A new adaptive control of dual-motor driving servo system with backlash nonlinearity

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MS received 26 August 2017; revised 17 January 2018; accepted 26 January 2018; published online 10 August 2018

Abstract. In order to weaken the influence of backlash nonlinearity on a dual-motor driving servo system, we first establish the state-space model of the system. We then propose a new adaptive controller combining a projection algorithm with backstepping control for the first time, to the best of our knowledge, and analyze its stability. In the simulation analysis, we respectively choose a triangular wave, sawtooth wave, and random signal as the input signal. Simulation results validate a higher tracking accuracy and stronger adaptability of the proposed control law than that of mere backstepping control. In the experimental tests, we respectively choose a step signal and sine signal and simultaneously apply a white noise signal to the system output after 3 s in each test. The test results validate a stronger adaptability and robustness than that of mere backstepping control.

Keywords. Adaptive control; backstepping control; dual-motor driving; projection algorithm; backlash nonlinearity.

1. Introduction

In mechanical transmission systems, the existence of backlash nonlinearity will affect the system's dynamic performance and steady-state precision [1–3]. Hence, it is necessary to study the control strategy of weakening the effect of backlash nonlinearity on the system. The commonly used models of backlash nonlinearity include the hysteretic model [4], the dead-zone model [5, 6], and the impact model [7]. In addition, some control strategies, such as the adaptive inverse method [8, 9], robust adaptive control [10–12], fuzzy logic control [13, 14], neural network control [15, 16], dynamic surface control [17], and the describing function method [18] have been successfully used by some scholars. However, backlash nonlinearity is still a challenging problem because of its non-differential characteristic.

Because of the effect of backlash nonlinearity, the system not only produces output error, but its stability is also reduced due to the limit cycle oscillation. In addition, the gear's mutual impact also produces serious oscillation and noise. In recent years, many scholars have also studied the control strategies of backlash nonlinearity. In [19], Tarbouriech *et al.* focused on the stability analysis and the control design for a linear system with a backlash operator

in the input based on Lyapunov techniques. In [20], an adaptive robust controller for a nonlinear system was designed in the case of unknown input backlash, parameter uncertainty, and external disturbances. Vörös [21] dealt with the parameter identification of nonlinear dynamic systems with input saturation and output backlash using three-block cascade models. An iterative least-squares-based parameter estimation algorithm with internal variables estimations was proposed and illustrated on examples of simulated nonlinear dynamic systems with saturation in the input block and with backlash in the output block. In [22], a robust method was applied to eliminate the influences of truncation errors and bounded errors in the backlash model on system performance. A nonlinear control system considering the uncertainties of the backlash of nonlinear plants was proposed in [23], and a robust nonlinear control system was designed. However, other scholars [19–23] have not used projection algorithms.

The projection algorithm by itself is mature. For the operation of variable-speed wind generators in [24], the adaptive control scheme was based on the affine projection algorithm, which provides faster convergence and less computational complexity than the least-mean-squares algorithm. In [25], to support fast adaptation to the acoustic echo environment, a linear filter was adapted using an affine projection algorithm. The simulation results confirm

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that the proposed algorithm achieves fast and stable convergence under saturation-type microphone distortion, and can achieve better performance than conventional algorithms. In [26], a projection algorithm was used in backstepping controller design, but the backstepping controller was conventional. An optimal variable step-size affine projection algorithm for modified filtered-x active noise control systems was introduced in [27]. The simulation results on the active noise control environments show both fast convergence rate and low steady-state error.

In this paper, we use a dual-motor driving servo system as the research object. We first establish the state-space model of the system, and then propose a new adaptive control strategy combining a projection algorithm with backstepping control for the first time. In the design process of the controller, we introduce the integration of position tracking error so as to ensure that the system tracking error is asymptotically stable and tends to zero better under the condition of an uncertain system model. Simulation analysis and experimental tests all validate the effectiveness of the proposed control strategy compared with mere backstepping control.

The rest of the paper is organized as follows. System modeling is detailed in Section 2 and design of the proposed controller in Section 3. Simulation analysis is presented in Section 4 to show the effectiveness of the proposed strategy. Section 5 details the experimental tests and Section 6 concludes the paper.

2. System modeling

The schematic of the dual-motor driving servo system is shown in Figure 1. O_0 is the driven-subsystem, and O_1 and O_2 are driving-subsystems. The block diagram of the system is shown in Figure 2.

In order to facilitate the study, we assume that the transmission ratio is 1 and the backlash is 2α . From the block diagram, we can obtain the dynamic equation of the dual-motor driving servo system as follows:

$$\begin{cases} J_i \ddot{\theta}_i(t) + b_i \dot{\theta}_i(t) = u(t) - \tau_i(t) + (-1)^{i-1} w, \\ J_m \ddot{\theta}_m(t) + b_m \dot{\theta}_m(t) = \sum_{i=1}^2 \tau_i(t), \end{cases} \quad (1)$$

where J_i and J_m are the rotational inertias of the driving wheel and driven wheel, respectively; b_i and b_m are the

Figure 1. Schematic of dual-motor driving servo system.

viscous friction coefficients of the driving wheel and driven wheel, respectively; θ_i and θ_m are the rotational angles of the driving wheel and driven wheel, respectively; $\dot{\theta}_i$ and $\dot{\theta}_m$ are the rotational speeds of the driving wheel and driven wheel, respectively; $u(t)$ is the input torque of the system; $\tau_i(t)$ is the transmitted torque when the driving wheel contacts the driven wheel; and w is the bias torque.

The dead-zone model of backlash can be expressed as follows [28]:

$$\tau_i = \begin{cases} k(z_i(t) + \alpha) + c\dot{z}_i(t) & z_i(t) < -\alpha, \\ 0 & |z_i(t)| \leq \alpha, \\ k(z_i(t) + \alpha) + c\dot{z}_i(t) & z_i(t) > \alpha, \end{cases} \quad (2)$$

where $z_i(t) = \theta_i(t) - \theta_m(t)$.

To simplify the controller design, we divide τ_i into three parts:

$$\tau_i = \tau_{i0} + \Delta\tau_{ia} + \Delta\tau_{ib}, \quad (3)$$

where τ_{i0} is the linear transmitted torque, $\Delta\tau_{ia}$ is the non-linear transmitted torque, $\Delta\tau_{ib}$ is the disturbance of transmitted torque τ_i , and their expressions are as follows, respectively:

$$\tau_{i0} = kz_i(t), \quad (4)$$

$$\Delta\tau_{ia} = \begin{cases} k\alpha & z_i(t) < -\alpha, \\ 0 & |z_i(t)| \leq \alpha, \\ -k\alpha & z_i(t) > \alpha, \end{cases} \quad (5)$$

$$\Delta\tau_{ib} = \begin{cases} c\dot{z}_i(t) & z_i(t) < -\alpha, \\ 0 & |z_i(t)| \leq \alpha, \\ c\dot{z}_i(t) & z_i(t) > \alpha. \end{cases} \quad (6)$$

In order to simplify the expression, let $\Delta\tau_i = \Delta\tau_{ia} + \Delta\tau_{ib}$.

If we choose the state variables of system as $x_1 = \theta_m$, $x_2 = \dot{\theta}_m$, $x_{3i} = \theta_i$, $x_{4i} = \dot{\theta}_i$, then the state-space equation model of the system is

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{k}{J_m}(x_{31} + x_{32}) - \frac{2k}{J_m}x_1 - \frac{b_m}{J_m}x_2 + \frac{\Delta\tau_1 + \Delta\tau_2}{J_m}, \\ \dot{x}_{3i} = x_{4i}, \\ \dot{x}_{4i} = \frac{u}{J_i} + \frac{(-1)^{i-1}w}{J_i} + \frac{k}{J_i}x_1 - \frac{k}{J_i}x_{3i} - \frac{b_i}{J_i}x_{4i} - \frac{\Delta\tau_i}{J_i}, \\ y = x_1, \end{cases} \quad (7)$$

where y is the output of system.

To facilitate the writing of the following expression, we choose $h_2 = \frac{k}{J_m}$, $h_{21} = \frac{2k}{J_m}$, $h_{22} = \frac{b_m}{J_m}$, $\Delta_2 = \frac{\Delta\tau_1 + \Delta\tau_2}{J_m}$, $h_{4i} = \frac{1}{J_i}$, $h_{4i1} = \frac{k}{J_i}$, $h_{4i3} = \frac{k}{J_i}$, $h_{4i4} = \frac{b_i}{J_i}$, $\Delta_{4i} = -\frac{\Delta\tau_i}{J_i}$.

The simplified state-space equation is expressed as follows:

Figure 2. Block diagram of dual-motor driving servo system.

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \hat{h}_2(x_{31} + x_{32}) - \hat{h}_{21}x_1 - \hat{h}_{22}x_2 + \Delta_2, \\ \dot{x}_{3i} = x_{4i}, \\ \dot{x}_{4i} = \hat{h}_{4i}u + \hat{h}_{4i}(-1)^{i-1}w + \hat{h}_{4i1}x_1 - \hat{h}_{4i3}x_{3i} - \hat{h}_{4i4}x_{4i} + \Delta_{4i}, \\ y = x_1. \end{cases} \quad (8)$$

In the actual system, the system parameters J_i , J_m , b_i , b_m , and k affected by lubrication, temperature, and material wear are uncertain, which causes the change of gear transmitted torque. The design goal of this paper is to design an adaptive controller [29, 30] that makes the system output y asymptotically stable when tracking the expected output y_d , namely $\lim_{t \rightarrow \infty} |y - y_d| = 0$.

3. Controller design

We can see that O_1 and O_2 have a parallel relationship from Eq. (8), and they are series-connected with O_0 . Moreover, Eq. (1) is a second-order nonlinear uncoupled differential equation of θ_i and $\dot{\theta}_m$.

Assumption 1 The system states θ_m , $\dot{\theta}_m$, θ_i , and $\dot{\theta}_i$ are all measurable.

Assumption 2 Suppose that the fourth derivative of y_d exists.

Assumption 3 System parameters J_i , J_m , b_i , b_m , k , and $\Delta\tau_{ib}$ are all bounded.

Then,

$$\begin{aligned} 0 < J_{i\min} &\leq J_i \leq J_{i\max}, \\ 0 < J_{m\min} &\leq J_m \leq J_{m\max}, \\ 0 < b_{i\min} &\leq b_i \leq b_{i\max}, \\ 0 < b_{m\min} &\leq b_m \leq b_{m\max}, \\ 0 < k_{\min} &\leq k \leq k_{\max}, \\ 0 < |\Delta\tau_{ib}| &\leq \Delta\tau_{ib\max}. \end{aligned}$$

3.1 Projection algorithm

A projection algorithm itself is mature, and there are three kinds of projection algorithms commonly used at present: successive projection [31], discontinuous projection [32], and smooth projection [33]. Since we use the backstepping

method to design the controller, we must take the derivative of the controller repeatedly, so we use the smooth projection method to ensure the smoothness of the controller.

For parameter estimation vector $\hat{\vartheta} = [\hat{\vartheta}_1, \dots, \hat{\vartheta}_p]^T$, we define the smooth projection mapping $\text{Pr } oj : R^p \rightarrow R^p$ as follows [33]:

$$\text{Pr } oj(\hat{\vartheta}) = [\text{Pr } oj_1(\hat{\vartheta}_1), \dots, \text{Pr } oj_p(\hat{\vartheta}_p)]^T, \quad (9)$$

where $\text{Pr } oj_i(\hat{\vartheta}_i)$ meets both of the following requirements:

- 1) $\text{Pr } oj_i(\hat{\vartheta}_i) = (\hat{\vartheta}_i), \forall \hat{\vartheta}_i \in \Omega_i = \{v : \vartheta_{i\min} \leq v \leq \vartheta_{i\max}\}$,
- 2) $\text{Pr } oj_i(\hat{\vartheta}_i) \in \{v : \vartheta_{i\min} - \varepsilon_{\vartheta_i} \leq v \leq \vartheta_{i\max} + \varepsilon_{\vartheta_i}\}, \forall \hat{\vartheta}_i \in R..$

According to the actual system requirements, we must ensure that the second derivative of the projection function exists, and we choose the projection function as follows [26]:

$$\text{Pr } oj_i(\hat{\vartheta}_i) = \begin{cases} \vartheta_{i\max} + \frac{2\varepsilon_{\vartheta_i} \arctan((\hat{\vartheta}_i - \vartheta_{i\max})/\varepsilon_{\vartheta_i} \times \frac{\pi}{2})}{\pi} & \hat{\vartheta}_i > \vartheta_{i\max}, \\ \hat{\vartheta}_i & \vartheta_{i\min} \leq \hat{\vartheta}_i \leq \vartheta_{i\max}, \\ \vartheta_{i\min} - \frac{2\varepsilon_{\vartheta_i} \arctan((\hat{\vartheta}_i - \vartheta_{i\max})/\varepsilon_{\vartheta_i} \times \frac{\pi}{2})}{\pi} & \hat{\vartheta}_i < \vartheta_{i\min}. \end{cases}$$

3.2 Backstepping design

Step 1 Define the position tracking error e_1 as

$$e_1 = x_1 - y_d. \quad (10)$$

We choose the Lyapunov function V_1 as follows:

$$V_1 = \frac{1}{2} e_1^2. \quad (11)$$

The derivative of V_1 is

$$\dot{V}_1 = e_1 \dot{e}_1 = e_1(\dot{x}_1 - \dot{y}_d) = e_1(x_2 - \dot{y}_d). \quad (12)$$

Let x_2 be a virtual control variable, suppose the expectation of x_2 is η_2 , and define the error between x_2 and η_2 as e_2 :

$$e_2 = x_2 - \eta_2. \quad (13)$$

and if

$$\eta_2 = -k_1 e_1 + \dot{y}_d - k_0 \int_0^t e_1(\tau) d\tau, \quad (14)$$

then

$$\dot{V}_1 = -k_1 e_1^2 + e_1 e_2 - k_0 e_1 A, \quad (15)$$

where $k_0 > 0$ and $k_1 > 0$ are adjustable parameters, and A is the integral of position tracking error, $A = \int_0^t e_1(\tau) d\tau$.

Step 2 Let $x_{31} + x_{32}$ be a virtual control variable. We choose the augmented control Lyapunov function (CLF) V_2 as follows:

$$V_2 = \frac{1}{2}e_1^2 + \frac{1}{2\hbar_2}e_2^2 + \frac{1}{2}k_0A^2. \quad (16)$$

Its derivative is

$$\dot{V}_2 = -k_1e_1^2 + e_2 \left(e_1 + x_{31} + x_{32} - \frac{\hbar_{21}}{\hbar_2}x_1 - \frac{\hbar_{22}}{\hbar_2}x_2 + \frac{\Delta_2}{\hbar_2} - \dot{\eta}_2 \right). \quad (17)$$

Letting

$$\vartheta_2 = \left[\frac{\hbar_{21}}{\hbar_2} \quad \frac{\hbar_{22}}{\hbar_2} \quad \frac{1}{\hbar_2} \right]^T, \varphi_2 = [-x_1 \quad -x_2 \quad -\dot{\eta}_2]^T,$$

Eq. (17) can then be simplified as follows:

$$\dot{V}_2 = -k_1e_1^2 + e_2 \left(e_1 + x_{31} + x_{32} + \vartheta_2^T \varphi_2 + \frac{\Delta_2}{\hbar_2} \right). \quad (18)$$

Supposing that the expectation of $x_{31} + x_{32}$ is η_3 , we use the adaptive robust control algorithm to design η_3 as follows [26]:

$$\begin{cases} \eta_3 = \eta_{3a} + \eta_{3s}, \\ \eta_{3s} = \eta_{3s1} + \eta_{3s2}, \\ \eta_{3s1} = -e_1 - k_2e_2, \\ \eta_{3a} = -\text{Pr } oj(\hat{\vartheta}_2)\varphi_2, \end{cases} \quad (19)$$

where $\hat{\vartheta}_2$ is the estimated value of ϑ_2 :

$$\dot{\hat{\vartheta}}_2 = \Gamma_2 \varphi_2 e_2. \quad (20)$$

In Eq. (19), η_{3a} is the compensating control of the adaptive model designed according to parameter estimation $\hat{\vartheta}_2$, η_{3s} is the robust control variable, η_{3s1} is the compensation for the Lyapunov function V_1 , η_{3s2} is the robust feedback control part, and $k_2 > 0$ is an adjustable parameter. In Eq. (20), Γ_2 is a diagonal matrix that can be replaced by a unit matrix.

Define the error between $x_{31} + x_{32}$ and η_3 as e_3 :

$$e_3 = x_{31} + x_{32} - \eta_3. \quad (21)$$

Define the error between ϑ_2 and the projection of $\hat{\vartheta}_2$ as follows:

$$\tilde{\vartheta}_{proj2} = \vartheta_2 - \text{Pr } oj(\hat{\vartheta}_2). \quad (22)$$

Suppose that η_{3s2} satisfies the following two conditions simultaneously:

$$\begin{cases} e_2 \cdot \eta_{3s2} \leq 0, \\ e_2 \left(\eta_{3s2} + \tilde{\vartheta}_{proj2}^T \varphi_2 + \frac{\Delta_2}{\hbar_2} \right) \leq \varepsilon_2, \end{cases} \quad (23)$$

where ε_2 is a positive number that can be required to take any small value. Substituting Eq. (21) into Eq. (18), we obtain

$$\dot{V}_2 = -k_1e_1^2 - k_2e_2^2 + e_2 \left(e_3 + \eta_{3s2} + \tilde{\vartheta}_{proj2}^T \varphi_2 + \frac{\Delta_2}{\hbar_2} \right). \quad (24)$$

Step 3 Letting $x_{41} + x_{42}$ be a virtual control variable, we choose the augmented CLF V_3 as follows:

$$V_3 = \frac{1}{2}e_1^2 + \frac{1}{2\hbar_2}e_2^2 + \frac{1}{2}e_3^2. \quad (25)$$

Its derivative is

$$\dot{V}_3 = -k_1e_1^2 - k_2e_2^2 + e_2 \left(\eta_{3s2} + \tilde{\vartheta}_{proj2}^T \varphi_2 + \frac{\Delta_2}{\hbar_2} \right) + e_3(x_{41} + x_{42} - \dot{\eta}_3 + e_2). \quad (26)$$

Supposing that the expectation of $x_{41} + x_{42}$ is η_4 , define the error between $x_{41} + x_{42}$ and η_4 as e_4 :

$$e_4 = x_{41} + x_{42} - \eta_4. \quad (27)$$

Substituting Eq. (27) into Eq. (26), we obtain

$$\dot{V}_3 = -k_1e_1^2 - k_2e_2^2 + e_2 \left(\eta_{3s2} + \tilde{\vartheta}_{proj2}^T \varphi_2 + \frac{\Delta_2}{\hbar_2} \right) - k_3e_3^2 + e_3e_4, \quad (28)$$

where $k_3 > 0$ is an adjustable parameter.

Step 4 Choose the augmented CLF V_4 as follows:

$$V_4 = \frac{1}{2}e_1^2 + \frac{1}{2\hbar_2}e_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2(\hbar_{41} + \hbar_{42})}e_4^2. \quad (29)$$

The derivative of V_4 is expressed as follows:

$$\begin{aligned} \dot{V}_4 = & -k_1e_1^2 - k_2e_2^2 + e_2 \left(\eta_{3s2} + \tilde{\vartheta}_{proj2}^T \varphi_2 + \frac{\Delta_2}{\hbar_2} \right) - k_3e_3^2 \\ & + e_4 \left(e_3 + u + \frac{\hbar_{41} - \hbar_{42}}{\hbar_{41} + \hbar_{42}}w + \frac{\hbar_{411} + \hbar_{421}}{\hbar_{41} + \hbar_{42}}x_1 \right. \\ & - \frac{\hbar_{413}}{\hbar_{41} + \hbar_{42}}x_{31} - \frac{\hbar_{423}}{\hbar_{41} + \hbar_{42}}x_{32} - \frac{\hbar_{414}}{\hbar_{41} + \hbar_{42}}x_{41} \\ & \left. - \frac{\hbar_{424}}{\hbar_{41} + \hbar_{42}}x_{42} + \frac{\Delta_{41} + \Delta_{42}}{\hbar_{41} + \hbar_{42}} - \frac{1}{\hbar_{41} + \hbar_{42}}\dot{\eta}_4 \right). \end{aligned} \quad (30)$$

Letting

$$\begin{aligned} \vartheta_4 = & \left[\frac{\hbar_{41} - \hbar_{42}}{\hbar_{41} + \hbar_{42}} \quad \frac{\hbar_{411} + \hbar_{421}}{\hbar_{41} + \hbar_{42}} \quad \frac{\hbar_{413}}{\hbar_{41} + \hbar_{42}} \quad \frac{\hbar_{423}}{\hbar_{41} + \hbar_{42}} \right. \\ & \left. \frac{\hbar_{414}}{\hbar_{41} + \hbar_{42}} \quad \frac{\hbar_{424}}{\hbar_{41} + \hbar_{42}} \quad \frac{1}{\hbar_{41} + \hbar_{42}} \right]^T, \\ \varphi_4 = & \left[w \quad x_1 \quad -x_{31} \quad -x_{32} \quad -x_{41} \quad -x_{42} \quad -\dot{\eta}_4 \right]^T, \end{aligned}$$

Eq. (30) can then be simplified as follows:

$$\begin{aligned} \dot{V}_4 = & -k_1 e_1^2 - k_2 e_2^2 + e_2 \left(\eta_{3s2} + \tilde{\vartheta}_{proj2}^T \varphi_2 + \frac{\Delta \tau_2}{\bar{h}_2} \right) - k_3 e_3^2 \\ & + e_4 \left(e_3 + u + \vartheta_4^T \varphi_4 + \frac{\Delta_{41} + \Delta_{42}}{\bar{h}_{41} + \bar{h}_{42}} \right). \end{aligned} \quad (31)$$

Next, we use the adaptive robust control algorithm to design u as follows:

$$\begin{cases} u = u_a + u_s, \\ u_s = u_{s1} + u_{s2}, \\ u_{s1} = -k_4 e_4 - e_3, \\ u_a = -\text{Pr } oj(\hat{\vartheta}_4) \varphi_4, \end{cases} \quad (32)$$

where $\hat{\vartheta}_4$ is the estimated value of ϑ_4 :

$$\dot{\hat{\vartheta}}_4 = \Gamma_4 \varphi_4 e_4. \quad (33)$$

In Eq. (32), u_a is the compensating control of the adaptive model designed according to the parameter estimation ϑ_4 . u_s is the robust control variable, u_{s1} is the compensation for the Lyapunov function V_3 , u_{s2} is the robust feedback control part, and $k_4 > 0$ is an adjustable parameter. In Eq. (33), γ_4 is a diagonal matrix that can be replaced by a unit matrix. Define the error between ϑ_4 and the projection of $\hat{\vartheta}_4$ as follows:

$$\tilde{\vartheta}_{proj4} = \vartheta_4 - \text{Pr } oj(\hat{\vartheta}_4). \quad (34)$$

Suppose that u_{s2} satisfies the following two conditions simultaneously:

$$\begin{cases} e_4 \cdot u_{s2} \leq 0, \\ e_4 \left(u_{s2} + \tilde{\vartheta}_{proj4}^T \varphi_4 + \frac{\Delta_{41} + \Delta_{42}}{\bar{h}_{41} + \bar{h}_{42}} \right) \leq \varepsilon_4, \end{cases} \quad (35)$$

where ε_4 is a positive number that can be required to take any small value. Substituting Eq. (32) into Eq. (31), we obtain

$$\begin{aligned} \dot{V}_4 = & -k_1 e_1^2 - k_2 e_2^2 + e_2 \left(\eta_{3s2} + \tilde{\vartheta}_{proj2}^T \varphi_2 + \frac{\Delta \tau_2}{\bar{h}_2} \right) - k_3 e_3^2 \\ & + e_4 \left(u_{s2} + \tilde{\vartheta}_{proj4}^T \varphi_4 + \frac{\Delta_{41} + \Delta_{42}}{\bar{h}_{41} + \bar{h}_{42}} \right) - k_4 e_4^2. \end{aligned} \quad (36)$$

3.3 Stability analysis

Theorem 1 The error variable $e = (e_1, e_2, e_3, e_4)$ of the closed-loop system containing controlled object (8), control law (32), and adaptive law (33) is bounded.

Simultaneously, e converges to a sufficiently small neighborhood of the origin according to the exponential law.

Proof From Eqs. (23), (35), and (36), we can obtain

$$\begin{aligned} \dot{V}_4 \leq & -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + \varepsilon_2 + \varepsilon_4 \\ \leq & -d_0 V_4 + \varepsilon_0, \end{aligned} \quad (37)$$

where

$$\begin{aligned} d_0 = & \min(2k_1, 2\bar{h}_2 k_2, 2k_3, 2(\bar{h}_{41} + \bar{h}_{42})/k_4), \\ \varepsilon_0 = & \varepsilon_2 + \varepsilon_4. \end{aligned}$$

Calculating the integral on both sides of Eq. (37), we obtain:

$$V_4(t) \leq e^{-d_0 t} V_4(0) + \frac{\varepsilon_0}{d_0} (1 - e^{-d_0 t}), \quad (38)$$

According to Eqs. (29) and (38), we have

$$e_1^2 \leq 2V_4(t), \lim_{t \rightarrow \infty} e_1(t) \leq \sqrt{\frac{2\varepsilon_0}{d_0}}, \quad (39)$$

$$e_2^2 \leq 2\bar{h}_2 V_4(t), \lim_{t \rightarrow \infty} e_2(t) \leq \sqrt{\frac{2\bar{h}_2 \varepsilon_0}{d_0}}, \quad (40)$$

$$e_3^2 \leq 2V_4(t), \lim_{t \rightarrow \infty} e_3(t) \leq \sqrt{\frac{2\varepsilon_0}{d_0}}, \quad (41)$$

$$e_4^2 \leq 2(\bar{h}_{41} + \bar{h}_{42}) V_4(t), \lim_{t \rightarrow \infty} e_4(t) \leq \sqrt{\frac{2(\bar{h}_{41} + \bar{h}_{42}) \varepsilon_0}{d_0}}. \quad (42)$$

Thus, the error variable $e = (e_1, e_2, e_3, e_4)$ is bounded. In addition, in the condition of adjustable parameters k_1, k_2, k_3, k_4 have been chosen, we can adjust ε_2 and ε_4 to ensure that ε_0 is small sufficiently, which can ensure that the error variable $e = (e_1, e_2, e_3, e_4)$ converges to a sufficient small neighborhood of the origin according to the exponential law.

4. Simulation analysis

The system physical parameters are as follows:

$$\begin{aligned} J_1 = J_2 = & 0.185 \text{ kg} \cdot \text{m}^2, b_1 = b_2 = 1.2 \text{ N} \cdot \text{m} \cdot \text{s/rad}, \\ J_m = & 0.028 \text{ kg} \cdot \text{m}^2, \alpha = 0.5 \text{ rad}, b_m = 1.3 \text{ N} \cdot \text{m} \cdot \text{s/rad}, \\ k = & [560 + 2 \sin(\pi t)] \text{ N} \cdot \text{m/rad}, w = 50 \text{ N} \cdot \text{m}, \\ c = & [0.15 + 0.01 \sin(\pi t)] \text{ N} \cdot \text{m/rad}, b_1 = b_2 \\ = & 1.2 \text{ N} \cdot \text{m} \cdot \text{s/rad}. \end{aligned}$$

The system initial conditions are as follows:

$$x_1 = 0, x_2 = 0, x_{3i} = 0, x_{4i} = 0, u(0) = 0.$$

The initial estimates of the system parameters are chosen as follows:

$$\hat{\vartheta}_2(0) = [8.2 \quad 0.0064 \quad 0.000749]^T,$$

$$\hat{\vartheta}_4(0) = [-0.039 \quad 486 \quad 52.65 \quad 55.76 \quad 0.42 \quad 0.48 \quad 0.034]^T.$$

The system simulation is completed in the MATLAB6.5 (MathWorks, USA) environment, and we choose the design parameters as follows:

$$k_1 = 32, k_2 = 12, k_3 = 9, k_4 = 4, \varepsilon_2 = 0.08, \varepsilon_4 = 25,$$

$$\Gamma_2 = \text{diag}[1, 1, 1], \Gamma_4 = \text{diag}[1, 1, 1, 1].$$

Figures 3 and 4 show the response curves of the system tracking triangular wave using mere backstepping control (MBC) and combination control of a projection algorithm and backstepping control (PABC), respectively. Figures 5 and 6 show the error curves of the system tracking triangular wave using MBC and PABC, respectively. Figures 7 and 8 show the response curves of the system tracking sawtooth wave using MBC and PABC, respectively. Figures 9 and 10 show the error curves of the system tracking sawtooth wave using MBC and PABC, respectively. Figures 11 and 12 show the response curves of the system tracking random signal using MBC and PABC, respectively. Figures 13 and 14 show the error curves of the system tracking random signal using MBC and PABC, respectively. From Figures 3–10, we can see that PABC has not only better dynamic tracking performance, but also smaller tracking error than MBC. From Figures 11–14, it is obvious that PABC not only has smaller tracking error, but also stronger adaptability than MBC.

5. Experimental tests

The block diagram of the dual-motor driving servo system is shown in Figure 15 and the experimental system is shown in Figure 16 [17]. The following graphics are the comparison charts of MBC and PABC.

Figure 3. Tracking triangular wave using MBC.

Figure 4. Tracking triangular wave using PABC.

Figure 5. Error of tracking triangular wave using MBC.

Figure 6. Error of tracking triangular wave using PABC.

Figure 7. Tracking sawtooth wave using MBC.

Figure 8. Tracking sawtooth wave using PABC.

Figure 12. Tracking random signal using PABC.

Figure 9. Error of tracking sawtooth wave using MBC.

Figure 13. Error of tracking random signal using MBC.

Figure 10. Error of tracking sawtooth wave using PABC.

Figure 11. Tracking random signal using MBC.

5.1 System hardware implementation

The dual-motor driving servo system consists of a computer control system, inverter, AC motor, gear drive chain, and digital signal processing (DSP) unit.

- 1) Computer control system. The computer control system completes signal sampling, system monitoring, and control quantity acquisition, and mainly includes a data sampling board, electric anti-backlash board, shaft encoder board, etc.

In this system, the PC/104 computer uses a SCM/SPT4 module (SBS Science & Technology Co., Ltd., China). The data sampling data uses a SysExpanModule/ADT700 type (SBS Science & Technology Co., Ltd.).

The electric anti-backlash board is custom built by us, and its main function is to generate the bias torque and speed-error negative feedback signal. At the same time, in order to solve the problem of system electromagnetic compatibility, the interface board also has a photoelectric isolation circuit. The board has a certain generality and can be applied to other systems.

The shaft encoder board uses an AD2S80AUD chip (AD Corp., USA) as well as a shaft encoder to convert the position of the system output axis to 16-bit natural binary code; the shaft encoder then sends it to the PC/104 computer to realize position feedback.

Figure 14. Error of tracking random signal using PABC.

Figure 15. Block diagram of dual-motor driving servo system.

- 2) Inverter and AC motor. The Inverter uses a CD/S series CR06250 driver (Kollmorgen, Inc., USA). The motor is a Kollmorgen M-405-C type.
- 3) Gear drive chain. The rotating transformer and the load gear are coaxially connected, and they are combined with the shaft encoder board to form the angular position feedback module. A 110XFSW008 rotating transformer was used.
- 4) DSP unit. A TMS320F2812 32-bit fixed-point DSP chip (Texas Instruments Corp., USA) was used for digital signal processing. The TMS320LF2812 chip formed a controller area network (CAN) interface card and CAN intelligent node, and the eCAN module of the TMS320LF2812 chip was used to connect the CAN bus driver, so that the node can be connected arbitrarily to any network with a CAN bus.

5.2 System software implementation

The system software framework is shown in Figure 17, mainly composed of a main program, interrupt handling routine, and algorithm subroutine. The interrupt handling routine contains a timer interrupt, serial port interrupt, CAN receive interrupt, and power protect interrupt. The algorithm subroutine contains a control algorithm and angular displacement planning. All the program code is

Figure 16. Experimental system.

Figure 17. System software framework.

implemented in the C language in the DSP development environment CCS3.3; an output file is then generated, which was burned to 2812 Flash for debugging.

5.3 System experiment

Figures 18 and 19 show the system response curves to a step signal using MBC and PABC, respectively. Figures 20 and 21 show the system response curves to a sine signal $y_d = \sin(0.4\pi t)$ using MBC and PABC, respectively. Simultaneously, we applied a white noise signal to the system output after 3 s in each experimental test. From Figures 18 and 19, we can see that the system response has larger overshoot and larger tracking error using MBC. However, the system response has almost no overshoot when using PABC, and the tracking error is also smaller than that of MBC. After 3 s, it is obvious that the system

Figure 18. System response to step signal using MBC.

Figure 19. System response to step signal using PABC.

Figure 20. System response to sine signal using MBC.

Figure 21. System response to sine signal using PABC.

response fluctuation amplitude of PABC is obviously smaller than that of MBC. From Figures 20 and 21, we can see that the tracking error and response fluctuation amplitude after 3 s of PABC are also smaller than those of MBC.

From the experimental tests, we can see that PABC has smaller tracking error and higher anti-interference ability than those of MBC.

6. Conclusions

Aimed at solving the problem of the parameter uncertainty of dual-motor driving servo systems, we proposed, for the first time, to the best of our knowledge, a new adaptive

control algorithm combining a projection algorithm with backstepping control. Simulation results show that the system can not only track the desired output asymptotically stably by using the proposed control algorithm, but that it also has higher tracking accuracy and stronger adaptability than that of mere backstepping control. Experimental test results show that the proposed control algorithm has better adaptability and stronger robustness than those of mere backstepping control, validating the effectiveness of the proposed control algorithm. However, even though friction nonlinearity also exists in dual-motor driving servo systems, we did not consider the coexistence of friction and backlash nonlinearity here, but have deferred it to planned future work.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant No. 61074023), the Natural Science Foundation in Anhui Province of China (Grant No. 1508085MF130), the Natural Science Research Key Project of Universities in Anhui Province of China (Grant No. KJ2015A297), the Engineering Technology Research Center of Optoelectronic Appliance of Anhui Province, China, and the Sichuan Institute of Aerospace System Engineering of Sichuan Province, China.

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