Formulation of new correlations in terms of extraterrestrial radiation by optimization of tilt angle for installation of solar photovoltaic systems for maximum power generation: case study of 26 cities in India

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Abstract. This paper deals with finding of optimum tilt angle for installation of optimized solar photovoltaic system in India. The optimization of tilt angles is performed using measured solar radiation data for 26 cities in India. The tilt angle is changed from 0° to 90° at step of 1° to find out optimum tilt angle for which monthly average solar radiation is maximum. It was found that the optimum tilt angle varies between 0° and 63° throughout the year in India. The monthly optimum tilt angle is maximum in December for all cities in India. It is found that increase in maximum solar radiation at monthly optimum tilt angle in comparison to latitude based tilt angle and annual optimum tilt angle varies from 5.85% to 8.08% and 5.95% to 8.34%, respectively, showing monthly optimum tilt angle is beneficial for maximum power generation for different cities in India. The novelty of this study is new correlations of optimum tilt angle in terms of extraterrestrial radiation are developed for estimating monthly optimum tilt angle for installation of photovoltaic systems at different sites in India. The root mean square error in correlation equations varies from 0.1256 to 0.9771.

Keywords. Solar photovoltaic system; optimum tilt angle; correlations.

1. Introduction

India has huge solar potential due to its location between the tropic of cancer and the equator. The mean daily global solar radiation of 5 kWh/m²/day is incident on 58% part of India showing good option for solar energy applications [1]. The Indian government has taken a major initiative to promote solar energy in the country by launching Jawaharlal Nehru National Solar Mission (JNNSM) in 2010 with a target of 175000 MW by 2022. The objective of National Solar Mission is to establish India as a global leader in solar energy [2]. The solar installed capacity up to 31 March 2017 is 12,288.83 MW and annual growth is 81.71% in India. The solar energy is utilized through standalone photovoltaic (SAPV) system which converts solar radiation (SR) directly into electricity. These systems are one of the best options for urban, rural and remote locations, including rooftop and building integrated applications. The SR is a function of days, site and time. Therefore to capture maximum SR for maximum power, SAPV system should be

installed at optimum tilt angle so that it faces the sun at all times. The optimum tilt angles (OPTA) are found by searching the angles from 0° to 90° for which the total incident radiation on the PV surface is maximum for the period under consideration [3].

The power generated from the SAPV system in India faces many difficulties such as non-availability of measured SR at different locations and optimum sizing i.e., system cost [4, 5]. Estimation of monthly incident SR on tilted surface is necessary for maximum power generation. The monthly average daily SR data (Indian cities) incident on horizontal surface is available in literature [6, 7]. However very few study related to radiation on tilted surface is reported. Thakur and Chandel [8] calculated optimum tilt angle and studied its effect on energy yield of 190 kWp grid-interactive solar power plant installed at Khatkar-Kalan in India. The energy yield at yearly, seasonally and monthly optimum tilt angle are found to be 25%, 28%, and 29% more in comparison to fixed tilt (25°) angle of the plant. Yadav and Chandel [9] analyzed different diffuse SR model for calculation of OPTA for Hamirpur, Himachal Pradesh (H.P.) and found that Liu and Jordan model give

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better result with 4.512% error between calculated and measured optimum tilt angle. Yadav *et al* [10] calculated OPTA and developed correlations of annual optimum tilt angle as a function of latitude for 15 cities of H.P. using predicted value of SR and found that OPTA varies from 0° to 65° for H.P. cities. Agarwal *et al* [11] calculated tilt angle for Delhi and Nandha in Haryana using Liu and Jorden, Reindel, Hay and Badescu models. The optimum tilt angles are 28.29°, 29.41°, 29.08° and 27.58° for Delhi and it is 30.61°, 31.66°, 31.20° and 29.58° for Nandha.

The objective of this study is to determine OPTA for installation of solar photovoltaic system in 26 different cities in India for maximizing incident SR and to obtain new correlations for estimating the OPTA that are useful for industry.

The paper is organized as follows: literature survey overview is given in section 2 and methodology is presented in section 3. The results and discussion are given in section 4 and conclusions are provided in section 5.

2. An overview of literature survey for OPTA calculation

A number of studies for OPTA calculation for various applications around the world have been presented. However, studies carried out in India are few in number for different sites and the general correlation giving OPTA for India are not obtained.

2.1 OPTA calculation for maximize solar radiation

The OPTA are found by searching the angles for which the total radiation incident on the PV surface is maximum for the period under consideration. Ulgen [12] computed OPTA for Izmir, Turkey as summer 4.3°, winter 55.7° and yearly 30.3°. The energy collected at monthly and yearly tilt angle are 6747.17 and 639 MJ/m²/year. Talebizadeh *et al* [13] used Genetic Algorithm for calculating hourly, daily, monthly, seasonally and yearly OPTA in Iran. The gain in SR on seasonally, monthly and daily OPTA are 5.13%, 6.05% and 6.89%, showing a gain at monthly and seasonal are almost same. In addition, the energy gain decreases by ignoring ground reflectivity. Pour *et al* [14] calculated minimum and maximum OPTA as 0.15° and 57.74° using the isotropic Liu model for Isfahan. The SR gain varies from 0.04% to 62% at a monthly OPTA. Kacira *et al* [15] determined monthly OPTA of 13° in June and 61° in December for Sanliurfa, Turkey. The PV panel mounted on a monthly tilt angle, obtained SR of 1.1% and 3.9% with respect to seasonal and latitude based tilt angle. Due to two-axis tracking system the gain in SR is 29.3%, resulting in 34.6% gain in power generation. Hartley *et al* [16] utilized Hays model for analyzing hourly deviation in OPTA for Spain. It is found that monthly average SR data is more accurate in obtaining OPTA in

comparison to hourly SR data. Mehleri *et al* [17] tested different types of diffuse models for OPTA formulation and found that “Liu and Jordan” model is accurate. The results demonstrate that changing the tilt angle in cold and hot months enhance generated power reliability. Siraki and Pillay [18] developed modified HDKR (Hay, Davies, Klucher, Reindl) model which consider building shading for calculating the OPTA in urban areas. For a similar clearness index the OPTA is close to latitude for small altitude and it is smaller than latitude for high altitude. Chang [19] used Particle Swarm Optimization technique for calculating the OPTA in Taiwan. The suggested tilt angle is Taipei area 18.16°, Taichung 17.3°, Tainan 16.15°, Kaosiung 15.79°, Hengchung 15.17°, Hualian 17.16° and Taitung 15.94°. The PV module temperature is high in Kaohsiung in comparison with Hengchung therefore the annual efficiency of a PV module is high in Hengchung. Chatterjee and Keyhani [20] utilized input parameters as latitude, ground reflectivity and 12 months irradiance for estimating OPTA and maximum solar radiation. The difference of 3° is observed between estimated and calculated OPTA, showing close estimation. This method can be used for estimation of energy output from PV station. Khorasanizadeh *et al* [21] used anisotropic model to determine OPTA of south-facing solar collectors and solar panels in Tabass city situated in sunny region of Iran. The monthly OPTA vary from 0° in June and July to 64° in December. Bakirci [22] developed correlations of OPTA in terms of declination angles for Turkey and correlation coefficients are found to be greater than 99%. The correlations are as follows.

$$\beta_{opt} = 32.619 - 1.3629\delta \quad (1)$$

$$\beta_{opt} = 34.784 - 1.3621\delta - 0.0081(\delta)^2 \quad (2)$$

$$\beta_{opt} = 34.783 - 1.4317\delta - 0.0081(\delta)^2 + 0.0002(\delta)^3 \quad (3)$$

The SR on SAPV system is maximized by OPTA and tracking system, resulting in maximum power generation. Abdallah [23] investigated the effect of two axes, one axis vertical, one axis east–west and one axis north–south tracking system on electrical characteristics of flat plate PV system. The gain in electrical power is increased by 43.87%, 37.53%, 34.43% and 15.69% for the two axes, east–west, vertical and north–south tracking respectively in comparison to fixed tilt angle of 32° for Amman, Jordan.

Beringer *et al* [24] investigated the performance of solar cell at OPTA in Germany. It is found that tilt angle effect on performance is minimum due to temperature effects. Bojić *et al* [25] used Hooke-Jeeves algorithm for computing OPTA at which electrical energy generation is maximized. Chang [26] used sequential neural-network approximation and orthogonal arrays (SNAOA) to estimate the OPTA for maximum output power energy of PV modules in seven regions of Taiwan. The annual OPTA is 23.251° for Taipei site.

Table 1. Latitude based OPTA correlations.

Study	Correlation of OPTA	Application
Heywood [28]	$\phi - 10^\circ$	The minus sign refers to the summer season and the positive sign is for the winter
Lunde [29]	$\phi \pm 15^\circ$	
Chinnery [30]	$\phi + 10^\circ$	
Kern and Harris [31]	$\phi + 10^\circ$	
Yellot [32]	$\phi + 20^\circ$	
Elminir <i>et al</i> [33]	$(\phi + 15^\circ) \pm 15^\circ$	
Tang and Wu [34]	$\phi - 3^\circ, \phi + 4^\circ, \phi - 5^\circ, \phi - 10^\circ$	
Talebizadeh <i>et al</i> [35]	1.073 ϕ + 10.3 (winter), 0.48850 ϕ - 10.27 (spring), 0.2637 ϕ + 4.961 (summer), 0.8966 ϕ + 23.81 (autumn), 0.6804 ϕ + 7.203 (yearly)	

Roux [27] performed study on solar tracker for locations in South Africa and suggested that for solar collector a fixed tilt angle of 30° collects 98% of maximum SR. The plots of contour are presented to show the effect of fixed tilt and azimuth angle on maximum SR.

Several authors have developed correlations of OPTA in terms of latitude (ϕ) as shown in table 1.

Moreover, in a comprehensive review by Yadav and Chandel [36], the OPTA calculation has been identified as an important research area for maximum power generation by maximizing SR.

3. Methodology

3.1 Optimum tilt angle determination

The extraterrestrial radiation is the SR received outside earth's surface and its intensity varies throughout the year. The average value of extraterrestrial radiation i.e., the solar constant is $S_0 = 1367 \text{ W/m}^2$.

The polar axis of earth is inclined at an angle of 66.55° to the elliptical plane and by 23.45° from perpendicular to elliptical plane. The rotation of the earth on its inclined polar axis with respect to elliptic plane is responsible for different seasons on earth. It causes lengthy days in summer and shorter days in winter. The angle made by the lines joining the centre of the earth to centre of the sun with its projection on the equatorial plane of the earth is called the declination angle. It varies due to the inclination of the earth's polar axis and its revolution around the sun. It varies between -23.45° and 23.45° .

The angular displacement of the sun about east or west of local meridian due to the rotary motion of earth on axis is called hour angle (w). It is an expression explaining the variation between local solar time and solar noon. After solar noon the hour angle measures time in duration of 1° for each 4 minutes or 15° per hour. The hour angle is positive after solar noon and it is negative before solar noon.

The incident radiation reaches the earth's surface without being absorbed or scattered, is called beam radiation. Some

of the radiation from the sun is scattered back to the atmosphere and a part of it is scattered to earth, this scattered radiation reaching the earth's surface is called diffuse radiation (H_d).

When the SR reaches the earth's surface, some of it is reflected by the ground and other objects on the ground. This radiation is called reflected radiation (H_r). The total SR on a horizontal surface is called global radiation (H_g) which is the sum of the beam, diffused and reflected radiations. For maximum utilization of SR on PV panel the tilt angle (β) is used which lies between 0° and 90° [37]. The angle between the plane of the PV panel surface and horizontal is called tilt angle (β) is shown in figure 1. The optimum tilt angle for maximizing incident SR (β_{opt}) is calculated by changing β between 0° to 90° at step of 1° for which H_T is maximum as shown in figure 2 [38].

4. Results and discussion

The monthly OPTA (β_{opt}) are calculated using developed program in MATLAB (R2011a) based on flow chart. This program utilizes monthly average SR (table 2) and for obtaining monthly OPTA, β is changed from 0° to 90° at an

Figure 1. PV panel installed at a tilt angle.

Figure 2. Flow chart for OPTA calculation.

interval of 1°. The tilt angle β which cause the maximum value of H_T is selected by the program as the optimum tilt angle β_{opt} for that month (table 2).

It was found that the OPTA changed between 0° (April, May, June, July and August) and 41° (December) for Minicoy, 0° (April, May, June, July and August) and 42°

(December) for Thiruvananthpuram, 0° (April, May, June, July and August) and 44° (December) for Port Blair, 0° (April, May, June, July and August) and 45° (December) for Bangalore, 0° (April, May, June, July and August) and 45° (December) for Chennai, 0° (April, May, June, July and August) and 48° (December) for Panjim, 0° (April, May,

Table 2. Monthly average daily global (measured), diffuse (calculated) and extraterrestrial SR on a horizontal surface, clearness index, optimum tilt-angle and monthly average daily global radiation on optimum tilted surface for 26 cities of India.

Cities	Months	H_g	H_d	H_0	K_t	β_{opt}	H_T	HL	HY
Minicoy	Jan.	4.93	0.92	9.48	0.52	39	6.05	5.35	5.55
	Feb.	5.61	1.04	9.82	0.57	28	6.21	5.91	6.04
	Mar.	6.05	1.07	10.37	0.58	12	6.16	6.15	6.16
	Apr.	5.93	1.13	9.85	0.60	0	5.93	5.81	5.69
	May	5.05	0.90	9.88	0.51	0	5.05	4.79	4.60
	Jun.	4.44	0.85	9.21	0.48	0	4.44	4.15	3.95
	Jul.	4.58	0.82	9.89	0.46	0	4.58	4.30	4.11
	Aug.	4.88	0.85	10.12	0.48	0	4.88	4.71	4.58
	Sep.	5.09	0.90	9.98	0.50	5	5.10	5.09	5.06
	Oct.	5.003	0.86	10.27	0.48	23	5.36	5.21	5.29
	Nov.	4.63	0.82	10.04	0.46	36	5.53	4.99	5.16
	Dec.	4.6	0.83	9.80	0.47	41	5.84	5.04	5.25
Thiruvananthapuram	Average	5.07	0.92	9.89	0.51	13.18	5.43	5.12	5.12
	Jan.	5.53	1.07	9.47	0.58	39	6.78	6.01	6.23
	Feb.	6.12	1.20	9.82	0.62	28	6.77	6.45	6.59
	Mar.	6.5	1.21	10.36	0.62	12	6.62	6.61	6.62
	Apr.	5.93	1.13	9.85	0.60	0	5.93	5.80	5.69
	May	5.44	0.99	9.88	0.55	0	5.44	5.15	4.96
	Jun.	4.82	0.93	9.21	0.52	0	4.82	4.50	4.29
	Jul.	4.95	0.88	9.89	0.50	0	4.95	4.65	4.45
	Aug.	5.27	0.92	10.12	0.52	0	5.27	5.09	4.94
	Sep.	5.7	1.04	9.97	0.57	5	5.72	5.71	5.67
	Oct.	5.04	0.86	10.26	0.49	23	5.41	5.26	5.34
	Nov.	4.6	0.82	10.03	0.45	36	5.50	4.96	5.13
Port Blair	Dec.	5.01	0.90	9.79	0.51	42	6.38	5.50	5.73
	Average	5.41	0.99	9.89	0.54	13.27	5.80	5.47	5.47
	Jan.	5.12	0.98	9.37	0.54	42	6.50	5.77	5.92
	Feb.	5.85	1.13	9.72	0.60	31	6.63	6.32	6.41
	Mar.	5.89	1.04	10.26	0.57	16	6.07	6.06	6.07
	Apr.	5.76	1.09	9.75	0.59	0	5.76	5.61	5.54
	May	4.37	0.80	9.77	0.44	0	4.37	4.08	3.98
	Jun.	3.87	0.78	9.10	0.42	0	3.87	3.54	3.43
	Jul.	3.82	0.76	9.78	0.39	0	3.82	3.53	3.43
	Aug.	4.02	0.76	10.01	0.40	0	4.02	3.85	3.78
	Sep.	4.3	0.79	9.88	0.43	8	4.33	4.32	4.31
	Oct.	4.48	0.80	10.16	0.44	26	4.90	4.77	4.82
Bangalore	Nov.	4.65	0.83	9.93	0.46	39	5.74	5.19	5.31
	Dec.	4.74	0.86	9.68	0.49	44	6.26	5.42	5.58
	Average	4.74	0.89	9.78	0.48	14.9	5.19	4.87	4.88
	Jan.	5.67	1.14	9.33	0.60	42	7.25	6.45	6.60
	Feb.	6.48	1.37	9.68	0.66	32	7.38	7.05	7.14
	Mar.	6.58	1.27	10.23	0.64	16	6.80	6.79	6.80
	Apr.	6.56	1.39	9.71	0.67	0	6.56	6.39	6.32
	May	6.35	1.30	9.73	0.65	0	6.35	5.91	5.78
	Jun.	4.92	0.97	9.06	0.54	0	4.92	4.47	4.35
	Jul.	4.64	0.84	9.74	0.47	0	4.64	4.26	4.15
	Aug.	4.48	0.81	9.98	0.44	0	4.48	4.28	4.21
	Sep.	5.24	0.94	9.84	0.53	9	5.30	5.29	5.27
	Oct.	5.11	0.89	10.12	0.50	28	5.64	5.48	5.54
	Nov.	4.84	0.86	9.89	0.49	40	6.05	5.47	5.58
	Dec.	4.81	0.88	9.64	0.49	45	6.44	5.57	5.72
	Average	5.47	1.05	9.75	0.56	15.45	5.98	5.62	5.62

Table 2 continued

Cities	Months	H_g	H_d	H_0	K_t	β_{opt}	H_T	HL	HY
Chennai	Jan.	4.89	0.93	9.31	0.52	43	6.32	5.61	5.73
	Feb.	5.85	1.14	9.66	0.60	32	6.71	6.40	6.48
	Mar.	6.51	1.25	10.21	0.63	17	6.75	6.73	6.74
	Apr.	6.6	1.41	9.69	0.68	0	6.60	6.42	6.36
	May	6.26	1.27	9.71	0.64	0	6.26	5.81	5.70
	Jun.	5.71	1.23	9.04	0.63	0	5.71	5.19	5.07
	Jul.	5.27	0.97	9.71	0.54	0	5.27	4.83	4.72
	Aug.	5.2	0.92	9.95	0.52	0	5.20	4.96	4.88
	Sep.	5.39	0.98	9.82	0.54	10	5.45	5.44	5.43
	Oct.	4.55	0.81	10.10	0.45	28	5.04	4.90	4.94
	Nov.	3.99	0.77	9.86	0.40	40	4.99	4.53	4.61
	Dec.	4.15	0.79	9.62	0.43	45	5.57	4.83	4.94
	Average	5.36	1.04	9.72	0.55	15.63	5.82	5.47	5.47
Panjim	Jan.	5.52	1.12	9.19	0.60	45	7.32	6.53	6.60
	Feb.	6.22	1.29	9.55	0.65	34	7.27	6.90	6.99
	Mar.	6.54	1.28	10.10	0.64	19	6.85	6.84	6.84
	Apr.	6.72	1.50	9.59	0.70	0	6.72	6.55	6.51
	May	6.56	1.43	9.59	0.68	0	6.56	6.05	5.99
	Jun.	4.63	0.91	8.92	0.51	0	4.63	4.14	4.09
	Jul.	4.1	0.78	9.59	0.42	0	4.10	3.71	3.67
	Aug.	4.4	0.80	9.84	0.44	0	4.40	4.18	4.15
	Sep.	5.38	0.99	9.72	0.55	12	5.48	5.48	5.47
	Oct.	5.42	0.97	9.99	0.54	30	6.11	5.95	5.98
	Nov.	5.32	0.97	9.74	0.54	43	6.87	6.22	6.28
	Dec.	5.16	0.97	9.49	0.54	48	7.18	6.23	6.31
	Average	5.5	1.09	9.61	0.57	16.9	6.12	5.73	5.74
Hyderabad	Jan.	5.45	1.12	9.09	0.59	46	7.41	6.62	6.65
	Feb.	6.11	1.28	9.45	0.64	36	7.28	6.96	6.98
	Mar.	6.72	1.38	10.00	0.67	21	7.11	7.09	7.10
	Apr.	6.9	1.62	9.49	0.72	2	6.91	6.73	6.71
	May	6.63	1.49	9.49	0.69	0	6.63	6.07	6.05
	Jun.	5.59	1.23	8.82	0.63	0	5.59	4.97	4.94
	Jul.	5.13	0.96	9.49	0.54	0	5.13	4.61	4.59
	Aug.	4.87	0.88	9.74	0.50	0	4.87	4.62	4.60
	Sep.	5.49	1.03	9.63	0.57	14	5.62	5.61	5.61
	Oct.	5.18	0.93	9.89	0.52	32	5.94	5.78	5.79
	Nov.	5.02	0.92	9.64	0.52	44	6.64	6.01	6.04
	Dec.	4.98	0.94	9.39	0.53	49	7.12	6.19	6.23
	Average	5.67	1.15	9.51	0.59	18	6.35	5.94	5.94
Vishakapatnam	Jan.	4.83	0.94	9.09	0.53	47	6.61	5.89	5.92
	Feb.	5.55	1.08	9.45	0.58	37	6.65	6.34	6.36
	Mar.	6.06	1.14	10.00	0.60	21	6.42	6.40	6.41
	Apr.	6.38	1.38	9.49	0.67	2	6.39	6.22	6.20
	May	6.16	1.29	9.48	0.64	0	6.16	5.63	5.61
	Jun.	4.85	0.99	8.82	0.55	0	4.85	4.31	4.28
	Jul.	4.45	0.83	9.48	0.46	0	4.45	3.99	3.97
	Aug.	4.54	0.83	9.74	0.46	0	4.54	4.30	4.28
	Sep.	4.73	0.87	9.62	0.49	14	4.85	4.85	4.84
	Oct.	4.89	0.87	9.89	0.49	32	5.61	5.45	5.47
	Nov.	4.55	0.84	9.64	0.47	44	6.03	5.46	5.49
	Dec.	4.53	0.85	9.38	0.48	49	6.48	5.63	5.67
	Average	5.13	0.99	9.51	0.53	18.09	5.75	5.37	5.37

Table 2 continued

Cities	Months	H_g	H_d	H_0	K_t	β_{opt}	H_T	HL	HY
Pune	Jan.	4.8	0.94	9.02	0.53	48	6.66	5.94	5.95
	Feb.	5.71	1.15	9.39	0.60	38	6.90	6.59	6.60
	Mar.	6.41	1.27	9.93	0.64	22	6.83	6.82	6.82
	Apr.	6.8	1.59	9.43	0.72	3	6.81	6.63	6.62
	May	6.99	1.70	9.42	0.74	0	6.99	6.40	6.39
	Jun.	5.36	1.16	8.75	0.61	0	5.36	4.74	4.74
	Jul.	4.47	0.84	9.41	0.47	0	4.47	4.00	3.99
	Aug.	4.35	0.81	9.67	0.45	0	4.35	4.12	4.11
	Sep.	5.2	0.97	9.56	0.54	15	5.35	5.34	5.34
	Oct.	5.34	0.97	9.83	0.54	33	6.18	6.02	6.02
	Nov.	4.9	0.90	9.57	0.51	45	6.58	5.97	5.98
	Dec.	4.57	0.86	9.31	0.49	50	6.64	5.78	5.79
	Average	5.41	1.10	9.44	0.57	18.72	6.09	5.69	5.70
Mumbai	Jan.	4.6	0.90	8.99	0.51	48	6.44	5.74	5.75
	Feb.	5.41	1.06	9.36	0.57	38	6.59	6.28	6.29
	Mar.	6.17	1.20	9.90	0.62	23	6.60	6.59	6.59
	Apr.	6.61	1.51	9.40	0.70	4	6.62	6.45	6.44
	May	6.48	1.46	9.38	0.69	0	6.48	5.92	5.91
	Jun.	4.85	1.00	8.72	0.55	0	4.85	4.28	4.27
	Jul.	3.73	0.76	9.38	0.39	0	3.73	3.34	3.33
	Aug.	4.03	0.77	9.64	0.41	0	4.03	3.81	3.81
	Sep.	4.54	0.84	9.53	0.47	16	4.68	4.67	4.67
	Oct.	5.003	0.90	9.79	0.51	34	5.82	5.66	5.66
	Nov.	4.61	0.85	9.53	0.48	46	6.24	5.66	5.67
	Dec.	4.29	0.82	9.27	0.46	51	6.28	5.48	5.49
	Average	5.03	1.01	9.41	0.53	19.27	5.70	5.32	5.32
Nagpur	Jan.	4.48	0.89	8.85	0.50	50	6.46	5.77	5.75
	Feb.	5.33	1.06	9.23	0.57	40	6.64	6.33	6.32
	Mar.	6.09	1.19	9.78	0.62	25	6.59	6.58	6.57
	Apr.	6.65	1.57	9.27	0.71	6	6.68	6.50	6.51
	May	6.55	1.53	9.25	0.70	0	6.55	5.95	5.97
	Jun.	5.23	1.15	8.58	0.60	0	5.23	4.58	4.60
	Jul.	4.11	0.80	9.24	0.44	0	4.11	3.65	3.66
	Aug.	4.1	0.79	9.51	0.43	0	4.10	3.88	3.89
	Sep.	4.87	0.91	9.41	0.51	18	5.06	5.06	5.06
	Oct.	5.18	0.95	9.66	0.53	36	6.14	5.98	5.97
	Nov.	4.54	0.85	9.39	0.48	48	6.32	5.74	5.72
	Dec.	4.27	0.42	9.13	0.46	54	6.80	5.82	5.79
	Average	5.12	1.01	9.28	0.55	20.63	5.89	5.49	5.48
Bhavnagar	Jan.	4.97	0.33	8.80	0.56	53	7.76	6.76	6.69
	Feb.	5.81	1.23	9.18	0.63	40	7.25	6.93	6.90
	Mar.	6.71	1.45	9.73	0.68	25	7.27	7.26	7.25
	Apr.	7.28	1.95	9.23	0.78	6	7.32	7.12	7.15
	May	7.37	2.02	9.20	0.80	0	7.37	6.71	6.76
	Jun.	6.19	1.61	8.53	0.72	0	6.19	5.44	5.49
	Jul.	4.52	0.87	9.19	0.49	0	4.52	4.00	4.03
	Aug.	4.48	0.84	9.46	0.47	0	4.48	4.24	4.26
	Sep.	5.53	1.09	9.37	0.59	18	5.76	5.75	5.76
	Oct.	5.85	1.14	9.62	0.60	36	6.96	6.78	6.75
	Nov.	5.09	0.97	9.34	0.54	48	7.15	6.50	6.45
	Dec.	4.59	0.38	9.08	0.50	55	7.48	6.38	6.31
	Average	5.7	1.16	9.23	0.61	20.72	6.62	6.16	6.15

Table 2 continued

Cities	Months	H_g	H_d	H_0	K_t	β_{opt}	H_T	HL	HY
Kolkatta	Jan.	3.75	0.46	8.76	0.42	53	5.75	5.07	5.04
	Feb.	4.35	0.84	9.14	0.47	42	5.51	5.26	5.24
	Mar.	5.27	0.97	9.69	0.54	27	5.76	5.75	5.75
	Apr.	5.85	1.24	9.19	0.63	8	5.88	5.73	5.74
	May	5.73	1.20	9.15	0.62	0	5.73	5.18	5.20
	Jun.	4.76	1.01	8.49	0.56	0	4.76	4.15	4.17
	Jul.	4.19	0.82	9.14	0.45	0	4.19	3.70	3.72
	Aug.	4.32	0.82	9.42	0.45	2	4.32	4.09	4.10
	Sep.	4.13	0.80	9.32	0.44	20	4.32	4.32	4.32
	Oct.	4.24	0.80	9.57	0.44	37	5.09	4.95	4.94
	Nov.	3.84	0.77	9.30	0.41	49	5.42	4.94	4.92
	Dec.	3.52	0.51	9.03	0.38	54	5.60	4.84	4.81
	Average	4.5	0.85	9.188	0.49	21.72	5.20	4.83	4.83
Ahmedabad	Jan.	4.53	0.37	8.71	0.52	53	7.19	6.28	6.25
	Feb.	5.43	1.11	9.09	0.59	43	6.91	6.60	6.58
	Mar.	6.34	1.32	9.64	0.65	28	6.94	6.93	6.93
	Apr.	6.95	1.79	9.14	0.76	9	7.00	6.81	6.82
	May	6.99	1.83	9.10	0.76	0	6.99	6.34	6.36
	Jun.	6.01	1.55	8.44	0.71	0	6.01	5.26	5.28
	Jul.	4.31	0.84	9.09	0.47	0	4.31	3.80	3.82
	Aug.	4.3	0.82	9.37	0.45	2	4.30	4.07	4.08
	Sep.	5.17	1.00	9.28	0.55	21	5.43	5.42	5.42
	Oct.	5.25	0.99	9.53	0.55	38	6.35	6.18	6.17
	Nov.	4.65	0.88	9.25	0.50	50	6.66	6.06	6.04
	Dec.	4.23	0.42	8.98	0.47	57	6.99	5.99	5.96
	Average	5.35	1.08	9.13	0.58	22.54	6.26	5.81	5.81
Bhopal	Jan.	4.38	0.38	8.69	0.50	54	6.95	6.09	6.02
	Feb.	5.2	1.04	9.08	0.57	42	6.63	6.34	6.30
	Mar.	6.23	1.28	9.63	0.64	27	6.84	6.83	6.82
	Apr.	7.03	1.85	9.13	0.77	8	7.08	6.89	6.9
	May	6.75	1.70	9.09	0.74	0	6.75	6.11	6.17
	Jun.	5.53	1.31	8.42	0.65	0	5.53	4.82	4.87
	Jul.	4	0.80	9.07	0.44	0	4.00	3.53	3.57
	Aug.	3.8	0.77	9.35	0.40	1	3.80	3.59	3.62
	Sep.	5.2	1.01	9.27	0.56	20	5.46	5.45	5.46
	Oct.	5.32	1.01	9.51	0.55	38	6.45	6.28	6.25
	Nov.	4.72	0.90	9.23	0.51	50	6.79	6.18	6.12
	Dec.	4.57	0.38	8.96	0.51	56	7.67	6.55	6.47
	Average	5.23	1.04	9.12	0.57	22	6.16	5.72	5.71
Ranchi	Jan.	4.34	0.39	8.69	0.49	54	6.89	6.03	5.96
	Feb.	4.91	0.96	9.07	0.54	42	6.29	6.00	5.96
	Mar.	5.78	1.12	9.62	0.60	27	6.35	6.34	6.33
	Apr.	6.16	1.39	9.12	0.67	8	6.21	6.05	6.07
	May	5.88	1.28	9.08	0.64	0	5.88	5.31	5.36
	Jun.	4.65	0.99	8.42	0.55	0	4.65	4.04	4.08
	Jul.	4.02	0.80	9.06	0.44	0	4.02	3.55	3.59
	Aug.	3.85	0.77	9.35	0.41	2	3.85	3.65	3.67
	Sep.	4.13	0.80	9.26	0.44	20	4.34	4.34	4.34
	Oct.	4.37	0.82	9.50	0.46	38	5.31	5.17	5.14
	Nov.	4.26	0.82	9.22	0.46	50	6.12	5.57	5.52
	Dec.	4.07	0.43	8.95	0.45	56	6.75	5.80	5.73
	Average	4.7	0.88	9.11	0.51	22.09	5.56	5.15	5.15

Table 2 continued

Cities	Months	H_g	H_d	H_0	K_t	β_{opt}	H_T	HL	HY
Shillong	Jan.	3.91	0.43	8.53	0.45	55	6.39	5.63	5.53
	Feb.	4.63	0.91	8.92	0.51	44	6.08	5.80	5.75
	Mar.	5.35	1.02	9.47	0.56	29	5.98	5.96	5.95
	Apr.	5.86	1.30	8.97	0.65	10	5.94	5.78	5.81
	May	5.11	1.04	8.92	0.57	0	5.11	4.6	4.65
	Jun.	4.56	0.99	8.25	0.55	0	4.56	3.93	4.00
	Jul.	4.46	0.88	8.90	0.50	0	4.46	3.91	3.97
	Aug.	4.14	0.81	9.19	0.45	4	4.15	3.92	3.96
	Sep.	3.89	0.78	9.11	0.42	22	4.13	4.13	4.13
	Oct.	4.21	0.81	9.35	0.45	40	5.22	5.09	5.05
	Nov.	4.34	0.41	9.06	0.47	53	6.79	6.09	5.99
	Dec.	4.008	0.43	8.78	0.45	57	6.91	5.95	5.84
	Average	4.54	0.82	8.959	0.50	23.54	5.48	5.07	5.05
Varanasi	Jan.	3.58	0.47	8.52	0.42	55	5.79	5.12	5.03
	Feb.	4.76	0.95	8.91	0.53	44	6.26	5.98	5.92
	Mar.	5.81	1.16	9.46	0.61	29	6.48	6.47	6.46
	Apr.	6.42	1.57	8.96	0.71	10	6.49	6.32	6.36
	May	6.39	1.58	8.91	0.71	0	6.39	5.77	5.84
	Jun.	5.79	1.51	8.24	0.70	0	5.79	5.03	5.11
	Jul.	4.35	0.86	8.89	0.48	0	4.35	3.82	3.88
	Aug.	4.8	0.92	9.18	0.52	4	4.81	4.55	4.59
	Sep.	4.54	0.88	9.11	0.49	22	4.82	4.82	4.82
	Oct.	4.76	0.90	9.34	0.51	40	5.92	5.76	5.72
	Nov.	4.01	0.44	9.05	0.44	53	6.25	5.61	5.52
	Dec.	3.37	0.52	8.77	0.38	57	5.68	4.93	4.84
	Average	4.8	0.98	8.95	0.54	23.54	5.75	5.35	5.34
Patna	Jan.	3.61	0.47	8.50	0.42	55	5.86	5.18	5.08
	Feb.	4.72	0.94	8.90	0.53	44	6.22	5.94	5.87
	Mar.	5.81	1.17	9.45	0.61	29	6.50	6.48	6.47
	Apr.	6.35	1.53	8.95	0.70	10	6.42	6.25	6.29
	May	6.29	1.53	8.90	0.70	0	6.29	5.67	5.75
	Jun.	5.63	1.42	8.23	0.68	0	5.63	4.88	4.96
	Jul.	4.36	0.87	8.87	0.49	0	4.36	3.83	3.89
	Aug.	4.64	0.89	9.17	0.50	4	4.64	4.39	4.44
	Sep.	4.55	0.88	9.09	0.50	22	4.84	4.83	4.84
	Oct.	4.64	0.88	9.33	0.49	40	5.78	5.63	5.58
	Nov.	4.08	0.44	9.04	0.45	53	6.38	5.73	5.63
	Dec.	3.29	0.53	8.76	0.37	57	5.54	4.82	4.72
	Average	4.83	0.96	8.93	0.54	23.54	5.70	5.32	5.30
Jodhpur	Jan.	4.31	0.38	8.45	0.51	56	7.24	6.36	6.23
	Feb.	5.05	1.04	8.85	0.57	45	6.69	6.40	6.32
	Mar.	6.04	1.26	9.40	0.64	30	6.77	6.76	6.74
	Apr.	6.73	1.76	8.91	0.75	11	6.82	6.63	6.68
	May	6.97	1.94	8.85	0.78	0	6.97	6.30	6.39
	Jun.	6.55	2.01	8.18	0.80	0	6.55	5.71	5.81
	Jul.	5.46	1.18	8.82	0.61	0	5.46	4.79	4.88
	Aug.	5.41	1.10	9.12	0.59	4	5.43	5.13	5.19
	Sep.	5.85	1.28	9.05	0.64	22	6.22	6.21	6.22
	Oct.	5.3	1.04	9.28	0.57	40	6.64	6.46	6.41
	Nov.	4.49	0.39	8.99	0.49	54	7.17	6.42	6.30
	Dec.	4.12	0.41	8.71	0.47	58	7.27	6.26	6.12
	Average	5.52	1.15	8.88	0.62	24	6.60	6.12	6.11

Table 2 continued

Cities	Months	H_g	H_d	H_0	K_t	β_{opt}	H_T	HL	HY
Lucknow	Jan.	4.44	0.36	8.43	0.52	56	7.53	6.61	6.48
	Feb.	5.43	1.17	8.83	0.61	45	7.19	6.88	6.80
	Mar.	5.97	1.24	9.38	0.63	30	6.70	6.69	6.68
	Apr.	6.76	1.79	8.89	0.76	11	6.85	6.66	6.71
	May	7.14	2.07	8.82	0.80	0	7.14	6.46	6.55
	Jun.	6.06	1.70	8.16	0.74	0	6.06	5.26	5.35
	Jul.	5.49	1.20	8.80	0.62	0	5.49	4.82	4.90
	Aug.	5.32	1.08	9.10	0.58	5	5.34	5.05	5.10
	Sep.	5.52	1.16	9.03	0.61	23	5.89	5.88	5.89
	Oct.	5.63	1.15	9.26	0.60	41	7.05	6.87	6.82
	Nov.	4.78	0.35	8.96	0.53	54	7.72	6.90	6.78
	Dec.	4.19	0.40	8.68	0.48	59	7.47	6.42	6.29
	Average	5.56	1.14	8.86	0.62	24.36	6.70	6.21	6.19
Jaipur	Jan.	4.25	0.38	8.39	0.50	57	7.24	6.36	6.21
	Feb.	5.01	1.03	8.79	0.56	45	6.69	6.40	6.31
	Mar.	6.11	1.30	9.34	0.65	30	6.88	6.87	6.85
	Apr.	7.08	2.01	8.85	0.80	11	7.18	6.98	7.04
	May	7.25	2.17	8.78	0.82	0	7.25	6.56	6.67
	Jun.	6.65	2.13	8.12	0.81	0	6.65	5.80	5.91
	Jul.	5.13	1.08	8.75	0.58	0	5.13	4.49	4.59
	Aug.	4.88	0.96	9.06	0.53	5	4.90	4.63	4.69
	Sep.	5.45	1.14	8.99	0.60	23	5.83	5.82	5.83
	Oct.	5.04	0.98	9.22	0.54	41	6.37	6.21	6.14
	Nov.	4.27	0.41	8.92	0.47	54	6.89	6.19	6.05
	Dec.	3.74	0.46	8.64	0.43	59	6.62	5.72	5.58
	Average	5.4	1.17	8.82	0.61	24.36	6.47	6.92	5.99
New Delhi	Jan.	3.7	0.44	8.26	0.44	58	6.39	5.65	5.52
	Feb.	4.56	0.93	8.67	0.52	47	6.23	5.96	5.88
	Mar.	5.73	1.19	9.22	0.62	32	6.54	6.53	6.51
	Apr.	6.68	1.82	8.73	0.76	13	6.81	6.62	6.67
	May	6.78	1.92	8.65	0.78	0	6.78	6.12	6.22
	Jun.	6.26	1.92	7.99	0.78	0	6.26	5.43	5.54
	Jul.	5.29	1.17	8.62	0.61	0	5.29	4.63	4.72
	Aug.	4.94	0.99	8.93	0.55	8	4.96	4.69	4.75
	Sep.	5.25	1.09	8.87	0.59	26	5.67	5.66	5.67
	Oct.	4.66	0.90	9.10	0.51	43	6.00	5.84	5.78
	Nov.	3.92	0.44	8.79	0.44	55	6.45	5.81	5.68
	Dec.	3.31	0.51	8.50	0.38	60	5.93	5.17	5.04
	Average	5.09	1.11	8.69	0.58	25.81	6.11	5.68	5.67
Dehradun	Jan.	3.58	0.44	8.08	0.44	59	6.44	5.71	5.52
	Feb.	4.4	0.91	8.51	0.51	48	6.18	5.91	5.80
	Mar.	5.47	1.13	9.05	0.60	34	6.35	6.34	6.31
	Apr.	6.35	1.69	8.57	0.74	15	6.51	6.33	6.40
	May	6.95	2.14	8.47	0.82	0	6.95	6.27	6.41
	Jun.	6.06	1.88	7.81	0.77	0	6.06	5.23	5.38
	Jul.	5.25	1.19	8.43	0.62	0	5.25	4.58	4.71
	Aug.	4.8	0.98	8.76	0.54	8	4.84	4.58	4.66
	Sep.	5.32	1.15	8.72	0.61	26	5.81	5.80	5.81
	Oct.	5.13	1.05	8.93	0.57	44	6.74	6.57	6.47
	Nov.	4.22	0.40	8.60	0.49	57	7.29	6.56	6.36
	Dec.	3.53	0.47	8.31	0.42	61	6.70	5.83	5.62
	Average	5.09	1.12	8.52	0.59	26.63	6.26	5.81	5.79

Table 2 continued

Cities	Months	H_g	H_d	H_0	K_t	β_{opt}	H_T	HL	HY
Hamirpur	Jan.	2.43	0.63	7.93	0.30	57	4.13	3.75	3.65
	Feb.	2.87	0.75	8.36	0.34	50	4.02	3.87	3.80
	Mar.	4.79	0.96	8.91	0.53	36	5.66	5.65	5.63
	Apr.	5.22	1.18	8.43	0.61	17	5.39	5.24	5.30
	May	6.14	1.67	8.32	0.73	0	6.14	5.52	5.64
	Jun.	4.95	1.28	7.65	0.64	0	4.95	4.23	4.35
	Jul.	4.06	0.86	8.27	0.49	0	4.06	3.53	3.63
	Aug.	3.48	0.76	8.61	0.40	11	3.52	3.33	3.38
	Sep.	3.98	0.82	8.58	0.46	29	4.40	4.39	4.40
	Oct.	4.21	0.85	8.78	0.47	47	5.66	5.52	5.44
	Nov.	3.16	0.53	8.45	0.37	59	5.40	4.91	4.78
	Dec.	2.85	0.56	8.15	0.35	63	5.37	4.73	4.58
	Average	4.01	0.90	8.37	0.47	28.36	4.89	4.56	4.55
Srinagar	Jan.	1.32	0.89	7.67	0.17	52	1.67	1.64	1.62
	Feb.	2.71	0.58	8.12	0.33	48	4.05	3.88	3.79
	Mar.	3.95	0.81	8.67	0.45	39	4.78	4.78	4.75
	Apr.	5.06	1.18	8.20	0.61	20	5.28	5.13	5.20
	May	5.62	1.48	8.06	0.69	3	5.62	5.05	5.19
	Jun.	6.18	2.24	7.39	0.83	0	6.18	5.33	5.51
	Jul.	5.6	1.49	8.00	0.69	0	5.60	4.88	5.04
	Aug.	5.2	1.19	8.37	0.62	13	5.29	5.00	5.10
	Sep.	5.06	1.14	8.35	0.60	31	5.68	5.67	5.68
	Oct.	3.85	0.81	8.54	0.45	49	5.35	5.22	5.13
	Nov.	2.56	0.62	8.18	0.31	59	4.40	4.05	3.92
	Dec.	1.94	0.73	7.87	0.24	61	3.36	3.07	2.96
	Average	4.09	1.10	8.12	0.50	29.72	4.77	4.48	4.49

June, July and August) and 49° (December) for Hyderabad, 0° (May, June, July and August) and 49° (December) for Vishakapatnam, 0° (May, June, July and August) and 50° (December) for Pune, 0° (May, June, July and August) and 51° (December) for Mumbai, 0° (May, June, July and August) and 54° (December) for Nagpur, 0° (May, June, July and August) and 55° (December) for Bhavnagar, 0°

(May, June and July) and 54° (December) for Kolkatta, 0° (May, June and July) and 57° (December) for Ahmedabad, 0° (May, June and July) and 56° (December) for Bhopal, 0° (May, June and July) and 56° (December) for Ranchi, 0° (May, June and July) and 57° (December) for Shillong, 0° (May, June and July) and 57° (December) for Varanasi, 0° (May, June and July) and 57° (December) for Patna, 0°

Figure 3. Variation of average annual OPTA with latitude.

(May, June and July) and 58° (December) for Jodhpur, 0° (May, June and July) and 59° (December) for Lucknow, 0° (May, June and July) and 59° (December) for Jaipur, 0° (May, June and July) and 60° (December) for New Delhi, 0° (May, June and July) and 61° (December) for Dehradun, 0° (May, June and July) and 63° (December) for Hamirpur, 0° (June and July) and 61° (December) for Srinagar. The variation of average annual OPTA (β_{opta}) with latitude is shown in figure 3.

The percentage increase in maximum SR at monthly optimum tilt angle in comparison to annual and latitude based tilt angles are shown in table 3. The increase in maximum SR at monthly OPTA in comparison to latitude based tilt angle and annual OPTA varies from 5.85% to 8.08% and 5.95% to 8.34%, respectively, showing monthly OPTA is beneficial for maximum power generation for different cities in India.

Additionally, the correlation equations of the monthly OPTA (β_{opt}) based on the extraterrestrial SR (H_o) are developed for different months for India. These correlation equations are given below

The accuracy of correlations equations are evaluated by using the most widely used statistical indicators which are Mean Bias Error (MBE), Root Mean Square Error (RMSE),

Table 3. Increase in SR at monthly OPTA in comparison to latitude and annual optimum based tilt angle.

Cities	% increase in comparison to latitude	% increase in comparison to annual OPTA
Minicoy	6.05	5.96
Thiruvananthapuram	5.85	5.95
Port Blair	6.43	6.25
Bangalore	6.40	6.40
Chennai	6.26	6.35
Panjim	6.63	6.53
Hyderabad	6.83	6.80
Vishakapatnam	6.91	6.87
Pune	6.85	6.84
Mumbai	7.002	6.982
Nagpur	7.266	7.305
Bhavnagar	7.450	7.572
Kolkatta	7.526	7.571
Ahmedabad	7.615	7.671
Bhopal	7.560	7.711
Ranchi	7.793	7.940
Shillong	8.086	8.343
Varanasi	7.436	7.577
Patna	7.385	7.547
Jodhpur	7.772	7.984
Lucknow	7.821	8.081
Jaipur	7.707	7.959
New Delhi	7.570	7.760
Dehradun	7.689	8.080
Hamirpur	7.213	7.378
Srinagar	6.473	6.117

$\beta_{opt} = -10.5686(H_o)^2 + 172.3649(H_o) - 646.0214$	for January (4)
$\beta_{opt} = -7.0820(H_o)^2 + 115.0357(H_o) - 418.2813$	for February (5)
$\beta_{opt} = -4.2248(H_o)^2 + 65.8137(H_o) - 215.0140$	for March (6)
$\beta_{opt} = 0.7043(H_o)^2 - 25.8401(H_o) + 184.9276$	for April (7)
$\beta_{opt} = (0.0003(H_o)^4 - 0.0128(H_o)^3 + 0.1753(H_o)^2 - 1.0691(H_o) + 2.4426) \times 10^4$	for May (8)
$\beta_{opt} = 0$	for June (9)
$\beta_{opt} = 0$	for July (10)
$\beta_{opt} = 5.2105(H_o)^2 - 104.6942(H_o) + 525.5833$	for August (11)
$\beta_{opt} = -5.3033(H_o)^2 + 82.3239(H_o) - 287.4305$	for September (12)
$\beta_{opt} = -4.3097(H_o)^2 + 67.1274(H_o) - 210.7192$	for October (13)
$\beta_{opt} = -4.6538(H_o)^2 + 72.3703(H_o) - 221.0626$	for November (14)
$\beta_{opt} = -5.9455(H_o)^2 + 94.5274(H_o) - 313.9101$	for December (15)

t -statistic (t-stat) and correlation coefficient (R) (table 4). The MBE, RMSE, t-stat and r are given by following equations.

$$MBE = \frac{1}{n} \sum_{i=1}^n (y_i - x_i) \quad (16)$$

$$RMSE = \left[\frac{1}{n} \sum_{i=1}^n (y_i - x_i)^2 \right]^{1/2} \quad (17)$$

$$t - stat = \left(\frac{(n-1)MBE^2}{RMSE^2 - MBE^2} \right)^{1/2} \quad (18)$$

$$R = \left[\frac{(Z_t - Z_r)}{Z_t} \right]^{1/2} \quad (19)$$

Table 4. Error evaluation of obtained correlation equations.

Equation no.	MBE	RMSE	t -statistic	R
(13)	1.92×10^{-13}	0.9771	6.53×10^{-13}	0.98
(14)	-7.21×10^{-14}	0.5496	4.35×10^{-15}	0.99
(15)	2.04×10^{-13}	0.6503	1.04×10^{-12}	0.98
(16)	-2.62×10^{-14}	0.7544	1.15×10^{-13}	0.91
(17)	-9.04×10^{-12}	0.1256	2.40×10^{-10}	0.97
(20)	3.49×10^{-14}	0.7036	1.64×10^{-13}	0.98
(21)	-4.37×10^{-15}	0.6700	2.16×10^{-14}	0.99
(22)	-6.77×10^{-14}	0.6062	3.70×10^{-13}	0.99
(23)	-9.40×10^{-14}	0.4229	7.37×10^{-13}	0.99
(24)	2.90×10^{-13}	0.6421	1.50×10^{-12}	0.99

Figure 4. Comparison of the results obtained in the literature for the city of New Delhi.

Table 5. Comparisons of OPTA calculation with PV Syst.

Cities	OPTA and maximum SR obtained in this study	OPTA and maximum SR given by PV Syst
Hamirpur	OPTA is 28.36° and maximum SR is 4.56 kWh/m ² /day	OPTA is 28° and maximum SR is 4.408 kWh/m ² /day
New Delhi	OPTA is 25.81° and maximum SR is 5.68 kWh/m ² /day	OPTA is 27° and maximum SR is 5.567 kWh/m ² /day

$$Z_t = \sum_{i=1}^n (x_o - x_i)^2 \quad (20)$$

$$Z_r = \sum_{i=1}^n (x_i - y_i)^2 \quad (21)$$

$$x_o = \frac{1}{n} \sum_{i=1}^n x_i \quad (22)$$

where x_i is i^{th} optimum tilt angle, y_i is i^{th} OPTA calculated by correlation equations.

The correlations equations 13 to 24 are used to find OPTA of New Delhi and is validated with results from previously published literature [45] shown in figure 4.

4.1 Comparison of optimum tilt angle calculation with PV Syst

The OPTA and maximum SR on tilted surface for Hamirpur and New Delhi cities are compared with PV Syst software are shown in table 5.

5. Conclusions

In this study optimum tilt angle (β_{opt}) of 26 different Indian cities are calculated by changing tilt angle (β) from 0° to 90° at step of 1°. The correlations of monthly optimum tilt

angles are developed in terms of extraterrestrial solar radiation. Based on the results of study, the following conclusions are drawn.

1. It was found that the optimum tilt angle changed between 0° (May, June, July) and 63° (December) throughout the year in India. The optimum tilt angle is maximum in December for all cities.
2. The months in which β_{opt} is 0° for different locations in India are April, May, June, July and August ($8.28^\circ\text{N} \leq \phi \leq 15.4^\circ\text{N}$); May, June, July and August ($17.3^\circ\text{N} \leq \phi \leq 21.7^\circ\text{N}$); May, June and July ($22.3^\circ\text{N} \leq \phi \leq 31.6^\circ\text{N}$); June and July ($\phi = 34^\circ\text{N}$).
3. The average global solar radiation on monthly optimum tilted surface varies from 4.77 to 6.7 kWh/m²/day throughout India.
4. The developed correlation Eqs. (4) to (15) in this study can be used to estimate the optimum tilt angle of a solar photovoltaic system in India. The root mean square error in correlation equations varies from 0.1256 to 0.9771.
5. The annual average optimum tilts angle increase with latitude (figure 3). The RMSE in correlation of annual optimum tilt angle is 0.21°.
6. The increase in maximum solar radiation at monthly optimum tilt angle in comparison to latitude based tilt angle and annual optimum tilt angle varies from 5.85% to 8.08% and 5.95% to 8.34%, respectively, showing monthly optimum tilt angle is beneficial for maximum power generation for different cities in India.

Future research work is focused on experimental analysis of maximum solar radiation on optimum tilt angles for different cities in India.

List of symbols

HL	Maximum solar radiation on latitude based tilt angle
HY	Maximum solar radiation on annual optimum based tilt angle
H_d	Diffused radiation on horizontal surface ($\text{kWh m}^{-2} \text{ day}^{-1}$)
H_g	Global solar radiation on horizontal surface ($\text{kWh m}^{-2} \text{ day}^{-1}$)
H_0	Extraterrestrial solar radiation ($\text{kWh m}^{-2} \text{ day}^{-1}$)
H_r	Reflected radiation on horizontal surface ($\text{kWh m}^{-2} \text{ day}^{-1}$)
H_T	Global solar radiation on tilted surface ($\text{kWh m}^{-2} \text{ day}^{-1}$)
K_t	Monthly Average Clearness index
R_b	Ratio of beam radiation on tilted surface to that on horizontal surface

Greek symbols

β	Tilt angle ($^\circ$)
β_{opt}	Monthly optimum tilt angle for maximizing incident solar radiation
β_{opta}	Annual optimum tilt angle
δ	Declination angle ($^\circ$)
ϕ	Latitude of a place ($^\circ$)
ρ	Reflectivity of ground (0.02)
w	Hour angle ($^\circ$)
w_{ss}	Sunset hour angle at tilted surface ($^\circ$)

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