Decomposition of a class of linear electrical networks for calculation of total power

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Abstract. A new method of decomposition of linear electrical networks for calculating total power is presented. By an iterative procedure, on the basis of decomposition of the electrical network, the branches containing power sources (voltage and/or current) are selected first until the remaining network becomes passive. Then, one calculates the power dissipated by the decomposed networks supplied by the corresponding power sources. Total power dissipated by impedances of the starting networks is equal to the sum of powers dissipated by impedances of the decomposed networks, under the condition that all impedances Z of the network are either resistive $\text{Im}(Z) = 0$ or reactive $\text{Re}(Z) = 0$ or mutually equal.

Keywords. Decomposition; iterative procedure; linear electrical network; Tellegen's theorem.

1. Introduction

Tellegen's theorem is a consequence of Kirchoff's law; therefore, it can be applied to electrical systems, either linear or nonlinear, reciprocal or nonreciprocal, time varying or time invariant [1–3]. Tellegen's theorem has also a significant application in performing the analyses of sensitivities of electrical networks, including electric power systems [4]. The use of variational principle for power system analysis is explained in [5], while several methods for calculation of power based on decomposition of electrical networks may be found in [6–9]. This work presents a new method of calculation of total power of the source by the method of decomposition or the total power dissipated in impedances in a linear electrical network (LEN), under the condition that all impedances Z of the network are either resistive $\text{Im}(Z) = 0$ or reactive $\text{Re}(Z) = 0$ or mutually equal. In performing these calculations in complex LENs, one applies either the basic or other methods derived through application of various transforms, either exact or approximative [1–3].

The significance of power calculations in electrical networks concerns primarily the analyses of electrical energy savings accomplished by application of suitable optimization techniques. In this paper a new simple method for calculating total power in a complex electrical network, supplied by voltage and/or current sources [10–12], based on the decomposition is presented.

2. Decomposition of LEN supplied by voltage sources

Let us formulate the theorem concerning total power dissipated by impedances in a LEN of harmonic current supplied by voltage sources, i.e., the total complex power of the source.

Theorem 1 (Decomposition of LEN supplied by voltage sources) *Let a given LEN N_0 consist of n branches conducting sinusoidal currents, as shown in figure 1. By decomposition a new unconnected network N , shown in figure 2, consisting of two networks N_1 and N_T is formed. Network N_1 represents network N_0 having j th branch disconnected and network N_T is Thevenin's circuit of j th branch of network N_0 . Then, total complex power of the voltage sources of network N_0 , S_{E,N_0} , under condition that all network impedances Z are either resistive $\text{Im}(Z) = 0$ or reactive $\text{Re}(Z) = 0$ or mutually equal, is equal to total power of the sources of unconnected network N , $S_{E,N}$:*

$$S_{E,N_0} = S_{E,N}. \quad (1)$$

Proof Let us write total complex power of the sources of the unconnected network N in the following form:

$$S_{E,N} = S_{E,N_1} + S_{E,N_T}, \quad (2)$$

where S_{E,N_1} is the total complex power of the sources of network N_1 and S_{E,N_T} is the total complex power of the sources of network N_T ; in the developed form, on the basis of figure 2

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Figure 1. Electrical network N_0 consisting of a separate complex branch j and network N_{1j} .

Figure 2. Network N as an equivalent to its decomposition to networks N_1 and N_T as regards the total complex power source.

$$S_{E,N_1} = \sum_{\substack{k=1 \\ k \neq j}}^n E_k \underbrace{(I_k + \Delta I_k)}_{I'_k}, \quad (3)$$

$$S_{E,N_T} = E_{Tj} I_j^* + E_j I_j^*, \quad (4)$$

I_k is the branch current of network N_0 , $k = \overline{1, n}$,

I'_k is the branch current of network N_1 , $k = \overline{1, n}$, $k \neq j$ upon disconnection of the j th branch of network N_0 (change of parameters of branch j) and superscript $*$ denotes conjugate value.

ΔI_k is the increment (change) of current in branch $k = \overline{1, n}$ upon change of parameters of network N_0 .

Using relations (2), (3) and (4), the complex power of the source of unconnected network N is obtained in the form

$$S_{E,N} = \underbrace{\sum_{k=1}^n E_k I_k^*}_{S_{E,N_0}} + \underbrace{\sum_{\substack{k=1 \\ k \neq j}}^n E_k \Delta I_k^* + E_{Tj} I_j^*}_A. \quad (5)$$

If we prove that $A = 0$, the proof of theorem 1 follows from relation (5). It is required to prove that

$$A = \sum_{\substack{k=1 \\ k \neq j}}^n E_k \Delta I_k^* + E_{Tj} I_j^* = 0. \quad (6)$$

Let us quote theorem of variation together with its derivation [2, 3], which will be used to prove relation (6). For the sake of transparency and simplicity of tracking, using

the present notation, with no loss of generality, let us restrict to branch j where a variation of impedance occurred. Let the change of impedance of j th branch amount to ΔZ_j ; then, according to the theorem of compensation, this change of impedance causes a change of the distribution of currents in the j th branch, which would have been obtained by the change of electromotive force of j th branch:

$$\Delta E_j = -\Delta Z_j I'_j, \quad (7)$$

where I'_j is the current of branch j , which corresponds to the current distribution in the network after the change of impedance in branch j :

$$I'_j = I_j + \Delta I_j. \quad (8)$$

According to the principle of superposition, current in branch j is equal to the sum of current I_j (current prior to variation ΔZ_j) due to all k sources E_k of network N_0 and source ΔE_j in relation (7); thus, one may write

$$I_j + \Delta I_j = I_j + y_{jj} \Delta E_j. \quad (9)$$

From relations (7), (8) and (9) one obtains

$$\Delta I_j = -\frac{y_{jj} \Delta Z_j}{1 + y_{jj} \Delta Z_j} I_j, \quad (10)$$

where y_{jj} is the input admittance of branch j with respect to voltage source E_j in branch j . Input admittance y_{jj} can be determined from the relation

$$y_{jj} = \frac{I_j}{E_j} \text{ for } E_k = 0, k \neq j. \quad (11)$$

From the property of linearity of network N_0 , the following relation concerning the change of current ΔI_k in branch k due to a change of current ΔI_j in branch j clearly follows:

$$\Delta I_k = \frac{y_{kj}}{y_{jj}} \Delta I_j, \quad (12)$$

where y_{kj} is the transfer admittance of branch k with respect to voltage source E_j in branch j . Transfer admittance y_{kj} can be determined from the relation

$$y_{kj} = \frac{I_k}{E_j} \text{ for } E_k = 0, k \neq j. \quad (13)$$

Finally, using relations (10) and (12) one obtains the law of change of current ΔI_k with variation of impedance ΔZ_j :

$$\Delta I_k = -\frac{y_{kj} \Delta Z_j}{1 + y_{jj} \Delta Z_j} I_j. \quad (14)$$

This demonstrates the theorem of parameter variation. In order to prove theorem 1, consider LEN N_0 , figure 3, for $E_g = 0$ and $g \neq k, j$, which corresponds to network N_1 with the j th branch separated.

Determine now the input y_{jj} and transfer admittance y_{kj} defined previously via reducing down the problem to input admittance y'_{jj} and transfer admittance y'_{kj} of network N'_1 . Since, by definition and using admittance parameters [10], for network N'_1 with $E_g = 0, g \neq k, j$ one can write exact relations

$$I_k = y'_{kk}E_k - y'_{kj}U_j, \quad (15)$$

$$I_j = y'_{jk}E_k - y'_{jj}U_j, \quad (16)$$

$$y'_{kj} = y'_{jk}, \quad (17)$$

based on figure 3 and relations (15)–(17), it is possible to express I_k as a function of E_j by the following relation:

$$I_k = y'_{kj} \frac{Z_{T_j}}{Z_{T_j} + Z_j} E_j = y'_{kj} \frac{1}{1 + y'_{jj}Z_j} E_j, \quad E_k = 0, \quad k \neq j. \quad (18)$$

Thus, we obtain y_{kj} via y'_{kj} since $y'_{jj} = 1/Z_{T_j}$ is input admittance of network N'_1 with respect to branch j :

$$y_{kj} = y'_{kj} \frac{1}{1 + y'_{jj}Z_j}. \quad (19)$$

Analogously, it is straightforward to show that input admittance is

$$y_{jj} = \frac{1}{Z_{T_j} + Z_j} = y'_{jj} \frac{1}{1 + y'_{jj}Z_j}. \quad (20)$$

Of interest for this proof is the ratio of transfer admittance y_{kj} to input admittance y_{jj} , i.e.

$$\frac{y_{kj}}{y_{jj}} = \frac{y'_{kj}}{y'_{jj}}. \quad (21)$$

Using expression (14), for a variation of impedance of j th branch $\Delta Z_j \rightarrow \infty$ one easily obtains increment of current in k th branch:

$$\Delta I_k = -\frac{y'_{kj}}{y'_{jj}} I_j. \quad (22)$$

Since mapping of the increment of current ΔI_j of j th branch to the increment of current ΔI_k of k th branch is in

Figure 3. Linear electrical network N_0 for $E_g = 0$ and $g \neq k, j$.

question, the same mapping of the conjugated values of the current applies; therefore, one can write the relation

$$\Delta I_k^* = -\left(\frac{y'_{kj}}{y'_{jj}}\right)^* I_j^*. \quad (23)$$

Introducing relation (23) in relation (6) one obtains

$$A = I_j^* \left(-\sum_{\substack{k=1 \\ k \neq j}}^n \left(\frac{y'_{kj}}{y'_{jj}}\right)^* E_k + E_{T_j} \right) = 0 \text{ for } \left(\frac{y'_{kj}}{y'_{jj}}\right)^* = \frac{y'_{kj}}{y'_{jj}}. \quad (24)$$

Since the disconnected j th branch is in question, value of the current is $I_j^* = 0$ and $(y'_{kj}/y'_{jj})^* = y'_{kj}/y'_{jj}$. Since parameters of network N'_1 are not dependent upon excitation, it clearly follows from relations (16) and (17) that

$$\sum_{\substack{k=1 \\ k \neq j}}^n \frac{y'_{kj}}{y'_{jj}} E_k = E_{T_j}, \quad (25)$$

which represents known equality of superposition of the sources of each k th branch at the place of disconnection of j th branch, i.e., Thevenin's source E_{T_j} . Alternatively, if $I_j^* = 0$, there follows a trivial solution of relation (24). Condition $(y'_{kj}/y'_{jj})^* = y'_{kj}/y'_{jj}$ is fulfilled when all impedances Z of the network are either resistive $\text{Im}(Z) = 0$ or reactive $\text{Re}(Z) = 0$ or mutually equal. This is a complete proof of theorem 1. $\square$

On the basis of theorem 1, by continuing decomposition of network N_1 , in the way of decomposition of network N_0 , until simple circuit N_{n-1} is obtained, it clearly follows that total power of the sources expressed in the form of the sum of all powers of the sources obtained by Thevenin's circuits, as shown in figure 2 in the part of the non-connected network N , i.e., iterative application of Tellegen's theorem under the condition that all impedances Z of the network are either resistive $\text{Im}(Z) = 0$ or reactive $\text{Re}(Z) = 0$ or mutually equal:

$$S_{E, N_0} = \sum_{k=1}^{n-1} (E_{T_k} + E_k) I_k^*. \quad (26)$$

3. Decomposition of LENs comprising current sources

In order to determine total power dissipated in impedances of a network comprising current sources, i.e., total complex source power, we shall, as the basis, use the Norton

theorem [2, 3] for the application and formulation of theorem 2, which is dual to the previous theorem 1; therefore, it will not be proved separately.

Theorem 2 (Decomposition of LEN comprising current sources) *Let there be given a LEN K_0 consisting of n branches conducting sinusoidal currents, as shown in figure 4. By decomposition a new unconnected network K , shown in figure 5, consisting of two networks K_1 and K_T is formed. Network K_1 represents network K_0 having j th branch disconnected and network K_N is Norton's circuit of j th branch of network K_0 . Then, total complex power of the current sources of network K_0 , S_{I,K_0} , under the condition that all network impedances Z are either resistive $\text{Im}(Z) = 0$ or reactive $\text{Re}(Z) = 0$ or mutually equal, is equal to total power of the sources of unconnected network N , $S_{I,K}$, i.e.*

$$S_{I,K_0} = S_{I,K}. \quad (27)$$

On the basis of theorem 2, by continuing further decomposition of network K_1 , like decomposition of K_0 , until simple electrical circuit K_{n-1} is obtained, it clearly follows that total power of the sources is expressed in the form of the sum of all powers of the sources obtained by Norton's circuits, as shown in figure 5. Iterative application of the proposed decomposition method, in the part of the non-connected network K , under the condition that all impedances Z of the network are either resistive $\text{Im}(Z) = 0$ or reactive $\text{Re}(Z) = 0$ or mutually equal, is

$$S_{I,K_0} = \sum_{k=1}^{n-1} U_k (I_k - I_{N_k})^*. \quad (28)$$

Figure 4. Electrical network K_0 consisting of a separate complex branch j and network K_{1j} .

Figure 5. Network as an equivalent to its decomposition to networks K_1 and K_N as regards the total complex power source.

Corollaries of theorem 1 and theorem 2. In order to obtain as low as possible number of the simple circuits created by decomposition of a network supplied by voltage/current sources, if there are branches containing no voltage/current source, the strategy should be such that active branches (branch containing a voltage/current source) are disconnected first. In this way the number of iterative decompositions M_g (number of active branches) is reduced. Analogously, the number of independent contours M_c in a given network supplied by voltage sources is equal to the number of iterations in the process of disconnecting a branch belonging to an independent contour. The number of required iterations M_I will be

$$M_I < \min[M_g, M_c] < n - 1. \quad (29)$$

4. Decomposition of linear complex electrical network

In order to determine total power dissipated in impedances of a complex network C_0 (a network supplied by both voltage and current sources) the property of decomposition of a complex network C_0 to a network supplied by voltage sources N_0 and a network supplied by current sources K_0 , as shown in figure 6, is applied first. For these networks the following rule applies [3]:

$$S_{G,C_0} = S_{E,N_0} + S_{I,K_0} \quad (30)$$

where S_{G,C_0} is the complex power of all sources (voltage and current) in a complex network, S_{E,N_0} is the complex power in a network supplied by voltage sources and S_{I,K_0} is the complex power in a network supplied by current sources. In the second step, theorems 1 and 2, i.e., iterative application of the proposed decomposition method to the decomposed network supplied by voltage sources and the decomposed network supplied by current sources, are applied. This completes the formulation of application of decomposition of a LEN for the purpose of application of Tellegen's theorem. Using the method described earlier, further investigations would refer to the adequate compensation as well as the definition of power [13–16] on certain parts of the network in power systems.

Figure 6. Network C_0 as an equivalent to its decomposition to networks N_0 and K_0 as regards the total complex power source.

Figure 7. DC electrical network supplied by voltage sources.

Example 1 For a DC network supplied by voltage sources, shown in figure 7, determine total power dissipated by all resistors in the network by applying decomposition of a linear network for the purpose of iterative application of Tellegen's theorem.

Using the previously described method, for decomposition of linear network of figure 7, e.g., branch 1, figure 8a shows the equivalent Thevenin's circuit [2, 3]. On the basis of the rule for calculating Thevenin's voltage it follows that $E_{T_1} = -U_{10} = 2$ [V]. Power of E_{T_1} source is $P_{E,N_{T_1}} = 2$ [W].

Now, branch 2 is disconnected, in accordance with theorem 1, and in a similar way one obtains $U_{20} = E_{T_2} = 2$ [V] and $R_{T_2} = 5/3$ [Ω]; therefore, a new Thevenin's circuit, shown in figure 8b, is obtained. It is obvious that total power of Thevenin's circuit N_{T_2} is $P_{E,N_{T_2}} = 6$ [W]. Finally, for the simple circuit N_{T_3} , shown in figure 8c, power of the source is easily calculated as $P_{E,N_{T_3}} = 12$ [W]. If now all obtained powers of the simple Thevenin's circuits are added together, one obtains total source power of the network of figure 7, i.e.

$$P_{E,N_T} = P_{E,N_{T_1}} + P_{E,N_{T_2}} + P_{E,N_{T_3}} = 20 \text{ [W]}. \quad (31)$$

Example 2 For AC network supplied by voltage and current sources, shown in figure 9, determine total apparent power S by applying decomposition of a LEN for the purpose of applying Tellegen's theorem. Values of the elements are $E_1 = (10 + j20)$ [V], $E_3 = (3 - j11)$ [V], $E_5 = (-3 + j11)$ [V], $I_g = j0.5$ [A], $Z_1 = j100$ [Ω], $Z_2 = -j10$ [Ω], $Z_3 = j20$ [Ω], $Z_4 = j20$ [Ω] and $Z_5 = -j40$ [Ω].

For a network supplied by both voltage and current sources, the network supplied exclusively by current sources (figure 10a), voltage sources are represented by short circuits, is analysed first and then the network supplied exclusively by voltage sources (figure 10b), current sources are disconnected, is analysed.

Figure 8. A decomposition of linear network of figure 7.

Figure 9. AC electrical network supplied by voltage and current sources.

A decomposition of these networks is shown in figure 11. Using the decomposition method as in the previous example, for the network supplied exclusively by current sources one obtains $Z_{T_2} = j20$ [Ω], as shown in figure 11a. The apparent power is now $S_{E,N_{T_1}} = Z_{T_1} I_g I_g^* = j5$ [VA]. For the network supplied exclusively by voltage sources by disconnecting, e.g., branches accommodating impedances Z_4 , Z_5 and source E_5 , the equivalent Thevenin's circuit is shown in figure 11b. On the basis of the rule for calculating Thevenin's voltage it follows that $E_{T_2} = (-1.8182 + j11.8182)$ [V], $Z_{T_2} = j9.0909$ [Ω] and $I_{N_2} = (0.0750 - j0.1083)$ [A]. The apparent power is now $S_{E,N_{T_2}} = E_5 I_{N_2}^* - E_{T_2} I_{N_2} = -j0.1894$ [VA]. By disconnecting this branch, the remaining circuit is shown in figure 11c. Now, $I_{N_3} = (0.0818 - j0.1182)$ [A] and the apparent complex power is $S_{E,N_{T_3}} = E_3 I_{N_3}^* + E_1 I_{N_3} = j2.2727$ [VA]. Adding the obtained powers of the sources of the simple Thevenin's circuits together, the total source power of the network of figure 11 is

$$S_{E,N_T} = S_{E,N_{T_1}} + S_{E,N_{T_2}} + S_{E,N_{T_3}} = j5 - j0.1894 + j2.2727 = j7.0833 \text{ [VA]}. \quad (32)$$

Solving the initial circuit shown in figure 9, one obtains that currents in the branches containing power sources as

Figure 10. Network supplied by (a) current sources only and (b) voltage sources only.

Figure 11. A decomposition of AC networks of figure 10.

follows: $I_3 = (0.1500 - j0.7167)[A]$, $I_1 = (0.0750 - j0.1083)[A]$ and $I_5 = (0.0750 - j0.6083)[A]$, while the voltage is $U_{24} = -10 [V]$. The total apparent source power is

$$S_{E,I} = E_1 I_1^* + E_3 I_3^* + E_5 I_5^* + U_{24} I_g^* = j7.0833[VA] . \quad (33)$$

The obtained values of the total complex power in expressions (32) and (33) are, as expected, equal.

Example 3 For the AC electrical network of figure 9 containing power sources $E_1 = (10 + j20)[V]$, $E_3 = (3 - j11)[V]$, $E_5 = (-3 + j11)[V]$, $I_g = j0.5[A]$ and all impedances are mutually equal, e.g. $Z_1 = Z_2 = Z_3 = Z_4 = Z_5 = (10 + j20)[\Omega]$, iterative application of the proposed decomposition method results in $S_{E,N_T} = (5 + j10)[VA]$.

Example 4 For the AC electrical network of figure 9 containing power sources $E_1 = (10 + j20)[V]$,

$E_3 = (3 - j11)[V]$, $E_5 = (-3 + j11)[V]$, $I_g = j0.5[A]$ and all impedances are resistive $Z_1 = 100[\Omega]$, $Z_2 = 70[\Omega]$, $Z_3 = 30[\Omega]$, $Z_4 = 20[\Omega]$ and $Z_5 = 80[\Omega]$, by the proposed iterative application of the proposed decomposition method one obtains $S_{E,N_T} = (13 + j0)[VA]$.

5. Conclusion

The proposed approach for total power calculation is conceptually simple and computationally effective, indicating that the proposed decomposition of the complex network to simple networks needs smaller processing time for total power calculation. The proposed method allows iterative decomposition of a complex LEN. The total power dissipated by impedances of the starting network is equal to the sum of powers dissipated by impedances of the decomposed networks under the condition that all network impedances Z are either resistive $\text{Im}(Z) = 0$ or reactive $\text{Re}(Z) = 0$ or mutually equal. A favourable feature of the proposed method is that it offers a flexible approach to any linear network, i.e., by application of the method at each subsequent iteration the network becomes simpler. The flexibility is related to the strategy in performing the basic steps of the method when analysing complex networks. In this way, one obtains a network consisting of simple elements involving calculations carried out through already developed algorithms. It is interesting to mention that in further investigations concerning the elaboration of the strategy of application of the method, it is justified that decomposition of networks should be considered from the optimization point of view, as well as the elimination of the limitations as regards impedances in linear electrical AC networks.

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