Simplified current minimization control of vector controlled Interior Permanent Magnet motor

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Abstract. This paper presents a simplified current minimizing technique for Interior Permanent Magnet (IPM) motor. This is primarily achieved by utilizing normalized 2D-Look Up Table (LUT) that is parameter independent except for saliency ratio. In addition, torque-flux reference frame is considered for implementation to reduce the complexity generally present in conventional methods utilizing $i_{ds} - i_{qs}$ current reference frame. The proposed algorithm also incorporates both the aspects that lead to field weakening operation, i.e., increase in speed and reduction of dc link voltage. A novel compensation method for incorporating saturation effect is also addressed. The current minimizing technique is analyzed in detail, supported by experimental results.

Keywords. IPM; current minimization; maximum torque per ampere; field weakening; dc-link variation; vector control; normalization.

1. Introduction

Recent advancements in the field of permanent magnets have enabled design of efficient permanent magnet motors. When compared to the surface mounted counterpart, the Interior Permanent Magnet (IPM) motor have higher power density and facilitate operation in deep-field weakening. These motors have gained vast importance especially in applications such as electric vehicles and wind-power generation. IPM motors have inverse-saliency which leads to the component of saliency torque in addition to the torque component developed due to Permanent Magnet (PM). This facilitates deep Field Weakening (FW) and a higher power density as the current component used for FW aids the torque developed in the form of saliency torque. At the same time, the presence of saliency torque complicates the motor control. The mathematical model for minimization techniques, such as current minimization involves quartic equations which are computational intensive and time consuming. Thus, to a wider-extent pre-made LUT based approach is generally followed.

The presence of both PM and saliency torque enables IPM motor operation under different techniques, one of them is current minimization or Maximum Torque Per Ampere (MTPA) control. This control technique is well-established and widely reported in literature [1–7]. Morimoto *et al* [1] demonstrated wide-speed operation of IPM motor utilizing high-performance current regulator. The current regulator saturation issue arising in field

weakening is addressed using a high-performance regulator. Uddin *et al* [2] investigated the performance of IPM motor over wide-speed range for industrial application. Here the drive operated in MTPA under normal operation and flux weakening in the constant power region. Jahns *et al* [3] proposed normalized MTPA. Bon-Ho *et al* [4] have established vector control using 2D-LUT.

Compensation of saturation is discussed in [5–7]. Morimoto *et al* [5] proposed compensation of saturation by use of corresponding increase in the current. Lee *et al* [6] looked into loss-minimization control of PM motors by use of polynomial approximations. They have proposed online solution rather than LUT based approach by use of order reduction and linear approximation to solve the fourth-order quartic equations involved. Sung-Yoon *et al* [7], on the other hand, utilized Ferrari's method of quartic to obtain online-solution for current minimization technique. They have explained in great detail the solution of various quartic equations involved under current minimization control. Consoli *et al* [8] have proposed scalar control for industrial drives using IPM motors. Bolonani *et al* [9] have used real-time tracking of MTPA based in AC-current injection.

Direct Torque Control (DTC) of IPM is also well-established [10–15]. Zhong *et al* [10] discussed detailed analysis of DTC for IPM motors. Rahman *et al* [11] demonstrated DTC for IPM motors including operation in FW. Lixin *et al* [12] proposed fixed switching frequency DTC with low torque and flux ripple.

It is evident that online estimation control techniques are computationally intensive. A simpler alternative is LUT based approach, accuracy of which depends on the

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resolution of the LUT data. Incorporating dc-link voltage variation and effects of saturation pose difficulty in LUT based implementation. Vector control techniques following current minimization utilize the $i_{ds} - i_{qs}$ reference frame for algorithm implementation and understanding. Similarly, the torque-flux reference frame is utilized when operating IPM motor under DTC. Though it may be seen that the operation trajectories of IPM motor become relatively simple, when seen in torque-flux reference frame as compared to $i_{ds} - i_{qs}$ frame of reference. The operating trajectories of constant torque hyperbola and voltage ellipse become straight lines in torque-flux frame of reference, thus current minimizing technique, especially the operation in FW becomes easier to interpret.

This paper aims to propose a simplified current minimizing technique using the insight as discussed above. This paper describes the current minimizing technique in a greater detail and utilizes the operation trajectories in torque-flux reference frame for its implementation. A simple algorithm is proposed to implement current minimization technique under all operating conditions. The operation in field-weakening both at higher speeds and at reduced dc-link voltage is included in the control algorithm. An all-parameters normalized 2D-LUT is developed such that it is independent of motor parameters as long as saliency ratio is constant. Also, the effect of saturation when using normalized parameters is automatically incorporated as long as the operation is within voltage-limit i.e., along MTPA trajectory. A simple method to compensate effect of saturation when operation is in FW is also proposed. The mathematical model and equations are established and the proposed algorithm is supported by experimental results.

This paper is organized as follows. Section 2 briefly discusses the mathematical model. The operation trajectories in normalized quantities are given in section 3. Section 4 establishes the control algorithm, various equations needed for its implementation and presents the proposed method to compensate effects of saturation. The implementation algorithm is supported by experimental results in section 5. Finally, the conclusions are provided in section 6.

2. Mathematical model of IPM motor

Mathematical model of motor is necessary to understand its operation. The IPM motor can be expressed using the following set of differential equations [3]. Equations (1), (2) and (3) define the mathematical model of a permanent magnet machine in synchronous rotating frame commonly referred to as dq frame of reference. The electrical torque developed is given by (6). The transformations used for converting variables in abc reference frame to dq reference frame are given in (4) and (5).

$$v_{qds} = r_s i_{qds} + \omega_e \lambda_{qds} + p \lambda_{qds} \quad (1)$$

$$(\lambda_{dqs})^T = [\lambda_{ds} \quad -\lambda_{qs}] \quad (2)$$

$$\lambda_{qds} = \begin{bmatrix} L_{ls} + L_{mq} & 0 \\ 0 & L_{ls} + L_{md} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \lambda_m \begin{bmatrix} 0 \\ 1.5 \end{bmatrix} \quad (3)$$

$$\begin{aligned} v_{qds} &= (v_{qs} \quad v_{ds})^T \\ &= \begin{bmatrix} -\sin(\theta_e) & \cos(\theta_e) \\ \cos(\theta_e) & \sin(\theta_e) \end{bmatrix} \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} \end{aligned} \quad (4)$$

$$\begin{aligned} i_{qds} &= (i_{qs} \quad i_{ds})^T \\ &= \begin{bmatrix} -\sin(\theta_e) & \cos(\theta_e) \\ \cos(\theta_e) & \sin(\theta_e) \end{bmatrix} \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \end{aligned} \quad (5)$$

$$T_e = \frac{2P}{3} \left[1.5 \lambda_m i_{qs} + (L_d - L_q) i_{qs} i_{ds} \right] \quad (6)$$

Note: A factor of 1.5 occurs due to the reference frame transformations used not being quantity equivalent.

2.1 Limitations imposed by motor ratings

The current rating of the motor is defined usually by the thermal capability of the machine and often varies with the operation duty of motor. In this case, continuous operation is assumed. Similar to the current rating, all machines are also designed for operation at an optimum flux or in other words voltage and speed. Machine nameplate contains details such as rated current, voltage, operating speed and machine parameters. It is very important to limit the operation of motor within the rated conditions and this is achieved by limiting and monitoring of current, voltage and flux in the controller designed.

The current limit of the motor in the dq-frame of reference can be given by

$$i_{dn}^2 + i_{qn}^2 \leq i_{sn_r}^2 \quad (7)$$

Voltage/Flux limit can be given as (resistance drop neglected):

$$\sqrt{\left(1 + \frac{i_{dn}}{\rho - 1}\right)^2 + \left(\frac{\rho i_{qn}}{\rho - 1}\right)^2} \leq \lambda_{sn} \quad (8)$$

(8) is obtained by normalizing the equation (9) which defines the operating flux.

$$L_q^2 i_{qs}^2 + [L_d i_{ds} + 1.5 \lambda_m]^2 \leq \lambda_s \quad (9)$$

It is to be noted that the expressions (7) and (8) have been normalized using (15)–(22). It is also clear that for a given rated current, the current limit locus as given by (7) would result in a circular trajectory in $i_{dn} - i_{qn}$ axes. Similarly, for a given operating voltage and speed (or flux) the voltage limit locus as given by (8) would result in an ellipse trajectory in $i_{dn} - i_{qn}$ axes. These trajectories are also shown in the following sections.

3. Operation trajectories of IPM motor

MTPA is the operation of IPM motor at maximum torque per current. Operation along MTPA results in current minimization and thus copper loss minimization. The MTPA is defined as the point of operation where the constant torque locus is tangential to a given current locus. MTPA [3] can be defined by:

$$T_{en} = \frac{i_{qn} + i_{qn}\sqrt{1 + 4i_{qn}^2}}{2} \quad (10)$$

$$T_{en} = \sqrt{i_{dn}(i_{dn} - 1)^3} \quad (11)$$

$$T_{en} = i_{qn}(1 - i_{dn}) \quad (12)$$

where i_{dn} and i_{qn} are normalized quantities.

Similarly, Maximum Torque Per Voltage (MTPV) can be described as:

$$i_{dn} = \frac{-(3\rho - 4) - \sqrt{(3\rho - 4)^2 - 8(\rho - 1)\{(\rho - 2) - (\rho - 1)\lambda_{sn}^2\}}}{4} \quad (13)$$

$$i_{qn} = \pm \left(\frac{\rho - 1}{\rho} \right) \sqrt{\lambda_{sn}^2 - \left(1 + \frac{i_{dn}}{\rho - 1} \right)^2} \quad (14)$$

Maximum Torque Per Voltage (MTPV) locus can also be referred as Maximum Torque Per Flux (MTPF). The MTPV trajectory is the locus along which the operation at a given voltage develops maximum torque. This is the operating point where the constant torque locus is tangential to a given voltage ellipse locus.

Where, the parameters are normalized as:

$$T_{eb} = \frac{2P}{3} 1.5\lambda_m^r i_b \quad T_{en} = \frac{T_e}{T_{eb}} \quad (15)$$

$$i_b = \frac{1.5\lambda_m^r}{L_q - L_d} \quad i_{sn} = \frac{i_s}{i_b} \quad (16)$$

$$i_{qn} = \frac{i_{qs}}{i_b} \quad i_{dn} = \frac{i_{ds}}{i_b} \quad (17)$$

$$\lambda_b = 1.5\lambda_m^r \quad \lambda_{sn} = \frac{\lambda_s}{\lambda_b} \quad (18)$$

$$\rho = \frac{L_q}{L_d} \quad (19)$$

$$\lambda_{sn} = \sqrt{\left(1 + \frac{i_{dn}}{\rho - 1} \right)^2 + \left(\frac{\rho i_{qn}}{\rho - 1} \right)^2} \quad (20)$$

$$\lambda_s = \frac{V_{dq}}{\omega_e} = \frac{\sqrt{(v_{qs})^2 + (v_{ds})^2}}{\omega_e} \quad (21)$$

$$i_s^2 = i_{ds}^2 + i_{qs}^2 \quad (22)$$

As seen in (7)–(22), the only machine parameter involved in the normalized equations is the saliency ratio ρ . Also, it is to be noted that MTPA (10)–(12) is independent of saliency ratio.

It is important to understand the locus of MTPA and MTPV trajectories for a given motor to understand the region of operation and for proper design of control. Also, as mentioned earlier the current and voltage limits of the motor cannot be violated during motor operation. To have a clear understanding of available region of operation and trajectories that can be traced during load disturbance or speed variation, the expressions (7) to (14) are plotted in $i_{qn} - i_{ds}$ axes as shown in figure 1. The flux limit locus becomes elliptical and current limit locus a circular trajectory. In any operating condition, the operating point is to be maintained within the current and voltage limits and therefore the operating point should be inside of current limit and voltage limit locus. The current limit locus is fixed for a given motor (unless temperature variation is taken into consideration) but the voltage ellipse locus shrinks with increase in speed. This also defines the maximum operating speed for the motor, which in this case is 523 rad/s at maximum developed torque of almost zero N-m.

Also it is to be noted that MTPV/MTPF trajectory for the chosen motor (“Appendix”) falls outside of its range

Figure 1. Operation trajectories in $i_{dn} - i_{qn}$ reference frame.

Figure 2. Operation trajectories in $T_{en} - \lambda_{sn}$ reference frame.

of operation defined by current limit locus and therefore operating along MTPV is not possible for the current design of motor. Operation along MTPA would require non-zero positive direct axis current in appropriate ratio with q-axis current based on the torque required (10)–(12).

The operation trajectories can also be transferred to Torque-Flux $T_{en} - \lambda_{sn}$ reference frame and are shown in figure 2. As seen in figure 2, the constant torque curves and the voltage ellipses in the current $i_{dn} - i_{qn}$ reference frame translate to straight lines, when seen in torque-flux reference frame, and are thus easier to understand and interpret. It is to be noted that subscript ‘n’ in the figures indicates normalized quantities i.e., per unit with respect to base quantities defined in (15)–(19).

4. Implementation algorithm

The current minimization technique [7] requires operation along the MTPA trajectory when operation has not entered field-weakening. With the onset of field weakening, the operation would be along MTPA whenever possible, i.e., as long as torque required is low and MTPA point of operation fall inside of voltage ellipse. If the point of operation falls outside of MTPA trajectory, then the operation would be along constant flux i.e., along the voltage ellipse corresponding to the operating speed and the dc-link voltage. A total of three different possible conditions for operation emerge as shown in figure 3(a) [conditions are shown in different colors]. These conditions are discussed for rated dc-link voltage and on-set of field weakening occurring at higher speed than rated.

- 1) At rated voltage V_s and rated speed $\omega_r = 230$ rad/s, the operation is along MTPA (trajectory shown in black color).
- 2) At a higher speed $\omega_r = 280$ rad/s, the operation is along MTPA upto $T_e = 8.2$ Nm and any higher load would require operation along voltage ellipse i.e., field-weakening (trajectory shown in blue color).

- 3) When operating at $\omega_r = 314$ rad/s, the operation would always be along voltage-ellipse as MTPA lies outside of the voltage-limit (trajectory shown in red color).

In addition, figure 3(a) also shows the operation at constant torque $T_e = 10$ Nm (trajectory shown in pink color), with increase in speed or decrease in the dc-link voltage, as both the conditions may require operation in field-weakening. The operation trajectories have also been shown in torque-flux reference frame in figure 3(b).

The proposed control algorithm (figure 4) for current minimization is simple and easy to implement. As seen in torque-flux reference frame shown in figure 3(b), operation is possible always to the left of flux-limit which is decided by dc-link voltage and operating speed. If the flux-limit at a given point of operation is lower than the flux along MTPA for required torque, then it indicates that MTPA falls outside of voltage-limit and thus operation would go along constant-flux i.e., voltage ellipse as shown in figure 3(a). The torque and flux references are initially generated based on the optimization technique, i.e., current minimization, which in-turn are used to generate current references using 2D-LUT. The method for saturation compensation is explained as a generalized case, though may be noted that saturation is found to be negligible for the chosen IPM motor.

4.1 Generation of 2D-LUT

The 2-D lookup table can be developed using equations (8) and (14), which leads to a quartic equation in i_{dn} as:

$$i_{dn}^4 + (2a - 2)i_{dn}^3 + (a^2 - c^2 + 1 - 4a)i_{dn}^2 + (2c^2 + 2a - 2a^2)i_{dn} + (a^2 - c^2 + bT_{en}^2) = 0 \quad (23)$$

where $a = \rho - 1$, $b = \rho^2$, and $c = a\lambda_{sn}$.

Equation (23) is used to develop a 2D-lookup table for different values of T_{en} and λ_{sn} giving corresponding i_{dn} and, i_{qn} can be similarly obtained using (12). Matlab/Simulink is used to solve the quartic equation and developed the 2D-lookup table.

4.2 Computation of torque limit

As shown in figures 1 and 2, the MTPV curve is outside the current limit, thus the current limit itself would become the limiting operation of the IPM motor which decides the maximum torque availability for given current and voltage rating.

The maximum torque curve can be either implemented by use of 1-D look-up table based on flux (voltage/speed) or computed online. Online computation of torque limit would also take care of saturation effects, if any. The torque limit expression is derived using (21) and (22) which gives the following quadratic equation in i_{ds} (the torque is calculated

Figure 3. Operation trajectories for IPM motor. (a) Operation trajectories in $i_{dn} - i_{qn}$ reference frame, and (b) operation trajectories in $T_{en} - \lambda_{sn}$ reference frame.

directly in actual parameters than normalized as it involves a quadratic equation which can be solved online with ease):

$$(L_d^2 - L_q^2)i_{ds}^2 + (3\lambda_m L_d)i_{ds} + L_q^2 i_{s_r}^2 - \lambda_{s_limit}^2 + (1.5\lambda_m)^2 = 0 \quad (24)$$

Solving equation (18) gives

$$i_{ds\lim} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad (25)$$

where $a = L_d^2 - L_q^2$, $b = 3\lambda_m L_d$, and $c = L_q^2 i_{s_r}^2 - \lambda_{s_limit}^2 + (1.5\lambda_m)^2$

$$i_{qs\lim} = \pm \sqrt{i_{s_r}^2 - i_{ds\lim}^2} \quad (26)$$

where i_{s_r} is the rated peak current in dq reference frame.

$$T_{\lim} = \frac{2P}{3} \left[1.5\lambda_m i_{qs\lim} + (L_d - L_q) i_{qs\lim} i_{ds\lim} \right] \quad (27)$$

Here, $\lambda_{s_limit} = V_{dq_r}/\omega_e$ is the limiting / maximum flux decided by the dc-link voltage or maximum phase voltage. $V_{dq_r} = KV_{dc}$, K depends on PWM technique used i.e., SPWM or SVPWM.

4.3 Reference flux computation

The current minimization algorithm follows the operation to be on MTPA [11] trajectory whenever possible and closest to MTPA trajectory otherwise (if operation enters field weakening either by increase in speed or reduction in dc-link voltage). A simple logic to select the operating flux reference based on the torque requirement and λ_{sn_limit} is shown in figure 5. The flux reference is chosen from MTPA as long as $\lambda_{sn_limit} > \lambda_{MTPA}$ for a given torque requirement. $\lambda_{sn_limit} > \lambda_{MTPA}$ indicates the operation of motor inside of the voltage limit locus when seen in $i_{dn} - i_{qn}$ reference frame (figure 1) or to the left of flux limit λ_{sn_limit} if seen in $T_{en} - \lambda_{sn}$ reference frame (figure 2). $\lambda_{sn_limit} > \lambda_{MTPA}$ simply means the voltage limit has not been violated and

Figure 4. Control block diagram for implementation of vector controlled IPM motor following current minimization.

Figure 5. Flux reference computation.

the operation has not entered field weakening region and therefore operation along MTPA trajectory is possible. If $\lambda_{sn_limit} < \lambda_{MTPA}$, indicating onset of field-weakening, then $\lambda_{ref} = \lambda_{sn_limit}$. λ_{sn_limit} is the flux limit defined by the operating speed and maximum voltage which limited by the dc-link of Voltage Source Converter and the modulation strategy used. The relation of λ_{sn_limit} is given in figure 5. The flux selection logic in figure 5. makes sure the operation is along MTPA as long as the voltage or flux limit has not been violated and along the voltage/flux limit in case of voltage saturation.

4.4 Proposed method for saturation compensation

The saturation correction algorithm can be divided into two parts, first is the saturation compensation during normal operation i.e., along MTPA and the second is the saturation compensation when the motor is operation under field weakening. The saturation compensation algorithm is shown in figure 6.

4.4a Saturation compensation for operation along MTPA: As seen in (10)–(12), the MTPA relation between torque and dq-axis currents in normalized parameters is independent of motor parameters. Thus, when operating along the MTPA trajectory the saturation effect can be compensated by simply using saturation-corrected base current as given by:

$$i'_b = \frac{1.5\lambda'_m}{L'_q - L_d} \quad (28)$$

It is seen [5–7] that the saturation has significant effect on the quadrature-axis inductance L_q , the direct-axis inductance is more or less constant.

$$L'_q = L_q(1 - \alpha i_{qs}),$$

L'_q being the saturated value of quadrature-axis inductance. The saturation coefficient ' α ' can be obtained experimentally. The saturation effect on the direct-axis inductance can also be incorporated in similar way. Also, if relation of saturated inductance and current is non-linear or complex, then look-up tables can also be used to obtain information of saturated inductances for the particular operating conditions.

4.4b Saturation compensation in field weakening: The effect of saturation when operating under field-weakening can be compensated in the following ways.

The modified/compensated q-axis current is taken as i'_{qs} . To maintain the flux developed equal to the reference flux command, mmf balance along q-axis should be maintained: (L_q being unsaturated quadrature axis inductance)

$$L_q i_{qs} = L_q(1 - \alpha i'_{qs}) i'_{qs} \quad (29)$$

where, α is the coefficient of saturation and i_{qs} is the uncompensated value of q-axis current. Solving (23), we have:

Figure 6. Saturation compensation.

$$i_{qs}' = \frac{1 - \sqrt{1 - 4\alpha i_{qs}}}{2\alpha} \quad (30)$$

Since the q-axis has been modified to i_{qs}' , the developed torque has also changed. The difference in torque due to saturation compensation i.e., modified i_{qs}' , can be given as a feed-forward term to the torque-reference as compensation (figure 6). This difference in torque developed is given by (31).

$$\Delta T_e = T_e^1 - T_e = \frac{2P}{3} \left[\lambda_{ds} (i_{qs}' - i_{qs}) \right] \quad (31)$$

The saturation compensation is explained, though the saturation effect is negligible for the chosen motor, the primary objective being current minimization. Also, the saturation of both L_d and L_q can also be incorporated by use of look-up tables to update the base current i_b' . L_d' , L_q' are the saturated values of dq-axes inductances. Other approaches to saturation correction are incorporation of saturation effect in the developed 2-d and 1-d LUTs used in control algorithm (figure 4) or use of online computation of 4th order polynomials as shown in [7].

5. Experimental results

The experiments have been performed in detail on an Interior Permanent Magnet motor by Moog Inc. It is a Moog Pitch Motor developed for wind-turbines, Model PMC6-030. The motor parameters are mentioned in the “Appendix”. The system is controlled using Texas Instruments floating point digital signal processor TMS320F28335. Sine Pulse Width Modulation (SPWM) is used with a maximum modulation index of 0.9 and switching at 10 KHz. Rated dc-link voltage is 425 V. The experimental set-up comprises of IPM motor coupled to a DC machine as shown in figure 7. The DC machine is operated as generator to load the coupled IPM motor.

The experimental results are shown in figures 8, 9, 10, 11, 12 and 13. The experiments are performed at three different operating speeds i.e., 230 rad/s, 280 rad/s and 314 rad/s, to demonstrate the current minimization algorithm. The operation with reduction in dc-link voltage is also demonstrated. The experimental results at each operating speed are divided into three parts i.e., the speed response, the load response and the $i_{ds} - i_{qs}$ trajectory traced for the load response.

5.1 Operation at speed of $\omega_r = 230$ rad/s

The speed response for $\omega_r = 230$ rad/s is shown in figure 8(a). Due to the limitation on the response time of resolver position estimator at high speeds during step change, the speed reference is not changed in step but in a ramp manner. This is justified by the aim of paper which is to test the current minimization algorithm i.e., proper selection of dq-axes current for the whole operation range which more or less can be easily interpreted in steady-state.

The speed reference is ramped up from 0 to 230 rad/s and reversed to -230 rad/s. The variation of direct axis current i_{ds} and quadrature axis current i_{qs} with the application of load is shown in figure 8(b). The load is ramped up from 3.5 Nm to $T_e = 18.13$ Nm. The operation on application of load follows MTPA, indicated by increase in both direct axis current i_{ds} and quadrature axis current i_{qs} . The $i_{ds} - i_{qs}$ trajectory traced with the application of load is shown in figure 8(c). This trajectory is the MTPA trajectory, as expected. The operation at rated speed of 230 rad/s would not require field-weakening as long as the dc-link is maintained to its rated value.

5.2 Operation at speed of $\omega_r = 280$ rad/s

Any operation above base speed of 230 rad/s under rated voltage may require operation in field-weakening else the voltage/flux limit locus is violated. The flux limit is reduced here due to an increase in operation speed beyond rated at rated dc-link voltage. The changeover of trajectory from MTPA to voltage-ellipse i.e., field weakening occurs at the point where MTPA trajectory falls outside the voltage-ellipse limit. As already discussed in section 4, at an operating speed of 280 rad/s and rated dc-link voltage, the operation is along MTPA for light loads which require $T_e \leq 8.2$ Nm. Higher loads than 8.2 Nm would result in field-weakening operation. The maximum load is decided as discussed in section 4.4b. The speed response of the controller when operating at 280 rad/s is shown in figure 9(a). The variation of direct axis current i_{ds} and quadrature axis current i_{qs} with the application of load is shown in figure 9(b). As expected, for developed torque $T_e = 18$ Nm, the operation at 280 rad/s is along the voltage-ellipse limit, thus the direct-axis current i_{ds} is higher as

Figure 7. Experimental set-up.

compared to the operation at 230 rad/s at the same load. The trajectory of operation with increase in load is shown in figure 9(c). The trajectory follows MTPA at lighter loads and operates along the voltage-limit (FW) at higher loads.

5.3 Operation at speed of $\omega_r = 314$ rad/s

The operation at speed $\omega_r = 314$ rad/s is shown in figure 10. The onset of field weakening depends both on the operating speed and the load (or developed torque T_e). As the speed reference is increased in a ramp, the developed torque required for acceleration is lower than the rated value. Therefore, the onset of field weakening operation is around $\omega_r = 285$ rad/s as marked by the increase in the stator current seen in figure 10(a). As discussed earlier, the operation at 314 rad/s and rated dc-link voltage is expected to be along the voltage-ellipse limit irrespective of the load, as the operation along MTPA lies outside of voltage-ellipse limit. This is clearly seen in figure 10(b), as even at no-load the direct-axis current $i_{ds} = -4.3$ A is high, as compared to lower direct-axis currents seen when operating at speeds of

Figure 8. Experimental results at $\omega_r = 230$ rad/s. (a) the speed response of the controller [ω_r -200 rad/div, T_e -6.8 Nm/div, i_{as} -10 A/div, and t -9 s/div], (b) the variation of i_{ds} and i_{qs} with application of load [ω_r -400 rad/div, T_e -13.5 Nm/div, i_{ds} -10 A/div, i_{qs} -10 A/div and t -4 s/div], and (c) the $i_{ds} - i_{qs}$ trajectory traced with increase in load [i_{ds} -2.4 A/div, i_{qs} -4 A/div].

Figure 9. Experimental results at $\omega_r = 280$ rad/s. (a) the speed response of the controller [ω_r -200 rad/div, T_e -6.8 Nm/div, i_{as} -10 A/div, and t -9 s/div], (b) the variation of i_{ds} and i_{qs} with application of load [ω_r -400 rad/div, T_e -13.5 Nm/div, i_{ds} -10 A/div, i_{qs} -10 A/div and t -4 s/div], and (c) the $i_{ds} - i_{qs}$ trajectory traced with increase in load [i_{ds} -2.4 A/div, i_{qs} -4 A/div].

Figure 10. Experimental results at $\omega_r = 314$ rad/s. (a) the speed response of the controller [ω_r -200 rad/div, T_e -6.8 Nm/div, i_{as} -10 A/div, and t -9 s/div], (b) the variation of i_{ds} and i_{qs} with application of load [ω_r -400 rad/div, T_e -13.5 Nm/div, i_{ds} -10 A/div, i_{qs} -10 A/div and t -4 s/div], and (c) the $i_{ds} - i_{qs}$ trajectory traced with increase in load [i_{ds} -2.4A/div, i_{qs} -4A/div].

Figure 11. Trajectories traced with increase in load (a) $i_{ds} - i_{qs}$ frame, and (b) $T_e - \lambda_s$ frame. [T_e -5.44 Nm/div, i_{qs} -4 A/div, i_{ds} -2.4 A/div, and λ_s -0.04 wb/div].

230 rad/s (figure 8(b)) and 280 rad/s (figure 9(b)). Higher i_{ds} indicates the operation in field-weakening. The variation of d-axis current i_{ds} and q-axis current i_{qs} under application of load is seen in figure 10(b).

As the motor is operating under field-weakening, the d-axis current i_{ds} is comparatively higher for same developed torque T_e in comparison to operation at lower speeds as seen in figures 8(b) and 9(b). The trajectory traced by i_{ds}

and i_{qs} with the application of load is shown in figure 10(c). As discussed, the trajectory is always along the voltage-ellipse limit (FW) at 314 rad/s. For the same developed torque $T_e = 18$ Nm, the direct-axis current drawn by the IPM motor is higher at speeds beyond rated. This is clearly seen in figure 8(b), figure 9(b), and figure 10(b). A comparison of current-minimization trajectories followed under the three different operating speeds with application of load is shown in figures 11(a) and 11(b). Both the $i_{ds} - i_{qs}$ and $T_e - \lambda_s$ trajectories are shown, respectively. The increased ripple in the trajectories during field weakening operation can be due to ripple present in the dc-link voltage and partly due to the size of LUTs used in control algorithm. The dc-link is obtained by an un-controlled diode rectifier with a capacitance filter and thus has 2nd harmonic ripple present in the dc-link voltage. During field weakening, the motor operates along the flux limit $\lambda_s = \lambda_{s_limit}$ (figure 5) and flux is related to dc-link voltage as $\lambda_{s_limit} = KV_{dc}/\omega_e$. Here, $V_{dq,r} = KV_{dc}$, K depends on PWM technique used i.e., SPWM or SVPWM..

As already elaborated in previous sections, the operation at 230 rad/s would be along MTPA trajectory. The operation at 280 rad/s at lower loads of about $T_e = 8.2$ Nm would be along MTPA and any higher load would require onset of field-weakening. The operation at 314 rad/s would always be along the voltage-ellipse or the constant flux trajectory (FW), as the MTPA falls beyond the flux limit.

The phase voltage V_{as} and phase current i_{as} when operated at 230 rad/s under no-load is shown in figure 12(a), and under load is shown in figure 12(b). Both, the PWM voltage measured at motor terminals V_{as} and the phase voltage v_{as} obtained from the dsp-controller using the reference wave are shown.

5.4 Operation with reduction in dc-link voltage

The effect of dc-link voltage variation on the current-minimization trajectory is shown in figure 13. The dc-link voltage is reduced from rated value of 425 V to 265 V, the

Figure 12. Phase voltage V_{as} and phase current i_{as} at $\omega_r = 230$ - rad/s and (a) $T_e = 3.5$ Nm (no-load), and (b) $T_e = 18$ Nm [time scale: 3.2 ms/div]. [i_{as} -10A/div, V_{as} -500 V/div, v_{as} -150 V/div, and t-3.2 ms/div].

IPM motor is operating at 230 rad/s and at load of $T_e = 10$ Nm. As the dc-link voltage is reduced, the onset of FW sets in and the trajectory moves long constant torque curve as shown in figure 13(b). Also, the onset of FW starts slightly at a later stage than the onset of the reduction of dc-link voltage, as the motor is operating at approximately half of rated load [figure 13(a)].

6. Concluding remarks

The current minimization control ensures operation with torque per minimum current at all operating conditions including field weakening. This control technique is also referred to as minimizing copper losses. For motor having negligible core losses as compared to copper losses as may be the case with electric vehicle based IPM motors, operation along current minimization would provide better efficiency than conventional control.

A simplified current minimization technique is proposed, analyzed and demonstrated with detailed experimental results. This is achieved by use of a normalized 2D-LUT which forms the basis of IPM motor model. A 1D-LUT is

Figure 13. Experimental results with reduction in dc-link voltage (a) variation of i_{ds} , i_{qs} , and V_{dc} [ω_r - 200 rad/s/div, i_q -10 A/div, i_{ds} -10 A/div, V_{dc} - 200 V/div, t-2 s/div], and (b) the $i_{ds} - i_{qs}$ trajectory traced [i_{ds} -2.64 A/div, i_{qs} -4 A/div].

used to relate MTPA trajectory in torqueflux reference frame and a torque limit calculator which works on a LUT or online computation to satisfy the current and voltage limits of the IPM motor.

Applications of IPM machine, such as electric vehicles and wind generation systems, or conditions such as fluctuation in supply voltage etc., may lead to variation in dc-link voltage. This feature is considered in the proposed algorithm, where field weakening operation, even due to variations in dc-link voltage is incorporated. Compensation of saturation effects in LUT based approach is also discussed. Though for the chosen IPM motor, saturation was found to be negligible.

Appendix

The motor parameters are: $L_d = 3.2$ mH, $L_q = 8$ mH, $r_s = 0.244$ Ω , $\lambda_m = 0.156$ wb, $P = 8$. The rms current and voltage rating is taken as: $i_s = 15$ A and $v_s = 130$ V. Rated speed $\omega_r = 230$ rad/s and rated torque $T_r = 23$ Nm [Moog make].

List of symbols

v_{ds}, v_{qs}	d-axis and q-axis components of stator voltage
i_{ds}, i_{qs}	d-axis and q-axis components of stator current
v_{ds}, v_{qs}	d-axis and q-axis components of stator voltage
$\lambda_{ds}, \lambda_{qs}$	d-axis and q-axis stator flux
r_s	stator resistance
λ_m	permanent magnet flux linkage per phase
ω_e	rotor speed in electrical rad/s
ω_r	rotor speed in mechanical rad/s
θ_e	rotor position in electrical rad
T_e	develop electrical torque

Subscript ‘b’ refers to base quantities and subscript ‘n’ refers to normalized quantities. ‘r’ indicates rated value.

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