The build-up and characterization of nuclear burn-up wave in a fast neutron multiplying medium

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Abstract. A study has been carried out on the build-up and characterization of nuclear burn-up wave in fast neutron multiplying media. The focus of the study is more on the transient part of the burn-up wave though the steady state is also studied. The characteristics of the transient part are expressed in terms of new parameters, i.e., Transient Length (TL) and Transient Time (TT) elapsed in establishing the asymptotic burn-up wave. The TT and TL are defined, respectively, as the time and length needed to establish 95% and 99% of the asymptotic neutron flux propagating in the media. From these parameters, the transient wave velocity is determined. The characteristics of asymptotic part of the wave are determined in terms of wave velocity, Full Width at Half Maximum (FWHM) and Full Width at 10% Maximum (FW10M) of neutron flux distribution in the space. A parametric study is also carried out to investigate the sensitivity of these parameters to some of the physical parameters of the ignition zone and the breeder zone of the reactor. The characterization parameters would be very meaningful to understand the transient characteristics of the self-sustaining nuclear burn-up wave and in evaluating the quality of the wave by the researchers working in the field of nuclear burn-up wave build-up and propagation.

Keywords. Nuclear burn-up wave; CANDLE; travelling wave reactor; characterization of wave.

1. Introduction

Nuclear energy is abundant and environment friendly. Hence, it is a source of energy for sustainable development. However, there are concerns relating to the nuclear safety, waste management and nuclear weapons proliferation associated with this source. Therefore, new reactor concepts are being studied internationally with emphasis on maximizing the resource utilization, intrinsic safety with self-regulation and proliferation resistance and reduction in nuclear waste generation. The resource utilization can be increased by transmutation of fertile materials (^{238}U and ^{232}Th) into fissile materials (^{239}Pu and ^{233}U). The intrinsic safety relies on the prevention of failures and mitigation of reactivity-initiated transients automatically by enhancing reactivity feedbacks. Long-term self-regulation is to be achieved by avoiding error and misuse through completely automatic means. Proliferation resistance is to be increased by minimization of the number of fuel cycles and fuel reprocessing to be integrated with the reactor. Waste generation can be reduced by burning the long-lived nuclear waste in fast reactors.

One of the concepts that meets some of these requirements is breed and burn concept [1]. This concept maximizes the fuel burn-up by appropriate reshuffling of the fuel. A concept called nuclear burn-up wave (NBW) development in fast neutron multiplying media has been proposed [2]. In the NBW concept, the fertile material block (called breeding zone) is ignited by a neutron source (called ignition zone). This results in breeding of fissile material from the fertile material. In the breeding zone, the fissile material burns and gets produced. However, the burn-up process is faster than the breeding process due to finite half-life of the intermediate elements involved in the breeding process. This reduces the concentration of fissile material and hence reduces fission rate. This reduction is compensated by the breeding process. Thus, the burning and breeding of fissile material progresses and run-away reaction in the reactor is unlikely due to sluggishness of the breeding process. Thus, in these reactors, once ignited, a self-sustaining fission wave will get established. In thermal reactors, it is achieved by fuel consisting of fissile material and burnable poison mixed in it. After ignition from one end, neutrons from fission will burn the poison. Hence, a self-sustaining nuclear fission wave will get established.

The reactors with the afore-mentioned concept are known mostly by two different names, that is, Travelling

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Wave Reactor (TWR) [3] and CANDLE (Constant Axial shape of Neutron flux, nuclide Densities and power shape during Life of Energy-producing reactor) [4, 5]. Other terminologies that have come into vogue are Self-Regulating Reactor [2], Self-organizing Nuclear Burning, NBW and Slow Nuclear Burning (SNB) reactors [6, 7]. The concept of automated nuclear reactor for long-term operation has been presented and elaborated [3]. The burn-up strategy is suggested in CANDLE concept. Some of the initial work on this concept carried out on the feasibility of development of burn-up wave in different media has been reported in references [8–12]. The work on the aspects like initiation of waves, effect of neutron source, burn-up wave in gas-cooled high-temperature reactors and studies in thorium fertile material is reported in references [13–19]. The work relating to start-up of CANDLE concept is reported in [20, 21]. The TerraPower company set up by Bill Gates in USA is carrying out studies on this concept [22–24]. The studies do establish the necessary and sufficient conditions for the establishment of soliton-like waves in fast neutron multiplying media. Simulation studies have established the validity of these criteria and the concept appears to be practically feasible.

The functioning of CANDLE fuel burn-up strategy is depicted in figure 1. This also resembles a normal candle that burns and emits light.

In this paper, we report the results of our investigations on the build-up phase of NBW in fast neutron multiplying media. The focus is on characterising the transient part of the burn-up wave and adding a new characterization parameter of the asymptotic part of the NBW, the aim being to specify new parameters that can be used to assess the

quality and practicality of the burn-up wave that is established.

The transient characteristics are expressed in terms of (i) Transient Time (TT) needed to establish the self-sustaining nuclear burning wave and (ii) Transient Length (TL) over which this transition develops. The speed at which the transient part establishes is determined. In the asymptotic state, the wave propagation speed is determined and breadth of the power producing region is measured by its FWHM (Full Width at Half Maximum) and the FW10M (width at 10% of the maximum). The changes in these characteristics with respect to critical plutonium concentration of breeder zone, strength of external neutron source, plutonium ignition zone width and ^{239}Pu concentration in ignition zone are investigated. The criterion employed for the establishment of the steady state wave is that the peak of the neutron density becomes constant as a function of time. Earlier, a study was carried out on the development of burn-up waves with emphasis on the investigations of the characteristics of transient part (in terms of TT and TL), in pure neutron absorbing media and scattering-dominant diffusive media [25]. The study established that the new parameters are extremely useful in quantitatively characterizing the transient part of the development of burn-up waves.

In section 2, the mathematical model used for the evolution of the burn-up wave is presented. Section 3 provides the results of the calculations and in section 4, conclusions are given.

Figure 1. CANDLE: the new fuel burn-up strategy [4, 21].

2. Mathematical model

The mathematical model is the same as given in reference [6] and is briefly reproduced below for the sake of completeness of the presentation. A one-dimensional slab geometry is considered. The right boundary of the slab is represented by $x = x_r$ and the left boundary is represented by $x = x_l$. The slab consists of two zones: ignition zone and fast breeding zone; j is the neutron current on the left boundary of the slab. The current could be generated from a neutron source as in a reactor (figure 2).

Single-group neutron diffusion approximation is used for calculations. The energy of neutrons produced from fission and neutrons participating in nuclear reactions are assumed to be the same. Effects of delayed neutrons, reactivity feedback effects and the presence of structural material are not taken into account. The change in the nuclide composition during wave progression is accounted for using burn-

Figure 2. Initial assembly of the fast reactor.

up and decay equations. The model is simple and has been introduced to gain insight in the meaningful understanding of the transient characteristics of the self-sustaining nuclear burning wave.

The transmutation reactions involved in uranium–plutonium cycle reactor are shown below in figure 3. The neutron capture and β -decay are represented by horizontal and vertical arrows, respectively.

A detailed study of uranium–plutonium cycle has to account for each nuclide and nuclear reactions in the above nuclear chain. However, for the sake of simplicity, to gain insight into the nuclear fission wave and assess the utility of the characterization, a simple model is used [2]. The reactor has been modelled using diffusion equation coupled with nuclear burn-up equations. In one-dimensional slab geometry, with x -axis representing space, the equations can be written as [6] follows:

$$\frac{\partial N}{\partial t} = vD \frac{\partial^2 N}{\partial x^2} - vN \sum_i \sigma_{ci} N_i + (v_f - 1) v \sigma_{fpu} N N_{pu} \quad (1)$$

$$\frac{\partial N_8}{\partial t} = -v \sigma_{c8} N N_8 \quad (2)$$

$$\frac{\partial N_9}{\partial t} = -\frac{1}{\tau_\beta} N_9 + v \sigma_{c8} N N_8 \quad (3)$$

$$\frac{\partial N_{pu}}{\partial t} = \frac{1}{\tau_\beta} N_9 - v(\sigma_{cPu} + \sigma_{fpu}) N N_{pu}. \quad (4)$$

Figure 3. Nuclear transmutation reactions (U–Pu cycle).

Here, N is neutron density, v is neutron velocity, D is diffusion coefficient of the medium, σ_c is microscopic capture cross-section, σ_f is microscopic fission cross-section and ν_f is number of neutrons produced per fission. The atomic densities of ^{238}U , ^{239}U and ^{239}Pu are represented by N_8 , N_9 and N_{pu} , respectively. The subscripts represent the particular atoms; τ_β ($= 3.46$ days) is the mean lifetime for the β -decay process.

2.1 Steady state

The equilibrium concentration of plutonium (n_{eq}) is obtained from steady state of Eqs. (3) and (4). Hence

$$\begin{aligned} \frac{1}{\tau_\beta} (N_9)_{eq} - v(\sigma_{cPu} + \sigma_{fPu}) N (N_{pu})_{eq} \\ = \frac{1}{\tau_\beta} (N_9)_{eq} - v\sigma_{c8} N (N_8)_{eq}. \end{aligned} \quad (5)$$

Thus, the ratio of N_{pu} to N_8 under equilibrium conditions (n_{eq}) can be written as

$$\left(\frac{N_{pu}}{N_8} \right)_{eq} = n_{eq} = \frac{\sigma_{c8}}{\sigma_{cPu} + \sigma_{fPu}}. \quad (6)$$

Equilibrium concentration of Pu normalized to U-238 represents the ratio of absorption of neutrons in ^{238}U to absorption of neutrons in fissile material, ^{239}Pu . In the initial stage, there is only fertile material and plutonium concentration is zero. As time progresses, plutonium will be produced and its concentration increases but never exceeds equilibrium concentration, n_{eq} . For transient phase, a more accurate formula has been determined and discussed [7, 9].

Plutonium critical concentration (n_{cr}) in steady state is obtained by putting time and spatial derivative zero in Eq. (1). The result is

$$\left(\frac{N_{pu}}{N_{tot}} \right)_{cr} = n_{cr} = \frac{\sum_i \sigma_{ci} \left(\frac{N_i}{N_{tot}} \right)}{(\nu_f - 1) \sigma_{fPu}}. \quad (7)$$

Indeed, the possibility of solitary burn-up wave propagation depends on the ratio, $\eta = n_{eq}/n_{cr}$. As η is constituted by a combination of several constants, η can be > 1 , η can be $= 1$ and η can be < 1 . The necessary condition for existence of a solitary wave is $\eta > 1$, i.e., $n_{eq} > n_{cr}$ [6, 7].

2.2 Time dependence

Now, we perform the time-dependent calculations. To make the calculations simple, the neutron capture cross-sections of ^{239}U and ^{239}Pu were taken to be the same and as that of ^{238}U ($\sigma_{c9} = \sigma_{cPu} = \sigma_{c8}$). Thus, Eqs. (2) and (3) can be written as [2] follows:

$$\frac{\partial N_8}{\partial t} = -v\sigma_{c8} N (N_8 - N_9 - N_{pu}) \quad (8)$$

$$\frac{\partial N_9}{\partial t} = -\frac{1}{\tau_\beta} N_9 + v\sigma_{c8} N (N_8 - N_9). \quad (9)$$

We normalize the nuclide concentration and neutron density as follows:

$$n_i = \frac{N_i}{N_{tot}} \quad (10)$$

$$n = N\tau_\beta v\sigma_{c8}. \quad (11)$$

One can write diffusion and burn-up equations in a dimensionless format as [6]

$$\beta \frac{\partial n(x, t)}{\partial t} = \frac{\partial^2 n(x, t)}{\partial x^2} + n(x, t) \left[\frac{n_{pu}(x, t)}{n_{cr}} - 1 \right] + q(x, t) \quad (12)$$

$$\frac{\partial n_8(x, t)}{\partial t} = -n(x, t) [n_8(x, t) - n_9(x, t) - n_{pu}(x, t)] \quad (13)$$

$$\frac{\partial n_9(x, t)}{\partial t} = n(x, t) [n_8(x, t) - n_9(x, t)] - n_9(x, t) \quad (14)$$

$$\frac{\partial n_{pu}(x, t)}{\partial t} = n_9(x, t) - \frac{1}{n_{eq}} n(x, t) n_{pu}(x, t) \quad (15)$$

where distance and time are measured, respectively, in terms of diffusion length l and decay time constant τ_β . The constant $\beta = \tau_a/\tau_\beta$ ($\approx 0.6 \times 10^{-12}$). The constants are represented by

$$\beta = \tau_a/\tau_\beta (\approx 0.6 \times 10^{-12}), \quad \tau_a = \frac{1}{v \sum_i N_i \sigma_{ai}}, \quad l = \sqrt{vD\tau_a} \quad (16)$$

where τ_a is mean neutron lifetime and l is neutron diffusion length of the medium. Here, $q(x, t)$ represents the external neutron source that is needed for burn-up wave ignition. The external neutron source can be modelled as indicated earlier or by allowing a neutron current to fall on boundary at $x = 0$ and then accounting for this through boundary conditions. To study the time evolution of solitary wave, Eqs. (12)–(15) should be solved. The boundary conditions for one-dimensional slab (see figure 1) $x_l \leq x \leq x_r$ are as follows.

- (i) At right boundary the neutron current coming into the slab is taken as zero:

$$\left[\frac{\partial n(x, t)}{\partial x} + \frac{1}{2\gamma} n(x, t) \right]_{x=x_r} = 0, \quad \text{the constant } \gamma = l/(v\tau_a). \quad (17)$$

- (ii) On the left boundary a neutron current of j_{ex} is applied to initiate the burn-up wave:

$$\left[\frac{\partial n(x, t)}{\partial x} - \frac{1}{2\gamma} n(x, t) + j_0 \right]_{x=x_l} = 0, \quad j_0 = \tau_\beta j_{ex}/Nl \quad (18)$$

In the present case, the initial medium consists of fertile material, ^{238}U with plutonium fuse. The Pu concentration is given by

$$n_{pu}(x) = a_0 n_{cr} \frac{[\cos(\frac{\pi x}{d}) + 1]}{2} \quad (19)$$

where d is the thickness of ignition zone.

2.3 Numerical method

The diffusion equation along with burn-up equations constitutes a coupled non-linear partial differential equation system. These kinds of systems are difficult to solve analytically. Generally, numerical schemes like finite difference method (FDM), finite element method (FEM) or Monte-Carlo methods are used to find the solution. Here, the Crank–Nicolson method is used for diffusion equation and simple FDM for burn-up equations. The simulation is carried out using MATLAB programming environment. The Crank–Nicolson method [26] is suitable for parabolic partial differential equations. For increasing the accuracy, the average of spatial derivatives of consecutive time steps is taken (instead of taking spatial derivative at a single time step). In this method, for the function ' $u(x,t)$ ',

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad (20)$$

is written as

$$\begin{aligned} \frac{1}{\Delta t} [u(i, k+1) - u(i, k)] &= \frac{1}{2\Delta x^2} [u(i-1, k) \\ &+ u(i+1, k) - 2u(i, k)] + \frac{1}{2\Delta x^2} [u(i-1, k+1) \\ &+ u(i+1, k+1) - 2u(i, k+1)] \end{aligned} \quad (21)$$

Using i (from 1 to i_{end}) and k (from 1 to k_{end}) for space and time index, respectively, the diffusion equation (12) can be written as the following equation; also notice that the second term in the right hand side of Eq. (12) is also time averaged in the following equation:

$$\begin{aligned} \beta \frac{n(i, k+1) - n(i, k)}{\Delta t} &= \frac{1}{2\Delta x^2} [n(i-1, k) + n(i+1, k) \\ &- 2n(i, k) \\ &+ \frac{1}{2\Delta x^2} [n(i-1, k+1) + n(i+1, k+1) - 2n(i, k+1)] \\ &+ \frac{1}{2} n(i, k) \left[\frac{n_{pu}(i, k)}{n_{cr}} - 1 \right] \\ &+ \frac{1}{2} n(i, k+1) \left[\frac{n_{pu}(i, k+1)}{n_{cr}} - 1 \right] \end{aligned} \quad (22)$$

Burn-up equations are discretized by a finite difference scheme. In FDM the nuclide burn-up equations (13)–(15) take the form

$$\frac{n_8(i, k+1) - n_8(i, k)}{\Delta t} = -n(i, k) [n_8(i, k) - n_9(i, k) - n_{pu}(i, k)] \quad (23)$$

$$\frac{n_9(i, k+1) - n_9(i, k)}{\Delta t} = n(i, k) [n_8(i, k) - n_9(i, k)] - n_9(i, k) \quad (24)$$

$$\frac{n_{pu}(i, k+1) - n_{pu}(i, k)}{\Delta t} = n_9(i, k) - \frac{1}{n_{eq}} n(i, k) n_{pu}(i, k). \quad (25)$$

The boundary conditions (Eqs. (17) and (18)) are also written in a simple finite difference form as follows:

$$\begin{aligned} \frac{n(i_{end}, k+1) - n(i_{end}-1, k+1)}{\Delta x} + \frac{1}{2\gamma} n(i_{end}-1, k+1) \\ = 0 \end{aligned} \quad (26)$$

$$\frac{n(2, k+1) - n(1, k+1)}{\Delta x} - \frac{1}{2\gamma} n(1, k+1) + j_0 = 0. \quad (27)$$

If the initial conditions ($k=1$) are known, the complete time evolution of solitary wave can be found by step by step time advance. Equations (23)–(26) can be used to calculate nuclide concentrations at the next time step:

$$n_8(i, k+1) = n_8(i, k) - \Delta t \{ n(i, k) [n_8(i, k) - n_9(i, k) - n_{pu}(i, k)] \} \quad (28)$$

$$\begin{aligned} n_9(i, k+1) &= n_9(i, k) \\ &+ \Delta t \{ n(i, k) [n_8(i, k) - n_9(i, k)] - n_9(i, k) \} \end{aligned} \quad (29)$$

$$\begin{aligned} n_{pu}(i, k+1) &= n_{pu}(i, k) \\ &+ \Delta t \left\{ n_9(i, k) - \frac{1}{n_{eq}} n(i, k) n_{pu}(i, k) \right\}. \end{aligned} \quad (30)$$

Once plutonium concentration is obtained, Eq. (22) along with Eqs. (26) and (27) provides flux distribution. The equations are rewritten by taking $n(k+1)$ terms to left hand side.

$$\begin{aligned} n(i-1, k+1) \left\{ -\frac{1}{2\Delta x^2} \right\} + n(i, k+1) \\ \left\{ \frac{\beta}{\Delta t} + \frac{1}{\Delta x^2} - \frac{1}{2} \left[\frac{n_{pu}(i, k+1)}{n_{cr}} - 1 \right] \right\} \\ + n(i+1, k+1) \left\{ -\frac{1}{2\Delta x^2} \right\} \\ = n(i, k) \left(\frac{\beta}{\Delta t} \right) + \frac{1}{2\Delta x^2} (n(i-1, k) + n(i+1, k) \\ - 2n(i, k)) + \frac{1}{2} n(i, k) \left(\frac{n_{pu}(i, k)}{n_{cr}} - 1 \right) \end{aligned} \quad (31)$$

$$n(i_{end}, k+1) \left\{ -\frac{1}{\Delta x} + \frac{1}{2\gamma} \right\} + n(i_{end}, k+1) \left\{ \frac{1}{\Delta x} \right\} = 0 \quad (32)$$

$$n(1, k+1) \left\{ -\frac{1}{\Delta x} - \frac{1}{2\gamma} \right\} + n(2, k+1) \left\{ \frac{1}{\Delta x} \right\} = -j_0. \quad (33)$$

Equation (31) can be written for $i = 2$ to $i_{end} - 1$, along with Eqs. (32) and (33). This completes total number of i_{end} equations for unknowns $n_1, n_2, n_3 \dots n_{i_{end}}$. Hence, we get a system of equations in the form

$$[A]_{i_{end} \times i_{end}} [n]_{i_{end} \times 1} = [C]_{i_{end} \times 1}. \quad (34)$$

Now the neutron density distribution at time ' $k+1$ ' can be obtained from the relation

$$[n] = [A]^{-1} [C]. \quad (35)$$

Equation (34) can be solved by the Gauss elimination method. Considering the large simulation time for this method, an inbuilt function for Eq. (35) is used.

3. Results and discussion

The dimensions of the one-dimensional slab and other data taken for calculations are [6] $x_l = 0$, $x_r = 20$, $j_0 = 0.001$, $\beta = 0.6 \times 10^{-12}$, $\gamma = 6 \times 10^{-2}$, $n_{eq} = 0.1$, $\eta = 2.0$, $a_0 = 1.65$ and $d = 8$. The simulation results show (figures 4 and 5) the ignition and propagation of solitary wave. The space profile of neutron density and nuclide concentrations at different times are shown in the figures. Figure 4 shows spatial profiles of neutron density at different times, i.e., $t = 15, 20, 32, 37$ and 46 . The transition to asymptotic state is clearly depicted in figure 5, where the spatial profile of neutron density, ^{239}Pu and ^{238}U concentrations are shown at each $t = 2$ unit interval.

The simulation is done up to $t = 50$. It is observed that the average velocity of burn-up wave in transient phase ($= 0.245$) is less than the wave velocity in equilibrium phase ($= 0.62$). The TT is 20 and TL is 4.9 (for wave development up to 99% of its equilibrium state). It should be noted that distance and time are measured in terms of l (diffusion length) and τ_β (mean lifetime of nuclide β -decay). Hence, for $l = 10$ cm (diffusion length of a U–Pu cycle reactor) and $\tau_\beta = 3.4$ days, the observed TT and TL are 76 days and 49 cm, respectively. Thus, the velocity of wave in transient phase is 2.63 m/year. The velocity in equilibrium phase works out to be 6.6 m/year. The shape of the neutron density distribution in asymptotic state can be observed from figure 4 at $t = 32$ and 37 . The slope at the far side is steeper than at near side, where external neutron current impinges the slab. The FWHM is found to be 6.28 (62.8 cm) and FW10M is 13.1 (130.9 cm).

Figure 4. Spatial profiles of neutron density at different times (x and t are expressed in l and τ_β , respectively).

Figure 5. Spatial profiles of neutron density and ^{239}Pu and ^{238}U concentrations at $t = 2$ unit intervals.

The plutonium critical concentration (n_{cr}) will take different values in different reactor configurations. As said earlier, plutonium equilibrium concentration n_{eq} depends only on microscopic neutron cross-sections. Hence, corresponding to different plutonium critical concentrations, different $\eta (= n_{eq}/n_{cr})$ values can be obtained. A parametric study is done by varying η to examine its effect on transient and asymptotic characteristics of solitary wave. Hence, in

Figure 6. Spatial profiles of neutron density for $\eta = 1.58, 1.59, 1.67$ and 1.75 (from top to bottom).

addition to $\eta = 2$, simulations are done for $\eta = 1.58, 1.59, 1.67$ and 1.75 . The neutron density profile depicted in figure 6 confirms that solitary wave fails to develop when $\eta = 1.58$. This is in conformity with the results of [6]. For all other values of η , solitary neutron fission wave develops into equilibrium state. From figure 6, it is observed that for lower values of η the neutron density is greater in transient phase than steady-state phase.

The characteristics of solitary wave for different η values are summarized in table 1 and figures 7–9. The values measured in normalized axis (i.e., in terms of diffusion length and mean lifetime of nuclide for β -decay) and converted in regular units are presented. TT and TL are given for two different criteria, i.e., wave developed up to 99% and 95% of the neutron density with respect to equilibrium state.

Following observations can be made from the simulation results given in table 1. The results are discussed for 99% criterion and the asymptotic state.

1. TT decreases when η increases. The change is large when $\eta = 1.59$; it takes about 1111 days to establish the

Figure 7. Transient time vs. η .

Figure 8. Transient length vs. η .

asymptotic solitary wave but for $\eta = 2$ it takes only 76 days. Thus the η value will play an important role in optimizing the design of transient and steady-state phase of the wave.

2. TL also decreases when η increases. It decreases from 124 to 49 cm when η is varied from 1.59 to 2.0. Here also, the importance of η is reflected as expected.

Table 1. Burn-up wave characteristics for different η values.

η	1.59		1.67		1.75		2	
Criterion for transient state (%)	99	95	99	95	99	95	99	95
Transient time (days)	1111.4	919.3	264.9	225.1	167.2	142.0	75.6	71.2
Transient length (cm)	124.0	112.0	97.0	78.0	84.0	62.0	49.0	41.0
Velocity (cm/year)	22.2		176.0		308.9		658.9	
FWHM (cm)	55.1		56.3		57.8		62.8	
FW10M (cm)	114.7		116.3		119.1		130.9	
n (asymptotic)	0.003		0.026		0.050		0.129

Figure 9. FWHM and FW10M vs. η .

3. The velocity of the equilibrium wave velocity increases from 0.22 to 6.6 m/year when η is increased from 1.59 to 2.0.
4. The widths of the reaction zone characterized by FWHM and FW10M remain almost the same. However, there is a slight increase with respect to increase in η .

5. The asymptotic neutron flux density increases as η increases. This is to be limited from the consideration of heat removal.

The interpretation of the results lies in the fact that the η ($= n_{eq}/n_{cr}$) variation is due to change in the plutonium critical concentration, n_{cr} . When the plutonium concentration required to sustain the nuclear chain reaction (n_{cr}) decreases, η increases. Hence, the TL and TT to attain asymptotic state, by producing enough plutonium, decrease with increase in η (i.e., a decrease in required critical plutonium concentration). The velocity of the wave will increase when the required plutonium critical concentration is decreased. It causes increase in velocity with increase in η .

Sensitivity of the transient parameters to the external neutron flux is examined by varying j_0 . Since it is seen from previous simulation that $\eta = 2$ gives optimal solitary wave development, it is taken as reference. From the simulation outputs it is observed that only TT is affected by variation in j_0 . Asymptotic parameters, including neutron flux density, remain constant as expected. Results are shown in table 2. The results for different values of 'd' and plutonium concentrations are shown in tables 3 and 4.

Table 2. Burn-up wave Transient Time for different j_0 values.

j_0	0.0005		0.001		0.002		0.003	
Criterion for transient state (%)	99	95	99	95	99	95	99	95
Transient time τ_β	26.0	24.7	21.84	20.6	19.0	17.7	17.6	16.4
Transient time (days)	89.9	85.4	75.6	71.2	65.7	61.3	61.1	56.7

Table 3. Burn-up wave transient parameters for different 'd' values.

d	6.5		7.5		8.0		8.3	
Criterion for transient state (%)	99	95	99	95	99	95	99	95
Transient time (days)	375.1	364.3	135.6	125.9	75.6	71.2	55.1	52.2
Transient length (cm)	74.0	55.0	63.0	45.3	49.0	41.0	44.0	39.0

Table 4. Burn-up wave transient parameters for different ' a_0 ' values

a_0	1.55		1.60		1.65		1.68	
Criterion for transient state (%)	99	95	99	95	99	95	99	95
Transient time (days)	199.9	194.5	124.2	119.3	75.6	71.2	57.9	53.7
Transient length (cm)	52.5	43.0	51.0	42.0	49.0	41.0	48.0	41.0

4. Conclusions

The solitary NBW build-up in a fast neutron multiplying medium is investigated. The main focus of the study is on the analysis of transient part of the burn-up wave and width of the steady-state part. The characteristics of the transient part are expressed in terms of new parameters, i.e., TL and TT that elapse, in establishing the asymptotic burn-up wave. From these parameters, the transient wave velocity is determined. The characteristics of asymptotic wave are determined in terms of wave velocity, FWHM and FW10M of neutron flux distribution in the space. The wave velocity in transient phase is lesser than the wave velocity in asymptotic burn-up wave.

A parametric study is carried out to investigate the sensitivity of these parameters to some of the physical parameters of the ignition zone and the breeder zone. A large η brings down TT and TL but it increases steady-state burn-up wave velocity. There is a large drop in TT in the range of η increasing from 1.6 to 1.7. The sensitivity of TT and TL is depicted with plutonium fuse width and ^{239}Pu concentration in plutonium fuse. However, both of these parameters also have upper limits, which causes system runaway. The external neutron source strength affects only TT as expected. The characterization parameters would be very meaningful to understand the transient characteristics of the self-sustaining NBW and in evaluating the quality of the wave by the designers.

These studies would be improved by considering radial buckling to account for neutron leakage and some other aspects.

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