Output-only damage localization technique using time series model

K LAKSHMI* and A RAMA MOHAN RAO

Academy of Scientific and Innovative Research, CSIR-Structural Engineering Research Centre,
Chennai 600113, India
e-mail: lakshmik@serc.res.in; arm@serc.res.in

MS received 2 February 2017; revised 11 August 2017; accepted 19 October 2017; published online 24 July 2018

Abstract. In this paper, we present a technique to detect the time instant and location of damage in civil structures using scalar time series models, by handling operational variability and measurement noise. The scalar Autoregressive (AR) and Autoregressive with exogenous inputs (ARX) models are used to obtain the time instant of damage and its spatial location. The spatial damage feature to locate the damage is obtained using a metric constructed from the probability density values of the prediction errors of AR–ARX model. The proposed method does not resort to any computationally expensive vector time series models to locate the damage and so highly preferable in smart wireless online continuous SHM schemes. Numerical simulation studies are carried out by using a simply supported beam model. The results of the studies indicate that the proposed technique is capable of identifying both the time instant and location of damage accurately using the proposed PDF based damage index. In order to validate the proposed technique with experimental results, the time-history data from the three-story bookshelf benchmark structure of EI-LANL is used. Finally, the laboratory experimental studies carried out on an RCC simply supported beam with inflicted damage are also presented. The experimental studies clearly indicate the effectiveness of the proposed damage index to detect the location of damage, by handling operational variability and measurement noise.

Keywords. Structural health monitoring; time series analysis; damage diagnosis; benchmark problem; operational variability; measurement noise.

1. Introduction

Structural health monitoring (SHM) which is the diagnosis of in-service structures on a continuous real-time basis, is of primary importance in aerospace, civil and mechanical engineering. SHM is essential for the optimal use of a structure, a reduced downtime and the avoidance of catastrophic failures and has several economic benefits. To achieve this, the measured data from the structure are converted, by a process of feature extraction, into a representation where variations due to damage are highlighted.

Vibration-based methods have been widely used for identification of various types of damages for several real and laboratory structures. Especially, vibration-based methods using modal information are popularly used for damage diagnostics [1–6]. However, methods using modal parameters like frequency, mode shapes, modal curvatures and strain energy, etc., have some limitations in general: (i) it is difficult to excite higher modes which are more sensitive to smaller damage, and (ii) mode shapes are also very sensitive to operational and environmental variability and measurement noise, (iii) do not work on complex structures.

In particular, the accuracy of mode shape curvature method is subject to numerical estimation difficulties resulting from the need for differentiation. Moreover, it is error sensitive because a small noise in the mode shapes will lead to a total discrepancy. The methods based on modal strain energy cannot be applied to two and three-dimensional frame type structures [7]. Also, the method cannot allow an efficient and robust detection of small damage due to measurement noise. According to Huth *et al* [8], the damage detection based on the modal flexibility matrix method poses problem in identifying damage at its earlier stage. Also, the main drawback is a change in dynamic flexibility shows less efficiency in the case of more complex and/or simultaneous damages when damages are close to joints or in the case of low quality of experimental data. The methods based on Frequency Response Functions depend on the principle that the structural properties and the variations in the mass or stiffness can be identified from the measured FRF [9]. The drawbacks of FRF based methods are: (i) There is a significant change in the FRFs due to the truncation effect of signal when there is no damage and masked the effects due to damage. (ii) The factors like time window size and decay rate are also to be considered. (iii) FRF-based methods can detect damage in

*For correspondence

simple composite structures but the potentially important information about damage type, size, location, and orientation are difficult to obtain as several combinations of these variables can yield identical response signatures. (iv) The presence of noise could corrupt the resolution in numerical studies. In order to detect, localize, and quantify damage in structures, many damage detection schemes have been utilized the neural networks [6, 10, 11]. The main drawback of such ANN is the requirement of excessive computational time and computer memory to train the ANN model, which increases dramatically with the increase in the number of structural degrees of freedom.

The wavelet transform [12, 13] and Hilbert-Huang transform (HHT) [14, 15] for system identification and damage detection based on signal processing techniques fall under the category of Time–frequency analysis [16]. The main drawback of these techniques is that the parameters of these algorithms need to be adjusted wisely for better performances. Their effectiveness also depends on the sensitivity to the measurement noise. The advancements in wireless smart sensors have led to the process of active diagnosis, where the majority of the damage diagnostic methods require spatial correlation of data to extract meaningful damage features. The spatial correlation generally involves enormous data communication among the sensors and also to the base station. The effective damage diagnostic techniques should be able to minimize the data communication and at the same time reveal the state of the structure with minimum computations.

On the other hand, among the vibration-based methods, time series analysis has been widely used in different fields including SHM. Time series-based SHM methods use and represent the output-only, stochastic and continuous acceleration time history signals, obtained from the sensors mounted on the structure, as outputs of a particular time series model. Time series models are based on the philosophy that, for a linear system, it is possible to predict the measured response at any time, based on the values of the linearly combined previous time and the random errors. In the context of vibration-based SHM, when the vibration response of the structure is fitted to the time series models, reflect the dynamic characteristics of the structure through its coefficients and residual errors. By monitoring the departure of these properties of the original model, the damage in the structure can be identified. Time series analysis based SHM provides unique advantage [17] over the other vibration-based techniques such as it does not require physics based or analytical, such as Finite Element (FE) models or complete structural models. Apart from the above, time series analysis has the advantages like, (i) uncertainties are inherently accounted as time series analysis is a data-based method, (ii) uses only the output vibration data from the structure and (iii) highly adaptability for wireless smart sensor networks.

During the past two decades, the attraction of the time series models for system identification and damage

detection techniques has tremendously increased. More details of the time series models can be obtained from Box *et al* [18] and Fuller [19]. Pandit and Wu [20] provided the application of these time series models for the problems of system identification and system analysis. Many researchers used univariate time series models to monitor structural condition and detect damage, using autoregressive (AR) model [21–23], autoregressive with exogenous input (ARX) model [24, 25], AR–ARX model [26–29], auto-regressive moving average (ARMA) model [30–33], autoregressive integrated moving average (ARIMA) model [34] and autoregressive moving average with exogenous input (ARMAX) model [35–37].

The majority of the damage detection methods based on time series analysis used the damage indices formulated from the coefficients/ the prediction errors of the times series models. Among them, the ratio of the standard deviation of prediction errors, D_f , obtained from a time series model, fitted to the data from the undamaged and damaged state of the structure is found to be popular. Even though damage indices of the univariate time series models work on individual sensor information and are undoubtedly a good candidate to specify the state of the structure, it has been shown that they have failed to provide the exact spatial location of damage in few applications [29, 38]. Alternatively, researchers have used, Vector AR (ARV) [20, 39] model which uses the spatial information to spatially locate the damage. While AR and ARX models use the time history data obtained from individual sensors, ARV model requires the spatially correlated time history data from all sensors for construction. This will be particularly disadvantageous on smart wireless sensor networks during continuous online monitoring, where the power consumption of the wireless sensors depends mainly on the weight and the frequency of the data packet transfers among the sensor nodes. The computations involved in ARV are obviously high as all the sensor data participate in the computations which result in matrices. The computational cost is bound to increase with the increase in a number of sensors. Apart from this, identifying multiple spatial damage locations are difficult using vector AR models. Further, ARV model, if directly used in the first phase (damage detection) poses the difficulty in locating the damage, as spikes might appear in the damage indices computed even from healthy current data set due to operational variability. Hence it is difficult to conclusively state that the spikes in the damage indices are the exact damage locations unless it is known *a priori* that the current data being processed through ARV model is from the damaged state of the structure.

Zhang [27] used a damage index called damage possibility (PD) for each node to identify the presence and location of damage. The percentage of damage possibility PD is found to show the higher values even for other nodes apart from the original node at which damage is simulated. Also, there exists a challenge in fixing a percentage value of

the damage index, PD, for the division between healthy and damaged conditions. Lakshmi and Rao [40] devised a damage index using the peaks of the Probability Density Function (PDF) of the prediction of AR–ARX model and they showed that the PDF constructed from the dataset collected after damage drifts away from its counterpart before the damage occurred. The proposed data compression using the Principal Component Analysis (PCA) allows monitoring the damage using single pattern vector, which is fitted to AR–ARX model to get the prediction errors. Even though this damage index is efficient in identifying the presence of damage, it cannot identify the location of damage as the signals from all the nodes are compressed into a single vector using PCA.

Therefore, considering the drawbacks of the various damage detection techniques and damage indices discussed above to locate damage, in this paper, a simple and statistically devised damage index is presented to detect and locate the damage. The damage index, to detect the spatial location of damage, is devised based on the peak and the spread of the probability distribution function (PDF) [41], constructed from the prediction errors of AR–ARX model. The evaluation of the damage index is simple and is effectively computed for every sensor node. Numerical and experimental simulations have been carried out to demonstrate the effectiveness of the damage index presented in this paper to locate the damage spatially in the structure.

2. Damage detection methodology

The damage identification technique discussed in this section requires only the acceleration data from the structure which is best fitted using autoregressive time series models to obtain their prediction errors. Initially, AR–ARX models, are constructed from the current data and the corresponding matched reference data to identify the presence of damage. A statistical measure of the prediction errors of AR–ARX model is used as the damage indicator. Once the presence of damage is identified, the probability density functions of the prediction errors of the test data are employed to extract the spatial damage feature. Since the nature of excitation of a structure is not available in most of the cases, the acceleration data is first fed into AR model and the prediction errors from the AR model is used as the input to the ARX model for further prediction. The procedure is discussed in detail as follows. For any real structural system, when the structural health monitoring system is installed, the present structural condition becomes the known structural condition and based on that the pool of baseline data is formed. The new data subsequently recorded, becomes the current data and subjected to diagnosis.

1. A pool of baseline data is formed incorporating the operational/environmental variability and measurement noise. Two-thirds of the baseline data is considered as

the reference data and the remaining one-third is taken as healthy data. The acceleration time history data of each node in the reference data is partitioned into several subsets. These subsets are generated essentially to have a good amount of data samples obtained under varying unknown loading conditions and also to include the effect of the statistical uncertainty while estimating the time series models.

2. Similarly, the current data containing acceleration time history data from the unknown state of the structure (i.e., healthy/damaged state) is formed. The subsets are generated by partitioning the current data. All the baseline and current data generated from all sensors placed on the structure will be standardized as in Eq. (1), prior to generation of smaller subsets of data.

$$\bar{y} = \frac{y - \mu_y}{\sigma_y} \quad (1)$$

where y is the acceleration time history signal, $\bar{y}$ is the standardized signal, μ_y and σ_y are the mean and standard deviation of y respectively. Since most of the damage detection methods use structural responses directly, variations in the operational/environmental conditions may alter the changes in the dynamic response caused by structural damage and significantly affect the damage diagnostic schemes. Therefore to avoid false positive damage indications, it becomes mandatory to normalize the data from different operational conditions. When environmental cycles influence the measured data, a temporal normalization scheme may be employed. The normalization strategy applied in the present work assumes that the measures of the varying operational/environmental conditions are unavailable. The normalization procedure outlined here is based on the assumption that a considerably large pool of data is available from a known structural condition (i.e., Healthy) of the system but with various unknown operational/environmental conditions. In other words, baseline data is readily available. The effectiveness of the normalization procedure is directly dependent on this data pool. This pool should consist of data measured in as many varying operational conditions as possible.

3. For each time series $x(t)$ in the reference database, an AR model with p AR terms is constructed. An AR(p) model [18] can be written as

$$x(t) = \sum_{j=1}^p \phi_j x(t-j) + e_x(t) \quad (2)$$

where p is the AR order, ϕ is the AR coefficients of reference subset and e_x is the prediction error. The partial autocorrelation analysis [18], is performed to fix the AR order. The AR(p) model has only one term for the residual $e_x(t)$ which is calculated at the current time step. In general, by modeling the output of the system through

AR models, it is possible to obtain the eigenvalues of the mechanical system in the discrete time domain. Then, mapping these eigenvalues to those in the continuous time domain, the natural frequencies of the system are finally obtained. AR models are only-poles functions. The autoregressive parameters are computed by using a least square minimisation and the autoregressive model is just an approximation of the response of a system subjected to a random excitation.

4. The procedure of fitting AR model is repeated for the entire signals in the reference database. Also, AR model is fitted to a new current data subset, $y(t)$, which is equal to the length of the signal, $x(t)$, obtained from the unknown state of the structure.

$$y(t) = \sum_{j=1}^p \phi_{yj} y(t-j) + e_y(t) \quad (3)$$

5. For each subset of the current data obtained from the unknown state of structure (can be either healthy or damaged) a subset is matched with a reference data subset, obtained with the known state of the structure (i.e. healthy). The reference data subset $x(t)$ 'matching' to the new current data subset $y(t)$ is defined as the one that minimizes the following difference of AR coefficients:

$$\text{Difference} = \sum_{j=1}^p (\phi_{xj} - \phi_{yj})^2 \quad (4)$$

6. Based on the above Euclidean distance measure, if the new current data subset is obtained when the structure is healthy and from an operational/environmental condition similar to one of the reference signal segments, then the dynamic characteristics (i.e., the AR coefficients) of the new unknown current data subset should be similar or 'matched' to that of the reference data subset. Therefore, if the new current data is matched to the reference signal, a time prediction model constructed from that particular reference data block should be able to appropriately predict the new current data subset. However, the prediction model estimated from even the 'closest' data subset in the reference database would not reproduce the new signal well, if it was obtained under a structural condition different from that of reference data subsets and the prediction errors are bound to be higher. With an assumption that the error ($e_x(t)$), between the measurement and the prediction obtained by AR model, is mainly caused by unknown external input, an ARX model is used to reconstruct the input-output relationship between $e_x(t)$ and $x(t)$ of the reference data subset which is closely matched with the current data subset.

$$x(t) = \sum_{i=1}^a \alpha_i x(t-i) + \sum_{j=1}^b \beta_j e_x(t-j) + \varepsilon_x(t) \quad (5)$$

where $\varepsilon_x(t)$ is the residual error after fitting the ARX(a,b) model to the $e_x(t)$ and $x(t)$ pair. The ARX orders a and b are chosen, keeping the sum of a and b smaller than AR order, p , such that $(a + b \leq p)$ as suggested by Ljung [42].

7. Once the parameters are set, it is investigated how accurately the ARX(a, b) model estimated in Eq. (5) reproduces the input/output relationship of $e_y(t)$ and $y(t)$ of the new current data subset.

$$\varepsilon_y(t) = y(t) - \sum_{i=1}^a \alpha_i y(t-i) - \sum_{j=1}^b \beta_j e_y(t-j) \quad (6)$$

where $e_y(t)$ is considered to be an approximation of the system input estimated from Eq. (3), α_i and β_j coefficients are associated with $x(t)$ obtained from Eq. (5). Thus, if the ARX model obtained from $x(t)$ and $e_x(t)$ of the matched reference data subset pair is not a good representation of the new current data subset $y(t)$ and $e_y(t)$, there is likely to be a significant change in the standard deviation of the residual error, $\varepsilon_y(t)$, compared to that of $\varepsilon_x(t)$. In view of this, the ratio of standard deviations of similar signals $\sigma(\varepsilon_y)/\sigma(\varepsilon_x)$ is considered as damage index.

$$D_f = \frac{\sigma(\varepsilon_y)}{\sigma(\varepsilon_x)}. \quad (7)$$

If the structure is free from damage, the value of D_f is expected to be close to 1, since the magnitudes of $e_x(t)$ and $e_y(t)$ will be approximately same. The presence of damage in the structure will alter the situation quite significantly and the value D_f will exhibit higher values as the variance of ε_y will be much higher than ε_x . This forms the first phase of the proposed algorithm where the presence of damage is indicated through the damage feature, D_f . Since the damage index D_f is shown to be capable only of detecting the presence of damage [29, 37], a simple yet effective damage index is presented in the next section to detect the spatial location of the damage.

2.1 Spatial damage location

The damage localization index presented in this paper is devised using the values at the peak and at 5% confidence interval of the Probability Density Function (PDF), obtained from the residuals of the ARX model, ε_y , based on the following.

- The residual errors of ARX model corresponding to the undamaged state of the structure are obtained as a difference between the actual undamaged vibration response and the undamaged predicted response using ARX model (Eq. 8).

$$\varepsilon_y^u(t) = y^u(t) - \hat{y}^u(t) \quad (8)$$

where $y^u(t)$ and $\hat{y}^u(t)$ denote the measured response and the predicted response of the structure in pristine condition.

- It is obvious that the residual errors corresponding to the undamaged state of the structure will not be significant (rather negligible) by virtue of its evaluation. Similarly, the residual error corresponding to the damaged structural vibration response at any spatial location on the structure is evaluated as the difference between the actual damaged response and the predicted damaged response using ARX model (Eq. 9)

$$\varepsilon_y^d(t) = y^d(t) - \hat{y}^d(t) \quad (9)$$

where $y^d(t)$ and $\hat{y}^d(t)$ denote the measured response and the predicted response of the structure with a change in the state of its pristine condition.

- As the proposed technique considers only the ambient vibrations on the structure, it is assumed to be excited with a band-limited white noise and therefore the structural response would follow a Gaussian distribution. So, the residuals of the ARX model constructed from these vibration responses of the structure, which is Gaussian in nature, would also follow a Gaussian distribution. Therefore, the residual errors can be fitted to a normal probability density function using Eq. (10) which should follow a bell-shaped curve, if truly Gaussian with mean μ and standard deviation σ .

$$f(\varepsilon_y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{(\varepsilon_y - \mu)^2}{2\sigma^2}\right)} \quad (10)$$

- The PDF curves constructed using the residual errors ($\varepsilon_y^u(t)$ and $\varepsilon_y^d(t)$) of AR-ARX model, fitted to the data, before and after the damage occurred, near a typical node are shown in figure 1(a). It can be verified from the figure 1(a) that the residual errors of the ARX

model, definitely follow a Gaussian distribution, irrespective of the state of the structure, during the collection of the acceleration data from it.

- Also, by comparing the two PDF curves of figure 1(a), the following valid observations are made:

- a) The PDF curve related to the prediction errors of the data after the damage is found to be more dispersed than that of the data before damage.
- b) Similarly, the peak value of the PDF curve related to data with structural damage is also much lesser in comparison with the healthy PDF curve.

Therefore, it can be observed clearly from figure 1(a), that the presence of structural damage, gives rise to a PDF with a wider spread and shorter peak than that of the PDF of the healthy condition. Based on this strong observation, the differences in the two PDF curves are exploited to arrive at a meaningful damage index [41]. Considering a significance level of ρ , a damage index, ‘DI_PDF’ can be expressed as

$$DI_PDF = \frac{f_{c(1-\rho/2)} - f_{c(\rho/2)}}{f_{c_max}} \quad (11)$$

where, f_{c_max} is the peak probability density value of a curve, $f_{c(1-\rho/2)}$ and $f_{c(\rho/2)}$ are the probability density values at the levels $(1 - \rho/2)$ and $(\rho/2)$, respectively. The elements of the damage index, DI_PDF are more clearly shown in figure 1(b).

Once the damage is alarmed using D_f of AR-ARX model, obtained from Eq. (7), the prediction errors of ARX model ε_y of that particular current data subset are considered for the construction of PDF. Therefore, a PDF is constructed for the prediction errors of each sensor node on the structure. The DI_PDF is evaluated for all the sensor nodes on the structure independently using Eq. (11) and the values are normalized. The highest magnitudes among the sensor nodes can be viewed as the indicators of the spatial locations of damage, near those nodes, on the structure.

Figure 1. (a) PDF curves of residual errors of ARX model related to the data obtained before damage, ε_y^u and after damage, ε_y^d and (b) elements of damage index, DI_PDF.

Figure 2. Simply supported beam.

The advantages of the proposed damage index are

1. It works on the individual sensor data and does not require the data from the adjacent sensors. This is highly appreciable in the framework of online continuous health monitoring using wireless smart sensor network platforms.
2. The proposed damage index does not have any parameter settings except the significance level, p . In this work, p is assumed as 0.05 with 95% confidence interval, to analyse the dispersion of the PDFs.
3. The spatial locations of damage are indicated clearly without any ambiguity, through the increased magnitude of the DI_PDF, as the peak probability density value

reduces and the density values at a defined confidence interval increase for the PDF related to damage data.

4. The evaluations involved are simpler and so can be faster during online monitoring.
5. There is no need of the healthy data at this stage because, already the presence of damage is ascertained by the damage index, D_f from Eq. (7). Therefore, the DI_PDF is calculated using the PDF curve related only to the current data which alarmed the damage.

The efficiency of the damage index, DI_PDF, is further validated using the numerical and experimental studies presented in the next section.

It should be mentioned here that the proposed damage index does not quantify the extent of the damage. However, once the location of the damage is known, the extent of damage can be evaluated using the inverse algorithms. One of the popular ways of solving the problem associated with quantification of the extent of damage is formulating the associated inverse problem into an optimization algorithm by devising a suitable objective function. We can solve the resulting optimization problem using meta-heuristic algorithms [43, 44].

Figure 3. Damage indices, D_f , extracted using AR–ARX models: (a) healthy data subset and 10th subset with SNR as 40, (b) healthy data subset and 21st subset of Scenario-1 (25% stiffness reduction at element no 4 after 10 s) with SNR as 40, (c) healthy data subset and 23rd subset of Scenario-2 (20% stiffness reduction at element no 4 after 11 s) with SNR as 40 and (d) healthy data subset and 25th subset of Scenario-3 (12% stiffness reduction at element no 4 after 12 s) with SNR as 40.

Figure 4. Normalised damage index, DI_PDF, indicating damage at 4th element of the SS beam. (a) PDFs of a healthy and damage subset, (b) 25% Damage at element 4, (c) 20% Damage at element 4, (d) 12% Damage at element 4.

3. Numerical studies

The performances of the proposed damage diagnostic technique using AR–ARX time series models to identify and locate the spatial damage using DI_PDF are evaluated using the results of a simulated example, a standard benchmark problem and an experimental investigation in this section.

3.1 Simply supported beam

A simply supported beam girder as shown in figure 2 with a span of 6 m is considered as the simplest example to verify the abilities of the proposed damage detection technique. For the purpose of numerical simulation studies, the beam is discretized into 20 elements with 21 nodes, including the support nodes, as shown in figure 2. The material and geometrical properties are also shown in the figure 2. The beam is excited using a random dynamic loading which is stochastic in nature at the middle and at the one-third portion of the either ends. The acceleration time history response is computed on every node using finite element analysis with Newmark's time marching scheme with random loads and environmental conditions. The sampling rate

is considered as 2000 samples per second. This simulates the baseline data collected on a bridge girder on time instant with varied operational conditions.

One of the main issues related to structural damage diagnostic techniques, when applied to real situations, is their sensitivity to noise. In view of this, it was decided to add white Gaussian noise to the acceleration time history response generated by the finite element code. The white Gaussian noise is added in the form of SNR (signal-to-noise ratio) to the acceleration time history before they are processed. SNR defines the amplitude of the noise with respect to that of the clean signal. When the noise level is given by a particular value of SNR, it means that a noisy signal with such an SNR has been added to the time series of each node. Moreover, the noisy sequences affecting different

Table 1. Optimal sensor locations for simply supported beam obtained using Efi.

Number of sensors	Sensor locations
11	[1,3,5,7,8,9,10,11,13,15,17,19]
9	[1,3,5,7,10,13,15,17,19]
7	[3,5,7,10,13,15,17]

Figure 5. Damage diagnosis of S.S. beam with 20% damage at element no. 4, using reduced measurements: (a) idealisation of SS beam along with the complete 19 sensors, (b) normalised damage index, DI_PDF, with 11 optimally placed sensors, (c) normalised damage index, DI_PDF, with 9 optimally placed sensors and (d) normalised damage index, DI_PDF, with 7 optimally placed sensors. *Note:* X-axis of (b)–(d) indicate the optimal spatial sensor locations and the values within the brackets indicate the original spatial sensor locations with full instrumentation as shown in (a).

nodes are uncorrelated, in this way severe experimental conditions are simulated.

Two-thirds of the baseline data generated with varied operational conditions is partitioned into 80 subsets of 1000 samples and becomes the reference data. Varying levels of noise (with SNR values ranging between 20 and 40) is added to the reference data of each subset to exactly simulate the measurement noise and other uncertainties in measurement. Similarly, the healthy data generated is partitioned into 60 subsets with varied SNR and random loads.

These subsets are used for false negative indication and also as a part of current data.

Similarly, data subsets, simulating the unknown conditions of the structure are generated with varying load levels, varied temperatures, and noise levels. This data is known as current data and it consists of both data of a healthy structure as well as data of a structure with simulated damage in some of the elements. Hypothetical damage scenarios are assumed by means of reduction of stiffness of some elements when generating the current data.

Figure 6. Three-storey frame structure.

A current data of 15 s, is simulated with varied load and noise levels, such that the damage occurs after 10 s. Damage (Scenario-1) is simulated by reducing the stiffness

of element 4 by 25%, varying both the operational loads and SNR. The current data is divided into 30 subsets with 1000 samples each. Out of 30 current subsets, last 10 subsets are considered to be obtained from the damaged condition of the structure.

AR-ARX model is employed on the current data subsets to identify the exact time instant of damage. The damage indices, i.e., D_f values are evaluated as explained in the earlier section. Figure 3(a) shows the details of the D_f values computed using one of the healthy data subset and one of the first 20 current data subsets (i.e., 10th subset), which correspond to the healthy condition of the structure. It can be seen from the figure that the damage indices of the current subset are in line with the damage indices of a healthy baseline subset and less than 1.0, ensuring the condition of normalcy in the structure. When the 21st subset of the current data is tested, the damage indices have shown an overall increase in their magnitude, indicating the change of dynamics, i.e. the time instant of damage. This is shown in figure 3(b) where the greater magnitudes of the current subset clearly indicate the presence of damage. At this event, the damage is alarmed and the time instant of damage is calculated from the index of the subset being investigated. This indicates that the AR-ARX model and the damage index D_f is capable of identifying the time instant of damage.

Similarly, in order to test the sensitivity of the proposed technique to alarm various levels of damage, two more damage scenarios with lesser damage intensities of Scenario-1 are simulated. Scenario-2 and Scenario-3, with 20% and 12% reduction of stiffness in element 4, after 11 s (i.e., 23rd subset), and 12 s (i.e., 25th subset), respectively, with

Figure 7. Bookshelf Structure: (a) test structure and its dimensions and (b) Floor layout as viewed from the top.

Table 2. Details of different damage scenarios related to bookshelf structure.

Damage scenario	Description of damage
1	Bolts removed between the bracket and the plate at location 1C
2	Bracket completely removed at location 1C
3	Bolts removed between the bracket and the plate at location 3A
4	Bracket completely removed at location 3A

SNR 40, are simulated. These datasets are divided into subsets as discussed above. The damage indices for this current data obtained from AR-ARX model prediction errors are shown in figure 3(c) and (d), respectively. It can be seen from figure 3 that the technique is able to clearly show the instant of damage in terms of subset number. It can also be seen that the damage indices obtained from the proposed technique provide only the qualitative judgement of the damage and do not give the quantitative assessment of the location/intensity of damage. Thus, the presence of damage is confirmed and the identification of the spatial location of the damage is invoked.

The PDFs corresponding to the prediction errors of AR-ARX model for a healthy and damage subsets (12% stiffness reduction) of an arbitrary sensor are shown in figure 4(a). Figure 4(a) clearly shows that the PDFs related to the ARX residuals of healthy data and the damage data are completely distinct with respect to the peak values and the values at 5% confidence level. The normalised damage index DI_PDF, calculated for the three damage scenarios are shown in figure 4(b) and (d). From these figures, it is clear that the DI_PDF is a good indicator of the spatial location of damage from the prediction errors of AR-ARX model. The damage index, DI_PDF, is not only capable of handling the operational variability and measurement noise efficiently but also indicates the spatial locations of different intensities (25%, 20% and 12% reduction in stiffness factor) of damage robustly.

3.2 Damage detection using limited measurements

Practically, due to cost and accessibility issues, the number of sensors placed on the structures will be rather limited. It is usually recommended to place the sensors at a limited number of optimal locations in order to capture the important structural responses with minimum cost for

Figure 8. Results of DB0 scenarios of Bookshelf structure: (a) Damage index, D_f indicating the presence of damage, (b) normalised damage index, DI_PDF for scenario-1 and (c) normalised damage index, DI_PDF for scenario-3.

Figure 9. Results of DBB scenarios of Bookshelf structure : (a) damage index, D_f indicating the presence of damage, (b) normalised damage indices of scenario-2 and (c) normalised damage indices of scenario-4.

Figure 10. Schematic representation of beam with sensors, actuator and damage locations.

structural health monitoring applications. Several optimal sensor placement techniques have been proposed in the literature [45]. In this paper, a popular optimal sensor placement technique based on the Effective independence approach [46, 47] is employed. The acceleration time history responses from the final sensor set, obtained using Efi, are generated to validate the proposed damage localization technique.

The optimal sensor locations for 11, 9 and 7 number of sensors for the simply supported beam are shown in table 1.

With the limited measurements obtained at the optimal sensor locations, damage diagnosis has been carried out using the proposed technique with DI_PDF to locate the damage. The normalized DI_PDF values obtained for the damage scenario of 20% stiffness reduction in element number 4 are shown in figure 5, for varied number of optimal sensors. It can be observed from figure 5 that the DI_PDF of the nearest sensor from the damage location (element no. 4), indicates the higher magnitude, which reveals the spatial location of damage.

Figure 11. Experimental set-up of “Test setup-1”.

3.3 Experimental verification using Bookshelf benchmark structure

The second example to validate the efficiency of the proposed algorithm to locate the damage is the experimental data of the Bookshelf SHM benchmark structure tested by EI-LANL [48] and published in the public domain. This test bed is a structure of a three-story frame model as shown in figure 6. The structure is built of Unistrut columns and aluminium floor plates. The floors are 1.3 cm (0.5 inch)

Table 3. Details of the different scenarios of vibration acceleration measurements from RCC beam.

Sl.no.	Dataset	Scenario	Damage location
Baseline Data			
1	Healthy	Pristine condition of the beam	Not applicable
Damage scenarios			
2	S_Damage	Single damage	Element No: 6
3	M_Damage	Multiple damage	Element No: 6 and 11

thick aluminium plates with two-bolt connections to brackets on the Unistrut. The base is a 3.8 cm thick (1.5 inch) aluminium plate. Support brackets for the columns are bolted to this plate and hold the Unistrut columns. The details of these joints and the dimensions of the test structure are displayed in figure 7. The structure was instrumented with 24 piezoelectric accelerometers with 2 accelerometers at each joint. One accelerometer is attached to the plate and the other accelerometer is attached to the uni-strut column. A shaker is connected to the structure by a stringer and the translational and torsional motions are simulated. The input excitation via shaker is band limited white noise (0 to 200 Hz).

In each test case, three separate data sets were collected with the shaker input levels at 3, 5 or 7 volts. The bandwidth of the shaker and response are also varied from 800 Hz to 3200 Hz in each test case to determine in which frequency bandwidth the damage would be most sensitive. In the damaged cases, the bolts at the joint indicated were loosened and then tightened again to hand tight allowing the plate to move relative to the column. All input from the shaker to the base was random. Each time signal gathered consisted of 8192 points and was sampled at 1600 Hz. Twenty reference data and twelve healthy data from different peak amplitude excitation levels (2 v, 5 v, and 8 v) were considered and each dataset is divided into 8 subsets each with 1024 time steps.

3.3a Damage scenario: Four damage scenarios were considered for this structure and the details are given in table 2. Totally 24 damage datasets, 6 from each of the 4 scenarios shown in table 2 are considered. For each scenario, there are three excitation levels (2 v, 5 v, and 8 v) and are divided into 8 subsets with 1024 samples.

Two scenarios with brackets removed (DBB) and two cases with bolts removed (DB0) are investigated. The operational variability is included in the datasets in the form of input voltages (2 v, 5 v, and 8 v) for excitation. The details of the damage scenarios considered for the

Figure 12. Load and acceleration time history data obtained from sensor 9 of the healthy structure.

Figure 13. Experimental set-up of “Test setup-2”.

Figure 14. Damage at 1/3rd span of left support.

study is presented in table 2. These datasets are subjected to the proposed algorithm and the damage indices obtained based on D_f and DI_PDF for DB0 scenarios are shown in figure 8. Similarly, for DBB scenarios the observations and the damage indices are shown in figure 9. From the plots in the above-said figures, it is clearly seen that the proposed technique is robust in identifying the location of damage precisely from the data with the operational variability.

3.4 Experimental simply supported RCC beam

The experimental test structure considered is a simply supported RCC beam with dimensions: span length

Figure 15. Multiple damage scenario.

300 cm, Width 16.5 cm and Depth 20 cm as shown in figure 10. The bottom longitudinal reinforcement was 2# 16 ϕ and the upper was 2# 12 ϕ with 25# 6 ϕ stirrups. The

Figure 16. Normalised DI_PDF of the experimental RCC beam: (a) S_Damage, (b) M_Damage.

Table 4. Optimal sensor locations for RCC beam obtained using Efi.

Number of sensors	Sensor locations
8	[2,4,6,8,10,12,14,16]

first four natural frequencies of the beam are 28 Hz, 112 Hz, 252 Hz and 448 Hz.

The beam is instrumented with 16 Uni-axial micro-electromechanical systems (MEMS) DC accelerometers (1000 mV/g, $\pm 2g$, +6 to 30 VDC excitation - PCB piezotronics make) placed spatially at equal distances along the beam to obtain the acceleration time history data. The beam is excited using a modal shaker (DP-M301 - SignalForce make) of 200 N sine peak force capacity. The test was performed in the 0–1000 Hz frequency range using both harmonic as well as random excitations using the shaker DP-M301. The experimental set-up is named as “Test setup-1” and is shown in figure 11. The loading frequencies, as well as amplitude, is varied during each set of measurements to simulate operational variability. Initially, vibration responses (acceleration time history) of the beam are measured at all the 16 sensor locations for the undamaged conditions of the beam. This scenario is named as ‘Healthy’. Each signal is measured for 19 s and is sampled at 3000 HZ so that the frequencies of the beam can be uncovered, in 0–1500 Hz frequency range.

Different scenarios simulated and their descriptions are shown in table 3. The loading and the acceleration time history obtained from sensor 10 for a typical set of applied random load for 1.5 s are shown in figure 12.

3.4a Damage scenario: The damage is inflicted at approximately one-third span from the left support of the beam (i.e., in element no. 6). For this purpose, the beam is mounted on a loading frame and in order to inflict cracks at 1/3 span of the beam, the span of the beam is reduced to 2/3rd of the total length of the beam by placing supports on

the loading frame accordingly. A hydraulic jack is used to apply static load. The complete experimental arrangement (Test setup-2) with a hydraulic jack, load cell, dial gauge to measure displacements at mid span (i.e., at the 1/3rd span of the overall beam) are shown in figure 13.

The static load is applied in small increments till the cracks are visible. The beam is unmounted from the loading frame at this stage and shifted back at the experimental set-up shown in figure 11. At this stage, the beam is having cracks developed approximately at 1/3rd span from the left side of the beam and is shown in figure 14. The experiment with dynamic excitation force (as discussed earlier for healthy beam) is repeated and the acceleration data are recorded as referred to as ‘S_Damage’.

Similarly, in order to inflict multiple damages on the test specimen, the damage is inflicted using the similar procedure at 1/3rd span from right support (element No: 12) as shown in figure 15 and the time history measurements recorded are referred to as ‘M_Damage’.

The proposed damage detection technique using D_f and DI_PDF for detecting and locating the damage respectively is validated using the acceleration time history data of 16 sensors obtained from the healthy condition of the beam and after inducing damage through static loading. The datasets considered to validate the technique based on AR-ARX are Healthy, S_Damage, and M_Damage, whose details can be found in table 3.

The normalized damage indices of the scenario, ‘S_Damage’ shown in figure 16(a) clearly show that the DI_PDF is able to identify and locate the single damage successfully at element 6. Further, it can be observed from figure 16(b) that the proposed damage index, DI-PDF, is able to identify the spatial locations (element No: 6 and Element No: 12) of the two damage scenario (M_Damage). Therefore, from the above investigations, it can be concluded that the DI_PDF is capable of locating the damage using the acceleration data with operational variability and measurement noise.

Figure 17. Normalised DI_PDF of the experimental RCC beam with limited measurement of 8 sensors: (a) S_Damage. (b) M_Damage. Note: X-axis of (a)–(b) indicate the optimal spatial sensor locations and the values within the brackets indicate the original spatial sensor locations with full instrumentation as shown in figure 10.

In order to test the efficiency of the proposed damage index to locate the damage with a limited number of sensor nodes, the sensor set is reduced to 8 using EFi and the final optimal sensor locations are shown in table 4. The acceleration data is generated at these optimally placed sensor nodes and are further used in AR–ARX model to derive the damage indices D_f and DI_PDF.

Accordingly, sensor number 2 and sensor number 6 of the limited sensors are expected to show the location of damage at 1/3rd from both the ends of the beam. The calculated damage indices are shown for single and multiple damage scenarios in figure 17. The x-axis shows the optimal sensor node number and the values within the brackets show the sensor node numbers with full instrumentation (as shown in figure 10). From figure 17, it can be clearly verified that the proposed damage technique is capable of locating the spatial location of the damages on the structure, even with the limited number of sensors.

4. Conclusion

In this paper, AR–ARX models are employed for the online health monitoring of structures. The proposed damage detection technique based on AR–ARX model and the damage index, D_f , is presented. It is shown that the damage index, D_f , of AR–ARX model is capable of clearly indicating the presence of damage. However, these features cannot give any conclusive indication of the spatial damage location because the AR–ARX model works on the acceleration data acquired from individual sensors to predict vibration responses. Keeping this in view, a new damage index based on the probability density function, DI_PDF, built from the prediction errors of the AR–ARX model is

used to locate the damage spatially on the structure. This damage index works on each sensor prediction errors and so there is no communication among the spatial sensors. The simple index of the ratio between the values of PDF at the confidence interval to the peak value of the PDF is easy to calculate and it reflects the damage location as the PDF extends and flattens out with the presence of damage. Therefore, the PDF-based damage index, DI_PDF, is found to be robust in identifying the spatial location of damage in the second phase. The investigation results of a numerical example of a simply supported beam, a benchmark problem of a three-storey bookshelf structure and the experimental studies on RCC beam clearly indicate the capability of the proposed damage index to locate the varied levels of damage from the acceleration data with operational variability and measurement noise. Also, the proposed damage diagnostic technique is found to be effective even with the limited number of optimally located sensors.

Acknowledgement

This paper is being published with the permission of the Director, CSIR-Structural Engineering Research Centre (SERC), Chennai.

References

- [1] Lifshitz J M and Rotem A 1969 Determination of reinforcement unbonding of composites by a vibration technique *J. Compos. Mater.* 3(3): 412–423
- [2] Plankis A 2012 *Structural health monitoring MEMS sensors using elasticity-based beam vibrations*. Ph.D thesis, Colorado State University, Fort Collins, CO, USA

- [3] West W M 1986 Illustration of the use of modal assurance criterion to detect structural changes in an orbiter test specimen In: *Proceedings of the Air Force Conference on Aircraft Structural Integrity*, pp. 1–6
- [4] Pandey A K, Biswas M and Samman M M 1991 Damage detection from changes in curvature mode shapes. *J. Sound Vib.* 145(2): 321–332
- [5] Mottershead J E and Friswell M I 1993 Model updating in structural dynamics: a survey. *J. Sound Vib.* 167(2): 347–375
- [6] Rytter A and Kirkegaard P 1997 Vibration based inspection using neural networks. In: *Structural Damage Assessment Using Advanced Signal Processing Procedures, Proceedings of DAMAS '97*, University of Sheffield, UK, 97–108
- [7] Farrar C R and Jauregui D A 1998 Comparative study of damage identification algorithms applied to a bridge: I. Experiment. *Smart Mater. Struct.* 7(5): 704–719
- [8] Huth O, Feltrin G, Maeck J, Kilic N and Motavalli M 2005 Damage identification using modal data: experiences on a pre-stressed concrete bridge. *J. Struct. Eng.* 131(12): 1898–1910.
- [9] Limongelli M P 2010 Frequency response function interpolation for damage detection under changing environment. *Mech. Syst. Signal Process.* 24(8): 2898–2913
- [10] Huang C C and Loh C H 2001 Nonlinear identification of dynamic systems using neural networks. *Comput. Aided Civ. Infrastruct. Eng.* 16(1): 28–41
- [11] Shu J, Zhang Z, Gonzalez I and Karoumi R 2013 The application of a damage detection method using artificial neural network and train-induced vibrations on a simplified railway bridge model. *Eng. Struct.* 52: 408–421
- [12] Kijewski T and Kareem A 2003 Wavelet transforms for system identification in civil engineering. *Comput. Aided Civ. Infrastruct. Eng.* 18(5): 339–355
- [13] Huang G, Su Y, Kareem A and Liao H 2015 Time-frequency analysis of nonstationary process based on multivariate empirical mode decomposition. *J. Eng. Mech.* 142(1): 04015065
- [14] Huang N, Shen Z, Long S, Wu M, Shih H, Zheng Q, Yen N, Tung C and Liu H 1998 The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* 454: 903–995
- [15] Giulio S, Francesco L, Riccardo T, Francesca G, Aurelio L C, Domenico L C, Mario C, Domenico G and Giovanni F 2016 A framework for the damage evaluation of acoustic emission signals through Hilbert–Huang transform. *Mech. Syst. Signal Process.* 75: 109–122
- [16] Pyayt A L, Kozionov A P, Mokhov I I, Lang B, Meijer R J, Krzhizhanovskaya V V and Sloot P M A 2014 Time-frequency methods for structural health monitoring. *Sensors* 14(3): 5147–5173
- [17] Fassois S D and Sakellariou J S 2007 Time-series methods for fault detection and identification in vibrating structures. *Philos. Trans. R. Soc. Math. Phys. Eng. Sci.* 365(1851): 411–448
- [18] Box G E P, Jenkins G M and Reinsel G C 1994 *Time series analysis: forecasting and control*, 3rd edn. Englewood Cliffs, NJ: Prentice-Hall
- [19] Fuller W A 2009 *Introduction to statistical time series*, vol. 428. Hoboken, New Jersey, United States: John Wiley & Sons
- [20] Pandit S M and Wu S M 1983 *Time series and system analysis with applications*. New York, USA: Wiley
- [21] de Lautour O R and Omenzetter P 2010 Damage classification and estimation in experimental structures using time series analysis and pattern recognition. *Mech. Syst. Signal Process.* ISSN 0888-3270, 24(5): 1556–1569
- [22] Wang Z and Ong K C G 2009 Structural damage detection using autoregressive-model-incorporating multivariate exponentially weighted moving average control chart. *Eng. Struct.* 31: 1265–1275
- [23] Yu L and Zhu J H 2015 Nonlinear damage detection using higher statistical moments of structural responses. *Struct. Eng. Mech.* 54(2): 221–237.
- [24] Gul M and Catbas F N 2011 Structural health monitoring and damage assessment using a novel time series analysis methodology with sensor clustering. *J. Sound Vib.* 330(6): 1196–1210
- [25] Peeters B and De Roeck G 2001 One-year monitoring of the Z24-Bridge: environmental effects versus damage events. *Earthq. Eng. Struct. Dyn.* 30(2): 149–171
- [26] Sohn H and Farrar C R 2001 Damage diagnosis using time series analysis of vibration signals. *Smart Mater. Struct.* 10(3): 446–451
- [27] Zhang Q W 2007 Statistical damage identification for bridges using ambient vibration data. *Comput. Struct.* 85(7–8): 476–485
- [28] Samuel da Silva, Júnior M D and Junior V L 2007 Damage detection in a benchmark structure using AR-ARX models and statistical pattern recognition. *J. Braz. Soc. Mech. Sci. Eng.* 29(2): 174–184
- [29] Lakshmi K and Rama Mohan Rao A 2015 Damage identification technique based on time series models for LANL and ASCE benchmark structures. *J. Non Destr. Test. Cond. Monit.* 57(10): 580–588
- [30] Nair K K, Kiremidjian A S and Law K H 2006 Time series based damage detection and localization algorithm with application to the ASCE benchmark structure. *J. Sound Vib.* 291(1–2): 349–368. <https://doi.org/10.1016/j.jsv.2005.06.016>
- [31] Zheng H and Mita A 2008 Damage indicator defined as the distance between ARMA models for structural health monitoring. *Struct. Control Health Monit.* 15(7): 992–1005
- [32] Carden E P and Brownjohn J M W 2008 ARMA modelled time-series classification for structural health monitoring of civil infrastructure. *Mech. Syst. Signal Process.* 22(2): 295–314
- [33] Samuel da Silva, Júnior M D, Junior V L and Brennan M J 2008 Structural damage detection by fuzzy clustering. *Mech. Syst. Signal Process.* 22(7): 1636–1649
- [34] Omenzetter P and Brownjohn J M W 2006 Application of time series analysis for bridge monitoring. *Smart Mater. Struct.* 15(1): 129–138. <https://doi.org/10.1088/0964-1726/15/1/041>
- [35] Xing Z and Mita A 2012 A substructure approach to local damage detection of shear structure. *Struct. Control Health Monit.* 19(2): 309–318
- [36] Lakshmi K and Rama Mohan Rao A 2016 Structural damage detection using ARMAX time series models and cepstral distances. *Sādhanā* 41(9): 1081–1097
- [37] Mei L, Mita A and Zhou J 2016 An improved substructural damage detection approach of shear structure based on ARMAX model residual. *Struct. Control Health Monit.* 23(2): 218–236. <https://doi.org/10.1002/stc.1766>
- [38] Mosavi A A, Dickey D, Seracino R and Rizkalla S 2012 Identifying damage locations under ambient vibrations utilizing vector autoregressive models and Mahalanobis

- distances. *Mech. Syst. Signal Process.* 26: 254–267. <https://doi.org/10.1016/j.ymssp.2011.06.009>
- [39] Mattson S G and Pandit S M 2006 Statistical moments of autoregressive model residuals for damage localization. *Mech. Syst. Signal Process.* 20(3): 627–645. <https://doi.org/10.1016/j.ymssp.2004.08.005>
- [40] Lakshmi K and Rama Mohan Rao A 2014 A robust damage-detection technique with environmental variability combining time-series models with principal components. *Non Destr. Test. Eval.* 29(4): 357–376
- [41] Tejasree G, Lakshmi K and Rama Mohan Rao A 2015 Comparison of damage indices employed for structural damage detection using time series analysis. In: *International Conference on Sustainable Energy and Built Environment, VIT-Chennai*, pp. 424–430
- [42] Ljung L 1998 *System identification. In Signal analysis and prediction*, pp 163–173, Birkhäuser, Boston, MA
- [43] Prawin J, Rama Mohan Rao A and Lakshmi K 2016 Non-linear parametric identification strategy combining reverse path and hybrid dynamic quantum particle swarm optimization. *Nonlinear Dyn.* 84(2): 797–815
- [44] Rama Mohan Rao A and Lakshmi K 2015 Damage diagnostic technique combining POD with time-frequency analysis and dynamic quantum PSO. *Meccanica* 50(6): 1551–1578
- [45] Mallardo V and Alibadi M H 2013 Optimal sensor placement for structural damage and impact identification: a review. *Struct. Durab. Health Monit.* 9: 287–323
- [46] Kammer D C 2005 Sensor set expansion for modal vibration testing. *Mech. Syst. Signal Process.* 19(4): 700–713
- [47] Rama Mohan Rao A and Anandakumar G 2008 Optimal sensor placement techniques for system identification and health monitoring of civil structures. *Smart Struct. Syst.* 4(4): 465–492
- [48] <http://institute.lanl.gov/ei/software-and-data>