, which poses a major challenge to character recognition. Writers write characters in different sizes. It has also been observed that a Gurmukhi character can be formed using different combinations of strokes, as illustrated in figure 4. This makes the character formation difficult.

Moreover, when a character is formed by combining the predicted online input strokes, the problem of strokes merging, strokes discarding and strokes sequencing arises, where it may be possible that online input strokes belong to same character but their sequence of input may be different, so these types of issues are handled by stroke merging strategy.

Apart from this, a problem of ambiguity between two or more strokes that may arise due to similar features of strokes that belong to different Unicode characters is illustrated in figure 5. It has been resolved by identifying the position of ambiguous strokes on x -axis and y -axis (horizontal and vertical projection, respectively) and then their assigned stroke labels are changed accordingly. In figure 5a, four strokes, s_2 , s_4 , s_5 and s_6 , are of similar shape. The predicted stroke label of these strokes is “121”, associated with stroke sequence number. This ambiguity has been resolved by replacing the stroke label of strokes s_2 , s_4 and s_5 with “163”, “101” and “101”, respectively, shown in figure 5b. Stroke classes for lower matras (i.e., ॐ, ॐ and ॐ) are then extracted from MiddleLower Zone.

In the literature, many researchers have used the rule-based approach to combine the recognized strokes

[10, 16, 24, 26, 27]. In the present study, Gurmukhi characters are formed by combining the recognized strokes using FSA-based scheme.

5. Data collection

In online handwriting recognition system, the creation of standard data for training and evaluation is the first challenge. In this section, we briefly describe the methodology adopted for creating the word level data set for Gurmukhi script.

- Identification of words: This is important to identify a minimal set of words that covers all the defined symbols and can be used as prompts for handwriting data collection. In this connection, we finalized a data set of 126 unique Gurmukhi words, composed of two–three characters.
- Source of data collection: Data have been collected from varied classes of writers. The writers are (i) familiar with Gurmukhi script writing, (ii) students studying in Government schools and Government colleges at diploma/degree level and (iii) professionals, working in Government offices are chosen for the data collection from different places in Punjab (a State of India). A total number of 175 writers, belonging to different age groups, contributed in data collection, shown in table 1.
- Data collection process: An application has been developed and used for collecting the handwriting samples corresponding to the identified data set. A touch-based device, Tablet-PC, has been used for capturing the handwriting samples. Proficiency with the Tablet-PC allows the writer to write in his/her natural handwriting style. Writing on the smooth glass surface using the stylus/digital-pen of the Tablet-PC feels very different from writing on paper. Therefore, a prior training about the writing with Tablet-PC is given to the writer before starting the data collection. Finally, the writer is asked to write the data set of 126 words.
- Annotation: A stroke is the building block in the online handwriting recognition systems as discussed in section 1. Therefore, handwriting information is stored at stroke level. The strokes information such as (i) stroke sequence number, (ii) stroke label, (iii) strokes count and (iv) x – y coordinates values for each point of a stroke is stored in a XML file with respect to a word. In

this way, the collected handwritten data are annotated by labelling every stroke with the respective stroke label, depicted in figure 3. We have annotated 65,946 strokes in the present study.

As described in section 3.1, a total of 12 unique stroke classes for Upper Zone and 81 unique stroke classes for MiddleLower Zone have been identified from the collected data. Hence, in order to train the classifiers for both the zones, we have taken 145–170 samples of each identified stroke class. To test the performance of online handwritten Gurmukhi Unicode character recognition system further, a new database was collected from 20 different writers in which each writer is asked to write 41 Gurmukhi *consonants* 10 times. Hence, in this way, a total of 8200 Gurmukhi characters were collected during this process and further used for testing the proposed system.

6. Pre-processing and feature extraction

Generally, the pre-processing operations are applied to remove the variability in the strokes due to the writing styles as well as due to the noise (duplicate points) in the collected data. In the present study, the captured handwritten strokes are partitioned into two zones (i.e., Upper Zone and MiddleLower Zone) [19]. This step helps in reducing the confusion between the similar shape of stroke classes included in Upper Zone and in MiddleLower Zone. After this step, size normalization and centring of the stroke are performed, where the online input stroke is scaled into a fixed size (300×300) of window and moved to the central location. While the strokes are being captured from the Tablet-PC, some of the points of a stroke may be missed due to the limitations of hardware or software and sometimes it may happen due to fast writing of a stroke. In order to retrieve these missing points, Bezier interpolation technique has been used [28]. Thereafter, re-sampling is performed to obtained a constant number of points with uniform distance for all the captured strokes. Therefore, after the interpolation step, the resulting N number of points of each stroke are re-sampled into a fixed size of 64 x – y coordinates (128 values). The resulting 128 values of x and y coordinates are considered as a feature set in the present study.

The length (x – y coordinates) of the stroke(s) may vary among different writers. Therefore, to make an efficient stroke classification, we have normalized all the strokes

Table 1. Writers information as age group and gender.

Writers	Age group (10–20)		Age group (20–35)		Age group (35–50)		Total
	Male	Female	Male	Female	Male	Female	
In training data	32	23	37	40	25	18	175
In test data	4	3	6	2	3	2	20

into uniform size, i.e., 64 points, and extracted 128 feature values (x and y coordinates for each point). Moreover, the efficiency of an online handwritten character recognition system mainly depends on the trained model. For a better recognition accuracy, it is important to identify suitable features. In the present study, we have considered the pre-processed x - y coordinates as features. One can, however, explore the suitability of other features for Gurmukhi script recognition.

7. Classification

We have used the LibSVM [29] classifier in our implementation. The key role of SVM classifier in this work is to classify the pre-processed strokes. This classifier takes the arrangement of features according to its standard input file format. The LibSVM classifier can be trained using a feature file consisting of all the classes in order to generate the model file, which can further be used to classify the test data at stroke level. The LibSVM classifier has four different kernels, namely, Linear Kernel, Polynomial Kernel, RBF Kernel and Sigmoid Kernel. It has been seen that effectiveness of the classifier depends on the kernel used, kernel parameters and soft margin or penalty parameter C . After carrying out these experiments, we decided to use the RBF Kernel in this work. To measure the robustness of the best predictive model, K -fold cross-validation is experimented. The parameters C and γ have been considerably varied in order to increase cross-validation accuracy. The model yielding maximum cross-validation accuracy has been used for performance evaluation.

8. Character formation

After the recognition of pre-processed strokes using SVM classifier, the recognized strokes are processed further for character generation in post-processing phase. A Gurmukhi character is written using a single stroke, generated easily by mapping the stroke-ID with respective Unicode character. However, when a character is written with more than one stroke, it is necessary to process those strokes in such a way to form a valid Gurmukhi character. As discussed in section 4, a Gurmukhi character can be written with varied number of strokes, use of different combinations of strokes and different sequences of strokes. In this connection, the recognized strokes are processed through several post-processing steps in order to form a valid Gurmukhi character. After the recognition phase the recognized strokes are stored zone-wise in two lists, namely, *UpperZoneList* and *MiddleLowerZoneList*. Each stroke in these lists contains the information on stroke-ID, stroke sequence number and stroke bounding-box knowledge. The two structures, namely, *structStrokes* and *structCharacter*, are used to store and process this information for post-processing operations.

The post-processing operations include (i) strokes merging, (ii) extracting lower matras, (iii) re-ordering of recognized strokes' lists and (iv) character formation using FSA, illustrated in figure 6. The recognized strokes from *UpperZoneList* and *MiddleLowerZoneList* are processed according to the original input sequence through these post-processing steps. The strokes from *UpperZoneList* are mapped to a new list for all the upper matras, referred to as *uzlist*. For efficient post-processing, the *MiddleLowerZoneList* is further partitioned into two sub-lists, namely, *lzlist-1* and *lzlist-2*, where *lzlist-1* contains all the strokes for Gurmukhi consonants and *lzlist-2* contains the lower matras strokes for vowels, and ੲ. Further, we explore the post-processing operations in detail.

8.1 Stroke merging

After the manual verification of handwritten collected data, we observed two strokes: one is *vertical-line* ('|') stroke and another one is *horizontal-line* ('—') stroke, used for writing most of the Gurmukhi characters. These two strokes have different meanings according to their positions with the respective partially written character. During the processing of recognized strokes, if any one of these two strokes occurs, its association (*vertical-line* association, *horizontal-line* association) is verified with the processed stroke(s).

8.1a Vertical-line association: The *vertical-line* stroke has five different types of association according to its position, with the respective partially written character illustrated in figure 7. The Gurmukhi characters involved in this association are ਅ, ਮ, ਜ, ਪ, ਖ, ਧ, ਸ, ਬ, ਘ, ਙ, ਗ, ਨ, ਠ, ਢ, ਟ and ਤ. In *vertical-line* association, the bounding box of the stroke *vertical-line* is checked for valid association with the bounding box of adjacent processed stroke(s) to form a valid Unicode character. For example some of the Gurmukhi characters such as ਅ, ਮ, ਜ, ਪ, ਖ, ਧ, ਸ, ਬ, ਘ and ਙ are usually written by two or more strokes along with a *vertical-line* stroke, where the position of *vertical-line* stroke is always at the right side to the bounding box of pre-written stroke(s), see figure 7a.

In figure 7b, two Gurmukhi characters ਗ and ਚ are written similarly as before using two or more strokes along

Figure 6. Flow chart of Gurmukhi character formation.

with a *vertical-line* stroke. These two characters used the same shapes of strokes, but their lengths of *vertical-line* strokes are different. Thus, in this case, the length of stroke is the key factor to decide these two different characters. If the length of the *vertical-line* stroke is greater than a threshold value (i.e., 75% of the length on y-axis of pre-written *consonant* ‘ੜ’), then a single Unicode *consonant* ‘ੜ’ is formed, shown in figure 8a; otherwise two separate Unicode characters *consonant* ‘ੜ’ and *vowel* ‘ੴ’ are formed and when these two Unicode characters are rendered in sequence, the character ‘ੜੴ’ is formed, demonstrated in figure 8b.

In this category, those characters in which the *vertical-line* stroke is involved are written at the top-centre position to the bounding box of the post-written stroke, as shown in figure 7c. Similarly, in other categories, those characters come in which the *vertical-line* stroke is written at the upper-left corner/bottom-right corner to the bounding box of the pre-written stroke(s), seen here in figure 7d and e.

8.1b Horizontal-line association: *Horizontal-line* stroke also plays an important role in Gurmukhi character recognition. This stroke is used for the formation of some of the *consonants* (ੜ, ਞ, ਯ and ਧ) and *vowels* (ਊ and ਊ). Two different cases have been found while associating the *horizontal-line* stroke with the partially written *consonant* on the basis of its position on x-axis and y-axis as shown in figure 9. In the first case, the Gurmukhi *consonants*

Figure 8. Decision of *vertical line* in formation of two different Gurmukhi characters, ੜ and ੜੴ. Different colours indicate different strokes.

ੜ, ਞ and ਧ are formed, when the *horizontal-line* stroke appears at the upper part of the bounding box of the *consonants* ੜ, ਞ and ਧ, respectively. In the second case, the *consonants* ਯ and ਧ are formed, when the *horizontal-line* stroke appears in the middle of bounding box of *consonants* ਯ and ਧ, respectively, by taking the decision of *horizontal-line* stroke on x-axis.

8.2 Lower matras extraction

In order to process the input stroke sequence more efficiently, stroke labels of three *vowels* (ਊ, ਊ and ਊ) have been extracted from *MiddleLowerZoneList* and moved to *lzlist-2* with their sequence number, depicted in figure 5b. This operation is performed in the Lower Matras Extraction module. This decision is carried out on the basis of x- and

(a)	(b)	(c)	(d)	(e)

Figure 7. Different cases of *vertical-line* association with Gurmukhi *consonants*, where different colours indicate different strokes: (a) examples, where *vertical line* is written to the right side of the bounding box of the *consonant*, (b) decision to identify two different characters is carried out on the basis of length of the *vertical-line* stroke on y-axis. Cases where the position of *vertical-line* stroke is at the top centre, upper left and bottom right are illustrated in (c), (d) and (e), respectively.

Figure 9. Cases of *horizontal-line* association with Gurmukhi *consonants*, where different colours indicate different strokes. The *horizontal-line* stroke in (a) is written above a *consonant* and in (b) at the middle to a *consonant*.

y-projections of these strokes. Now, the lists *uzlist*, *lzlist-1* and *lzlist-2* are added into a single list, *alist*. This decision of addition is also taken at an intermediate stage when the stroke label of vowel ‘ਫਿ’ or ‘ਫੀ’ is encountered.

8.3 Strokes’ lists re-ordering

The *alist* consisting of three sub-lists: *uzlist*, *lzlist-1* and *lzlist-2* is now ready to form a Unicode character. These strokes’ lists need to be rendered in a fixed sequence in order to form a valid Unicode character. In this connection, these strokes’ lists are arranged/re-ordered according to the minimum sequence number in each list, as illustrated in figure 10. Finally, these three sub-lists in *alist* are processed in re-ordered sequence and a valid Unicode character is

Algorithm 1 Re-ordering of recognized strokes’ lists

Input: Sequence of predicted strokes, $PS \leftarrow \{ps_0, ps_1, \dots, ps_N\}$ RO_l for re-ordering the lists in *alist*

Output: *alist*

▷ This list (*alist*) is passed further to Algorithm 2 for Character formation

```

1: Initializations:  $i = 0$ 
2:  $l \leftarrow \text{NULL}$ 
3:  $uzlist \leftarrow \text{NULL}$ 
4:  $lzlist-1 \leftarrow \text{NULL}$ 
5:  $lzlist-2 \leftarrow \text{NULL}$ 
6:  $alist[] \leftarrow \text{NULL}$ 
7: while  $i \leq N$  do
8:   if ( $ps_i$  belongs to set of Upper Zone stroke-labels) then
9:     Add  $uzlist \leftarrow ps_i$ 
10:  end if
11:  if ( $ps_i$  belongs to set of Lower Zone stroke-labels) then
12:    if ( $lzlist-1 = \text{IsEmpty}$ ) then
13:      Add  $lzlist-1 \leftarrow ps_i$ 
14:    else if ( $(Xmin \text{ of } ps_i > \text{MAX}(Xmax \text{ of strokes in } lzlist-1)) \text{ and } (ps_i \neq \text{label of stroke '—'})$ ) then
15:      if ( $uzlist \neq \text{NULL}$ ) then
16:        Add  $RO_l \leftarrow \text{MIN}(\text{position of strokes in } uzlist)$ 
17:      end if
18:      if ( $lzlist-1 \neq \text{NULL}$ ) then
19:        Extract Lower Zone matras stroke-label(s) from  $lzlist-1$ , if exist and move in  $lzlist-2$ 
20:        Add  $RO_{l+1} \leftarrow \text{MIN}(\text{position of strokes in } lzlist-1)$ 
21:      end if
22:      if ( $lzlist-2 \neq \text{NULL}$ ) then
23:        Add  $RO_{l+2} \leftarrow \text{MIN}(\text{position of strokes in } lzlist-2)$ 
24:        Add  $uzlist, lzlist-1$  and  $lzlist-2$  in  $alist[i]$  according to min to max index value, stored in  $RO_l$ 
25:         $uzlist \leftarrow lzlist-1 \leftarrow \text{NULL}$ 
26:        Add  $lzlist-1 \leftarrow ps_i$ 
27:      end if
28:    else if ( $(ps_i = \text{label of stroke 'l'}) \text{ OR } (ps_i = \text{label of stroke '—'})$ ) then
29:      if ( $ps_i$  Has a valid association with already processed strokes) then
30:        Add  $lzlist-1 \leftarrow ps_i$ 
31:      else
32:        Delete ( $ps_i$ )
33:      end if
34:    else
35:      Add  $lzlist-1 \leftarrow ps_i$ 
36:    end if
37:  end if
38:   $i \leftarrow i + 1$ 
39:  Repeat steps 14 to 24
40: end while
41: return alist

```

▷ Predicted stroke counter

▷ Subscript to RO

▷ *UpperZoneList*

▷ *MiddleLowerZoneList*

▷ Lower matras

▷ Containing re-ordered lists of recognized strokes

▷ Separate character

▷ Association of *vertical-line* and *horizontal-line*

Algorithm 2 Gurmukhi Unicode character formation using Deterministic Finite Automata**Input:** Sequence of predicted stroke-labels in *alist*, $PSL \leftarrow psl_0, psl_1, \dots, psl_N$;**Output:** *finalState* number

```

    struct NextStateInfo
    {
        int nextState;
        int strokeLabel;
        NextStateInfo* next;
    };
    struct StateInfo
    {
        int stateNo;
        NextStateInfo* head;
        NextStateInfo* tail;
        StateInfo* next;
    };
    } q;
    struct StateLocation
    {
        int state;
        int location;
        StateLocation* next;
    };
    } qFinal;

1: Initializations:  $s \leftarrow 0$  ▷ State counter
2: load(amFile) ▷ Read CharactersMachine file
3: load(fsuFile) ▷ Read finite State to Unicode mapping file
4:  $q \rightarrow stateNo[] \leftarrow \text{NULL}$  ▷ For storing all states information
5:  $qFinal \rightarrow state[] \leftarrow \text{NULL}$  ▷ For storing all final states information
6:  $nextState[] \leftarrow \text{NULL}$  ▷ next-state number
7:  $currentState \leftarrow 0$  ▷ current-state initialization
8:  $prevState \leftarrow 0$  ▷ Previous state
9:  $tranCount \leftarrow 0$  ▷ Transitions count
10:  $finalState \leftarrow 0$  ▷ Final state number
11: while ( $(q \rightarrow stateNo[s] \leftarrow amFile.ReadLine()) \neq \text{NULL}$ ) do ▷ Reading all the next-states and stroke labels with reference to the state number.
12:    $s \leftarrow s + 1$ 
13: end while
14: while ( $(qFinal \rightarrow state[f] \leftarrow fsuFile.ReadLine()) \neq \text{NULL}$ ) do ▷ Reading all the final states number with reference to their Unicode Index
15:    $f \leftarrow f + 1$ 
16: end while
17: for  $i \leftarrow 0$  to  $N$  do
18:   for  $j \leftarrow 0$  to  $\text{sizeof}(q \rightarrow stateNo)$  do
19:     if ( $q \rightarrow stateNo[j] = currentState$ ) then
20:        $nextState[i] \leftarrow currentState$ 
21:       for  $n \leftarrow 0$  to  $\text{sizeof}(q \rightarrow stateNo)$  do
22:         if ( $q \rightarrow stateNo[nextState[i]] \rightarrow strokeLabel[n] = psl_i$ ) then
23:            $nextState[i] \leftarrow q \rightarrow stateNo[j] \rightarrow nextTransitionState[n]$ 
24:           break
25:         end if
26:       end for
27:        $tranCount \leftarrow tranCount + 1$ 
28:     end if
29:     if ( $prevState \neq nextState[i]$ ) then
30:       for  $k \leftarrow 0$  to  $\text{sizeof}(qFinal \rightarrow state)$  do
31:         if ( $nextState[i] = qFinal \rightarrow state[k]$ ) and ( $N = tranCount$ ) then
32:            $finalState \leftarrow nextState[i]$ 
33:           break
34:         end if
35:       end for
36:     else

```

```

37:      if ((N = tranCount) and (finalState = 0)) then
38:          t ← N
39:          while (t > 0) do
40:              for m ← 0 to sizeof(qFinal->state) do
41:                  if ((nextState[t] = qFinal->state[m]) and (N = tranCount)) then
42:                      finalState ← nextState[m]
43:                      break
44:                  end if
45:              end for
46:              if (finalState ≠ 0) then
47:                  break
48:              else
49:                  t ← t - 1
50:              end if
51:          end while
52:      end if
53:  end if
54:  prevState ← nextState[i]
55:  currentState ← nextState[i]
56: end for
57: end for
58: if (finalState ≠ 0) then
59:     return finalState
60: else
61:     if (N > 0) then
62:         for s ← 0 to N do
63:             psl[s] ← psl[s + 1]
64:         end for
65:         goto step number 17
66:     end if
67: end if

```

Figure 10. Strokes' lists (*uzlist*, *lzlist-1* and *lzlist-2*) re-ordering. Different colours indicate different strokes. The zone-wise strokes lists are arranged on the basis of minimum stroke sequence number from the grouped strokes in each list.

formed corresponding to each processed list using FSA-based Gurmukhi character formation algorithm.

8.4 Character formation using FSA

The key role of FSA in this work is to form the Gurmukhi Unicode characters from the recognized strokes. A Gurmukhi character can be written using varied number of strokes, different shapes of strokes and different sequences of strokes. Therefore, FSA is implemented in the present work to form a Gurmukhi character by covering all these variations. This section elaborates the character formation process, from the recognized strokes to a formed Gurmukhi

character using the proposed FSA-based character formation algorithm. Before employment of FSA, we discuss the basic terminology used in creation of a Finite State Machine (FSM), as an automaton is a study of abstract machines. FSMs are commonly used to organize and represent an execution flow, which is useful for implementation in various applications of AI. The implementation of an FSM begins with the states and transitions it has. The machine has only one state at a time. The state it is in at any given time is called the *current state*. It can change from one state to another state by triggering an event or a condition, called a transition. A particular FSM is defined by a set of its states, and triggering condition for each transition. Following notations are used to model an FSM.

$$\text{Automata} = (Q, \Sigma, \delta, q_0, F) \quad (1)$$

where Q is a finite set of states, Σ is the alphabet (of input symbols), δ is the transition function, $q_0 \in Q$ is the initial/start state and $F \subseteq Q$ is the final state set.

The transition function (Eq. (2)), where δ denotes the transition function, q_i and q_j are states $\in Q$ and the second parameter (i.e., a) is an alphabet $\in \Sigma$ takes two input parameters, *current state* of the machine and input symbol a . This function returns *next state* after processing the input symbol in set Σ . The *next state* then becomes the *current state* when it is processed again with the next symbol in set Σ . The same process is repeated again and again until the set Σ becomes empty.

$$\delta(q_i, a) = q_j. \quad (2)$$

This state transition function helps in formation of Gurmukhi Unicode character. Initially, the value of the *current state* is set to zero ($q_i = 0$). When the first predicted stroke label is processed along with the *current-state* value, the function returns the *next state*, q_j . This q_j may be either a final state or a non-final state. If q_j is a non-final state and there exists any unprocessed predicted stroke label, then it is processed again along with the

Table 2. Gurmukhi character machine file format.

0	1:161 2:159 3:162 7:154 8:160 9:195 11:121 12:141 12:142 13:143 14:144 14:145 17:150 16:182 16:146 18:148 ...
1	2:162 4:121 5:163 9:154 2:147
2	9:121 8:163
3	2:161 6:163 55:207 7:121 35:174 49:198 50:149 17:146 44:360 44:148 53:202
4	9:162
5	8:162
6	8:161
7	9:161 35:174 49:197 55:207 53:202
8	10:121
9	10:163
10	
11	4:161 7:162 9:159 12:141 12:142 13:143 19:189 26:157 26:158 16:146 18:148 21:147 23:152 23:151 23:206 ...
12	12:121
13	13:121
14	14:121 14:162 14:147
:	

next predicted stroke label in the sequence set Σ . This process is repeated for all the predicted strokes in Σ to get a final state. After the completion of this process the final state number is mapped with Unicode Gurmukhi character-set file's index in order to get the respective Unicode character. On the other hand, if the transition function returns a non-final state at the end, the processed set Σ is re-submitted to the transition function by eliminating the last predicted stroke label from the sequence set Σ . This process is followed to search a final state by processing the sub-set of Σ . The process of eliminating the predicted stroke label starting from the last index in the sequence set Σ and re-submission is continued $n-1$ times.

The FSA-based Gurmukhi Unicode character formation algorithm works with three associated files, namely, CharactersMachine file, FinalStateToUnicodeMapping file and UnicodeCharacterSet file. In the CharactersMachine file, all the stroke labels defined in figure 3 are arranged along with their assigned state number (i.e., '0' or initial state to the n^{th} or final state). Formation of a Unicode character always starts from the initial state; hence, here all the possible stroke labels, which are usually written as the first stroke for the formation of any Unicode character, are written along with their *next state*, which is written prior to stroke labels in this file (i.e., *next-state: stroke label*). Table 2 illustrates the arrangements of states and stroke labels in CharactersMachine file. In the first column of this file, all the final and non-final states have been written and in the front of each state, the movement information to next possible state on the occurrence of stroke label is given. Further, each state has different transitions either to a non-final state or final state on the basis of processed predicted stroke label from the sequence of set Σ . A total of 53 final states have been identified in this implementation. Figure 11 also describes the format of CharacterMachine file.

Similarly, table 3 shows some samples of final states written in FinalStateToUnicodeMapping file. The values of the final states in the second column are mapped with the indices of Unicode characters' indices that are shown in table 4.

8.5 Validation and working of FSA algorithm

Validation is possibly the most important step in model building process. Online unconstrained handwritten Gurmukhi characters recognition is a real-time application. Online

<Initial-state> $_{i,j}$ <next-state: class-label> $_{i,j+1}$ <next-state: class-label> $_{i,j+2}$... <next-state: class-label> $_{i,j+n}$
 <next-state> $_{i+1,j}$ <next-state: class-label> $_{i+1,j+1}$ <next-state: class-label> $_{i+1,j+2}$... <next-state: class-label> $_{i+1,j+n}$
 .
 .
 .
 <next-state> $_{i+m,j}$ <next-state: class-label> $_{i+m,j+1}$ <next-state: class-label> $_{i+m,j+2}$... <next-state: class-label> $_{i+m,j+n}$

Figure 11. Machine file's structure of Gurmukhi characters, where i, j, m and n are positive integers.

Table 3. Final state to Unicode index mapping.

Sl. no.	Final state	Mapping index
1	12	1
2	14	2
3	17	3
4	23	4
5	25	5
6	26	6
7	8	7
8	28	8
9	29	9
10	31	10
⋮	⋮	⋮

Table 4. Indexing of Unicode Gurmukhi character set.

Index	Unicode character set
1	ੳ
2	ਅ
3	ੲ
4	ਸ
5	ੴ
6	ਕ
7	ਖ
8	ਗ
9	ਘ
10	ਙ
⋮	⋮

handwritten Gurmukhi Unicode character formation process using Deterministic Finite Automata (DFA) 2 has been explained by testing 8200 characters collected from 20 different writers. This algorithm takes the predicted strokes information (stroke label, stroke sequence number and bounding box information) as an input. The *current-state* number and stroke label (psl_i) are passed as parameter q_i and ' a ', respectively, to the state transition function, $\delta(q_i, a) = q_j$, which returns the *next-state* number (q_j). Further, this *next-state* number q_j is processed as *prevState* along with the next stroke label psl_{i+1} in sequence set Σ , to get the *next-state* number q_{j+1} . Similarly, this process is repeated for all the stroke labels in Σ with the updated value of *next state*, which is produced by the previous iteration. Finally, after processing all the strokes from Σ , the last value *prevState* in the *next state* is checked as to whether it is a final state number or not. If this *prevState* value is matched with the final state value, then the corresponding Gurmukhi Unicode character is returned by mapping the final state value to the index value of the Gurmukhi Unicode character set, defined in table 4. On the other side, the line numbers [37–52] and [58–67] are executed only

if the *prevState* value does not match with the final state value. This snippet of code searches the final state by matching the traversed states in *next state* in reverse order, with all the final states described in table 3. If found, it returns a respective Unicode character; otherwise, this strokes set Σ is processed again to this state transition function after eliminating the first stroke label from the sequence set Σ in order to get the Gurmukhi Unicode character output, similar in shape to the input. Figure 12a shows the formation process of Gurmukhi Unicode characters ੳ, ਅ, ਘ and ਙ using an FSA-based finite machine. A finite set of stroke labels are defined in this figure to form these Gurmukhi Unicode characters. Hence, the representation model for this machine is deterministic. Following notations have been used in the implementation of this FSM.

$$Q = S0, S1, S2, \dots, S11,$$

$$\Sigma = 121, 159, 160, 161, 162, 163, 195,$$

$$q_0 = S0,$$

$$F = S2, S8, S9, S10.$$

In this FSM, we have covered all the possible combinations of stroke labels that help in the formation of these Unicode characters. For example 'ੳ' can be formed using two different ways from the initial state S0: (i) with stroke label '159' and (ii) with stroke labels '161' and '162'. Gurmukhi character 'ਅ' is formed in six different ways. Similarly 'ਘ' is formed in seven different ways. Finally, the character 'ਙ' has totally 13 variations. State transition, from the initial state to the final state on the occurrence of stroke labels, has been discussed in the state transition table (table 5). The states in bold text are the final states (i.e., S2, S8, S9 and S10). With the same idea, we have implemented totally 35 machines that are used for the formation of 53 Gurmukhi Unicode characters. The FSM structure helps in reducing both memory and execution time during the formation of Gurmukhi Unicode characters. Similarly, figure 12b illustrates the formation of Gurmukhi lower matra Unicode character 'ੲ' and figure 12c shows the formation of Gurmukhi upper matra Unicode character 'ੴ' in three different ways.

9. Experimentation and comparative study

This section briefly describes about the experiments carried out for training the zone-wise classifiers, testing the performance of the proposed recognition system, comparison of the present work to state-of-the-art and error analysis for online handwritten character recognition.

9.1 Training the classifier

To implement the online handwritten Gurmukhi character recognition system, two zone-wise classifiers

(a) Illustration of all the possible variations to form the Gurmukhi characters ੲ, ੳ, ੴ and ੵ from strokes.

(b) Formation of lower matrix, ੲ.

(c) Formation of upper matrix, ੳ on the occurrence of different classes of stroke.

Figure 12. Examples of Gurmukhi Unicode characters formation using Finite State Automata's machines, where a single circle represents a non-final state and double circles represent the final state.

UpperZoneClassifier and *MiddleLowerZoneClassifier* are trained using SVM classifier. The pre-processed strokes in their respective zones are classified with these two zone-wise classifiers. A total of 1,872 samples of stroke classes are considered for training the *UpperZoneClassifier* and 13,197 samples of stroke classes are considered for *MiddleLowerZoneClassifier*. In order to make a better training model, a few experiments have been carried out on these data sets, by tuning the penalty parameter (C) and learning rate (γ) of RBF kernel on different folds. The value of $\log_2(C)$ and $\log_2(\gamma)$ varies over the interval $(-5 \text{ (1) } 15)$ and $(3 \text{ (-1) } -15)$, respectively. Table 6 contains the values of parameters and cross-validation accuracy achieved by the proposed system for two zones. The highest cross-validation accuracy of 99.08% ($k = 7$, $\log_2(C) = 6$,

$\log_2(\gamma) = -7$) is achieved for *UpperZoneClassifier* and 97.38% ($k = 8$, $\log_2(C) = 7$, $\log_2(\gamma) = -9$) for *MiddleLowerZoneClassifier*. These two classifiers are further used for predicting the pre-processed strokes in Upper Zone and MiddleLower Zone, respectively.

9.2 Testing

For testing the performance of the proposed character recognition approach, a data set of 8,200 samples of online handwritten Gurmukhi Unicode characters, written by 20 new writers, has been considered to test the utility of the proposed character recognition system. A "Dell-latitude XT-3" Tablet-PC device was used to collect these data. Initially, the online handwritten character's stroke(s) was

Table 5. State transition table for Gurmukhi Unicode characters.

Transition	Input symbols						
	121	159	160	161	162	163	195
S0	S11	S2	S8	S1	S3	–	S9
S1	S4	–	–	–	S2	S5	–
S2	S9	–	–	–	–	S8	–
S3	S7	–	–	–	–	S6	–
S4	–	–	–	–	S9	–	–
S5	–	–	–	–	S8	–	–
S6	–	–	–	S8	–	–	–
S7	–	–	–	S9	–	–	–
S8	S10	–	–	–	–	–	–
S9	–	–	–	–	–	S10	–
S10	–	–	–	–	–	–	–
S11	–	S9	–	–	S7	–	–

(were) processed through the pre-processing steps in order to get the feature vector of 64 x - y co-ordinates values against each processed stroke. After this, the pre-processed strokes are processed further for the recognition with the respective zone classifier. The zone-wise classifiers produce a unique stroke identification number (i.e., stroke label) corresponding to the processed pre-processed stroke. This process is repeated for all the pre-processed strokes. Finally, the recognized strokes are processed for character formation with some additional information about each strokes, such as (i) stroke label, (ii) stroke sequence number and (iii) stroke bounding box information. The proposed Gurmukhi character formation approach forms a valid Unicode character after processing the recognized strokes.

9.3 Comparison with state-of-the-art

Table 7 illustrates the character-wise result obtained for 41 basic Gurmukhi characters, including 6 additional modified

characters ਘ, ਖ, ਧ and ਞ. The proposed recognition system achieved an accuracy of 97.3% for these 41 Gurmukhi characters. The average character recognition time in milliseconds is also shown in the table. The character recognition time is computed as total time taken by the recognition system from submission time of input strokes to the final character generation.

The proposed algorithm achieved a good amount of accuracy of 95.8% for characters ਞ, which is better than the achieved accuracy of 88.5% using the proposed work by Verma and Sharma [12]. The proposed system also overcomes the limitations of the work proposed by Kumar and Sharma [10] as follows.

- The problem related to dealing with unknown sequence of strokes and characters has been resolved in the present work. It is fixed by restricting the transition for the unknown stroke. Figure 13 illustrates the example of formation of the Gurmukhi character ਞ from input sequence of stroke labels “148”, “146” and “147”. The stroke label “147” is an extra/unknown stroke in the current stroke sequence set. Therefore, no transition will take place on processing of stroke label “147”. After processing first two stroke labels, the current state will be a final state (i.e., double circled) and from this state there is a self-loop for stroke label “147”. Hence, the output character here is (i.e., ਘ, ਖ, ਧ, ਞ, ਟ and ਠ).
- The problem of writing the same stroke more than once and problem of occurrence of the noisy stroke at the end or at the beginning in a stroke sequence are also resolved by restricting the transition to the same state on the occurrence of such stroke label, as depicted in figure 13.

The study on online handwritten Gurmukhi script recognition in the recent past is summarized in table 8, where the authors have reported character recognition accuracy using

Table 6. Values of parameters and stroke-level accuracy achieved for two zones.

Zone	Number of classes	Number of samples	Value of k in k -fold cross-validation	$\log_2(C)$	$\log_2(\gamma)$	Accuracy (%)
Upper	12	1872	3	4	–7	98.67
			4	6	–7	98.58
			5	6	–7	98.83
			6	6	–7	99.00
			7	6	–7	99.08
			8	4	–7	98.91
			9	4	–7	99.00
			3	4	–9	96.96
			4	5	–9	97.11
MiddleLower	81	13197	5	7	–11	97.21
			6	7	–11	97.31
			7	7	–9	97.31
			8	7	–9	97.38
			9	4	–11	97.15

Table 7. Character-wise recognition accuracy.

Gurmukhi characters	Recognition accuracy	Confusing characters	Gurmukhi Characters	Recognition Accuracy	Confusing Characters
ੳ	100.0		ਤ	97.5	ਭ
ਅ	100.0		ਥ	97.5	ਖ
ੲ	94.0	ੲ, ੲ	ਦ	100.0	
ਸ	96.0	ਮ	ਧ	98.5	ਪ
ਹ	96.5	ਰ	ਨ	96.5	ਟ
ਕ	100.0		ਪ	98.5	ਧ
ਖ	98.5	ਥ	ਫ	100.0	
ਗ	95.5	ਰਾ, ਗਾ	ਬ	96.5	ਵਾ
ਘ	96.5	ਪ	ਭ	98.5	ਤ
ਙ	100.0		ਮ	96.5	ਸ
ਚ	100.0		ਯ	97.5	ਪ
ਛ	100.0		ਰ	99.0	ਹ
ਜ	98.5	ਕ	ਲ	95.0	ੲ
ਝ	94.5	ਕੱ	ਵ	100.0	
ਞ	93.5	ਾਵ, ੲ	ੜ	96.5	ਡ
ਟ	100.0		ਸ਼	93.0	ਮ
ਠ	98.5	ਟ	ਖ਼	92.5	ਖ਼
ਡ	100.0		ਗ਼	91.5	ਰਾ
ਢ	100.0		ਜ਼	93.5	ਚੁ, ਕਾ
ਣ	95.5	ੲ	ਫ਼	97.5	ਫ਼
			ਲ਼	94.5	ਲੁ, ੲ
Average recognition rate (%)				97.3	
Average recognition time (in ms) (%)				27.9

Figure 13. Example of handling the occurrence of same stroke more than once and extra stroke(s).

varied number of stroke classes and character formation approaches considered. Sharma *et al* [21] have achieved 90.1% accuracy at character level using the elastic matching scheme. Later on they used HMM classifier in their work and achieved 91.9% accuracy. Kumar and Sharma [10] used the rule base character formation strategy in post-processing and achieved 95.6% accuracy. Verma and

Sharma [12] proposed the character recognition system using voting-based classifiers and employed 96.7% accuracy. In their work, a single classifier is used to classify the captured strokes. Due to this reason, their system encountered the issues related to strokes misclassification. However, in the present study, the captured strokes are classified into two zones to overcome the problem of strokes

Table 8. Comparison analysis with earlier approaches for Gurmukhi character recognition.

References	Stroke classes	Classification techniques	Character accuracy (%)
Sharma <i>et al</i> [20]	40	Elastic matching	90.1
Sharma <i>et al</i> [22]	40	HMM	91.9
Kumar and Sharma [10]	114	SVM, Rule based	95.6
Kumar <i>et al</i> [11]	114	SVM, Rule based	93.3
Verma and Sharma [24]	102	SVM	92.2
Verma and Sharma [12]	74	SVM, HMM	96.7
Current work	93	SVM, FSA based	97.3

Character	Standard writing order	Observed order and output
ਰਿ	ਰ + ਿ	ਿ + ਰ → ਿਰ
ਕਿ	ਕ + ਿ	ਿ + ਕ → ਿਕ
ਸੋ	ਸ + ੋ + ੇ	ਸ + ੋ + ੇ → ਸੋ

Figure 14. Standard and observed rendering order of *consonant* and *vowel(s)*.

misclassification. Moreover, an efficient FSA-based Gurmukhi character formation algorithm has been employed in post-processing, which produces a higher recognition accuracy of 97.3%. As observed, the character recognition accuracy achieved using FSA is more than those from the other earlier proposed schemes.

9.4 Error analysis

Though the proposed FSA-based formation of online handwritten Gurmukhi character performs sufficiently well, it still has some errors. The errors are mostly of two types.

- Misclassification of strokes: Incorrect Gurmukhi character formation due to processing of one or more misclassified stroke(s). The present system achieved an average recognition accuracy of 93.8% for six additional modified characters ਗ, ਝ, ਞ and ਥ. The reason for this low recognition accuracy is misclassification of strokes, such as stroke labels “102” ‘ੲ’ and “222” ‘ੲ’, confused with “101” ਸ, ਖ, ਗ, ਜ, ਫ, and ਲ and 158 (.), respectively. The main issue associated with writing the *vowels* (ੲ) is that mostly very few number of points (i.e., 5–10 points) are captured while writing them, which are insufficient to construct the feature vector of size 64 x - y points during pre-processing.
- Incorrect rendering order of *consonant* and *vowel(s)*: The order in which *consonant* and *vowel(s)* constituting a single character are written varies across

individuals. Some variations observed during the testing phase are shown in figure 14.

10. Conclusions and future directions

This paper presents an online handwritten Gurmukhi character recognition using FSA theory. The issues that are probably encountered in the formation of handwritten characters have been discussed in this study. A total of 93 stroke classes have been identified for Gurmukhi character-set formation. Initially, the online handwritten character is processed through pre-processing steps: normalization, duplicate points removal, interpolation and re-sampling. Two SVM-based classification models, namely, *UpperZoneClassifier* and *MiddleLowerZoneClassifier*, have been used for stroke classification in their respective zones. In post-processing phase the stroke merging, lower matras extraction, re-ordering the strokes lists and character formation steps are carried out in order to form a valid Unicode character. In online handwritten Gurmukhi character formation, we have covered all the variations used to write a Gurmukhi character by different individuals, using the proposed FSA-based character formation algorithm. The proposed work presents a higher recognition accuracy of 97.3% for Gurmukhi Unicode character recognition using FSA-based theory. It is observed that the character recognition accuracy achieved using FSA is more than those from the other earlier proposed schemes, shown in table 8.

Our future attempts will be towards improving the stroke classification by adding different classifiers based on deep learning, enhancing the recognition performance at word level, development of an android-based recognition system and incorporating a language model for improving the post-processing at word-formation level.

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