Effect of flapping kinematics on aerodynamic force of a flapping two-dimensional flat plate

G SENTHILKUMAR* and N R PANCHAPAKESAN

Indian Institute of Technology Madras, Chennai 600 036, India
e-mail: gsk.mach23@gmail.com; g.senthilkumar@cemilac.drdo.in; panch@iitm.ac.in

MS received 16 March 2016; revised 6 May 2017; accepted 7 September 2017; published online 10 May 2018

Abstract. Potential applications of flapping-wing micro-aerial vehicles (MAVs) have prompted enthusiasm among the engineers and researchers to understand the flow physics associated with flapping flight. An incompressible Navier–Stokes solver that is capable of handling flapping flight kind of moving boundary problem is developed. Arbitrary Lagrangian–Eulerian (ALE) method is used to handle the moving boundaries of the problem. The solver is validated with the results of problems like inline oscillation of a circular cylinder in still fluid and a flat plate rapidly accelerating at constant angle of attack. Numerical simulations of flapping flat plate mimicking the kinematics of those like insect wings are simulated, and the unsteady fluid dynamic phenomena that enhance the aerodynamic force are studied. The solution methodology provides the velocity field and pressure field details, which are used to derive the force coefficients and the vorticity field. Time history of force coefficients and vortical structures gives insight into the unsteady mechanism associated with the unsteady aerodynamic force production. The scope of the work is to develop a computational fluid dynamic (CFD) solver with the ALE method that is capable of handling moving boundary problems, and to understand the flow physics associated with the flapping-wing aerofoil kinematics and flow parameters on aerodynamic forces. Results show that delayed stall, wing–wake interaction and rotational effect are the important unsteady mechanisms that enhance the aerodynamic forces. Major contribution to the lift force is due to the presence of leading edge vortex in delayed stall mechanism.

Keywords. Insect flight; micro-aerial vehicles; flapping aerofoil; arbitrary Lagrangian–Eulerian method; unsteady forces.

1. Introduction

Insects have played vital role in the design and development of micro-aerial vehicles (MAVs). Insect flight seems impossible according to the conventional aerodynamic theory, because to support an insect weight the wing must produce two to three times more lift than that predicted by the conventional fixed wing theory [1]. Wang [2] indicated that the lift produced by flight like insect flapping is predominantly higher than expected from the quasi-steady aerodynamics results. As stated in Shyy *et al* [3], insects have been experimenting successfully with wing design, aerodynamics, control and sensory system for millions of years. They mastered the art of flying around 350 million years ago [1]; den Berg [4] and Ellington [1] hinted that the flight like insect flapping could be a very successful design for MAVs because they have much better aerodynamic performance than conventional fixed-wing and rotary-wing MAVs. One of the primary design challenges in design and development of flapping-wing MAV has been the understanding of the unsteady fluid mechanics associated with the flapping wing.

Early attempts to explain the force production during flapping flight, pioneered by Weis-Fogh [5] and Jensen [6], relied on the quasi-steady-state model, which assumes that the steady-state forces are produced by the wing at each instantaneous position throughout a full stroke cycle.

Freymuth [7] did experiments on a hovering apparatus over a limited parameter range. He also observed the thrust-producing vortical structure and calculated thrust coefficient from the velocity profile of the thrust producing vortical structures. He concluded that an aerofoil in hovering can produce large thrust by full utilization of dynamic stall vortices for thrust generation. The vortical signature of this thrust is a simple vortex street with the character of a jet stream.

During the early development of flapping-wing design, the time-dependent forces were correlated with the wing kinematics [8]. Recent advancement in instrumentation and high-speed video cameras provide the capability to capture the wing kinematics of flapping wing birds and insects and to measure the flow field around a flapping wing. This kinematics is being used by most of the authors for computational fluid dynamic (CFD) simulations to understand the flow physics. This necessitated the development of an

*For correspondence

incompressible Navier–Stokes flow solver that is capable of handling moving-boundary problems.

Insects alter their wing kinematics and geometry during flight to attain the efficient flight conditions. According to the requirements, they alter their wing kinematics whether to produce more lift or to fly with better aerodynamic efficiency. Stroke deviation plays a major role in aerodynamic force generation. During horizontal stroke plane motion, the drag forces produced during up and down stroke cancel each other so the stroke-averaged drag force is zero. Curved stroke plane is another mode of flapping in which the drag plays a major role in the vertical force generation, because of the asymmetry in the cycle. Potential applications of MAVs in the field of military are reconnaissance, surveillance and remote observation of hazardous environments. Also they can be used for civil applications like weather observation, traffic monitoring and hazardous places inaccessible to human beings.

2. Numerical methodology

Flows with moving boundaries are encountered in various practical applications like flapping-wing MAVs, free-surface flows, flow through blood vessels, etc. The unsteadiness of these flows arises from the flow pattern, shape of the boundary and the time dependence of the boundary conditions. The Arbitrary Lagrangian–Eulerian (ALE) method is an effective method to handle large boundary movement. In the ALE description, the nodes of the computational mesh may be moved with the continuum in normal Lagrangian fashion, or be held fixed in Eulerian manner or be moved in some arbitrary specified way to give a continuous re-zoning capability. Due to the flexibility in using this method, it is called as the ALE method.

A CFD solver is developed using the semi-implicit pressure-linked equations revised (SIMPLER) algorithm of Patankar [9], which provides pressure–velocity coupling. Two-dimensional Cartesian co-ordinate systems are used for formulation of equations and grid generation of the computational domain. The governing equations employed are incompressible Navier–Stokes equation in ALE formulation for an arbitrary region of volume V , bounded by a closed surface S and can be written as follows. Space conservation equation

$$\frac{\partial}{\partial t} \int_V dV - \int_S u_j^b n_j dS = 0 \quad (1)$$

continuity equation

$$\int_S \rho u_j n_j dS = 0 \quad (2)$$

momentum equation

$$\begin{aligned} \frac{\partial}{\partial t} \int_V \rho u_i dV + \int_S \rho (u_i - u_i^b) u_j n_j dS \\ = - \int_S p n_i dS + \int_S \tau_{ij} n_j dS + \int_S q_i dV \end{aligned} \quad (3)$$

where q_i is the source term.

The governing equations are discretized using finite-volume technique over non-orthogonal cells. These non-orthogonal quadrilateral cells are used to map/approximate the boundaries of the moving body. Convective and diffusive fluxes are discretized using central-differencing scheme, which has second-order accuracy. Unsteady terms are discretized using explicit forward Euler method. The well-known SIMPLER algorithm of Patankar is used for pressure–velocity coupling. In order to get the grid-independent results, an optimized square computational domain of 31 times the chord length is considered. Grid independence studies were carried out with different grid sizes. Variation in the results was found to be negligible with the grid sizes of 332×332 and 552×552 . Hence, the grid size of 332×332 was used for all simulations.

2.1 Interpolation of variables for new time step

In explicit time marching schemes, the solution from previous time step is needed to compute the surface and volume integral; some interpolation scheme is needed to compute the flow field variables of old time step at new time step locations. One possibility is to compute the gradient vector at the centre of each old control volume and then, for each new control volume centre, finding the nearest centre of an old control volume and using linear interpolation to obtain the old value at the new control volume centre.

$$\phi_{C_{new}}^{old} = \phi_{C_{old}}^{old} + (\nabla \phi)_{C_{old}}^{old} \cdot (r_{C_{new}} - r_{C_{old}}) \quad (4)$$

This interpolation scheme is adopted from Ferziger and Perić [10].

3. Validation studies

The solver is validated with the benchmark results of fluid dynamic problems like lid-driven cavity, flow past a circular cylinder, transient evolution of Couette flow, inline oscillation of circular cylinder in still fluid and rapidly accelerating flat plate. For all the numerical cases tested in this section, the second-order central-differencing scheme has been used for discretizing the convective and diffusive terms. Euler explicit scheme is used for temporal derivative. Results of flow past a stationary circular cylinder, inline oscillation of a circular cylinder in still fluid and

rapidly accelerating flat plate are presented in the following sections.

3.1 Flow past a stationary circular cylinder

Before getting into the simulation of flapping aerofoil, the solver's capability to simulate the unsteady flow field is assessed with the benchmark problems. Flow past a stationary circular cylinder at $Re = 100$ (based on free-stream velocity, kinematic viscosity and the cylinder diameter) is simulated. The Strouhal number (St) of the vortex shedding is 0.165. This value agrees with the experimental results (0.164–0.165) of Tritton [11].

3.2 Inline oscillation of a circular cylinder

To assess the capability of the solver, the flow field around a moving boundary, inline oscillation of a circular cylinder in still fluid is simulated. The force coefficients of this flow depend on the Reynolds number $Re = (U_{max}D)/\nu$ and Keulegan–Carpenter number $KC = U_{max}/(fD)$ of the flow, where U_{max} is the maximum velocity of the cylinder in motion, D is the diameter of the cylinder, ν is the fluid kinematic viscosity and f is characteristic frequency of oscillation. Harmonic motion [$x(t) = -h_a \sin(2\pi ft)$ and $y(t) = 0$] used by Dütsch *et al* [12] is used for this simulation. Drag coefficient of a circular cylinder in this kind of motion is compared to computational results of Dütsch *et al* [12] and shown in figure 1. The percentage temporal variation of the drag coefficient is 0.3%.

The agreement between our result and the available literature result is good. Only the force peaks are slightly under-predicted. $Re = 100$ and $KC = 5$ are used for the simulation. The resulting flow field is characterized by

Figure 1. Time history of drag coefficient of an oscillating circular cylinder in still fluid [$Re = 100$ and $KC = 5$].

stable, symmetric and periodic vortex shedding as shown in figure 2. When the oscillating cylinder moves in forward direction, boundary layer is developed in the bottom and top of the cylinder wall. The separated flow produces two counter-rotating vortices of apparently same magnitude of strength, resulting in the same shape.

This vortex formation comes to an end when the cylinder moves to the forward-most point. Then the cylinder moves backwards, resulting in the same vortex formation on the other side of the cylinder. The backwards motion of the cylinder causes a splitting of the vortex pair, which was produced by forward motion, and finally wake reversal occurs. Vorticity and pressure isolines corresponding to different phase angles (0° , 96° , 192° and 288°) of cylinder motion are shown in figures 2 and 3. Vorticity and pressure contours are similar to the experimental and numerical results of Dütsch *et al* [12]. These validation studies indicate that the solver is capable of predicting the time course of aerodynamic forces throughout the stroke cycles of flapping flight.

3.3 Rapid acceleration of a flat plate at constant angle of attack

This is an another validation case, which attempts to quantify the time dependence of aerodynamic forces for a simple yet important motion, rapid acceleration of a flat plate from rest to a constant velocity at a fixed angle of attack ($\alpha = 18^\circ$). Parameters used by Dickinson and Gotz [13] for their experiments are used for this validation case ($Re = 192$ and $\alpha = 18^\circ$).

Results of the present simulation are compared to experimental results of Dickinson and Gotz [13] and CFD results of Knowles *et al* [14]. Figure 4 shows the comparison of coefficient of lift. The results are comparable to the experimental results. The discrepancy in the initial peak between the computational and experimental results has been noticed by Knowles *et al* [14] also. Reason for the discrepancy in the initial peak can be explained by considering the fact that the physical wing (hardware) used in the experiments had inertia, so it would not respond instantly to instantaneous changes in lift. The non-physical CFD/numerical simulation of flat plate had no inertia and thus any changes in lift—no matter how rapid—are captured.

4. Horizontal stroke plane kinematics

Horizontal stroke plane hovering of a thin (thickness is 0.08 times the width of the flat plate) sharp-edged massless flat plate is considered for the simulation and the flat plate executes a plunging and pitching motion simultaneously in still air as shown here. The following equation shows the kinematics followed for simulation:

Figure 2. Vorticity isolines of an oscillating circular cylinder in still fluid [Re = 100 and KC = 5].

non-dimensional displacement

$$H(t) = h(t)/h_a = \sin(\omega t) \quad \text{and} \\ \theta(t) = \alpha(t)/\alpha_a = (\alpha_o/\alpha_a) + \sin(\omega t + \varphi)$$

non-dimensional velocity

$$H'(t) = h'(t)/h'_a = \cos(\omega t) \quad \text{and} \\ \theta'(t) = \alpha'(t)/\alpha'_a = \cos(\omega t + \varphi)$$

where

$H(t)$	non-dimensional linear displacement
$\theta(t)$	non-dimensional angular displacement
$H'(t)$	non-dimensional linear velocity
$\theta'(t)$	non-dimensional angular velocity
$h(t)$	location of flat plate at different times
$h'(t)$	linear velocity of flat plate at different times
h_a	amplitude of linear translation

h'_a	amplitude of linear velocity ($2\pi f h_a$)
f	frequency of oscillation
$\alpha(t)$	pitch angle of flat plate at different times
$\alpha'(t)$	pitch velocity of flat plate at different times
α_o	mean pitch angle
α_a	pitch amplitude
α'_a	amplitude of pitch velocity ($2\pi f \alpha_a$)
φ	phase angle
T	time period
Ω	angular frequency

This simulation was carried out for normal hovering mode, in which $\alpha_o = 90^\circ$ and $\varphi = 90^\circ$, and Reynolds number of 100. The frequency of oscillation (f) is 0.1136. The flapping amplitude (α_a) of the aerofoil is 45° . Figure 5 shows the time history of non-dimensional velocity profile.

Figure 3. Pressure isolines of an oscillating circular cylinder in still fluid [$Re = 100$ and $KC = 5$].

Figure 4. Coefficient of lift vs chords of travel [$Re = 192$ and angle of attack $\alpha = 18^\circ$].

During both up and down strokes there are two leading edge vortices at the top surface of the aerofoil; the suction pressure produced over the wing is very high and the net lift coefficient is also high. The time history of lift coefficient indicates the different mechanisms involved in vertical

force generation. Figures 6(a) and (b) show the time history of drag and lift coefficient for one complete cycle.

Figure 7 shows the instantaneous vorticity contour of a complete flapping cycle. A large leading edge vortex (LEV) is formed at the beginning of each half stroke and remains attached to the flat plate till the beginning of next stroke. As the wing translates, the LEV grows in size and increases the aerodynamic forces. The LEV grows to the maximum possible size according to the flow conditions, prior to the shedding. Despite the LEV shed from the aerofoil, the lift coefficient is much higher than the steady-state value. During the stroke reversal the aerodynamic forces are enhanced due to the rotation of the aerofoil.

The complicated reciprocating wing motion of insects suggests that they try to interact with the shed vortices. The shed vortices augment the aerodynamic AoA of the wing and in turn produce high lift. Because of the interaction of the aerofoil with shed vortices, this mechanism is called wing–wake interaction. This mechanism also contributes to

Figure 5. Time history of non-dimensional velocities.

the increased aerodynamic forces of flapping wing. The lift peak generated during the second half stroke is almost 40% higher than that in the first half stroke. The vorticity contour plots in figures 7(a)–(h) show that during each half stroke, a pair of counter-rotating vortices is shed. In the second half of the stroke (figure 7f) the aerofoil encounters the existing pair of counter-rotating vortices present in the field, and momentum from the wake is transferred to the aerofoil. These findings are consistent with those reported by Wang [2]. This phenomenon is evidenced by increased lift curve peak in the second half stroke.

The two force peaks in the lift coefficient curve are due to the delayed stall mechanism. Growing nature of the LEV delays the stall and the force coefficients increase until the flow attachment is no longer possible. This phenomenon is

called delayed stall. Delayed stall is one of the important unsteady mechanisms that contribute to the enhanced aerodynamic forces of flapping wing. The two force peaks in figure 6(b) are due to the presence of LEV during up and down strokes.

In the normal hovering mode, at the beginning of the forward stroke, the flat plate accelerates and pitches down. Rotation of the leading and trailing edge leads to the suction effect on the top surface and high pressure stagnation area on the lower surface due to the previous stroke vortex. When the flat plate is in the middle of the forward and backward stroke, the flat plate moves at almost constant pitching angle; a vortex bubble is formed on the top surface and increases the lift and drag to their maximum value. During the translation of the flat plate, the strength of the LEV is increased due to the growth of the vortex size. The LEV is attached to the flat plate till the beginning of the next stroke.

Vortex shedding plays a major role in the variation of the lift throughout the stroke. During rotation both the LEV and trailing edge vortex (TEV) are shed, which form a counter-rotating vortex pair in the flow field at every cycle like a dipole. TEV of the upstroke combines with the starting vortex of the up stroke and the dipole is formed (figure 7h). The dipole moves upwards in the flow field. By virtue of the upward momentum carried by the dipole, the downward forces are produced. The kinematics of the aerofoil is such that it produces an upward jet. Hence, the net vertical force acts downwards. The kinematics with the mirror image of the results shown in figures 7(a)–(h), which produces a downward jet and produces a net upward force to balance the weight of the body. Due to the complications in the instrumentation of the experiments, Freymuth [6] has used kinematics of the wing in such a way that it produces an upward jet. The same kinematics is adopted for this simulation.

Figure 6. (a) Time history of drag coefficient [$Re = 100$, $h_a = 1.4$, $f = 0.1136$]. (b) Time history of lift coefficient [$Re = 100$, $h_a = 1.4$, $f = 0.1136$].

Figure 7. Instantaneous vorticity contours at different stages of flapping cycle [$Re = 100$, $h_a = 1.4$, $f = 0.1136$]. (a) $0.0T$ (starting point), (b) $0.08T$ (translation to right side and pitching anti-clockwise), (c) $0.17T$ (translation to right side and pitching anti-clockwise), (d) $0.25T$ (translation to right side and pitching anti-clockwise), (e) $0.31T$ (translation to right side and pitching anti-clockwise), (f) $0.45T$ (translation to right side and pitching clockwise), (g) $0.60T$ (stroke reversed translation to left side and pitching clockwise) and (h) $0.80T$ (translation to left side and pitching clockwise), where T is time period of one cycle.

4.1 Stroke kinematics

Lift generation for sustained hovering flight in still air is accomplished by many insects and small birds. Experimental and CFD models, combined with modern flow visualization techniques, have revealed that the fluid dynamic phenomena underlying flapping flight are different from those of non-flapping. The mechanism of vertical force generation by flapping flat plate in curved stroke plane kinematics is studied. The insights gained from the simulations help in investigating the extent of the significance of unsteady mechanism in enhancement of aerodynamic forces for sustained hovering flight.

For a rigid wing, the kinematics of the wing may be uniquely described as the time sequence of three angles: stroke position $\phi(t)$, AoA $\alpha(t)$ and stroke deviation $\theta(t)$ (see figure 8). In all the simulations, the angular position of the

flat plate with the stroke plane and AoA is described by harmonic functions, in which velocity and AoA vary throughout the stroke. We have used two different harmonic functions to describe the stroke deviation: an oval pattern, in which the flat plate deviates from the stroke plane according to a half-sine wave per stroke period, and a figure of eight pattern in which the stroke deviation varied as a full sine-wave. The kinematics discussed in this section is about the mid-chord point of the flat plate. A massless sharp-edged flat plate with thickness of 0.08 times the width is used for the simulation. Figures 9(a), (b) and (c) show the horizontal stroke plane, oval stroke plane and figure of eight stroke plane, respectively.

The simulations are conducted for different amplitudes of pitching oscillation, phase differences between the pitching and plunging motions, mean AoA and Reynolds

Figure 7. continued

numbers. Then, the effect of these parameters are investigated and addressed.

The kinematics equations of motion of the figure like eight are

$$x(t) = A \cos(\omega t) \quad \text{and} \quad X(t) = (x(t)/A) = \cos(\omega t)$$

$$y(t) = 0.5A \sin(2\omega t) \quad \text{and} \\ Y(t) = (y(t)/A) = 0.5 \sin(2\omega t)$$

$$\alpha(t) = \alpha_o + \alpha_a \sin(\omega t + \varphi) \quad \text{and} \\ \theta(t) = \alpha(t)/\alpha_a = (\alpha_o/\alpha_a) + \sin(\omega t + \varphi)$$

where A is amplitude of stroke position and ω is angular frequency; $X(t)$ and $Y(t)$ are non-dimensional displacements; $x(t)$ and $y(t)$ indicate the horizontal and vertical positions of the flat plate at mid-chord, respectively; $\theta(t)$ is non-dimensional pitch angle and $\alpha(t)$ is the instantaneous pitch angle; α_a , α_o , φ and f are the amplitude of pitch oscillation, mean AoA, phase difference between pitch and

plunging and frequency of oscillations, respectively. The figure of eight and oval shape kinematics are very inclined patterns in comparison with the horizontal flat patterns. In horizontal stroke plane motion, the vertical motion is negligible with respect to the horizontal stroke plane motion.

4.2 Reference velocity and force coefficient calculation

The reference velocity $\bar{U}$ in the computation is based on the average velocity during one period of the cycle:

$$\bar{U} = \frac{1}{T} \int_0^T \sqrt{u^2 + v^2} dt$$

where u and v are the velocity components of the mid-chord point in x and y directions, respectively, and T stands for the period of oscillations. The vertical and

Figure 8. Time history of stroke parameters.

horizontal force coefficients are obtained from the following equations:

$$C_v = \frac{F_y}{\bar{q}S}, \quad C_H = \frac{F_x}{\bar{q}S}$$

where C_v and C_H are instantaneous force coefficients in x and y directions, respectively; F_x and F_y are the forces in x and y directions, respectively, and $\bar{q} = 0.5\rho\bar{U}^2$ stands for dynamic pressure.

5. Results and discussion

5.1 Effect of stroke deviation on force production

Insects and birds alter their wing kinematics and geometry during flight to attain the efficient flight conditions. According to the requirements, they modulate their wing kinematics, whether to produce more lift or to fly with better aerodynamic efficiency. The stroke deviations may play a major role in force generation. The stroke plane may be a horizontal stroke plane or curved stroke plane. The presence of an attached LEV due to the delayed stall mechanism is the most important aerodynamic mechanism acting on the flapping insect wings. Though the importance of delayed stall for horizontal stroke plane hovering is addressed in many experimental and computational studies, the significance of the same is not directly interpreted for curved stroke plane like figure of eight and oval shape of hovering motion. A complete understanding of insect flight emerges only when the wing kinematic patterns and the corresponding aerodynamic forces acting on the wings are clear. The stroke

deviation from the mean stroke plane plays a major role in force generation.

Stroke deviations from mean stroke plane like figure of eight, oval shape and horizontal stroke plane kinematics are simulated and the time histories of the force coefficients are studied. The stroke deviation used for all the three stroke deviation cases is 0.7 times of chord length of the aerofoil. The stroke amplitude is 1.4 times the chord length of the aerofoil. In oval pattern, upward deviation at the start of the down stroke requires a downward deviation at the start of the upstroke and vice versa. In case of figure of eight pattern, the two half strokes are mirror images of each other. Horizontal stroke plane is also a kind of figure of eight motion, but without the stroke deviation. Reynolds number used for this simulation is 75. Kinematic parameters $\alpha_a = 45^\circ$, $\varphi = 0^\circ$ and $\alpha_0 = 90^\circ$ are considered for the simulation.

Wang [2] addressed the effect of drag on vertical force using inclined stroke plane hovering kinematics and concluded that the drag enhances the vertical component of the force to balance the weight of the flying object. Figure of eight motion has near-inclined stroke plane motion during both upstroke and down stroke. During this period, a large magnitude of drag is produced. Drag force acts almost normal to the flat plate surface and increases the vertical component of the force, which balances the weight.

Even though the net vertical force produced by the figure of eight kinematics is less than that by the horizontal stroke plane kinematics, towards the end of the figure of eight kinematics, thrust is produced. This provides the net horizontal force to propel the flying object; the figure of eight motion is used to produce net horizontal force (thrust). This thrust reduces the cycle-averaged drag coefficient. This enhances the aerodynamic performance ($\bar{C}_v/\bar{C}_H$) of the figure of eight motion, which is shown in table 1. Figures 10(a) and (b) shows the variation of horizontal and vertical force coefficients with time resulting from the three kinds of kinematics of the aerofoil. The force peaks in figures 10(a) and (b) are due to the presence of LEV during the translation and the oscillation in the lift coefficient curve is due to the development and the shedding of the LEV and TEV.

The trends of the horizontal stroke plane and figure of eight motion are similar because the horizontal stroke plane motion is also a kind of figure of eight motion, but without the stroke deviation. The stroke-averaged vertical, horizontal force coefficients and the ratio of vertical to horizontal force coefficients for the three stroke deviation cases are listed in table 1. The net horizontal force (drag) is comparatively less in figure of eight kinematics. The power required to overcome the drag is less compared with the other two deviation

Figure 9. (a) Horizontal stroke plane kinematics. (b) Oval shape kinematics. (c) Figure of eight kinematics, where the dot represents the leading edge of the aerofoil.

cases. Hence, the figure of eight motion is considered for the parametric study in the subsequent sections. Fruit fly and hummingbird wing kinematics are also near figure of eight shape kinematics during hovering motion.

5.2 Effect of amplitude of pitch oscillation on force production

Unsteady mechanisms like delayed stall, rotational circulation and wake capture are the fluid dynamic phenomena

Table 1. Stroke-averaged vertical force coefficients for different stroke deviations [$Re = 75$, $A = 1.4$, $f = 0.1136$].

Sl. no.	Figure of eight	Oval shape	Horizontal stroke
$\bar{C}_V$	1.087	0.964	1.841
$\bar{C}_H$	1.204	1.402	2.602
$\bar{C}_V/\bar{C}_H$	0.902	0.678	0.707

Figure 10. (a) Horizontal force coefficient vs time history [$Re = 75$, $A = 1.4$, $f = 0.1136$]. (b) Vertical force coefficient vs time history [$Re = 75$, $A = 1.4$, $f = 0.1136$].

that account for most of the aerodynamic force production by flapping flight. Effect of amplitude of pitch oscillation on the vertical and horizontal force generation is studied for pitching angles of 45° , 30° and 15° by keeping all other parameters the same. The change in amplitude of pitch oscillation affects the flow attachment pattern and alters the

Figure 11. (a) Horizontal force coefficient vs time history [$Re = 75$, $A = 1.4$, $f = 0.1136$]. (b) Vertical force coefficient vs time history [$Re = 75$, $A = 1.4$, $f = 0.1136$].

force coefficients. Figures 11(a) and (b) show the time course of vertical and horizontal forces for the afore-mentioned pitching oscillations. These figures correspond to the 6th flapping cycle; after this the curves attain a periodic steady-state condition. The flow structures obtained and the time variation of force coefficients are used to study their effect. The mean vertical force coefficient is averaged over a complete cycle, but horizontal force coefficient is averaged between the two half strokes as the flat plate changes its direction at the end of the down stroke.

The resultant force is almost perpendicular to the flat plate direction during the entire flapping cycle due to the strong contribution of pressure forces. The pitch oscillation determines the contribution of total force to vertical and horizontal forces. The result shows that most part of the lift is produced during down stroke, while thrust is produced at the end of upstroke. Variation in amplitude of pitch oscillation (45° , 30° , 15°) produces AoA of 135° , 120° and 105°

Table 2. Stroke-averaged vertical force coefficients for different pitch oscillations [$Re = 75$, $A = 1.4$, $f = 0.1136$].

A	45°	30°	15°
$\bar{C}_V$	1.087	0.756	0.569
$\bar{C}_H$	1.204	1.418	2.306
$\bar{C}_V/\bar{C}_H$	0.902	0.533	0.246

during down stroke and 45° , 60° and 75° during upstroke. The time history of vertical force coefficient shows that the decrease in pitch amplitude reduces the net vertical force. However, the force peaks are the same for all the pitch oscillations. Time history of horizontal force shows that the pitch oscillation of 15° (AoA of 75°) does not produce thrust towards the end of the upstroke. The stroke-averaged vertical, horizontal force coefficients and their ratio for the three stroke deviation cases are listed in table 2.

5.3 Effect of stroke rotation (phase angle) on force production

Another possible means for aerodynamic force enhancement in flapping is that the circulation around the wing is enhanced by the quick rotation of the wing at the end of the down stroke. Large rotational forces generated during rotation induce a net lift force that is analogous to the Magnus effect seen in the case of a spinning baseball. Rotation of the leading and trailing edge leads to the suction effect on the top surface and high pressure stagnation area on the lower surface due to the previous stroke vortex.

Simulations are carried out to study the effect of phase difference between pitching and plunging angle of the stroke on force generation. Effect of advanced ($\varphi = 30^\circ$), symmetric ($\varphi = 0^\circ$) and delayed ($\varphi = -30^\circ$) stroke rotation is studied in this section. The stroke-averaged vertical and horizontal force coefficients are provided in table 3. The time history of horizontal and vertical force coefficient for one complete cycle is shown in figures 12(a) and (b). The stroke-averaged vertical force is better during advanced rotation ($\varphi = 30^\circ$), but the aerodynamic

Table 3. Stroke-averaged vertical force coefficients for advanced, symmetric and delayed rotation history [$Re = 75$, $A = 1.4$, $f = 0.1136$].

Stroke rotation	Advanced rotation	Symmetric rotation	Delayed rotation
$\bar{C}_V$	1.392	1.205	0.621
$\bar{C}_H$	1.684	1.389	1.580
$\bar{C}_V/\bar{C}_H$	0.827	0.867	0.392

Figure 12. (a) Horizontal force coefficient vs time history [$Re = 75$, $A = 1.4$, $f = 0.1136$]. (b) Vertical force coefficient vs time history [$Re = 75$, $A = 1.4$, $f = 0.1136$].

performance is the best in symmetric rotation ($\varphi = 0^\circ$) case. The first peak in C_V decreases from 7.3 at ($\varphi = 30^\circ$) to 6.9 at ($\varphi = 0^\circ$). Similarly, the first peak in C_V decreases from 6.9 at ($\varphi = 0^\circ$) to 5.5 at ($\varphi = -30^\circ$). Hence, the percentage decrease from advanced to symmetrical rotation is 7%. The percentage decrease from symmetric to delayed rotation is 25%. Stroke-averaged vertical force coefficient in advanced stroke is higher than in the other two cases. However, the aerodynamic performance ($\bar{C}_V/\bar{C}_H$) is the best in symmetric rotation case, which conforms to the results of Sane and Dickinson [15]; hence, the power required for the symmetric rotation case will be less than those in the other two cases. In this study the effect of phase angle is not much pronounced because the rotation is continuous throughout the cycle; if the rotation is restricted to the end of the stroke, the rate of rotation will be high and the contribution will be significant to the variation in force coefficients.

Table 4. Effect of Re on force production for Re = 25, Re = 50, Re = 75 and Re = 100 with $A = 1.4$ and $f = 0.1136$.

Re	25	50	75	100
$\bar{C}_V$	1.084	1.146	1.205	1.258
$\bar{C}_H$	1.200	1.281	1.389	1.457
$\bar{C}_V/\bar{C}_H$	0.903	0.894	0.867	0.863

Figure 13. (a) Vertical force coefficient vs time history for Re = 25, Re = 50, Re = 75 and Re = 100 with $A = 1.4$, $f = 0.1136$. (b) Horizontal force coefficient vs time history for Re = 25, Re = 50, Re = 75 and Re = 100 with $A = 1.4$, $f = 0.1136$.

5.4 Effect of Reynolds number on force production

Due to the highly inclined stroke during both the down stroke and upstroke of figure of eight motion the vertical force is enhanced by the drag. Wang [2] stated that hovering motion along a horizontal stroke plane the aerodynamic drag does not make any contribution to the vertical force. However, some of the best hover flies like

dragon flies and hoverflies employ inclined stroke plane, where the drag during down stroke and upstroke does not cancel each other and part of the drag enhances the vertical force component. In this section our aim is to analyse whether the drag mechanism acting on the figure of eight motion can augment the vertical force production in low-Reynolds Number flapping wings. Reynolds number was varied from 25 to 100 in steps of 25 and the effect of Re on force production was studied, where the Re is defined using stroke-averaged velocity and chord length of the aerofoil. In low-Reynolds-number flapping motion, the most common means of force generation are different from that of the conventional flight vehicles in terms of fluid dynamic phenomenon.

The stability of the LEV depends on the Reynolds number. As the Re increases, the LEV is more stable and attaches closer to the aerofoil. Hence, the core region of the LEV comes closer to the flat plate surface and increases the average vertical force to some extent. If we see the results globally, there is not much difference in the average vertical force coefficient. Stroke-averaged vertical force coefficient $\bar{C}_V$ for different Re is shown in table 4. Figures 13(a) and (b) show the variation of vertical force and horizontal force coefficient with time for one complete cycle. From the results, it is understood that the drag due to earlier flow separation from the leading edge at low Re does not contribute much to the vertical force coefficient. At low Re, the LEV is unstable and quickly separates from the top surface during the stroke reversal. Figure 13(b) shows the variation in the horizontal force coefficient due to the early flow separation in low Reynolds number. However, in case of high Re, the LEV is more stable and attached for longer time.

6. Conclusions

The insights gained from our simulations help in understanding the extent of the contributions of delayed stall, stroke reversal and wing-wake interaction in enhancement of aerodynamic forces in flapping kinematics. The effects of changing the wing kinematic parameters are studied and aerodynamic force enhancement is addressed using the vortical structures around the aerofoil. However, the results may get altered if the three-dimensional simulations are carried out.

At the beginning of the forward stroke, the flat plate accelerates and pitches down. Rotation of the leading and trailing edge leads to the suction effect on the top surface and high pressure stagnation area on the lower surface due to the previous stroke vortex. When the flat plate is in the middle of the forward and backward stroke, the flat plate moves at almost constant pitching angle; a vortex bubble is formed on the top surface and increases the lift and drag to their maximum value. During the translation of the flat plate the strength of the LEV is increased due to the growth

of the vortex size. The LEV is attached to the flat plate till the beginning of the next stroke and during the stroke reversal the flat plate interacts with the already shed vertical structures, which increases the effective AoA and the lift. Hence, delayed stall, wing–wake interaction and rotational effect are the important unsteady mechanisms enhancing the vertical force produced by the flapping flight. A major part of the contribution is due to the presence of LEV in delayed stall mechanism.

The stroke deviation from the mean stroke plane plays a major role in force generation. Figure of eight motion has near-inclined stroke plane motion during both upstroke and down stroke. During this period, a large magnitude of drag is produced. Drag force acts almost normal to the flat plate and increases the vertical component of the force, which balances the weight. In wing kinematics like that of figure of eight, there is an imbalance in the horizontal force, due to the thrust produced towards the end of the cycle. Even though the stroke-averaged vertical force is higher in horizontal stroke plane motion, the figure of eight motion has better aerodynamic efficiency, which reduces the power required for flight.

The change in amplitude of pitch oscillation affects the flow attachment pattern and alters the force coefficients. As the pitch amplitude decreases below 45° , the vertical force as well as the aerodynamic efficiency of the figure of eight motion decreases. When the pitch oscillation is 15° , there is no thrust at the end of the stroke. Stroke-averaged vertical force coefficient in advanced rotation is higher than the symmetric rotation and delayed rotation. Hence, time of rotation plays a major role in the enhancement of vertical force. Advanced rotation produces better vertical force but symmetric rotation provides better aerodynamic efficiency.

The stability of the LEV depends on the Reynolds number. As the Re increases, the LEV is more stable and attached closer to the leading edge of the flat plate. Hence, the core region of the LEV comes closer to the flat plate and increases the average vertical force. The drag at low Re does not contribute much to the vertical force coefficient.

References

- [1] Ellington C P 1999 The novel aerodynamics of insect flight: applications to micro-air vehicles. *J. Exp. Biol.* 202: 3439–3448
- [2] Wang Z J 2004 The role of drag in insect hovering. *J. Exp. Biol.* 207: 4147–4155
- [3] Shyy W, Berg M and Ljungqvist D 1999 Flapping and flexible wings for biological and micro air vehicles. *Prog. Aerosp. Sci.* 35: 455–505
- [4] den Berg C V 1997 The vortex wake of a hovering model hawkmoth. *Philos. Trans. R. Soc. London Ser. B: Biol. Sci.* 352: 317–328
- [5] Weis-Fogh T 1973 Quick estimates of flight fitness in hovering animals, including novel mechanisms of lift production. *J. Exp. Biol.* 59: 169–230
- [6] Jensen M 1956 Biology and physics of locust flight. III. The aerodynamics of locust flight. *Philos. Trans. R. Soc. London Ser. B: Biol. Sci.* 239: 511–552
- [7] Freymuth P 1990 Thrust generation by an airfoil in hovering modes. *Exp. Fluids* 9: 17–24
- [8] Sane S P 2003 The aerodynamic of insect flight. *J. Exp. Biol.* 206: 4191–4208
- [9] Patankar S V 1980 Numerical heat transfer and fluid flow. New York: Hemisphere Publishing Corporation
- [10] Ferziger J H and Perić M 2002 *Computational methods for fluid dynamics*. New York: Springer
- [11] Tritton D 1959 Experiments on the flow around a circular cylinder at low Reynolds numbers. *J. Fluid Mech.* 6: 547
- [12] Dutsch H, Durst F, Becker S and Lienhart H 1998 Low-Reynolds-number flow around an oscillating circular cylinder at low Keulegan–Carpenter numbers. *J. Fluid Mech.* 360: 249–271
- [13] Dickinson M H and Gotz K G 1993 Unsteady aerodynamic performance of model wings at low Reynolds numbers. *J. Exp. Biol.* 174: 45–64
- [14] Knowles K, Wilkins P, Ansari S and Zbikowski R 2007 Integrated computational and experimental studies of flapping-wing micro air vehicle aerodynamics. In: *Proceedings of the 3rd International Symposium on Integrating CFD and Experiments in Aerodynamics*, U.S. Air Force Academy, CO, USA
- [15] Sane S P and Dickinson M H 2001 The control of flight force by a flapping wing: lift and drag production. *J. Exp. Biol.* 204: 2607–2626