BER performance analysis of compound chaotic sequence (CCS)-based NR-DCSK system under multipath fading channel

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Abstract. This paper presents the performance analysis of compound chaotic sequence (CCS)-based noise reduction differential chaos shift keying (NR-DCSK) system under multipath Rayleigh fading channel conditions. The special characteristics of chaotic sequences are their deterministic randomness behaviour that adds security and multipath immunity to the data when used as a carrier in communication systems. In this paper, the chaotic sequences are generated by combining the outputs of chaotic maps, such as logistic map, Chebyshev map, Bernoulli shift map, tent map, etc., leading to new complex sequences known as CCSs. This sequence possesses more randomness, overcomes severe interference levels encountered during transmission and provides higher multipath immunity compared with those of pseudo-noise (PN) codes. Since NR-DCSK is a spread spectrum technique, its performance in wireless multipath fading channels has important considerations. The CCS is used as a carrier in NR-DCSK systems, which leads to improved bit error rate (BER) performance. Comparisons of simulation results to theoretical BER expressions of additive white Gaussian noise (AWGN) and Rayleigh fading channels have been carried out to test the efficiency of the proposed CCS-based NR-DCSK system.

Keywords. Bit error rate (BER); compound chaotic sequence (CCS); moving average filtering; noise reduction differential chaos shift keying (NR-DCSK) system; non-coherent chaos-based system; Rayleigh fading.

1. Introduction

Chaotic signals are non-periodic, random-like bounded signals that can be generated in a deterministic manner using non-linear iterative systems known as chaotic maps. These signals are very sensitive towards the initial conditions used for the generation of chaotic signals. A slight change in the initial condition produces a different chaotic sequence [1].

Unlike conventional systems, chaotic communication methodology offers several coherent and non-coherent chaotic modulation schemes, such as chaos shift keying (CSK), chaotic on-off keying (COOK), correlation delay shift keying (CDSK), differential chaos shift keying (DCSK) and quadrature chaos shift keying (QCSK), and the performances of these schemes have been assessed in the recent years [2–5].

Among these non-coherent schemes, DCSK is the most suitable one in wireless communications due to its excellent performance over multipath fading channels with simple correlation-type receiver structures. In [6], frequency-modulated DCSK (FM-DCSK) architecture was proposed for single input multiple output (SIMO) systems, which eliminates the need for channel state information, exact synchronization and complex rake receiver, making the proposed algorithm simpler and yet more efficient than the vertical Bell laboratories layered space-time (VBLAST) scheme. In [7], M-ary DCSK scheme was implemented for multiple input multiple output (MIMO) communications and two transceiver structures employing spatial diversity and adaptive beam forming techniques were proposed.

Nevertheless, the limitation of conventional DCSK is its inferior performance due to the fact that the reference and information-bearing signals are corrupted by the channel noise, and a noisy reference signal is correlated with a noisy

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information bearing signal at the receiver, which deteriorates the performance of the DCSK system.

To overcome the acknowledged defects of the DCSK scheme and to reinforce its spectral efficiency, a large volume of research has been carried out to come up with new non-coherent DCSK systems. A high-efficiency DCSK (HE-DCSK) was proposed in [8]. In [9], an improved DCSK (I-DCSK) was proposed, where a time-reversal operation was used to generate an orthogonal reference signal to the data carrier signal, and then these two sequences were added in one time slot, prior to transmission. Short reference (SR) DCSK [10] schemes were suggested for enhancing the data rate and improving system performance. Eventually, noise reduction (NR) DCSK (NR-DCSK) was proposed by Kaddoum and Soujeri [11] where the reference signal of the NR-DCSK system uses β/P different chaotic samples only, instead of β samples as done in classical DCSK schemes. Therefore, each chaotic sample is repeated P successive times to make a reference signal of β samples in total, for improved bit error rate (BER) performance.

1.1 Contributions and paper outline

The majority of the proposed schemes use one dimensional (1-D) logistic map for the generation of chaotic sequences and provide optimum performance in a non-coherent system. Increased randomness in the nature of chaotic sequences plays a vital role in enhancing the BER performance and signal-to-noise ratio (SNR) of both coherent and non-coherent systems. The main motivation of this paper is to propose a robust chaotic sequence generation method that enhances the BER performance of both coherent and non-coherent chaotic systems. The research contributions of this paper are as follows.

- 1) A robust method for the generation of compound chaotic sequence (CCS) using conventional maps like logistic, Bernoulli shift, Chebyshev and tent map is proposed. This CCS is used as a reference chaotic sequence in an NR-DCSK system and the performance is evaluated for various combinations of the chaotic maps.
- 2) Computing and analysing the BER performance of CCS-based NR-DCSK systems over Rayleigh fading channel and additive white Gaussian noise (AWGN) channels.

The rest of this paper is organized as follows. In section 2, the CCS-based NR-DCSK system is briefly discussed. The method of obtaining CCS and the performance analysis of CCS-based NR-DCSK system is carried out in section 3. Simulation results and discussion are presented in section 4 and concluding remarks are presented in section 5.

2. System model

2.1 CCS-based NR-DCSK system

The NR-DCSK system was introduced by Kaddoum and Soujeri [11], where the reference chaotic carrier uses β/P

different chaotic sequences instead of β chaotic sequences used in the conventional DCSK system. Each β/P chaotic sequence in NR-DCSK is repeated P successive times to get a chaotic reference signal of length β in total. The chaotic sequences are generated from a 1-D logistic map.

In CCS-based NR-DCSK systems, the CCSs are used for improved BER performance and SNR gain of the NR-DCSK system. A block diagram of the proposed CCS-based NR-DCSK system; NR-DCSK transmitter and receiver is shown in figure 1. As shown in figure 1, the compound chaotic generator generates β/P samples at a frequency of $f_s = \frac{1}{PT_c}$ per sample. Each compound chaotic sample is repeated P times to obtain the reference signal of length β . If $+1$ is transmitted, the reference signal is repeated in the information-bearing time slot and if -1 is transmitted, an inverted copy of the reference signal is used in the information-bearing time slot. Therefore, during the i th bit time interval, the transmitted discrete baseband signal $S_{i,k}$ of the NR-DCSK system would be given by

$$S_{i,k} = \begin{cases} x_{i,\lceil \frac{k}{P} \rceil}, & 0 < k \leq \beta \\ b_i x_{i,\lceil \frac{k-\beta}{P} \rceil}, & \beta < k \leq 2\beta \end{cases} \quad (1)$$

where $\lceil a \rceil$ is the ceiling operator, which represents the smallest integer not less than a ; $x_{i,\lceil \frac{k}{P} \rceil}$ is the β/P distinct compound chaotic sample.

The received signal corresponding to the i th bit is given by

$$r_{i,k} = \sum_{l=1}^L \alpha_{l,i} S_{i,k-\tau_l} + n_{i,k}, \quad (2)$$

where $\alpha_{l,i}$ and τ_l are, respectively, the channel gain and the delay of the l th path, L is the number of paths and $n_{i,k}$ is the AWGN with zero-mean complex number and variance $N_0/2$. At the receiver, the received signal is first averaged using a moving average filter with window size P . The resultant averaged signal is given by

$$r_{avg,i,k} = \frac{1}{P} \sum_{p=1}^P r_{i,k+p}, \quad (3)$$

where P is the repetition factor in a CCS-based NR-DCSK system. The purpose of the moving average filtering operation at the receiver is to reduce the noise variance of the CCS. The resultant averaged signal is passed to a correlator, where it is correlated with its delayed replicas by a factor of $\frac{\beta}{P}$ and then summed over the duration $\frac{\beta T_c}{P}$, where T_c is the chip duration. In our analysis, we assume that the delay τ is much shorter than the reference duration $(0 < \tau \ll \frac{\beta T_c}{P})$, a case in which the presence of Inter-Symbol Interference (ISI) is neglected. Further, for mathematical simplification, we consider unity chip duration, i.e., $T_c = 1$. Based on the assumptions, the decision variable at the output of the correlator of the i th bit is

Figure 1. Block diagram of (a) CCS-based NR-DCSK communication system, (b) transmitter and (c) receiver.

$$\begin{aligned}
 U_i &= \sum_{k=1}^{\frac{\beta}{P}} \left(\sum_{l=1}^L \alpha_{l,i} x_{i,k-\tau_l} + \frac{1}{P} \sum_{p=1}^P n_{i,p+k} \right) \\
 &\quad \left(\sum_{l=1}^L \alpha_{l,i} b_i x_{i,k-\tau_l} + \frac{1}{P} \sum_{p=1}^P n_{i,p+k+\frac{\beta}{P}} \right) \\
 &= \sum_{k=1}^{\frac{\beta}{P}} \left(\sum_{l=1}^L \alpha_{l,i}^2 b_i x_{i,k-\tau_l}^2 + \sum_{l=1}^L \alpha_{l,i} x_{i,k-\tau_l} \frac{1}{P} \sum_{p=1}^P n_{i,p+k+\frac{\beta}{P}} \right. \\
 &\quad \left. + \sum_{l=1}^L \alpha_{l,i} b_i x_{i,k-\tau_l} \frac{1}{P} \sum_{p=1}^P n_{i,p+k} \right. \\
 &\quad \left. + \frac{1}{P} \sum_{p=1}^P n_{i,p+k} \frac{1}{P} \sum_{p=1}^P n_{i,p+k+\frac{\beta}{P}} \right). \quad (4)
 \end{aligned}$$

In (4), the first term is the useful signal; the remaining terms designate the interference caused by Gaussian noise. Define

$$A = \sum_{k=1}^{\frac{\beta}{P}} \left(\sum_{l=1}^L \alpha_{l,i}^2 b_i x_{i,k-\tau_l}^2 \right), \quad (5)$$

$$B = \sum_{k=1}^{\frac{\beta}{P}} \left(\sum_{l=1}^L \alpha_{l,i} x_{i,k-\tau_l} \frac{1}{P} \sum_{p=1}^P n_{i,p+k+\frac{\beta}{P}} \right), \quad (6)$$

$$C = \sum_{k=1}^{\frac{\beta}{P}} \left(\sum_{l=1}^L \alpha_{l,i} b_i x_{i,k-\tau_l} \frac{1}{P} \sum_{p=1}^P n_{i,p+k} \right), \quad (7)$$

$$D = \sum_{k=1}^{\frac{\beta}{P}} \left(\frac{1}{P} \sum_{p=1}^P n_{i,p+k+\frac{\beta}{P}} \frac{1}{P} \sum_{p=1}^P n_{i,p+k} \right). \quad (8)$$

2.2 Generation of CCS

In this work, CCS is used as the carrier for data transmission due to its enhanced security and complexity in the nature of the chaotic sequences even for a finite length. The method of obtaining CCS is discussed below.

For a given initial condition, the outputs of the different chaotic maps (for example, logistic map, Chebyshev map, Bernoulli shift map, tent map, etc.) are summed to generate a new complex chaotic sequence called CCS. We can combine any combination of map outputs to get the

Table 1. Conventional chaotic maps [12].

Type of map	Definition
Logistic map (L)	$f_1(x_k) = 1 - 2x_{k-1}^2$
Chebyshev map (C)	$f_2(x_k) = 4x_{k-1}^3 - 3x_{k-1}$
Bernoulli shift map (B)	$f_3(x_k) = \begin{cases} \frac{3}{2}x_{k-1} - \frac{1}{2}, & x_{k-1} > 0, \\ \frac{3}{2}x_{k-1} + \frac{1}{2}, & x_{k-1} \leq 0, \end{cases}$
Tent map (T)	$f_4(x_k) = \begin{cases} \frac{x_{k-1}}{0.6}, & \text{when } 0 \leq x_{k-1} \leq 0.6, \\ \frac{(1-x_{k-1})}{0.4}, & \text{when } 0.6 \leq x_{k-1} \leq 1. \end{cases}$

compound signal provided the initial condition to those maps will be the same.

Definition:

Let

$$x_{i,k} = f_i(x_{i,k-1}) \quad (9)$$

be the discrete chaotic map. For $k = 1, 2, 3, \dots, \beta$, where β is the spreading length, (9) generates the chaotic sequences $x_{i,1}, x_{i,2}, x_{i,3}, \dots, x_{i,\beta}$. To generate the CCS, we consider the form

$$x_{i,k} = \sum_{i=1}^N f_i(x_{k-1}), \quad (10)$$

where N is the number of discrete maps used to generate CCS. In this paper, we will use four chaotic maps ($N = 4$) shown in table 1 corresponding to the state variables fluctuating in $[-1, 1]$.

3. Performance analysis of CCS-based NR-DCSK

In this section, the performance of CCS-based NR-DCSK system is evaluated under AWGN and Rayleigh fading channels. Towards this end, the mean and the variance expressions of the decision variable U_i are determined. The signal components A – D in Eq. (4) are independent because the channel coefficients are independent of each other, of the chaotic sequences and of the Gaussian noise. In order to find the mean and the variance of the decision variable U_i , we assume $b_i = +1$ and $b_i = -1$, respectively. Using (5)–(8), (4) can be written for $b_i = +1$ and $b_i = -1$, as

$$U_i|(b_i = +1) = A + B + C + D, \quad (11)$$

and

$$U_i|(b_i = -1) = -A + B - C + D, \quad (12)$$

respectively. To proceed with the analysis, the following assumptions are considered.

- 1) Chaotic sequence $x_{i,k}$ and noise $n_{i,k}$ are independent of each other.

- 2) Furthermore, $n_{i,k}$ is statistically independent of $n_{j,k}$ for any $i \neq j$.

With these assumptions, it can be shown that

$$\begin{aligned} E[U_i|(b_i = +1)] &= E[A] + E[B] + E[C] + E[D], \\ E[U_i|(b_i = -1)] &= -E[A] + E[B] - E[C] + E[D], \\ \text{Var}[U_i|(b_i = +1)] &\approx \text{Var}[A] + \text{Var}[B] + \text{Var}[C] + \text{Var}[D], \\ \text{Var}[U_i|(b_i = -1)] &\approx \text{Var}[A] + \text{Var}[B] + \text{Var}[C] + \text{Var}[D]. \end{aligned} \quad (13)$$

It can be shown that the relevant means and variances are

$$E[A] = \frac{\beta}{P} E[x_{i,k-\tau_l}^2] \sum_{l=1}^L \alpha_{l,i}^2 \equiv \frac{\beta}{P} P_s \sum_{l=1}^L \alpha_{l,i}^2, \quad (14)$$

where $P_s = E[x_{i,k-\tau_l}^2]$. $E[B] = E[C] = E[D] = 0$.

$$\text{Var}[A] = \frac{\beta}{P} \text{Var}[x_{i,k}^2] \cdot \sum_{l=1}^L \alpha_{l,i}^2 \quad (15)$$

The Gaussian noises present in B , C and D are the average of the P independent and identically distributed (i.i.d.) Gaussian samples. Therefore, the variance of the noise power is reduced to $\frac{N_0}{2P}$. Hence, the variance of B , C and D will become

$$\begin{aligned} \text{Var}[B] &= \frac{\beta}{P} \text{Var}[x_{i,k}] \text{Var}\left[\frac{1}{P} \sum_{p=1}^P n_{i,p+k+\frac{\beta}{P}}\right] \\ &= \frac{\beta}{P} E[x_{i,k}^2] \frac{N_0}{2P} \sum_{l=1}^L \alpha_{l,i}^2, \\ \text{Var}[B] &= \frac{\beta N_0}{2P^2} P_s \sum_{l=1}^L \alpha_{l,i}^2, \\ \text{Var}[B] &= \text{Var}[C], \\ \text{Var}[D] &= \frac{\beta N_0^2}{4P^3}. \end{aligned} \quad (16)$$

Substituting (14)–(16) in (13), we obtain

$$\begin{aligned} E[U_i|(b_i = +1)] &= \frac{\beta}{P} P_s \sum_{l=1}^L \alpha_{l,i}^2, \\ E[U_i|(b_i = -1)] &= -\frac{\beta}{P} P_s \sum_{l=1}^L \alpha_{l,i}^2, \\ \text{Var}[U_i|(b_i = +1)] &= \frac{\beta}{P} \text{Var}[x_{i,k}^2] \sum_{l=1}^L \alpha_{l,i}^2 \\ &\quad + \frac{\beta N_0}{P^2} P_s \sum_{l=1}^L \alpha_{l,i}^2 + \frac{\beta N_0^2}{4P^3}, \\ \text{Var}[U_i|(b_i = -1)] &= \frac{\beta}{P} \text{Var}[x_{i,k}^2] \sum_{l=1}^L \alpha_{l,i}^2 \\ &\quad + \frac{\beta N_0}{P^2} P_s \sum_{l=1}^L \alpha_{l,i}^2 + \frac{\beta N_0^2}{4P^3}. \end{aligned} \quad (17)$$

Since bit energies (or chaotic chips) are deterministic variables, by virtue of central limit theorem the decision variable at the output of the correlator follows a Gaussian distribution. Therefore, the bit error probability is represented as

$$BER = \frac{1}{2} P(U_i < 0 | b_i = +1) + \frac{1}{2} P(U_i > 0 | b_i = -1). \quad (18)$$

This can be simplified as

$$BER = \frac{1}{2} \operatorname{erfc} \left(\left[\sqrt{\frac{E[U_i]^2}{2\operatorname{Var}[U_i]}} \right] \right), \quad (19)$$

where $\operatorname{erfc}(x)$ is the complementary error function [13] defined as $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du$. By virtue of Eq. (17), the BER for the CCS-based NR-DCSK scheme can be determined and stated as

$$BER = \frac{1}{2} \operatorname{erfc} \left(\left[\frac{\left(\frac{\beta}{P} \right)^2 P_s^2 \left(\sum_{i=1}^L \alpha_{l,i}^2 \right)^2}{\frac{2\beta}{P} \operatorname{Var}[x_{i,k}^2] \sum_{i=1}^L \alpha_{l,i}^2 + 2 \frac{\beta N_o}{P^2} P_s \sum_{i=1}^L \alpha_{l,i}^2 + \frac{\beta N_o^2}{2P^3}} \right]^{1/2} \right). \quad (20)$$

If the transmitted energy is $E_b = 2P \left(\frac{\beta}{P} P_s \right)$, then $P_s = \frac{E_b}{2\beta}$. Substituting $P_s = \frac{E_b}{2\beta}$ and $\psi = \frac{\operatorname{Var}[x_{i,k}^2]}{P_s^2}$, we have

$$BER = \frac{1}{2} \operatorname{erfc} \left(\left[2 \frac{\psi P}{\beta} + \frac{4N_o}{\sum_{i=1}^L \alpha_{l,i}^2 E_b} + \frac{2\beta N_o^2}{P E_b^2 \left(\sum_{i=1}^L \alpha_{l,i}^2 \right)^2} \right]^{-1/2} \right). \quad (21)$$

$$\begin{aligned} BER &= \frac{1}{2} \operatorname{erfc} \left(\left[2 \frac{\psi P}{\beta} + \frac{4}{\sum_{i=1}^L \gamma_l^2} + \frac{2\beta}{P \left(\sum_{i=1}^L \gamma_l^2 \right)^2} \right]^{-1/2} \right) \\ &= \frac{1}{2} \operatorname{erfc} \left(\left[2 \frac{\psi P}{\beta} + \frac{4}{\gamma} + \frac{2\beta}{P \gamma^2} \right]^{-1/2} \right), \end{aligned} \quad (22)$$

where $\gamma_l = \frac{E_b}{N_o} \alpha_l^2$ is the instantaneous SNR of the l th path. Equation (22) gives the theoretical BER of the CCS-based NR-DCSK system over multipath fading channels. However, it is essential to note that the channel keeps changing at every transmission instant; hence, the average lower bound on the BER expression becomes

$$\overline{BER} = \frac{1}{2} \int_0^\infty \operatorname{erfc} \left(\left[2 \frac{\psi P}{\beta} + \frac{4}{\gamma} + \frac{2\beta}{P \gamma^2} \right]^{-1/2} \right) f(\gamma) d\gamma, \quad (23)$$

where $f(\gamma)$ is the Probability Density Function (PDF) of instantaneous SNR.

3.1 PDF of SNR

The instantaneous SNR at the output of the correlator can be given from (22) as

$$\gamma = \sum_{l=1}^L \gamma_l, \quad (24)$$

where $\gamma_l = \frac{E_b}{N_o} \alpha_l^2$. From (24), the average instantaneous SNR can be defined as $\bar{\gamma} = E[\gamma] = \frac{E_b}{N_o}$, assuming $\sum_{l=1}^L \alpha_l^2 = 1$. For L i.i.d. Rayleigh-fading channels, the PDF of the instantaneous γ can be written as [14]

$$f(\gamma) = \frac{1}{(L-1)! (\bar{\gamma})^L} \gamma^{L-1} \exp \left(-\frac{\gamma L}{\bar{\gamma}} \right). \quad (25)$$

Substituting (25) in (23) we get

$$\begin{aligned} \overline{BER} &= \frac{1}{2} \int_0^\infty \operatorname{erfc} \left(\left[2 \frac{\psi P}{\beta} + \frac{4}{\gamma} + \frac{2\beta}{P \gamma^2} \right]^{-1/2} \right) \\ &\quad \frac{1}{(L-1)! (\bar{\gamma})^L} \gamma^{L-1} \exp \left(-\frac{\gamma L}{\bar{\gamma}} \right) d\gamma. \end{aligned} \quad (26)$$

In this expression, the erfc has a quadratic argument function; it is difficult to solve the integration in (26) and get a closed-form expression. Hence the average BER is numerically evaluated using the Gauss–Laguerre numerical integration technique.

4. Simulation results and discussion

To begin the investigation on CCS-based NR-DCSK system proposed in this work, and to compare it to the existing NR-DCSK schemes, BER performance is first analysed for

Table 2. System parameters.

Symbol	Quantity	Value
M	No. of information bits	10,000
β	Spreading length	300
P	Repetition factor	20
β/P	Reduction factor	15
SNR	Signal-to-noise ratio	0–20 dB
T_c	Chip duration	1
$T_b = 2\beta T_c$	Bit duration	600
A	Amplitude	2 V

Figure 2. BER performance of NR-DCSK with $\beta = 300$ for $P = 20$ over an AWGN channel.

an NR-DCSK system using chaotic sequences from a 1-D logistic map for various bifurcation parameters (r) under an AWGN channel. The average BER expression derived in (26) is numerically evaluated and compared to simulation results. Monte Carlo simulations are carried out to evaluate the performance of CCS-based NR-DCSK system. The parameters and its specific values used for performing simulation and analytical results are listed in table 2.

Figure 2 shows the BER performance of NR-DCSK system for varying r of the logistic map and the CCS generated by combining the sequences obtained from logistic map for varying r . For a slight change in the bifurcation parameter r to 1.8, 1.9 and 2, we obtain a completely different sequence. A CCS is now generated by combining the chaotic sequence obtained from logistic map with $r = 1.8, 1.9$ and 2 and is then used as carrier sequence for transmitting the data bits in NR-DCSK system. Here the CCS is a sum sequence with stronger initial sensitivity and a high level of complexity. To prove this, the nature of the sequences generated from logistic map with $r = 1.8, 1.9$ and 2 for an initial condition of 0.1304 and the CCS obtained from the sequences are shown in table 3.

Thus, the CCS is a kind of spread spectrum sequence which can be easily generated and bears a high level of randomness and good spread spectrum performance. These sequences overcome severe interference levels encountered during transmission, and provide an improvement in BER of the NR-DCSK system as in figure 2. Further enhancement in BER performance of NR-DCSK system could be

Table 3. Nature of conventional chaotic sequence and compound chaotic sequence.

$f(x_k) = 1 - rx_{k-1}^2$, $r = 1.8, 1.9, 2$, initial condition = 0.1034			
$r = 1.8$	$r = 1.9$	$r = 2$	CCS
0.1034	0.1034	0.1034	0.3102
0.9786	0.9797	0.9808	2.9391
-0.9154	-0.8236	-0.7314	-2.4704
-0.6758	-0.2888	0.0371	-0.9275
0.0865	0.8416	0.9975	1.9255
0.9851	-0.3456	-0.7911	-0.1516
-0.9406	0.7730	-0.1264	-0.2940
-0.7696	-0.1354	0.9712	0.0662
-0.1847	0.9652	-0.6979	0.0826
0.9318	-0.7699	0.1233	0.2852
-0.7365	-0.1262	0.9726	0.1100
-0.0848	0.9697	-0.7028	0.1821
0.9856	-0.7868	0.1108	0.3097
-0.9429	-0.1761	0.9779	-0.1411
-0.7782	0.9411	-0.7213	-0.5585

achieved if we concentrate on the type of chaotic maps used to generate the CCSs.

Figure 3 shows the nature of the CCS generated by combining the chaotic sequences generated using Bernoulli shift map and tent map for a given initial condition. Figures 4 and 5 show the nature of the CCS generated by

Figure 3. (a) Compound chaotic sequence (CCS) obtained by combining the outputs of Bernoulli shift map and tent map (CCS-BT) and (b) waveform of the compound chaotic carrier sequence used in NR-DCSK system ($\beta = 300$, $P = 20$, $\beta/P = 15$).

Figure 4. (a) Compound chaotic sequence (CCS) obtained by combining the outputs of logistic map, Chebyshev map and the Bernoulli shift map (CCS-LCB). (b) Waveform of the compound chaotic carrier sequence used in NR-DCSK system ($\beta = 300$, $P = 20$, $\beta/P = 15$).

Figure 5. (a) Compound chaotic sequence (CCS) obtained by combining the outputs of logistic map, Chebyshev map and Tent map (CCS-LCT). (b) Waveform of the compound chaotic carrier sequence used in NR-DCSK system ($\beta = 300$, $P = 20$, $\beta/P = 15$).

Figure 6. BER performance of CCS-based NR-DCSK system over AWGN channel for $\beta = 300$ with $P = 20$. Performance is analysed for 100 random initial conditions. CCS-LCT performs better compared with all other combinations of CCS.

Table 4. BER performance comparison between conventional and compound chaotic maps.

Type of the map	SNR at BER of 10^{-4} (dB)
Logistic map (L)	15
Chebyshev map (C)	16
Bernoulli shift map (B)	25
Tent map (T)	21
Bernoulli shift + tent (BT) map	19
Logistic + Chebyshev + Bernoulli shift (LCB) map	13
Logistic + Chebyshev + tent (LCT) map	11

Figure 7. BER performance of the CCS-based NR-DCSK system in Rayleigh fading channel.

combining the outputs of logistic, Chebyshev and Bernoulli shift map and logistic, Chebyshev and tent map, respectively.

In figure 6, the BER performance of CCS-based NR-DCSK is analysed for various combinations of CCS like Bernoulli shift and tent map (CCS-BT), logistic, Chebyshev and Bernoulli shift map (CCS_LCB), logistic, Chebyshev and tent map (CCS-LCT). It is observed that at 10 dB, the simulated BER of NR_DCSK system using Bernoulli shift and tent map is 0.1491 and 0.0982, respectively, whereas the BER of CCS-BT-based NR-DCSK system is 0.0697. There is a considerable reduction in the BER of the proposed system. Further enhancement in BER is witnessed in CCS-LCB system and CCS-LCT system with BERs 0.0070 and 0.0012 at SNR = 10 dB, respectively.

Moreover, BER of 10^{-4} is achieved at 15 dB in an NR-DCSK system with $P = 20$ using 1-D logistic map proposed in [11], whereas the same BER is obtained at 11 dB in the case of CCS-LCT, and at 13 dB in the case of CCS-

Figure 8. BER performance of CCS-based NR-DCSK over multipath Rayleigh fading channels for $\beta = 300$ with $P = 20$, $L = 2, 3, 4, 6$ and exponential decay path gains.

LCB. Thus, the gain achieved by CCS-based NR-DCSK system is 4 dB. This enhancement is due to the fact that the amount of randomness created in CCS is more and the effect of noise is considerably reduced.

In order to achieve a BER of 10^{-4} , the required SNR estimated from the simulation results of an NR-DCSK system using conventional chaotic sequences and different combination of CCS is listed in table 4.

The performance of the CCS-based NR-DCSK is analysed in Rayleigh fading channel. The average BER expression derived in (26) is numerically evaluated and compared to the MATLAB simulation results. The CCS-LCT map is used to generate the chaotic sequence. In the simulation we use $\beta = 300$, $P = 20$, $\beta/P = 15$ and multipath $L = 2$. The two paths have average power gain of $E[\alpha_1^2] = 0.7$ and $E[\alpha_2^2] = 0.3$. The time delay between two paths is assumed to be $\tau = 5T_c$.

Average BER of the CCS-based NR-DCSK is evaluated by numerical integration of Eq. (26). In figure 7, average BER is plotted against the average SNR. The figure demonstrates the validation of numerical and simulation results of the proposed system. For example, at 14 dB average SNR, average BER obtained through numerical integration is 0.0318 whereas the BER obtained through simulation result is 0.0310. This shows that the simulation results are in close agreement with the numerical results.

To demonstrate the effect of multipath on the system performance, we have simulated the CCS-based NR-DCSK system for different numbers of paths $L = 2, 3, 4$ and 6 with average power gain over the number of multipath L satisfying $\sum_{l=1}^L \alpha_{l,i}^2 = 1$. For an example, for $L = 4$, the average power gain chosen for the first through four paths $E[\alpha_1^2] = 0.5$, $E[\alpha_2^2] = 0.3$, $E[\alpha_3^2] = 0.15$ and $E[\alpha_4^2] = 0.05$,

in accordance with the delays $\tau_1 = 0$, $\tau_2 = T_c$, $\tau_3 = 2T_c$ and $\tau_4 = 5T_c$ that exist between the paths, respectively. The results of this specific simulation shown in figure 8 indicate that the CCS-based NR-DCSK system takes advantage of the space diversity offered by the multipath environment to enhance the BER performance when the number of paths increases.

5. Conclusions

A CCS-based NR-DCSK system is proposed in this paper. The aim of this work is to improve the BER performance of the NR-DCSK system with the use of CCSs while preserving the advantages of conventional DCSK, NR-DCSK system and being easy to implement. For a given initial condition, chaotic samples from distinct chaotic maps are summed up to obtain the compound chaotic sample possessing the combined nonlinear dynamics of the various maps. This CCS helps in providing more randomness, overcomes severe interference levels encountered during transmission and provides higher multipath immunity compared with that of pseudo noise (PN) codes. Having presented the method of generating the CCS, the BER expression for CCS-based NR-DCSK system is derived. The mathematically obtained BER expressions are numerically evaluated and plotted, and the performances of the NR-DCSK system for various combinations of CCS are studied. From the simulated results, CCS generated by combining logistic, Chebyshev and tent map (CCS-LCT) provides better results compared with other CCS inheriting the basic characteristics of the individual maps in the compound sequence. With CCS-LCT, the performance analysis of NR-DCSK system over L-path Rayleigh fading channels is presented. The numerical results match closely with the simulation results. The proposed CCS provides an SNR gain of 4 dB with respect to the conventional chaotic sequence. In conclusion, taking BER performance under multipath channels into consideration, the performance of the proposed CCS-based NR-DCSK system may pave the way for the establishment of a new method for generation of chaotic sequence for both coherent and non-coherent chaos-based communication systems.

Appendix

This appendix provides the procedure to obtain simulation results. The BER performance of the CCS-based NR-DCSK system is tested for 100 random initial conditions used for the generation of compound chaotic carrier sequences. Hence, the simulation has one main program that runs the subroutine 100 times. Every time we obtain a

BER, it is averaged to obtain the final BER for one particular E_b/N_o . The second program is the simulation of NR-DCSK system that takes the initial conditions as the input, and returns the BER for one particular value of E_b/N_o . The third program is again a subroutine used for generating NR structure of the compound chaotic carrier sequences that take the initial conditions and reduction factor $\frac{\beta}{P}$ from the second program as the inputs and return the NR structure of the chaotic carrier sequences to the second program, which then performs the simulation of the CCS-based NR-DCSK system. The simulation of CCS-based NR-DCSK system comprises the following steps:

Algorithm 1: Computation of Average BER using No. of trails

Input : No. of trials
Output: Average_BER

```
01: in=Generate (random initial condition)
02: CALL (Simulation of NR_DCSK m file with initial condition generated in Step (01))
03: Let B be an array to store BER
04: B=GET BER
05: Repeat (01) to (03) for number of trials
06: Calculate Average_BER
```

Algorithm 2: Simulation of CCS based NR-DCSK System

Input : A, T_b, M, β , P, T_c, SNR, Initial Condition (in)
Output: BER

// Process at NR-DCSK Communication System //

```
01: Calculate Reduction factor  $\beta/P$  and assign to variable N
02: d= GENERATE (Information bits)
03: C=CALL (Generation of CCS m file)
04: a= d × C;
05: for k =1: M do
06:   S(k,:)= [ C, a(k,:)]
07: end
08: Received signal= S+AWGN Noise
09: for i=1: M do
10:   Initialize q=1;
11:   for h=1: p: 2*L_c do
12:     rr(i, q)=Moving average of the received signal
13:   end
14: end
15: for ii=1: M do
16:   r1(ii,:)= rr(ii, N+1:2N);
17:   r2(ii,:)= rr(ii, 1:N);
18:   Correlator output= ADD (r1, r2);
19: end
20: // BER Computation //
21: Initialize COUNT to be zero
22: for kk = 1: length (d) do
23:   de (kk, :) = Determine the sign of the real part of the correlator output,
    multiplied by the amplitude A
24:   if (de not equal to d) then
25:     INCREMENT the COUNT by 1
26:   end
27: end
28: BER= COUNT/M;
```

Algorithm 3: Generation of compound chaotic sequence (CCS)**Input :** N; in; P;**Output:** Z

// Process at Transmitter Side //

```

01: To GENERATE (Chebyshev Map of degree 3)
02: for j=1: N do
03:   if (j==1) then
04:     x(j)=in;
05:   else
06:     x(j)=(4*(x(j-1)^3)-(3*x(j-1)))
07:   end
08: end
09: x;
10: To GENERATE (Tent Map)
11: Initialize system parameter of map A=0.5; B=1.95;
12: y(1)=A-(B*in);
13: for it= 2:1: N do
14:   y(it)=A-(B*abs(y(it-1)));
15: end
16: y;
17: To GENERATE (Logistic Map)
18: for i=1: N do
19:   if (i==1)
20:     z(i)=in;
21:   else
22:     z(i)=1-2*(z(i-1))^2;
23:   end
24: end
25: Seq=ADD (x, y, z) to get compound sequence
26: To GENERATE (Noise Reduction Structure of Compound Chaotic Sequence)
27: for jj=1 : N do
28:   S(jj,:)=repelem(Seq(jj), 20);
29: end
30: Z=[S(1,:) S(2,:) S(3,:) ..... S(N, :)]

```

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