Boundary layer transition experiments with embedded streamwise vortices

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Abstract. Experiments were conducted with a counter-rotating, streamwise vortex pair embedded in flat plate boundary layers, in a low-turbulence wind tunnel, to understand the role of local separation on transition. Steady, streamwise vortices were generated downstream of gaps in spanwise-uniform, smooth hills (of height h) affixed to the plate, 175 mm from its leading edge. The flow between is directed away from the plate. At the four tunnel speeds 1.8–3.5 m/s considered, the Reynolds numbers based on displacement thickness at this location varied from 248 to 346. Small, medium and large gaps of 2, 4 and 8 mm, respectively, were set up; they were about a third to twice the boundary layer thickness ($2/3 < b/h < 8/3$). With the closest vortex pairs, transition was observed at all freestream speeds considered. With larger spacing, transition occurred at the highest speed only. The vortex pair caused the flow to separate in all but the largest-gap cases. Separation was steady and re-attachment unsteady in all cases. Velocity fluctuations grew slightly upstream of re-attachment in transitional cases. No evidence was found for separation or re-attachment as a direct cause for transition; transition occurred even without separation. Instead, whenever transition was observed, its origin could be traced to instability of a streak of sufficient amplitude that had been created by the vortex pair. Streak instability appeared as fluctuations growing along its sides and spreading. Anomalous behaviour was also observed with moderate spacing, where transition did not occur in spite of flow separation and streak amplitudes in excess of known thresholds for streak instability.

Keywords. Boundary layer transition; boundary layer separation; streak instability; streamwise vortex.

1. Introduction

Theoretical and experimental studies had revealed many aspects of boundary layer transition. Over the past two decades, computational studies have added to our understanding. Experimental confirmation of the linear instability theory had remained elusive but was obtained, eventually [1]: two-dimensional Tollmien–Schlichting modes of the linear theory were observed to grow, and develop into orderly, three-dimensional structures before the turbulent flow became established. Such a sequence is obtained only when disturbance levels are small; otherwise, for example, when the freestream turbulence (FST) level exceeds 1%, transition occurs more quickly without any feature of linear instability. This faster process, termed bypass transition, has been harder to understand, but a broad classification into a small number of routes, based on some distinctive features, exists. Of these, there is now considerable clarity on bypass

transition induced by FST. Schlatter *et al* [2] assembled results of a sequence of studies, experiments and computations from different institutions, and additional simulations, to provide a unified picture of the stages leading to the formation of a turbulent spot. Streak instability plays a central role in this route to transition; quasi-streamwise vortices appear when these streaks break down, evolve into hairpin vortices and thence into turbulent spots. Bypass transition may also be induced by surface roughness. Then, invariably, streamwise vortices are present immediately downstream of the roughness element. Protrusions support horseshoe vortices wrapped around themselves. The wake has a central low-speed region, and a pair of low-speed and high-speed streaks may be induced by the legs of the horseshoe vortex. When there are commonalities in flow structure (streamwise vortices and streaks) that develop from different forcings such as FST, roughness, etc., it is useful to determine what aspects of the subsequent development are shared, even if a single universal process cannot be identified.

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1.1 Streamwise vortices and streaks in boundary layer transition

While there have been several experiments with streamwise vortices and streaks in boundary layers, here we recount those that are connected with streak instability. Klebanoff *et al* [3] detected streamwise vortices from measurements of spanwise velocities. Breakdown was attributed to a sudden secondary instability, which was supposed to be the appearance of hairpin eddies. Wall-normal profiles of the streamwise velocity component were inflectional, and this was considered the origin of this instability. In a study in the same spirit as ours, Tani and Komoda [4] observed the development of the three-dimensional stage with an array of thin wings that generated tip vortices just outside a flat plate boundary layer. A vibrating ribbon was placed downstream to excite T–S waves, similar to several other experiments [1, 5–7].

Breakdown was observed slightly to the sides of the spanwise location where the boundary layer had thickened the most. These are the flanks of the low-speed streak (location D in their paper). Significantly, the flow did not break down without the downstream ribbon excitation.

Bakchinov *et al* [5] mounted a regular array of rectangular blocks as roughness elements, along the span, on a flat plate mounted in a wind tunnel. Element height was about half the boundary layer thickness and generated counter-rotating streamwise vortex pairs that provoked transition when the freestream speed was about 10 m/s. At a smaller speed, transition was effected by a vibrating ribbon. They observed that the instability grew where the spanwise derivative of the streamwise velocity was the largest—at the flanks of where low-speed streaks should be. The maximum response was at the critical layer, and frequencies were much higher than those of unstable T–S waves (above upper branch).

The optimal perturbation to a subcritical boundary layer—one that results in the maximum growth of perturbation kinetic energy over a given distance—consists of streamwise vortices without streamwise fluctuations that undergo transient algebraic growth, resulting in alternating (high-speed and low-speed) streaks with little cross-flow [8–10]. Andersson *et al* [11] refer to the formation of streaks as the primary instability, and their breakdown as the secondary instability. Since streamwise vorticity essentially disappears during this algebraic growth, Schlatter *et al* [2] describe this type of bypass transition as streak instability—the breakdown of these streaks. Of course, as the instability progresses, streamwise vorticity reappears in quasi-streamwise vortices that meander along the flanks of the low-speed streaks [2, 12, 13, provide a closer examination]. Further downstream, these vortices connect above interspersed high-speed streaks, forming lifted hairpins, which in turn break down, spawning spots. Quasi-streamwise vortices are not just natural features of streak instability; they have also been found to be the

optimal, finite-amplitude perturbation that develops into hairpins in a short target time [14].

In the studies discussed earlier, streamwise vorticity is initially present, is responsible for creating streaks by the lift-up mechanism and vanishes downstream in this process. Asai *et al* [15] created an isolated low-speed streak directly, with very little streamwise vorticity, by mounting a rectangular mesh in a boundary layer. Since the streak by itself could not initiate transition, periodic suction and blowing was applied through small ports downstream of the mesh. Sinuous or varicose oscillation of the streak could be created followed by transition. Although Asai *et al* [15] have not mentioned it, the unsteady flow through the ports will create streamwise vorticity. It would appear then that streamwise vorticity was indeed present when transition occurred. Brandt [13] studied this experiment by performing numerical simulations of the linearized Navier–Stokes equations with a base flow that agreed closely with that in the experiment. The simulations showed that the varicose mode had larger growth rates for the wider streak, and the sinuous mode for the narrow streak as had been observed in the experiment. Asai *et al* [15] had identified the varicose mode to arise from inflectional instability of the wall-normal profile of the streamwise velocity, and the sinuous mode to be due to the spanwise profile. Brandt [13], however, found the production associated with wall-normal shear to be larger for both varicose and sinuous modes. This unexpected result is dependent on the base flow, since the spanwise shear has the dominant role for sinuous instability of the optimal streak [11]. Loiseau *et al* [16] have shown from global stability analyses and DNS that, in the wake of a cylindrical roughness element, the sinuous mode occurs when element diameter is on the order of its height and the varicose mode occurs for larger diameter elements.

Zaki and Durbin [17] showed that transition results when low-frequency modes of the continuous spectrum can penetrate the boundary layer to create streaks (backward jets). Subsequently, streaks lift up towards the edge of the boundary layer, where they break down when perturbed by smaller scale FST. A train of cat's eye structures towards the edge of the boundary layer were observed, similar to the end state of Kelvin–Helmholtz instability of plane mixing layers. A single, low-frequency (penetrating) and a single, high-frequency (non-penetrating) mode, acting together, effect transition. By repeating this simulation, and from visualizations, Schlatter *et al* [2] showed that the developments in the different studies and the eventual breakdown to spots are quite similar, whether initiated by a couple of selected modes or their collection. In a subsequent study, Liu *et al* [18] demonstrated transition by the interaction of a single continuous and a single discrete (Tollmien–Schlichting (T–S) wave) mode. Here, the T–S mode by itself would not destabilize because it travels well past the upper branch. However, the penetrating continuous mode creates the streak, which is unstable and can now break down without lifting up to the edge of the boundary layer,

because the T–S wave is a perturbation present within the boundary layer itself.

1.2 Present experiments

The aim of our studies was to clarify the role of streamwise vortices in bypass transition, because they are a common feature. In the first set of experiments, reported before [19], a single streamwise vortex was generated by the side of half-span, smooth hills of height $h = O(\delta_0)$, where δ_0 is the local boundary layer thickness without the hill. A single feature of late stages of transition was set up—the streamwise vortex in a boundary layer—with no other forcing. Hence, tunnel FST levels were lower than those required in studies, and there was no shedding from the hills. Two types of hills, steep and shallow, were affixed to the flat plate upstream of the location where linear instability would occur. Although the action of the streamwise vortex caused spanwise and wall-normal velocity profiles to be inflectional, transition occurred only after the freestream velocity was increased above a threshold. Two routes to transition were observed: in one, hotwire signals showed a dominant, single frequency for some distance downstream of the hill ($O(10\delta_0)$) before becoming irregular. In the second route, only irregular (broadband) signals were obtained as turbulence developed without a regular oscillation stage. In the latter case, the initial streamwise vortex formed closer to the plate downstream of the shallow hill. We suppose that the regular oscillation stage of the other route to be a precursor that brought the vortex closer to the plate before transition; then, perhaps, flow separation, and subsequent instability of the separating shear layer, serves as a distinguishing characteristic of flows that would transition—an idea with a long history going back to Taylor [20]. We supposed that transition was not completed and any disturbance would still subside unless flow separated. There have been several studies of vortex interactions with walls (see, for example the review by Doligalski *et al* [21] and discussion of vortex-induced separation in section 2.3 therein). In particular, an effect of the presence of a vortex close to a wall is a strong eruption [22]. Goldstein and Leib [23] have shown this separation and sharp eruption to occur even when a small amplitude streamwise vortex is present outside the boundary layer on a flat plate. It is thus conceivable that such a strong response could be the crucial trigger for transition.

The subject of this paper is the subsequent study using pairs of streamwise vortices. With a pair that generates a common flow between themselves that is directed away from the wall, small, separated regions would form. When the common flow is towards the wall, the vortices would also be brought closer to the wall; if the wall interaction takes some other form, it should be provoked by the latter arrangement of vortices. These experiments can be thought of as another setting that isolates a feature of the late stages

of transition—primarily, small local separation in the vicinity of streamwise vortices.

Our experiments do not provide any support for separation as a direct, crucial stage in transition. Even when there is separation, fluctuations do not always grow. Even when there is unsteady re-attachment, the process of transition starts before the re-attachment. Unsteady re-attachment is due to the transition, rather than it provoking transition. Instead, streak instability, as observed in other studies with arrays of streaks/vortices (spanwise periodicity in simulations), is indicated; separation merely provides more favourable conditions for the instability to progress. Separation is not needed for transition. Neither is the mere presence of streaks, with streak amplitude above what has been assumed to be thresholds for transition, sufficient.

2. Apparatus

Experiments were performed in a low-turbulence wind tunnel that has been used for many studies of transition over the past several decades. Velocity measurements reported here were obtained by PIV. Descriptions of the tunnel and measurements have been documented before [24]; a summary of salient features is provided here. The horizontal, open-circuit tunnel has a square test section of size 500 mm $\times$ 500 mm $\times$ 3000 mm. The maximum speed in the test section is 22 m/s. FST level is less than 0.05% for speeds less than 10 m/s. The PIV unit (IDT piv, USA) consists of a double-cavity Nd:YAG laser (100 mJ) and a CCD camera (*sharpVISION^{TM}* 1400DE; 1360 $\times$ 1036 pixels) that captures five image pairs per second in the double-exposure mode. A commercial fog generator (HP Line and Antari) produces smoke particles (diameter of about 1 μ m). The data processing software (*proVISION^{TM}II*) was the second-order accurate mesh-free algorithm of Lourenco and Krothapalli [25]. Field of view, correlation window size, spatial resolution, the laser sheet thickness, etc. were comparable to those reported by other investigators [26–28]. The PIV data provide velocity fields down to 1 mm from the wall. As validation, the velocity measured at $x = 750$ mm with freestream velocity at 7.5 m/s is compared to the Blasius profile in figure 5a.

Hamilton and Abernathy [29] showed that a single streamwise vortex can be generated by the side of a smooth hill mounted at the bottom of a water table. Manu *et al* [19] affixed the same types of hills to generate isolated streamwise vortices in flat plate boundary layers. Figure 1 shows a configuration use in the present set of experiments. The Cartesian coordinate system has its origin on the plate surface with x directed downstream, y being normal to the plate and z being spanwise. Instantaneous velocity components are u , v , w , and means are U , V , W . The hill has a Gaussian shape with elevation $y_h(x) = h \exp[-(x/a)^2]$,

Figure 1. Small-gap-hill configuration.

($|x| \leq c/2$); the origin is at the symmetry plane of the hill profile and not at the plate leading edge; x_0 is the distance of the hill midplane to the plate leading edge. The mechanism of generation is explained in Manu *et al* [19]. Briefly, there are two processes. Due to the curvature of streamlines of the flow that goes over the hill, there must be pressure gradients, and velocity gradients that tilt the spanwise vorticity near the edges of the hill ($z = \pm b/2$) to generate streamwise vorticity. The second mechanism is due to the spanwise pressure gradient and viscous diffusion. Streamwise vorticity organizes itself into a streamwise vortex downstream of the edge of the hill. Its width is about that of the hill height h and centred at about $h/2$. In Manu *et al* [19] the hill occupied only half the span ($z < 0$). In the configuration shown, with the gap, a pair of counter-rotating streamwise vortices form, with the common flow between them directed away from the plate. In the second configuration, a small-span hill was used. Here, the streamwise vortex pair has the opposite sense of rotation, and the common flow between them is directed to the plate. This downwash brought higher-speed fluid (higher streamwise velocity) from the upper part of the boundary layer nearer to the plate. Transition was not detected in hot-wire surveys as far downstream as $x = 375$ mm with hills of spans $b = 2, 4$ and 8 mm. The common flow to the plate seems to have a stabilizing effect, and this configuration was not studied further.

In all the experiments discussed henceforth, the following configuration parameters are the same: chord $c = 30$ mm, height $h = 3$ mm, $x_0 = 175$ mm and $a = 7.1$ mm. Hill gap b and freestream speed U_0 were varied. This is the steep-hill profile of Manu *et al* [19]. To make x - y plane PIV visualization feasible, acrylic windows are attached to the side of the wind tunnel test section that covers the distance from zero to 450 mm. Hence, in order to capture the transition process initiated by the hill, we have selected the location of the hill centre as 175 mm from the leading edge, so that the transition process can be visualized using the attached window.

3. Measurements and analyses

In the small-gap configuration, the common flow between the counter-rotating vortices is directed away from the wall. The upwash creates a low-speed region downstream of the gap. Away from the gap, the velocity returns gradually to its unperturbed upstream value. The spanwise profile of the mean streamwise velocity component is similar to that in the experiment of Asai *et al* [15]. The main difference is that streamwise vorticity is present here, but not in their experiment until triggered artificially. Transition was observed with small-gap hills even at $U_0 = 1.8$ m/s. In single-vortex experiments with the same hill, at the same location, higher freestream speeds were needed for transition; though a precise value was not determined, there was no transition at 3.5 m/s [19].

Parameters in the experiments with the small-gap configuration are listed in table 1, grouped together by gap widths (SG: small, MG: medium, LG: large) at four freestream speeds 1.8 – 3.5 m/s. Boundary layer thickness δ_0 and displacement thickness δ_0^* are for profiles obtained with the same freestream speed at $x = 0$ on bare plate. The Reynolds number $Re_{\delta_0^*}$ is based on freestream speed and displacement thickness. Transition was observed at all speeds for the smallest gap (SG0, SG1, SG2, SG3), and at the highest speed for larger gaps (MG3, LG3). Although experiments were conducted at the lower speeds with the large gap, they will not be considered any further: these flows did not undergo transition, and there was no separation. Note that there is nevertheless some effect of the presence of the vortex pair even with the large gap; transition now occurs at a lower freestream speed than that for an isolated vortex. The development of streamwise velocity fluctuations along $z = 0$ for transitional and non-transitional flows is shown in figure 2; the maximum of the fluctuation rms across the boundary layer is plotted. When there is no transition, fluctuation rms level remains at about 2% of freestream speed. For the transitional cases, the rms level reaches 19%, similar to that in the [30] numerical experiments, but much higher than in the experiments with a single streamwise vortex. Transition is increasingly faster

Table 1. Parameters in small-gap-hill experiments; $h = 3$ mm, $c = 30$ mm.

Case	b [mm]	U_0 [m/s]	b/δ_0^*	h/δ_0	$Re_{\delta_0^*}$	State of flow
SG0	2	1.8	0.97	0.50	248	Transitions
SG1	2	2.5	1.14	0.58	292	Transitions
SG2	2	3.0	1.25	0.63	320	Transitions
SG3	2	3.5	1.35	0.69	346	Transitions
MG0	4	1.8	1.94	0.50	248	No transition
MG1	4	2.5	2.28	0.58	292	No transition
MG2	4	3.0	2.5	0.63	320	No transition
MG3	4	3.5	2.7	0.69	346	Transitions
LG3	8	3.5	5.4	0.69	346	Transitions

Figure 2. Maximum, over the boundary layer at each station $x, z = 0$, of rms of streamwise velocity fluctuations (PIV data).

at higher freestream speeds, and delayed as gap width is increased. In every case, fluctuation levels remain roughly constant and then suddenly grow. The growth rate is nearly the same at all speeds U_0 with the same gap (cases SG0–SG4), and is lower for the wider gaps.

It is useful to recall that Hamilton and Abernathy [29] had compared single-vortex-induced transition and that by multiple vortices. They found that the single vortex showed signs of instability first, and produced a larger number of turbulent spots. Here we found the opposite: the vortex pair past the 2 mm gap provoked transition at a significantly lower speed and close to the hill. On increasing the value of b ($= 4.8$ mm) the transition point (location x where u_{rms} is the maximum) moved downstream, and the flow remained laminar for speeds of 1.8–3 m/s.

The structure of the flow immediately downstream of the gap can be understood from contour plots such as figure 3 on the wall-parallel plane $y/h = 0.67$ for the lowest free-stream speed, smallest-gap-case SG0. The mean streamwise velocity shows the flow to have separated downstream of the gap and then re-attached at the end of a thin, long bubble (figure 3a). Its width and height (not shown here)

are on the order of the boundary layer thickness. This reverse-flow and low-speed region is flanked by relatively higher speed regions. Figure 3b of the velocity field $u(x, z, t)$ at an instant shows variations of a symmetric (varicose) instability mode (see, for example, figures 15 and 18 in Asai *et al* [15]). Figure 3c and d shows development of velocity fluctuations.

The shear layer that lies above the separation bubble rolls up near the re-attachment region. Contours of spanwise vorticity ω_z in figure 4a show the rolled-up end about to re-attach. A part of the shear layer close to the wall re-attaches, and is convected downstream (figure 4b). However, this rolled-up shear layer has a short spanwise extent; it is not present in data on the plane $z/h = 2$ (not shown here). As can be seen in figure 3c and d, the stronger fluctuations grow to the sides, around $z/h \approx \pm 2.5$.¹ Breakdown is restricted to the region $-4 < z/h < 4$. The rest of the region behind the hill remains comparatively calm. Transition is initiated solely by the streamwise vortices generated at the gap, and not from any shedding because the hill has a smooth profile unlike that of roughness elements.

Wall-normal profiles $U(y)/U_0$ at three streamwise stations along the midplane $z = 0$ are shown in figure 5 for one non-transitional and three transitional cases. The Blasius profile for the flat plate without hills is included for reference in all four graphs. Transition is indicated when profiles become fuller with a sharp gradient close to the wall. The velocity profiles at $x/c = 4.5$ are not fully turbulent yet, but do become so downstream.

For the small gap at the smallest speed (case SG0), the mean profile shows the reverse flow immediately downstream of the gap (figure 5a). Although the reverse flow is over a thicker region at the larger speed SG3, the mean velocity is nearly zero and not significantly negative. The case MG0 does not exhibit transition, but the profile immediately downstream of the gap indicates the reverse flow in the separation bubble (figure 5c). The initial profiles at $x/c = 1$ above the reverse flow are like that of free shear layers and are inflectional for both the transitional and non-transitional cases SG0, SG3 and MG0 shown here; it is quite pronounced for MG3. Neither is separation sufficient for transition. At the higher speed and medium-gap (MG3) configuration, there is considerable distortion just downstream of the gap ($x/c = 1$), but reverse flow is found further downstream ($x/c \approx 3$, figure 5d). Separated flow was not detected for the large-gap cases when $b = 8$ mm. If present, the bubbles were too thin to be detected.

¹Although figure 3 shows fluctuations to begin to grow off-centre, the curves in figure 2 of the maximum of fluctuations along the centre plane $z = 0$ suffice to distinguish between transitional and non-transitional cases because there was no off-centre growth either for non-transitional cases.

Figure 3. Velocity distributions on wall parallel plane $y/h = 0.67$, case SG0.

3.1 Unsteady separation

These experiments were conceived from the hypothesis that transition follows some kind of interaction of the

Figure 4. Instantaneous spanwise vorticity $\omega_z \delta_0 / U_0$ on wall-normal plane $z/h = 0$, case SG0.

streamwise vortices with the wall. Further, since transition occurs only when the freestream speed (interaction strength) exceeds a threshold, it was supposed that the point of no return might be incipient or unsteady separation. The velocity data such as in figure 3 do not suggest that separation effected by the interaction of the streamwise vortices with the wall causes transition directly. It is more revealing to examine the unsteady separation using a parameter that will be called the reverse-flow index, and defined as

$$\mathcal{R} = \sum_i R_i / N_f,$$

$$R_i(\mathbf{x}, t) = \begin{cases} 1, & (u(\mathbf{x}, t) < 0) \\ 0, & (u(\mathbf{x}, t) \geq 0). \end{cases}$$

$N_f = 400$ is the number of PIV frames. The index $\mathcal{R}(\mathbf{x})$ is an estimate of the fraction of the time that there is reverse flow at position $\mathbf{x}$. Though not a separation index, it is, nevertheless, a useful, simple, quantitative marker of unsteady separated flow for the present study.

Figure 6a shows the variation of the index $\mathcal{R}(x)$ at different heights y on the midplane $z = 0$ for case SG0. Curves for $0.5 < y/h < 1.33$ are similar. Flow separates immediately downstream of the gap, and there is always reverse flow near the wall for some distance. As can be expected

Figure 5. Mean velocity $U(y)/U_0$ along midplane $z = 0$.

for unsteady re-attachment, $\mathcal{R}(x)$ falls to zero over quite some distance, from about 2.2 to 3.4 units at $y/h = 0.5$. The distance over which the reverse-flow boundary fluctuates near re-attachment reduces with distance from the wall; the

mean position of this boundary also moves upstream with height. Figure 6b shows the corresponding growth of streamwise velocity fluctuations; the maximum of the rms across the boundary layer at every position ($x, z = 0$) has been plotted. In this case, fluctuations grow steadily till $u_{rms,max}/U_0$ is about 10% and then grow faster as flow transitions. Fluctuations begin to grow well upstream of re-attachment. The reverse-flow extent at $y/h = 1.67$ (near bubble top) varies noticeably as the fluctuations grow, whereas there is less variation at intermediate heights $y/h = 1.33$, and 1.00. At $y/h = 0.67$, for a small fraction of the time (small $\mathcal{R}$), the extent of the reverse flow varies significantly. Close to the wall, the extent varies much more, by $x/c \approx 1$, which is about a third of the bubble size. Velocity fluctuations are already quite large, about 10% by $x/c = 2.25$, well upstream of re-attachment.

With increase in speed U_0 , transition moves upstream towards the hill. Development of $\mathcal{R}(x)$ and fluctuations for small-gap cases SG1–SG3 are shown in figure 7. Note that, for SG1 and SG2, fluctuations remain small (about 0.03%) as in non-transitional cases over the initial reverse-flow region, and begin to rise a little upstream of the re-attachment region. Curves for SG3 support this observation, though it is not as clear because transition and re-attachment begin earlier. Also, $\mathcal{R}(x)$ falls rapidly at first, and then very slowly, indicating a less frequent occurrence of reverse flow beyond $x/c \approx 1$. Observe also that velocity fluctuation levels grow at roughly the same rate; it seems likely that the unsteadiness in re-attachment is not strongly connected to the growth of fluctuations—to transition. The process of transition may not be provoked by unsteadiness in re-attachment, but transition causes unsteady re-attachment.

Extensive separated flow regions exist at all speeds for medium-gap cases MG0–MG3 (figure 8a). Separation occurs further downstream at higher speeds (MG2 and MG3). The rise in $\mathcal{R}$ is quite sharp, and velocity fluctuations remain small, indicating essentially steady separation. The fall over a larger extent for the transitional case MG3 is due to unsteady re-attachment. Fluctuation levels remain small and comparable in all cases for some distance downstream of separation, though they are larger everywhere for the transitional case MG3 (figure 8b). Thereafter, for MG3 alone, there is a steady increase in fluctuations, beginning a little upstream of re-attachment. The large-gap cases did not exhibit any separation; growth of fluctuations for LG3 is similar to that for MG3.

In every case, separation is *steady* as indicated by a rapid rise in reverse-flow index and low fluctuation levels. For transitional cases, re-attachment is unsteady as indicated by a gradual fall in reverse-flow index. However, fluctuation levels begin to increase slightly upstream of that where the index begins to fall. Thus we find no evidence for either separation or re-attachment causing transition. Of course, the separation bubble lifts up the oncoming boundary layer and enhances the spanwise velocity difference between

Figure 6. Reverse-flow index at different heights from the wall on midplane $z = 0$, and growth of velocity fluctuations, case SG0.

adjacent low-speed and high-speed streaks. Hence, separation may contribute indirectly by enhancing conditions for streak instability.

3.2 Streak instability

Figure 9 shows spanwise profiles $U(z)$ from all cases. Profiles in (a) and (c) are at $x/c = 1.66$ and $y/h = 1$. Those in (a) alone, for the smallest gap, have been plotted against z/δ_0 . At the smaller speeds (SG0–SG2), streak width scales with δ_0 . The low-speed streak in the middle has a width of about $2\delta_0$ and is flanked by high-speed regions that are about half as wide. Profiles are smoother and very nearly symmetric on lines further away from the wall (figure 9b). The three non-transitional medium-gap cases (MG0–MG2) are similar and slightly asymmetric (figure 9c). The asymmetry diminishes considerably away from the wall even for the transitional case MG3 (figure 9d).

Andersson *et al* [11] have determined the linear instability of a boundary layer with streamwise-uniform alternating high-speed and low-speed streaks. Streak amplitude was defined as the maximum difference of the excess of streamwise velocity over the Blasius value at the same

Figure 7. Reverse-flow index and velocity fluctuations for cases SG1–SG3 at $y/h = 0.4$ on midplane $z = 0$.

Figure 8. Reverse-flow index and velocity fluctuations for medium- and large-gap cases at $y/h = 0.4$ on midplane $z = 0$.

Figure 9. Spanwise profiles $U(z)/U_0$. (a, c) At $x/c = 1.66$, $y/h = 1$; (b) case SG0 at $x/c = 1.66$, $0.67 < y/h < 2.53$; (d) case MG3 at $x/c = 1.66, 2.61, 3.54$, $y/h = 1$.

location. Streak amplitudes grew initially and then fell. When the amplitude exceeded 26% of freestream velocity, sinuous oscillations grew and the flow broke down to

Figure 10. Development of streak amplitudes $A_s(x)$. (a) Small gap and (b) medium gap.

turbulence. Varicose oscillations appeared, but with a lower growth rate when streak amplitude exceeded 37%. They concluded that the sinuous mode was more probable in natural flows since it was triggered at a lower amplitude and had a larger growth rate.

Streak amplitude $A_s = (U_{max} - U_{min})/(2U_0)$ was measured taking an adjacent minimum and maximum from spanwise profiles of $U(z)$ at constant y . Velocity differences $\Delta U = U_{max} - U_{min}$ used for each case are marked on the profiles in figure 9a and d. Fransson *et al* [31] used a similar definition, but considered the maximum of A_s at any given constant- x -plane. The development of streak amplitudes for all cases can be seen in figure 10. It is helpful to compare these curves of A_s with those in figure 2b of the development of fluctuations. After an initial growth close to the hills, we can expect streak amplitude to decay as in the theoretical studies of Andersson *et al* [11], followed by a levelling off when fluctuation levels also level off. Thus we see the fall in A_s for cases SG0 and SG1, since fluctuations peak at around $x/c \approx 3-3.5$ in those cases, around 2 for SG2 and 1.5 for SG3. Although the definition for streak amplitude used here differs

slightly from that in Andersson *et al* [11], the small- and large-gap cases seem to have streak instability characteristics as in their stability study: all small-gap cases undergo transition with symmetric (varicose) structures (see figure 3 for SG0) and A_s is of the order required. Streak amplitude is large enough for instability to have occurred for at least MG1 and MG2 of the sinuous type. Perhaps the symmetry permits only the varicose mode, which requires a larger streak amplitude. For the transitional case MG3, streak amplitude grows till around $x/c \approx 2.5$. Velocity fluctuations change little during this stage and thereafter grow rapidly (figure 8b). In our experiments with a single streamwise vortex, generated by a half-span hill of the same shape, sinuous instability developed when streak amplitude was about 0.4 (case ST4 with $U_0 = 6.5$ m/s in Manu *et al* [32]); with $U_0 = 3.5$ m/s, streak amplitude grew to about 0.2 only (non-transitional case in [19]).

According to the sequence of events summarized by Schlatter *et al* [2], from their simulation of the Zaki–Durbin model problem [17], streak instability is manifest as quasi-streamwise, meandering vortices that appear on the flanks of low-speed streaks. As the spanwise extent of this meandering increases, these vortices connect above the adjacent high-speed streak, forming a hairpin that breaks down rapidly into spots. In other simulations [13, for example], the hairpins form above the low-speed streak. The interface between adjacent low-speed and high-speed streaks is a sheet of wall-normal vorticity. Although the wall-normal gradient of streamwise velocity will tilt this to give rise to streamwise vorticity as well, the quasi-streamwise vortices are likely end states of a Kelvin–Helmholtz instability of this inter-streak vortex sheet that has rolled up into vortices. In the non-transitional medium-gap cases, it is possible that such a roll-up did not occur so that further progress to transition did not occur because of diffusion of the shear layer on the flanks of the low-speed streak. Brandt [13] found instability of the varicose mode to be sensitive to diffusion of the flank shear layer.

4. Conclusions

In the experiments described earlier, we observed transition effected by a counter-rotating vortex pair, embedded in a flat plate boundary layer. The lift-up due to the sense of rotation of the vortex pair created a low-speed streak between them when the distance between the vortices was not too large; else, a pair of low-speed streaks with interspersed high-speed ones were obtained. Fluctuations grew along the sides of this low-speed streak when transition occurred. When the vortices were close together (hill gap $b = 2$ mm), transition occurred at all freestream speeds greater than 1.8 m/s. When the spacing was larger, transition was observed at $U_0 = 3.5$ m/s. In this speed range, and at the location of the hill, the range of Reynolds numbers

$Re_{\delta_0^*}$ is 248–346. Transition occurs at about $10\delta_0$ – $20\delta_0$ downstream, and is always subcritical.

Flow separates immediately downstream when the gap is small; when larger, separation occurs slightly further downstream. Separation is always steady as indicated by the fairly sharp rise in the reverse-flow index, and low fluctuation levels in the neighbourhood of separation. It is reasonable to expect the separating shear layer to become unstable. However, surprisingly, there is no indication that transition is initiated immediately by separation (figures 6–8). Even when the separated shear layer has rolled up near the re-attachment region, stronger fluctuations grow by the flanks of the separation bubble, and a slight distance away ($\approx 3h$, figure 3). The origin of transition does not appear to be unsteady re-attachment: non-transitional cases also show unsteady re-attachment but velocity fluctuations continue to drop; and, when transition occurs, fluctuation levels grow some distance ahead of re-attachment.

We suppose that the long, thin separation bubble can promote streak instability since the incoming stream is lifted above this bubble, resulting in larger streak amplitudes. In all cases that resulted in transition, streak amplitudes had exceeded the threshold for varicose instability found in previous theoretical and experimental studies. The lower speed medium-gap cases, MG0–MG2, exhibited streak amplitudes that exceeded the threshold for sinuous instability but did not transition. Perhaps the symmetry of the arrangement inhibited this mode. Schlatter *et al* [2] describe the initial development of streak instability as wall-normal vorticity at the flanks of low-speed streaks sides tilting into quasi-streamwise vortices. These vortices connect as hairpins and break down. While the present arrangement produces a single vortex pair and streak, rather than the streak arrays in the simulations, a similar process can be inferred from figure 3. The PIV snapshot (figure 3b) suggests the legs of a symmetric succession of vortices growing in strength as they travel downstream. In the non-transitional medium-gap cases, we suppose that transition has been prevented because the vortex sheet with wall-normal vorticity at the streak flanks did not roll up into vortices. Kelvin–Helmholtz instability is convective (requires initiating perturbations); the wavelength of the unstable mode depends on the thickness of the shear layer, and the growth rate depends on the velocity difference.

We found no support for our initial hypothesis that local separation acts as the qualitatively different, wall-interaction process that is a distinguishing point-of-no return in transition. This experiment and the previous one with a single vortex should be considered as ones that set up a common element of late stages of different routes to transition free of other features such as FST fluctuations or shedding from roughness elements. Whenever transition was observed the origin could be traced to streak instability. In a recent study, a hairpin packet evolved from a counter-rotating vortex pair in a shear layer without any wall [33]. Nevertheless, it was surprising that even after local

separation was induced, transition was not triggered by the separating shear layer.

When a feature has been isolated, there can be similarities and differences in aspects of transition compared with those in other configurations. Analyses of the transition provoked by a single vortex had revealed qualitative (break-up of high-shear layer) and quantitative differences (frequency scaling, Reynolds stress events) with FST- or roughness-induced transition [32]. Yet, it appears that streak instability has been the common process in these experiments with vortex pairs or isolated vortices. Other features, like local separation, may have an influence, but may not be crucial.

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