Effect of couple stresses on static and dynamic characteristics of MHD wide tapered land slider bearing

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Abstract. This paper presents an analytical study on the effect of couple stress on static and dynamic characteristics of wide tapered land slider bearing in the presence of applied magnetic field. A non-dimensional modified Reynolds equation is derived for the bearing under consideration. Closed-form expressions for the steady fluid film pressure, steady load carrying capacity and dynamic characteristics, viz., dynamic stiffness and dynamic damping coefficients are obtained. Numerical computations of the results revealed that magnetohydrodynamics tapered land slider bearing lubricated with couple stress fluid provides higher steady load carrying capacity, dynamic stiffness coefficient and dynamic damping coefficient as compared with the corresponding Newtonian case. It is observed that the presence of applied magnetic field characterized by the Hartmann number in the couple stress lubrication of tapered land slider bearing provides the improved static and dynamic characteristics as compared with the non-magnetic case.

Keywords. Tapered land bearing; MHD; couple stress fluids; dynamic stiffness; dynamic damping.

1. Introduction

Design of bearings plays an important role in the efficient performance of machines. To ensure high efficiency and reliability of bearings, the use of not only correct size, functional type and the specific material but also proper lubricant is important. In many of the classical hydrodynamic theories, the lubricant is considered to be a Newtonian fluid but for the practical application the assumption of fluid to be Newtonian is not satisfactory. A desirable lubricant that is a non-Newtonian fluid is obtained from a Newtonian fluid by adding a small amount of long chained polymers to it. In recent years, the study of lubrication with non-Newtonian fluids has attracted many researchers. The Stokes [1] couple stress theory is the simplest generalization of classical theory that allows for polar effects such as the presence of couple stresses and body couples. This theory has been used by several investigators to study the effect of couple stresses such as composite inclined step bearing by Sinha and Singh [2], slider bearing by Ramanaiah and Sarkar [3], circular stepped plates by Naduvinamani and Siddangouda [4], porous stepped composite bearing by Apparao *et al* [5], porous parallel stepped plates by Biradar [6] and rough parallel plates by Naduvinamani *et al* [7]. They concluded that, the use of a

couple stress fluid for lubrication increases the load carrying capacity and reduces the coefficient of friction as compared with the corresponding Newtonian case.

The study of dynamics of flow of conducting fluid in the presence of applied magnetic field is known as magnetohydrodynamics (MHD). Application of magnetic field provides many uses of lubricants at high temperature. The effect of strength of applied magnetic field on bearing characteristics has been studied by many researchers, such as by Hughes [8] for inclined slider bearing, Malik and Singh [9] for finite journal bearing and Dudzinsky *et al* [10] for journal bearing. According to these studies, the increase in the strength of applied magnetic field increases the load carrying capacity of bearing compared with the non-magnetic case. In view of enhanced results obtained with the effect of couple stress and MHD individually, the researchers have shown attention towards the study of their combined effect. Many researchers studied the combined effect of MHD and couple stress such as slider bearing by Das [11], squeeze film characteristic between sphere and plane surface by Naduvinamani and Rajashekar [12], parallel rectangular plates by Lin *et al* [13] and different types of finite plates by Fathima *et al* [14]. According to these results, the effect of couple stress and MHD enhances the bearing characteristics compared with Newtonian and non-magnetic case.

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In the design of bearing, their static and dynamic characteristics play an important role. Static characteristics provide basic reference whereas dynamic characteristics provide stability behaviour. Taylor and Dowson [15] studied the use of existing lubrication theories in the design of bearings. Dynamic characteristics of plane inclined slider bearing in the presence of applied magnetic field have been studied by Lin *et al* [16], who concluded that the applied magnetic field enhances the dynamic characteristics. Dynamic characteristics of wide tapered land bearing in the absence of magnetic field have been studied by Lin *et al* [17] and in the presence of a magnetic field by Lin [18]. The objective of this paper is to study the effect of couple stress on static and dynamic characteristics of wide tapered land bearing in the presence of an external magnetic field. The results obtained are compared to the Newtonian magnetic case by Lin [18] and Newtonian non-magnetic case by Lin *et al* [17]. The present results show close agreement with the results obtained by Lin *et al* [17] and Lin [18].

2. Mathematical analysis

The geometry of a wide tapered land slider bearing of length L is shown in figure 1. Couple stress fluid is to be considered in the film region and a uniform transverse magnetic field B_0 is applied perpendicular to the bearing. The lower surface of bearing moves with sliding velocity U in the x -direction and the upper surface has a squeezing effect $\partial h/\partial t$.

For the formulation of this problem, it is assumed that the fluid film is thin and fluid inertia is negligible as compared with viscous force. It is also assumed that the body couples and body forces are negligible except the Lorentz force. The induced magnetic field is smaller as compared with the

externally applied magnetic field. Under these assumptions, the governing equations for the flow of conducting couple stress fluid in the presence of applied magnetic field described by Das [11] are considered:

$$\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u = \frac{\partial p}{\partial x} + \sigma E_y B_0 \quad (1)$$

$$\frac{\partial p}{\partial z} = 0 \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

$$\int_{z=0}^h (E_y + B_0 u) dz = 0 \quad (4)$$

where μ is the lubricant classical viscosity, η is the material constant responsible for couple stresses, σ is the conductivity of the lubricant, p is the film pressure and u, w are the velocity components in the x and z directions, respectively.

The boundary conditions for velocity components are as follows.

$$\text{At the upper surface } z = h: u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad w = \frac{\partial h}{\partial t}. \quad (5)$$

$$\text{At the lower surface } z = 0: u = U, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad w = 0. \quad (6)$$

With the use of boundary conditions (5), (6) and Eq. (4) the solution of Eq. (1) is obtained as

$$u = \begin{cases} -\frac{U}{2}(f_{11} - f_{12}) - \frac{h_{ms}^2 h}{2l\mu M^2} \frac{\partial p}{\partial x} (f_{13} - f_{14}) & \text{for } \frac{4M^2 l^2}{h_{ms}^2} < 1 \\ -\frac{U}{2}(f_{21} - f_{22}) - \frac{h_{ms}^2 h}{2l\mu M^2} \frac{\partial p}{\partial x} (f_{23} - f_{24}) & \text{for } \frac{4M^2 l^2}{h_{ms}^2} = 1 \\ -\frac{U}{2}(f_{31} - f_{32}) - \frac{h_{ms}^2 h}{2l\mu M^2} \frac{\partial p}{\partial x} (f_{33} - f_{34}) & \text{for } \frac{4M^2 l^2}{h_{ms}^2} > 1 \end{cases} \quad (7)$$

where $M = B_0 h_{ms} (\sigma/\mu)^{1/2}$ is the Hartmann number and $l = \sqrt{\eta/\mu}$ is the couple stress parameter. The associated relations in (7) are given in Appendix A as Eqs. (A1a)–(A11).

With the use of boundary conditions (5), (6) and the expression (7) for u , the integration of continuity equation (3) over the film thickness gives the modified Reynolds equation in the form

$$\frac{\partial}{\partial x} \left\{ g(h, l, M) \frac{\partial p}{\partial x} \right\} = 6\mu U \frac{\partial h}{\partial x} + 12\mu \frac{\partial h}{\partial t} \quad (8)$$

Figure 1. Geometry of tapered land slider bearing.

where

$$g(h, l, M) = \begin{cases} \frac{6h_{ms}^2 h^2}{lM^2} \left\{ \frac{\alpha^2 - \beta^2}{\frac{\alpha^2}{\beta} \tanh\left(\frac{\beta h}{2l}\right) - \frac{\beta^2}{\alpha} \tanh\left(\frac{\alpha h}{2l}\right)} - \frac{2l}{h} \right\} & \text{for } \frac{4M^2 l^2}{h_{ms}^2} < 1 \\ \frac{6h_{ms}^2 h^2}{lM^2} \left\{ \frac{2 \left(\cosh\left(\frac{h}{\sqrt{2}l}\right) + 1 \right)}{3\sqrt{2} \sinh\left(\frac{h}{\sqrt{2}l}\right) - \frac{h}{l}} - \frac{2l}{h} \right\} & \text{for } \frac{4M^2 l^2}{h_{ms}^2} = 1 \\ \frac{6h_{ms}^2 h^2}{lM^2} \left\{ \frac{M(\cos \beta_1 h + \cosh \alpha_1 h)}{h_{ms}(\alpha_2 \sin \beta_1 h + \beta_2 \sinh \alpha_1 h)} - \frac{2l}{h} \right\} & \text{for } \frac{4M^2 l^2}{h_{ms}^2} > 1 \end{cases} \quad (9)$$

$$\begin{aligned} \alpha &= \left\{ \left(1 + \sqrt{1 - (4l^2 M^2 / h_{ms}^2)} \right) / 2 \right\}^{1/2}, \\ \beta &= \left\{ \left(1 - \sqrt{1 - (4l^2 M^2 / h_{ms}^2)} \right) / 2 \right\}^{1/2}, \\ \alpha_1 &= \sqrt{M / l h_{ms}} \cos(\varphi/2), \quad \beta_1 = \sqrt{M / l h_{ms}} \sin(\varphi/2), \\ \varphi &= \tan^{-1} \left(\sqrt{(4l^2 M^2 / h_{ms}^2) - 1} \right), \\ \alpha_2 &= (\beta_1 - \alpha_1 \cot \varphi), \quad \beta_2 = (\alpha_1 + \beta_1 \cot \varphi). \end{aligned}$$

The modified Reynolds type equation (8) is applicable to one-dimensional slider bearings lubricated with couple stress fluid in the presence of transverse magnetic field with the squeezing effect $\partial h / \partial t$. The film thickness in the flow region of bearing is given by

$$h(x, t) = h_m(t) + h_1(x) \quad (10)$$

where $h_m(t)$ is the minimum film thickness and $h_1(x)$ is the known profile for the slider bearing. For wide tapered land bearing, $h_1(x)$ is given by

$$h_1(x) = \begin{cases} d(1 - x/KL) & \text{for } 0 \leq x \leq KL \\ 0 & \text{for } KL \leq x \leq L \end{cases} \quad (11)$$

where $d = h_1 - h_m$ is the difference between the inlet and outlet film thickness and K is the ratio of the inclined part length to the length of the bearing, known as the geometric parameter.

Defining the non-dimensional quantities

$$\bar{x} = \frac{x}{L}, \quad \bar{p} = \frac{ph_{ms}}{\mu UL}, \quad \bar{h}_m = \frac{h_m(t)}{h_{ms}}, \quad \bar{h} = \frac{h}{h_{ms}}, \quad \bar{h}_1 = \frac{h_1}{h_{ms}}, \\ \bar{t} = \frac{Ut}{L}, \quad \bar{l} = \frac{2l}{h_{ms}}$$

and using these in Eq. (8), the non-dimensional modified Reynolds equation is obtained in the form

$$\frac{\partial}{\partial \bar{x}} \left[\bar{g}(\bar{h}, \bar{l}, M) \frac{\partial \bar{p}}{\partial \bar{x}} \right] = 6 \frac{\partial \bar{h}}{\partial \bar{x}} + 12 \frac{\partial \bar{h}}{\partial \bar{t}} \quad (12)$$

where

$$\bar{g}(\bar{h}, \bar{l}, M) = \begin{cases} \frac{12\bar{h}^2}{\bar{l}M^2} \left\{ \frac{\bar{\alpha}^2 - \bar{\beta}^2}{\frac{\bar{\alpha}^2}{\bar{\beta}} \tanh\left(\frac{\bar{\beta}\bar{h}}{\bar{l}}\right) - \frac{\bar{\beta}^2}{\bar{\alpha}} \tanh\left(\frac{\bar{\alpha}\bar{h}}{\bar{l}}\right)} - \frac{\bar{l}}{\bar{h}} \right\} & \text{for } M^2 \bar{l}^2 < 1 \\ \frac{12\bar{h}^2}{\bar{l}M^2} \left\{ \frac{1 + \cosh\left(\frac{\sqrt{2}\bar{h}}{\bar{l}}\right)}{3\sqrt{2} \sinh\left(\frac{\sqrt{2}\bar{h}}{\bar{l}}\right) - \frac{\bar{h}}{\bar{l}}} - \frac{\bar{l}}{\bar{h}} \right\} & \text{for } M^2 \bar{l}^2 = 1 \\ \frac{12\bar{h}^2}{\bar{l}M^2} \left\{ \frac{M(\cos \bar{\beta}_1 \bar{h} + \cosh \bar{\alpha}_1 \bar{h})}{\bar{\alpha}_2 \sin \bar{\beta}_1 \bar{h} + \bar{\beta}_2 \sinh \bar{\alpha}_1 \bar{h}} - \frac{\bar{l}}{\bar{h}} \right\} & \text{for } M^2 \bar{l}^2 > 1 \end{cases} \quad (13)$$

$$\begin{aligned}\bar{\alpha} &= \left\{ \left(1 + (1 - \bar{l}^2 M^2)^{1/2} \right) / 2 \right\}^{1/2}, \\ \bar{\beta} &= \left\{ \left(1 - (1 - \bar{l}^2 M^2)^{1/2} \right) / 2 \right\}^{1/2}, \\ \bar{\alpha}_1 &= \sqrt{2M/\bar{l}} \cos(\bar{\varphi}/2), \\ \bar{\beta}_1 &= \sqrt{2M/\bar{l}} \sin(\bar{\varphi}/2), \quad \bar{\varphi} = \tan^{-1} \left(\sqrt{\bar{l}^2 M^2 - 1} \right), \\ \bar{\alpha}_2 &= (\bar{\beta}_1 - \bar{\alpha}_1 \cot \bar{\varphi}), \quad \bar{\beta}_2 = (\bar{\alpha}_1 + \bar{\beta}_1 \cot \bar{\varphi})\end{aligned}$$

$\bar{h}$ is the non-dimensional film thickness and is given by

$$\bar{h}(\bar{x}, \bar{t}) = \bar{h}_m(\bar{t}) + \bar{h}_1(\bar{x}) \quad (14)$$

$$\text{where } \bar{h}_1(\bar{x}) = \begin{cases} \delta(1 - \bar{x}/K) & \text{for } 0 \leq \bar{x} \leq K \\ 0 & \text{for } K \leq \bar{x} \leq 1 \end{cases}$$

$\delta = d/h_{ms}$ is the profile parameter.

The pressure boundary conditions are given by

$$\bar{p} = 0 \quad \text{at } \bar{x} = 0 \text{ and } \bar{x} = 1 \quad (15)$$

Integrating the non-dimensional Reynolds equation (12) with respect to $\bar{x}$ twice and applying the pressure boundary conditions given in Eq. (15), the non-dimensional pressure is obtained as

$$\bar{p} = \begin{cases} 6\bar{\zeta}_A(\bar{x}, \bar{h}_m) + 12\bar{V}\bar{\zeta}_B(\bar{x}, \bar{h}_m) + C_1(\bar{V}, \bar{h}_m)\bar{\zeta}_C(\bar{x}, \bar{h}_m) & \text{for } 0 \leq \bar{x} \leq K \\ 12\bar{V}\bar{\zeta}_D(\bar{x}, \bar{h}_m) + C_1(\bar{V}, \bar{h}_m)\bar{\zeta}_E(\bar{x}, \bar{h}_m) & \text{for } K \leq \bar{x} \leq 1 \end{cases} \quad (16)$$

where $C_1(\bar{V}, \bar{h}_m) = \frac{12\bar{V}\bar{\zeta}_{DK}(\bar{h}_m) - 12\bar{V}\bar{\zeta}_{BK}(\bar{h}_m) - 6\bar{\zeta}_{AK}(\bar{h}_m)}{\bar{\zeta}_{CK}(\bar{h}_m) - \bar{\zeta}_{EK}(\bar{h}_m)}$

The associated relations are given in Appendix A through Eqs. (A2a)–(A2j).

On integration of non-dimensional film pressure over the film region, the film force in non-dimensional form is obtained as

$$\begin{aligned}\bar{F} &= 6\chi_{AK}(\bar{h}_m) + 12\bar{V}[\chi_{BK}(\bar{h}_m) + \chi_{DK}(\bar{h}_m)] \\ &\quad + C_1(\bar{V}, \bar{h}_m)[\chi_{CK}(\bar{h}_m) + \chi_{EK}(\bar{h}_m)].\end{aligned} \quad (17)$$

The associated relations are given in Appendix A through Eqs. (A3a)–(A3e).

2.1 Steady-state characteristics

The steady film pressure and the steady load capacity in non-dimensional form are obtained from (16) and (17) by setting the non-dimensional minimum film height to be a constant and the non-dimensional squeezing velocity to be zero.

$$\bar{p}_s = \begin{cases} 6[\bar{\zeta}_A(\bar{x}, \bar{h}_m)]_s + C_1(0, \bar{h}_m)[\bar{\zeta}_C(\bar{x}, \bar{h}_m)]_s & \text{for } 0 \leq \bar{x} \leq K \\ C_1(0, \bar{h}_m)[\bar{\zeta}_E(\bar{x}, \bar{h}_m)]_s & \text{for } K \leq \bar{x} \leq 1 \end{cases} \quad (18)$$

$$\bar{W}_s = 6[\chi_{AK}(\bar{h}_m)]_s + C_1(0, \bar{h}_m)[\chi_{CK}(\bar{h}_m) + \chi_{EK}(\bar{h}_m)]_s \quad (19)$$

where subscript 's' denotes the steady state.

2.2 Dynamic characteristics

The non-dimensional dynamic stiffness coefficient is obtained by calculating the partial derivative of film force with respect to non-dimensional minimum film thickness and thereafter taking the value in steady state:

$$\begin{aligned}\bar{S}_d &= -6 \left[\frac{\partial \chi_{AK}}{\partial \bar{h}_m} \right] - C_1(0, \bar{h}_m) \left[\left\{ \frac{\partial \chi_{CK}}{\partial \bar{h}_m} \right\}_s + \left\{ \frac{\partial \chi_{EK}}{\partial \bar{h}_m} \right\}_s \right] \\ &\quad - \left[\frac{\partial C_1}{\partial \bar{h}_m} \right]_s [\chi_{CK}(\bar{h}_m) + \chi_{EK}(\bar{h}_m)]_s.\end{aligned} \quad (20)$$

The non-dimensional dynamic damping coefficient is obtained by calculating the partial derivative of film force with respect to non-dimensional squeezing velocity and thereafter taking the value in steady state:

$$\begin{aligned}\bar{C}_d &= -12[\chi_{BK}(\bar{h}_m) + \chi_{DK}(\bar{h}_m)]_s \\ &\quad - \left[\frac{\partial C_1}{\partial \bar{V}} \right]_s [\chi_{CK}(\bar{h}_m) + \chi_{EK}(\bar{h}_m)]_s.\end{aligned} \quad (21)$$

The associated relations of (20) and (21) are given in Appendix A through Eqs. (A4a)–(A4e).

3. Results and discussion

The effect of couple stress on the performance characteristics of wide tapered land slider bearing in the presence of applied magnetic field is studied analytically in the present paper. The performance of bearing is influenced by the couple stress parameter $\bar{l}$, the Hartmann number M , profile parameter δ and geometric parameter K . Some of the previous results can be recovered from the present study by taking specific values for the parameters influencing the performance of bearings as follows.

- (i) $\bar{l} = 0$: MHD tapered land slider bearing lubricated with Newtonian fluid studied by Lin [18].
- (ii) $\bar{l} = 0, M = 0$: tapered land slider bearing (non-conducting Newtonian case) studied by Lin *et al* [17].

3.1 Steady film pressure

The non-dimensional pressure $\bar{p}_s$ is a function of the non-dimensional coordinate $\bar{x}$ and the corresponding variations for different values of $\bar{l}$ and M with $\delta = 1$ and $K = 0.8$ is presented in figure 2. The use of couple stress fluid in the film region increases the steady-state pressure as compared with the corresponding Newtonian case ($\bar{l} = 0$). Compared with the non-magnetic case, the application of magnetic field increases the steady-state pressure even; bearings are lubricated with couple stress fluid, and these effects are more pronounced for larger values of $\bar{l}$. This increase in the steady pressure is observed because of reduced velocity of fluid in the film region on the application of magnetic field. With $K = 0.8$, $\bar{x} = 0.65$ and $\delta = 1$, for couple stress fluid ($\bar{l} = 0.4$), nearly 24% of increase is observed in steady film pressure compared with Newtonian fluid, whereas this increase is about 68% in the presence of magnetic field with $M = 5$ and $\bar{l} = 0.4$ compared with the non-magnetic case.

3.2 Steady load carrying capacity

For different values of M and $\bar{l}$ with $\bar{h}_{ms} = 1$ and $K = 0.8$, the variation of non-dimensional steady load carrying capacity $\bar{W}_s$ with profile parameter δ is presented in figure 3 and its variation with the geometric parameter K is presented in figure 4. It is observed that the steady load carrying capacity increases with increase in profile parameter and geometric parameter and attains a maximum; thereafter, $\bar{W}_s$ decreases. It is also observed that the effect of couple stress and applied magnetic field increases the steady load carrying capacity as compared with Newtonian and non-magnetic cases. This increase in steady load capacity is observed as a result of increased pressure on the application of magnetic field due to reduced velocity of fluid. The steady load carrying capacity attains its maximum value at geometric parameter $K = 0.8$ irrespective of the values of couple stress parameter and the Hartmann number, which implies that the tapered land slider bearing gives maximized performance when $K = 0.8$. With $K = 0.8$ and $\delta = 1$, for couple stress fluid ($\bar{l} = 0.4$), nearly 21% of increase is observed in steady load capacity compared with the Newtonian fluid whereas this increase is about 76% in the presence of magnetic field with $M = 5$ and $\bar{l} = 0.4$ compared with the non-magnetic case.

3.3 Dynamic stiffness coefficient

For different values of M and $\bar{l}$ with $\bar{h}_{ms} = 1$ and $K = 0.8$, the variation of non-dimensional dynamic stiffness coefficient $\bar{S}_d$ with profile parameter δ is presented in figure 5

Figure 2. Variation of non-dimensional steady film pressure $\bar{p}_s$ with $\bar{x}$ for different values of M and $\bar{l}$ with $\delta = 1$, $\bar{h}_{ms} = 1$, $K = 0.8$.

Figure 3. Variation of non-dimensional steady load carrying capacity $\bar{W}_s$ with δ for different values of M and $\bar{l}$ with $\bar{h}_{ms} = 1$, $K = 0.8$.

Figure 4. Variation of non-dimensional steady load carrying capacity $\bar{W}_s$ with K for different values of M and $\bar{l}$ with $\delta = 1$, $\bar{h}_{ms} = 1$.

Figure 5. Variation of non-dimensional dynamic stiffness coefficient $\bar{S}_d$ with δ for different values of M and $\bar{l}$ with $\bar{h}_{ms} = 1$, $K = 0.8$.

Figure 6. Variation of non-dimensional dynamic stiffness coefficient $\bar{S}_d$ with K for different values of M and $\bar{l}$ with $\delta = 1$, $\bar{h}_{ms} = 1$.

and its variation with the geometric parameter K is presented in figure 6. The dynamic stiffness coefficient increases with increase in the values of the profile parameter and the geometric parameter till it reaches its maximum value and thereafter the trend reverses. It is observed that effect of couple stress and applied magnetic field increases the dynamic stiffness coefficient as compared to Newtonian and non-magnetic cases. The dynamic stiffness coefficient attains the maximum value when $K = 0.8$ for any value of M and $\bar{l}$. Also observed that the point of maximum value of $\bar{S}_d$ shifts towards larger value of δ for higher values of M and $\bar{l}$. With $K = 0.8$ and $\delta = 1$, for couple stress fluid ($\bar{l} = 0.4$), nearly 36% of increase is observed in dynamic stiffness coefficient compared with the Newtonian fluid whereas this increase is about 39% in the presence of magnetic field with $M = 5$ and $\bar{l} = 0.4$ compared with the non-magnetic case.

Figure 7. Variation of non-dimensional dynamic damping coefficient $\bar{C}_d$ with δ for different values of M and $\bar{l}$ with $\bar{h}_{ms} = 1$, $K = 0.8$.

Figure 8. Variation of non-dimensional dynamic damping coefficient $\bar{C}_d$ with K for different values of M and $\bar{l}$ with $\delta = 1$, $\bar{h}_{ms} = 1$.

3.4 Dynamic damping coefficient

For different values M and $\bar{l}$ with $\bar{h}_{ms} = 1$ and $K = 0.8$, the variation of non-dimensional dynamic damping coefficient $\bar{C}_d$ with profile parameter δ is presented in figure 7 and its variation with the geometric parameter K is presented in figure 8. The dynamic damping coefficient decreases with increase in the values of the profile parameter and the geometric parameter. Compared with Newtonian and non-magnetic cases, the dynamic damping coefficient increases with the effect of couple stresses and applied magnetic field. With $K = 0.8$ and $\delta = 1$, for couple stress fluid ($\bar{l} = 0.4$), nearly 23% of increase is observed in dynamic damping coefficient compared with the Newtonian fluid whereas this increase is about 72% in the presence of magnetic field with $M = 5$ and $\bar{l} = 0.4$ compared with the non-magnetic case.

Table 1. Design example of slider bearing lubricated with couple stress fluid in the presence of magnetic field considering squeezing effect irrespective of the bearing shape.

Physical quantity	Symbol	Value of physical quantity
Length of the bearing	L	1×10^{-1} m
Inlet film thickness	h_1	$(1.5, 2.0, 2.5, 3.0, 3.5) \times 10^{-4}$ m
Outlet film thickness	h_{ms}	1.0×10^{-4} m
Profile parameter	δ	0.5, 1.0, 1.5, 2.0, 2.5
Electrical conductivity	σ	1.07×10^6 mho/m
Lubricant viscosity	μ	1.55×10^{-3} Pa s
Applied magnetic field	B_0	0, 0.95, 1.90 Wb/m ²
Couple stress material constant	η	(0, 0.3875, 1.55, 3.4875, 6.2) $\times 10^{-13}$ Ns

4. Design example

For the use of present study in the design of bearing, an example of tapered land slider bearing lubricated with a couple stress fluid in the presence of applied external magnetic field is considered with values for physical quantities as mentioned in table 1. With these values of physical quantities, the corresponding values of Hartmann number and couple stress parameter are obtained as $M = 0, 2.5, 5$ and $\bar{l} = 0, 0.1, 0.2, 0.3, 0.4$. Using the expressions obtained in the previous section, the non-

dimensional steady load carrying capacity, dynamic stiffness and damping coefficients are calculated and the considered physical quantities are tabulated in tables 2–4. These results obtained for bearing characteristics are compared to the non-magnetic Newtonian case by Lin *et al* [17] and MHD Newtonian case by Lin [18]. It is observed that all the results are in close agreement to the previous results and it provides a support for the present study.

The relative percentage increase due to couple stress effect in the non-dimensional steady load carrying capacity $R_{\bar{W}_s}$, dynamic stiffness coefficient $R_{\bar{S}_d}$ and dynamic damping coefficient $R_{\bar{C}_d}$ are given by the relations

$$R_{\bar{W}_s} = \left[\left\{ (\bar{W}_s)_{\text{Couplestress}} - (\bar{W}_s)_{\text{Newtonian}} \right\} / (\bar{W}_s)_{\text{Newtonian}} \right] \times 100$$

$$R_{\bar{S}_d} = \left[\left\{ (\bar{S}_d)_{\text{Couplestress}} - (\bar{S}_d)_{\text{Newtonian}} \right\} / (\bar{S}_d)_{\text{Newtonian}} \right] \times 100$$

$$R_{\bar{C}_d} = \left[\left\{ (\bar{C}_d)_{\text{Couplestress}} - (\bar{C}_d)_{\text{Newtonian}} \right\} / (\bar{C}_d)_{\text{Newtonian}} \right] \times 100$$

The values of $R_{\bar{W}_s}$, $R_{\bar{S}_d}$ and $R_{\bar{C}_d}$ for various values of the couple stress parameter and the Hartmann number are computed with $\bar{h}_{ms} = 1$, $\delta = 1$ and $K = 0.8$ and are presented in table 5. From table 5, nearly 26.12% increase in $\bar{W}_s$, nearly 38.25% increase in $\bar{S}_d$ and nearly 26.88% increase in $\bar{C}_d$ are observed. They are the encouraging results for the design engineers in the design of efficient bearing systems.

Table 2. Values for non-dimensional steady state load carrying capacity $\bar{W}_s$ with $\bar{h}_{ms} = 1$.

M	δ	Lin <i>et al</i> [17] $\bar{l} = 0$	Lin [18] $\bar{l} = 0$	Present analysis				
				$\bar{l} = 0$	$\bar{l} = 0.1$	$\bar{l} = 0.2$	$\bar{l} = 0.3$	$\bar{l} = 0.4$
0	0.5	0.153	0.1534	0.1534	0.1564	0.1649	0.1784	0.1969
	1.0	0.190	0.1897	0.1897	0.1926	0.2006	0.2135	0.2309
	1.5	0.190	0.1902	0.1902	0.1926	0.1993	0.2098	0.2240
	2.0	0.179	0.1794	0.1794	0.1813	0.1867	0.1952	0.2065
	2.5	0.165	0.1655	0.1654	0.1670	0.1714	0.1783	0.1874
	3.0	0.151	0.1515	0.1515	0.1528	0.1564	0.1621	0.1694
2.5	0.5	–	0.1771	0.1771	0.1809	0.1907	0.2055	0.2250
	1.0	–	0.2320	0.2320	0.2362	0.2467	0.2621	0.2820
	1.5	–	0.2472	0.2472	0.2512	0.2610	0.2752	0.2929
	2.0	–	0.2475	0.2475	0.2512	0.2601	0.2728	0.2885
	2.5	–	0.2417	0.2418	0.2451	0.2532	0.2646	0.2786
	3.0	–	0.2336	0.2336	0.2366	0.2440	0.2543	0.2669
5	0.5	–	0.2332	0.2332	0.2409	0.2567	0.2769	0.3008
	1.0	–	0.3227	0.3227	0.3326	0.3528	0.3780	0.4070
	1.5	–	0.3590	0.3590	0.3696	0.3909	0.4173	0.4474
	2.0	–	0.3718	0.3718	0.3823	0.4036	0.4298	0.4594
	2.5	–	0.3730	0.3730	0.3832	0.4039	0.4294	0.4578
	3.0	–	0.3682	0.3682	0.3781	0.3981	0.4225	0.4496

Table 3. Values for non-dimensional dynamic stiffness coefficient $\bar{S}_d$ with $\bar{h}_{ms} = 1$.

M	δ	Lin <i>et al</i> [17]	Lin [18]	Present analysis				
		$\bar{l} = 0$	$\bar{l} = 0$	$\bar{l} = 0$	$\bar{l} = 0.1$	$\bar{l} = 0.2$	$\bar{l} = 0.3$	$\bar{l} = 0.4$
0	0.5	0.379	0.3788	0.3788	0.3911	0.4256	0.4809	0.5566
	1.0	0.403	0.4032	0.4032	0.4137	0.4428	0.4887	0.5502
	1.5	0.359	0.3585	0.3585	0.3665	0.3883	0.4224	0.4674
	2.0	0.306	0.3062	0.3061	0.3121	0.3286	0.3539	0.3871
	2.5	0.260	0.2600	0.2599	0.2644	0.2770	0.2964	0.3216
	3.0	0.221	0.2216	0.2215	0.2252	0.2351	0.2503	0.2700
2.5	0.5	—	0.4029	0.4029	0.4168	0.4534	0.5102	0.5872
	1.0	—	0.4458	0.4458	0.4586	0.4918	0.5417	0.6071
	1.5	—	0.4139	0.4139	0.4244	0.4513	0.4910	0.5415
	2.0	—	0.3689	0.3689	0.3775	0.3993	0.4309	0.4702
	2.5	—	0.3260	0.3260	0.3332	0.3510	0.3766	0.4081
	3.0	—	0.2886	0.2886	0.2945	0.3094	0.3306	0.3563
5	0.5	—	0.4726	0.4726	0.4930	0.5377	0.6003	0.6810
	1.0	—	0.5545	0.5545	0.5763	0.6256	0.6876	0.7666
	1.5	—	0.5415	0.5415	0.5617	0.6051	0.6633	0.7336
	2.0	—	0.5030	0.5030	0.5210	0.5596	0.6109	0.6724
	2.5	—	0.4594	0.4595	0.4754	0.5094	0.5546	0.6080
	3.0	—	0.4177	0.4177	0.4318	0.4619	0.5016	0.5483

Table 4. Values for non-dimensional dynamic damping coefficient $\bar{C}_d$ with $\bar{h}_{ms} = 1$.

M	δ	Lin <i>et al</i> [17]	Lin [18]	Present analysis				
		$\bar{l} = 0$	$\bar{l} = 0$	$\bar{l} = 0$	$\bar{l} = 0.1$	$\bar{l} = 0.2$	$\bar{l} = 0.3$	$\bar{l} = 0.4$
0	0.5	0.569	0.5686	0.5685	0.5800	0.6121	0.6636	0.7342
	1.0	0.364	0.3643	0.3643	0.3702	0.3868	0.4132	0.4489
	1.5	0.253	0.2526	0.2525	0.2561	0.2660	0.2817	0.3028
	2.0	0.185	0.1854	0.1854	0.1877	0.1943	0.2047	0.2186
	2.5	0.142	0.1421	0.1421	0.1438	0.1485	0.1560	0.1659
	3.0	0.112	0.1128	0.1128	0.1141	0.1177	0.1233	0.1309
2.5	0.5	—	0.6540	0.6540	0.6685	0.7056	0.7615	0.8356
	1.0	—	0.4416	0.4416	0.4499	0.4710	0.5021	0.5423
	1.5	—	0.3227	0.3227	0.3283	0.3421	0.3622	0.3878
	2.0	—	0.2489	0.2489	0.2529	0.2629	0.2772	0.2953
	2.5	—	0.1996	0.1996	0.2027	0.2103	0.2212	0.2349
	3.0	—	0.1648	0.1648	0.1673	0.1734	0.1821	0.1929
5	0.5	—	0.8576	0.8576	0.8861	0.9450	1.0204	1.1104
	1.0	—	0.6086	0.6086	0.6275	0.6664	0.7154	0.7722
	1.5	—	0.4620	0.4620	0.4759	0.5042	0.5395	0.5801
	2.0	—	0.3667	0.3667	0.3774	0.3992	0.4263	0.4572
	2.5	—	0.3005	0.3005	0.3091	0.3265	0.3482	0.3728
	3.0	—	0.2522	0.2522	0.2594	0.2737	0.2916	0.3118

Table 5. Variation of $R_{\bar{W}_s}$, $R_{\bar{S}_d}$ and $R_{\bar{C}_d}$ for various values of M and $\bar{l}$ with $\bar{h}_{ms} = 1$ and $\delta = 1$.

M	$\bar{l}$	$R_{\bar{W}_s}$	$R_{\bar{S}_d}$	$R_{\bar{C}_d}$
0	0.2	5.7459	9.8214	6.1762
	0.4	21.7185	36.4583	23.2226
2.5	0.2	6.3362	10.3185	6.6576
	0.4	21.5517	36.1821	22.8034
5	0.2	9.3276	12.8224	9.4972
	0.4	26.1233	38.2507	26.8814

5. Conclusions

The effect of couple stress on static and dynamic characteristics of tapered land slider bearing in the presence of applied magnetic field is analysed. In accordance with the results and discussion, the following conclusions can be drawn.

1. Compared with the Newtonian case, the couple stress on MHD tapered land slider bearing effects

higher values for steady film pressure, steady load carrying capacity, dynamic stiffness coefficient and dynamic damping coefficient. Applied magnetic field also improves bearing characteristics and bearings are even lubricated with couple stress fluid.

2. Tapered land slider bearing lubricated with couple stress fluid in the presence of applied magnetic field shows higher performance for higher values of Hartmann number and couple stress parameter.

Nomenclature

B_0	applied magnetic field (Wb/m ²)
$\bar{C}_d$	non-dimensional dynamic damping coefficient
d	difference between the inlet and outlet film thickness (m)
F	frictional force (N)
$\bar{F}$	non-dimensional frictional force, $\bar{F} = \frac{Fh_{ms}^2}{\mu UL^2 B_0}$
$h(x, t)$	film thickness (m)
$\bar{h}$	non-dimensional film thickness, $\bar{h}(\bar{x}, \bar{t}) = h(x, t)/h_{ms}$
$h_m(t)$	minimum squeezing film thickness (m)
$\bar{h}_m(\bar{t})$	non-dimensional minimum squeezing film thickness $\bar{h}_m(\bar{t}) = h_m(t)/h_{ms}$
h_{ms}	steady-state reference minimum film thickness at outlet (m)
h_1	inlet film thickness (m)
L	length of the bearing (m)
l	couple stress parameter (m)
$\bar{l}$	non-dimensional couple stress parameter, $\bar{l} = 2l/h_{ms}$
M	Hartmann number, $M = B_0 h_{ms}(\sigma/\mu)^{1/2}$
p	film pressure (Pa)
p_s	steady film pressure (Pa)
$\bar{p}$	non-dimensional film pressure, $\bar{p} = ph_{ms}^2/\mu UL$
$\bar{p}_s$	non-dimensional steady film pressure
$\bar{S}_d$	non-dimensional dynamic damping coefficient
$t, \bar{t}$	time, $\bar{t} = Ut/L$ (s)
U	sliding velocity of lower part (m/s)
$\bar{V}$	non-dimensional squeezing velocity, $\bar{V} = d\bar{h}_m/d\bar{t}$
u, w	velocity components in x and z directions
W_s	steady load carrying capacity (N)
$\bar{W}_s$	non-dimensional steady load carrying capacity,
x, z	Cartesian coordinates
$\bar{x}$	non-dimensional coordinate $\bar{x} = x/L$
δ	profile parameter $\delta = d/h_{ms}$
η	material constant responsible for couple stress parameter (Ns)

μ	lubricant viscosity (Pa s)
σ	conductivity of the lubricant (mho/m)

Appendix A

$$f_{11} = \frac{\beta^2}{(\alpha^2 - \beta^2)} \left\{ \frac{\sinh\left(\frac{\alpha h}{l}\right) - \sinh\left(\frac{\alpha z}{l}\right) + \sinh\left(\frac{\alpha(h-z)}{l}\right)}{\sinh\left(\frac{\alpha h}{l}\right)} \right\} \quad (A1a)$$

$$f_{12} = \frac{\alpha^2}{(\alpha^2 - \beta^2)} \left\{ \frac{\sinh\left(\frac{\beta h}{l}\right) - \sinh\left(\frac{\beta z}{l}\right) + \sinh\left(\frac{\beta(h-z)}{l}\right)}{\sinh\left(\frac{\beta h}{l}\right)} \right\} \quad (A1b)$$

$$f_{13} = \frac{\beta^2}{\frac{\beta^2}{\alpha} \tanh\left(\frac{\alpha h}{2l}\right) - \frac{\alpha^2}{\beta} \tanh\left(\frac{\beta h}{2l}\right)} \left\{ \frac{\sinh\left(\frac{\alpha h}{l}\right) - \sinh\left(\frac{\alpha z}{l}\right) - \sinh\left(\frac{\alpha(h-z)}{l}\right)}{\sinh\left(\frac{\alpha h}{l}\right)} \right\} \quad (A1c)$$

$$f_{14} = \frac{\alpha^2}{\frac{\beta^2}{\alpha} \tanh\left(\frac{\alpha h}{2l}\right) - \frac{\alpha^2}{\beta} \tanh\left(\frac{\beta h}{2l}\right)} \left\{ \frac{\sinh\left(\frac{\beta h}{l}\right) - \sinh\left(\frac{\beta z}{l}\right) - \sinh\left(\frac{\beta(h-z)}{l}\right)}{\sinh\left(\frac{\beta h}{l}\right)} \right\} \quad (A1d)$$

$$f_{21} = \frac{\sinh\left(\frac{z-h}{\sqrt{2}l}\right) + \sinh\left(\frac{z}{\sqrt{2}l}\right) - \sinh\left(\frac{h}{\sqrt{2}l}\right)}{\sinh\left(\frac{h}{\sqrt{2}l}\right)} \quad (A1e)$$

$$f_{22} = \frac{z \cosh\left(\frac{z-h}{\sqrt{2}l}\right) + y \cosh\left(\frac{z}{\sqrt{2}l}\right) - h \coth\left(\frac{h}{2\sqrt{2}l}\right) \sinh\left(\frac{z}{\sqrt{2}l}\right)}{2\sqrt{2}l \sinh\left(\frac{h}{\sqrt{2}l}\right)} \quad (A1f)$$

$$f_{23} = \frac{z \sinh\left(\frac{z-h}{\sqrt{2}l}\right) + z \sinh\left(\frac{z}{\sqrt{2}l}\right) - h \sinh\left(\frac{z}{\sqrt{2}l}\right)}{(6l \sinh\left(\frac{h}{\sqrt{2}l}\right) - \sqrt{2}h)} \quad (A1g)$$

$$f_{24} = \frac{2 \cosh\left(\frac{z-h}{\sqrt{2}l}\right) + 2 \cosh\left(\frac{z}{\sqrt{2}l}\right) - 2 \cosh\left(\frac{h}{\sqrt{2}l}\right) - 2}{(3\sqrt{2} \sinh\left(\frac{h}{\sqrt{2}l}\right) - \frac{h}{l})} \quad (A1h)$$

$$f_{31} = \frac{\cosh \alpha_1 z \cos \beta_1 (z-h) - \cos \beta_1 z \cosh \alpha_1 (z-h)}{(\cosh \alpha_1 h - \cos \beta_1 h)} \quad (A1i)$$

$$f_{32} = \frac{\cot \varphi \{ \sinh \alpha_1 z \sin \beta_1 (z - h) - \sin \beta_1 z \sinh \alpha_1 (z - h) \} + (\cosh \alpha_1 h - \cos \beta_1 h)}{(\cosh \alpha_1 h - \cos \beta_1 h)} \quad (\text{A1j})$$

$$f_{33} = \frac{M}{h_{ms}} \left\{ \frac{\cot \varphi \{ \sinh \alpha_1 z \sin \beta_1 (z - h) + \sin \beta_1 z \sinh \alpha_1 (z - h) \} + (\cos \beta_1 h + \cosh \alpha_1 h)}{(\beta_1 - \alpha_1 \cot \varphi) \sin \beta_1 h + (\alpha_1 + \beta_1 \cot \varphi) \sinh \alpha_1 h} \right\} \quad (\text{A1k})$$

$$f_{34} = \frac{M}{h_{ms}} \left\{ \frac{\cos \beta_1 z \cosh \alpha_1 (z - h) + \cosh \alpha_1 z \cos \beta_1 (z - h)}{(\beta_1 - \alpha_1 \cot \varphi) \sin \beta_1 h + (\alpha_1 + \beta_1 \cot \varphi) \sinh \alpha_1 h} \right\} \quad (\text{A1l})$$

$$\chi_{AK}(\bar{h}_m) = \int_{\bar{x}=0}^K \int_{\bar{x}=0}^{\bar{x}} \frac{\bar{h}_1(\bar{x}, t)}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} d\bar{x} \quad (\text{A3a})$$

$$\xi_A(\bar{x}, \bar{h}_m) = \int_{\bar{x}=0}^{\bar{x}} \frac{\bar{h}_1(\bar{x}, t)}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2a})$$

$$\chi_{BK}(\bar{h}_m) = \int_{\bar{x}=0}^K \int_{\bar{x}=0}^{\bar{x}} \frac{\bar{x}}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} d\bar{x} \quad (\text{A3b})$$

$$\xi_B(\bar{x}, \bar{h}_m) = \int_{\bar{x}=0}^{\bar{x}} \frac{\bar{x}}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2b})$$

$$\chi_{CK}(\bar{h}_m) = \int_{\bar{x}=0}^K \int_{\bar{x}=0}^{\bar{x}} \frac{1}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} d\bar{x} \quad (\text{A3c})$$

$$\xi_C(\bar{x}, \bar{h}_m) = \int_{\bar{x}=0}^{\bar{x}} \frac{1}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2c})$$

$$\chi_{DK}(\bar{h}_m) = \int_{\bar{x}=K}^1 \int_{\bar{x}=1}^{\bar{x}} \frac{\bar{x}}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} d\bar{x} \quad (\text{A3d})$$

$$\xi_D(\bar{x}, \bar{h}_m) = \int_{\bar{x}=1}^{\bar{x}} \frac{\bar{x}}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2d})$$

$$\chi_{EK}(\bar{h}_m) = \int_{\bar{x}=K}^1 \int_{\bar{x}=1}^{\bar{x}} \frac{1}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} d\bar{x} \quad (\text{A3e})$$

$$\xi_E(\bar{x}, \bar{h}_m) = \int_{\bar{x}=1}^{\bar{x}} \frac{1}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2e})$$

$$\frac{\partial \chi_{AK}}{\partial \bar{h}_m} = - \int_{\bar{x}=0}^K \int_{\bar{x}=0}^{\bar{x}} \frac{\bar{h}_1(\bar{x})}{\{\bar{g}(\bar{h}, \bar{l}, M)\}^2} \frac{\partial \bar{g}}{\partial \bar{h}_m} d\bar{x} d\bar{x} \quad (\text{A4a})$$

$$\xi_{AK}(\bar{h}_m) = \int_{\bar{x}=0}^K \frac{\bar{h}_1(\bar{x}, t)}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2f})$$

$$\frac{\partial \chi_{CK}}{\partial \bar{h}_m} = - \int_{\bar{x}=0}^K \int_{\bar{x}=0}^{\bar{x}} \frac{1}{\{\bar{g}(\bar{h}, \bar{l}, M)\}^2} \frac{\partial \bar{g}}{\partial \bar{h}_m} d\bar{x} d\bar{x} \quad (\text{A4b})$$

$$\xi_{BK}(\bar{h}_m) = \int_{\bar{x}=0}^K \frac{\bar{x}}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2g})$$

$$\frac{\partial \chi_{EK}}{\partial \bar{h}_m} = - \int_{\bar{x}=K}^1 \int_{\bar{x}=1}^{\bar{x}} \frac{1}{\{\bar{g}(\bar{h}, \bar{l}, M)\}^2} \frac{\partial \bar{g}}{\partial \bar{h}_m} d\bar{x} d\bar{x} \quad (\text{A4c})$$

$$\xi_{CK}(\bar{h}_m) = \int_{\bar{x}=0}^K \frac{1}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2h})$$

$$\left(\frac{\partial C_1}{\partial \bar{h}_m} \right)_s = -6 \left[\frac{\{ \xi_{CK} - \xi_{EK} \} \frac{\partial \xi_{AK}}{\partial \bar{h}_m} - \xi_{AK} \left(\frac{\partial \xi_{CK}}{\partial \bar{h}_m} - \frac{\partial \xi_{EK}}{\partial \bar{h}_m} \right)}{\{ \xi_{CK} - \xi_{EK} \}^2} \right] \quad (\text{A4d})$$

$$\xi_{DK}(\bar{h}_m) = \int_{\bar{x}=1}^K \frac{\bar{x}}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2i})$$

$$\left(\frac{\partial C_1}{\partial V} \right)_s = 12 \left[\frac{\xi_{DK} - \xi_{BK}}{\xi_{CK} - \xi_{EK}} \right] \quad (\text{A4e})$$

$$\xi_{EK}(\bar{h}_m) = \int_{\bar{x}=1}^K \frac{1}{\bar{g}(\bar{h}, \bar{l}, M)} d\bar{x} \quad (\text{A2j})$$

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