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Design optimization of deep groove ball bearings using crowding distance particle swarm optimization

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Abstract. This paper presents a crowding distance particle swarm optimization technique to optimize the design parameters of deep groove ball bearings. The design optimization problem is multi-objective in nature. The considered objectives are maximizing dynamic and static load bearing capacities and minimizing elasto-hydrodynamic film thickness. The technique is applied to bearings used in transmission system of a tractor for the purpose of farming. Pareto optimal solutions are obtained using the proposed technique. The results obtained from the technique are found to be superior compared with NSGA-II and catalogue values.

Keywords. Mechanical design; design optimization; deep groove ball bearings; multi-objective optimization; particle swarm optimization; crowding distance.

1. Introduction

Deep groove ball bearings are dynamic mechanical components that minimize the frictional losses during power and motion transfer processes. They belong to the class of rolling element bearings [1] and are used extensively in industrial applications occupying a major market share [2]. Bearings can sustain radial and moderate amount of thrust loads exerted by the rotating shafts on a small area, giving rise to high magnitude contact stress. If a bearing element is provided with proper lubrication and alignment, without any abrasive particles, moisture or corrosion-causing elements with appropriate load, it is unlikely to fail except due to contact fatigue [3]. Bearings look trivial but their failure is costly. This has led to the increasing interest in research on bearings design and other operating parameters.

The design optimization of bearing literature mentions application of traditional optimization techniques [4, 5]. The limitations of these techniques are well documented. To overcome these drawbacks of traditional techniques, researchers have proposed different artificial intelligence (AI) techniques [6, 7]. In the field of mechanical design, different types of AI techniques have been used [8–12].

Particle swarm optimization (PSO) [13] is an AI technique that explores the best solution by sharing the information about global or local best. There are many variants of PSO proposed by the researchers [14–19]. They have wide and varied applications in mechanical design [20–26]. The variant of PSO that deals with multiple objectives is known as multi-objective PSO (MOPSO) [27]. Crowding distance PSO (PSOCD) is a class of MOPSO. It ensures an even spread of non-dominated solutions to cover the entire Pareto front [28]. Researchers have implemented the concept of crowding distance to obtain better results for a variety of problems [29, 30]. PSOCD is a little more time consuming than MOPSO. However, it is competitive for selecting the global best [31, 32]. However, in the field of ball bearing research, PSOCD has not been explored much.

In the present work, the application of deep groove ball bearings in farm equipment like tractors has been considered. An attempt has been made to increase the load bearing capacity of deep groove ball bearings using PSOCD. A modified mutation strategy is used to maintain the diversity in swarm for including the entire range of design variables. The obtained results were compared with the NSGA-II solutions and catalogue values. Also, the possible increased reliability of tractors in harsh soil and environmental conditions is discussed.

The paper is organized as follows. The problem statement, presented in section 2, gives a detailed description of the objective functions, constraints and their conditions. The PSOCD is presented in section 3. An application of the deep groove ball bearings is presented in section 4. Results

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and discussion are presented in section 5. Conclusions are presented in section 6.

2. Problem statement

We focus on deep groove ball bearings, which are used to carry radial load in the transmission system of a tractor. Figure 1 indicates the details of the internal geometry of the system. The design variables and objective functions are explained in the following sub-sections.

2.1 *Design variables*

The design parameters of this study include pitch diameter of bearing (D_1), ball diameter (D_2), curvature coefficient of the inner raceways (f_1), curvature coefficient for outer raceway (f_2) and the number of rolling elements (Z). These parameters form the internal geometry of the bearings. Also, the boundary dimensions are essential to define the major geometry of the bearings, which are standardized to facilitate interchangeability. These standardized dimensions include bearing bore diameter, outside diameter, width and chamfer. Thus, to optimize the overall life and performance of the bearing, variables that define the internal geometry are used as these variables directly influence the study objectives.

2.2 *Objective functions*

The objective functions considered are maximizing the static and dynamic load bearing capacities and minimizing the elasto-hydrodynamic film thickness. They are defined as follows.

2.2a *Static load bearing capacity (C_{or})*: The static load bearing capacity is related to the loads applied to non-rotating bearings. It is expressed as a function of variables corresponding to the internal geometry of the system. C_{or} is the maximum load that any bearing can resist before reaching permanent deformation of the component. If the applied load exceeds C_{or} then brinelling will take place, which is furthermore influential for failure. The static load rating is influenced by the highest contact stress that occurs between the balls and either of the inner or outer raceways. It is affected by the material used for bearing, number of balls used and their size, curvature of inner and outer raceway and their depth along with the contact angles [1].

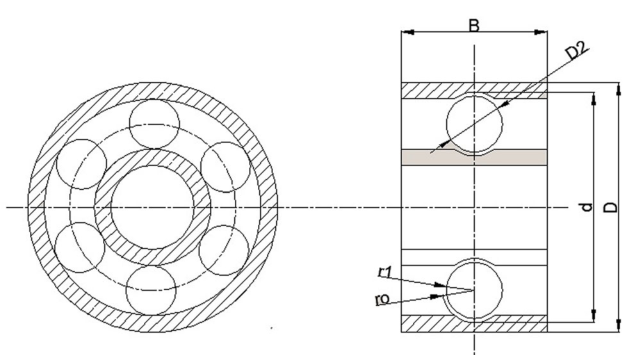

Figure 1. Deep groove ball bearing.

2.2b *Dynamic capacity (C_r)*: The dynamic load bearing capacity is defined using the maximum shear stress theory. The theory states that the total strain energy in the complex state of stress system reaches the shear strain at the sub-surface near the area of contact. Dynamic load ratings are found by using the variables like geometry of the bearing, number of balls, size of the balls used, pitch diameter of bearing and the material used for ring and the balls. This load rating is used along with the actual applied radial load for calculating bearing fatigue life. It has been observed that the dynamic capacity of the bearing varies with the number of rollers and the diameter of the ball [10].

2.2c *Minimum elasto-hydrodynamic film thickness*: The elasto-hydrodynamic film thickness is related to the maximum wear life of the bearing. Theoretical and experimental results have proved that the elasto-hydrodynamic lubrication is a manipulating factor for the basic pattern of stresses developed in the contacting surfaces that intensifies the failure due to fatigue. It has been proved that the bearing life decreases when the film thickness decreases [33–35]. This is due to the increase in friction and thereby increase in temperature. This process enhances the lubrication loss and damages the raceways. As a consequence, the sliding metal to metal contact damages the surfaces in contact and the bearing life decreases rapidly. Hence, an optimum elasto-hydrodynamic film thickness must be maintained for smooth and pure rolling in the contact surfaces.

2.3 *Constraints*

The constraints defined in Chakraborty *et al* [8] and Gupta *et al* [10] are abridged and adopted for this study. Ranges of the design variables are defined as follows [36]:

i. the mean diameter ranges between $0.5(D+d)$ and $0.6(D+d)$;

ii. the ball diameter varies between $0.5(D-d)$ and $0.6(D-d)$;

iii. the number of balls is provided by the condition $(4 \leq Z \leq 50)$;

iv. the inner and outer raceways vary according to the inequalities

$$0.515 \leq f_1 \leq 0.6, \; 0.515 \leq f_2 \leq 0.6.$$

Bearings should be assembled properly to obtain better life. To ensure proper assembly with required assembly angle, the number of balls should satisfy the following constraints:

$$2(Z-1)\sin^{-1}\left(\frac{D_b}{D_m}\right) \le \varphi_0, \tag{1}$$

$$g_1(x) = \frac{\varphi_0}{2\sin^{-1}\left(\frac{D_b}{D_m}\right)} - Z + 1 \ge 0, \tag{2}$$

where φ_0 is the maximum tolerable assembly angle. This is influenced by the bearing geometry.

The diameter of balls should vary between the following limits:

$$K_{Dmin}\frac{D-d}{2} \le D_b \le K_{Dmax}\frac{D-d}{2}, \tag{3}$$

where K_{Dmin} and K_{Dmax} are the minimum and maximum values of the ball diameter, respectively. This is associated with the diametric series of bearings and the ball strength.

The constraints related to these conditions can be constructed as follows:

$$g_2(x) = 2D_b - K_{Dmin}(D-d) \ge 0, \tag{4}$$

$$g_3(x) = K_{Dmax}(D-d) - 2D_b \ge 0. \tag{5}$$

In order to ensure the running mobility of bearings, the difference between the pitch diameter and the average diameter should be less than a given value. Therefore, we write

$$g_4(x) = D_m - (0.5-e)(D+d) \ge 0, \tag{6}$$

$$g_5(x) = (0.5+e)(D+d) - D_m \ge 0, \tag{7}$$

where e is a constant and its value is obtained based on the mobility conditions of the balls. In the design optimization of bearings, the pitch diameter is typically greater than the related average diameter of a bearing. Therefore, for the bearing ring at the outer raceways, the thickness should not be lower than εD_b, where ε is a constant that depends on simple strength consideration of the outer ring. The corresponding constraint is defined as follows:

$$g_6(x) = 0.5(D - D_m - D_b) - \varepsilon D_b \ge 0. \tag{8}$$

For the inner and outer raceways of a bearing, the groove curvature radius should be designed in such a way that it does not exceed $0.515Db$. There is no upper limit for the groove curvature radius, but when it is greater than $0.52Db$ and $0.53Db$, respectively, for the inner and outer raceways, the dynamic load rating of the bearing reduces. Therefore

$$g_7(x) = 0.52 \ge f_i \ge 0.515, \tag{9}$$

$$g_8(x) = 0.53 \ge f_o \ge 0.515. \tag{10}$$

3. Crowding distance PSO (PSOCD)

The proposed PSOCD is presented here. The algorithm selects the global best particle and deletes the external archive after selection. The mutation operator is used to maintain diversity of the solution. Crowding distance will be computed for each non-dominated solution separately and the non-dominated solution with highest crowding distance will act as the swarm leader. The pseudo-code of the approach is presented below. The notations used to describe the pseudo-code are presented first.

N: swarm size

$x[i]$: particle

t: iteration count

$v[t]$: velocity of the particle

AR: external archive where the non-dominated solutions will be stored

m: objective value

$R_1, R_2 \sim U(0,1)$.

The pseudo-code is presented as follows:

```
BEGIN
    FOR i = 1 to N
        Initialize x[i]
        Initial velocity v[i] = 0
        pBest [i]= x[i]
        gBest = Best position of x[i]
    END FOR

    FOR t = 1 to t_max
        Count non-dominated solutions in AR as N = |A|
        Initialize the distance
            FOR d = 1 to D
                S[d].distance = 0
                    FOR  j= 1 to (n–1)
                        S[d].distance = S[d].distance + S[j+1].m – S[j-1].m
                    END FOR
            END FOR
        Sort the solutions by the crowding distance in descending order
        gBest = Top 15% of x[t]
        v[t] = w * v[t] + (C1 R1 × pBest − x[t]) + (C2R2 × AR(gBest) − x[t])
        AR(gBest) = max{particle in the repository}
            x[t] = x[t] + v[t]
                IF particle goes beyond the boundary
                    v[t] = - v[t]
                END IF
        IF (t < (t_max×pmut)
            perform mutation on x[t]
            Calculate x[t].
        END IF
        IF (t < t_max)
            INCREMENT t
            Continue
        ELSE
            Report gBest
    END FOR
END
```

4. Application

Tractors and farm implements should be able to function smoothly in huge farms under harsh working conditions that place severe demands on the bearing in the equipment. In such working conditions the customers expect bearings to be highly reliable while offering superior cost performance. Various components of a tractor, including engine, transmission system and wheel hub, are embedded with bearings. The present work is focussed on the transmission system that invloves heavy duty radial load bearings. These bearings should withstand high amount of meshing loads that are caused by the engagement and disengagement of gears. For this purpose, the bearings should be designed in such a way that they should be reliable in the highly variable loading conditions. An attempt is made to increase the load bearing capacity by applying the proposed PSOCD so as to attain the best solution. Figure 2 shows a sliding mesh gear transmission system. The ball bearings are employed to work under the radial load that arises out of the meshing of gears. Generally, the deep groove ball bearing with bearing number 6204 will be used in this particular application.

5. Results and discussion

In this study, the optimum design parameters of deep grove ball bearing used in transmission systems of mechanical applications like tractor have been investigated. NSGA II and PSOCD algorithms have been applied simultaneously to analyse the performance of both algorithms. A set of Pareto optimum solutions have been obtained by implementing both the algorithms considering any two of the stated objectives concurrently. The boundary values for each of the variable have been kept identical according to proper consideration of the application and constraints applicable to the case under study. The static and dynamic load capacities have been mentioned in N and the elasto-hydrodynamic film thickness has been calculated in μm.

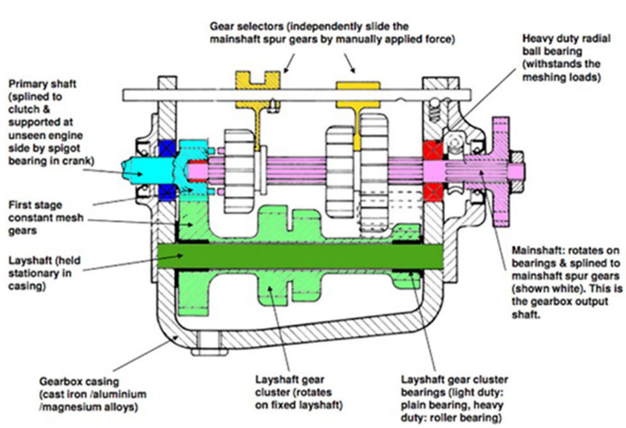

Figure 2. Transmission system of a tractor.

5.1 *NSGA II results*

For finding the NSGA II results, we have used the MATLAB MOGA toolbox. In figures 3, 4 and 5, the Pareto fronts between static and dynamic load capacity, static load capacity and elasto-hydrodynamic film thickness, and dynamic load capacity and elasto-hydrodynamic film thickness are shown, respectively.

The NSGA II results have been found out to be reasonably practical for both static load capacity (25000–40200 N) and elasto-hydrodynamic film thickness (0.02–0.17 μm). However, the dynamic load capacity obtained by NSGA II is within a range of 36.5–39 kN, which is quite low. Even the number of Pareto optimal solutions obtained by NSGA II algorithm is fewer (within a range of 5–12), resulting in more restricted design options available to the decision makers.

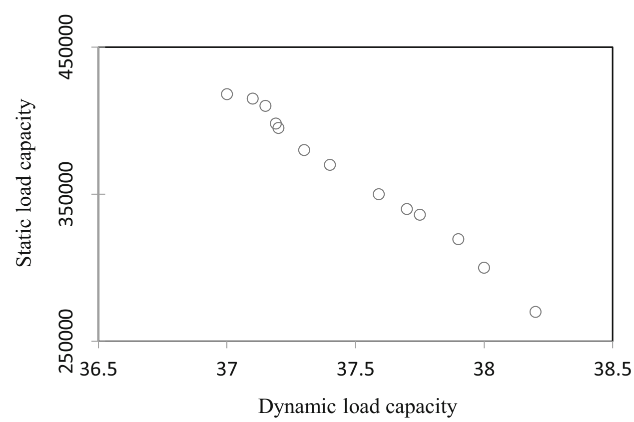

Figure 3. Pareto front for dynamic load capacity and static load capacity using NSGA II.

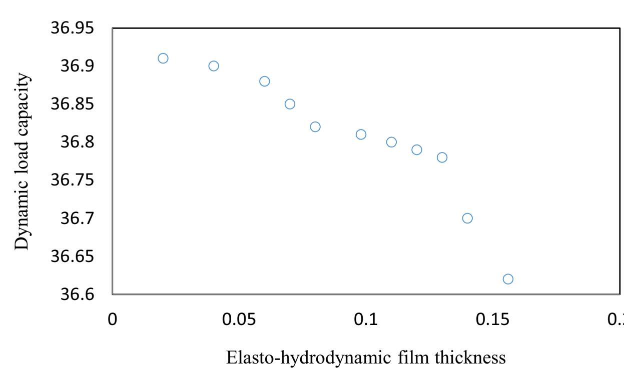

Figure 4. Pareto front of static load capacity and elasto-hydrodynamic film thickness using NSGA II.

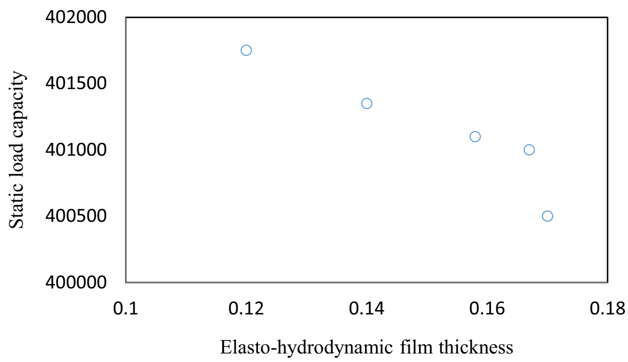

Figure 5. Pareto front for dynamic load capacity and elasto-hydrodynamic film thickness using NSGA II.

5.2 *PSOCD results*

When the optimization is carried out comprehensively for an archive size of 1000 and a maximum of 50 generations, the maximum value of static load bearing capacity is obtained as 65261.55 N. The values of the design variables corresponding to this highest static capacity are D_1: 33.56480; D_2: 16.00000; Z: 50 and f_1: 0.5674568, f_2: 0.5673589 as per PSOCD algorithm. The highest value of the dynamic load bearing capacity is 30578.67 N and the corresponding design variables are D_1: 40.00000; D_2: 15.99996; Z: 50 and f_1: 0.5495156, f_2: 0.5677343. The variation of static load bearing capacity along with the dynamic load bearing capacity is shown in figure 6.

When the optimization is carried out between the elasto-hydrodynamic film thickness and static load bearing capacity, the maximum static load bearing capacity obtained is 10787.678 N and the corresponding value of the design parameters are D_1: 39.99737; D_2: 16.00000; Z: 50 and f_1: 0.5159298, f_2: 0.5274896. The elasto-hydrodynamic film thickness obtained is 0.2713662 µm and the values of the design variables corresponding to the thickness are D_1: 39.99165, D_2: 13.57279, Z: 50 and f_1: 0.5150475, f_2: 0.5284733. The trend in the variation of static load bearing capacity along with the elasto-hydrodynamic film thickness can be observed from figure 7.

The graph indicates that with a raise in load the film thickness decreases. The minimum film thickness that keeps the balls is always advantageous. However, the minimum film thickness occurs at lower static load capacity than that obtained without considering the film thickness. When dynamic load bearing capacity and the elasto-hydrodynamic film thickness are under consideration the dynamic capacity obtained is 30586.36 N and the corresponding elasto-hydrodynamic film thickness is 0.5233867 µm, whereas the highest value obtained in the optimization is 0.5431483 µm. Though the value of the highest dynamic load bearing capacity is not compromised, the value of the film thickness has increased from its minimum obtainable value. However, the trend is acceptable as the theory also confirms that with increase in the load the film thickness also increases. The variation of dynamic load bearing capacity and the elasto-hydrodynamic film thickness is indicated in the graph given in figure 8.

When the optimization is carried out for all three objectives simultaneously, the graph shown in figure 9 is obtained.

The afore-mentioned solutions achieved by PSOCD are more implementable and diverse when compared with NSGA II. Thus, PSOCD provided the decision makers with more design choices with more viable opportunities to set parameters for designing the deep grove ball bearing.

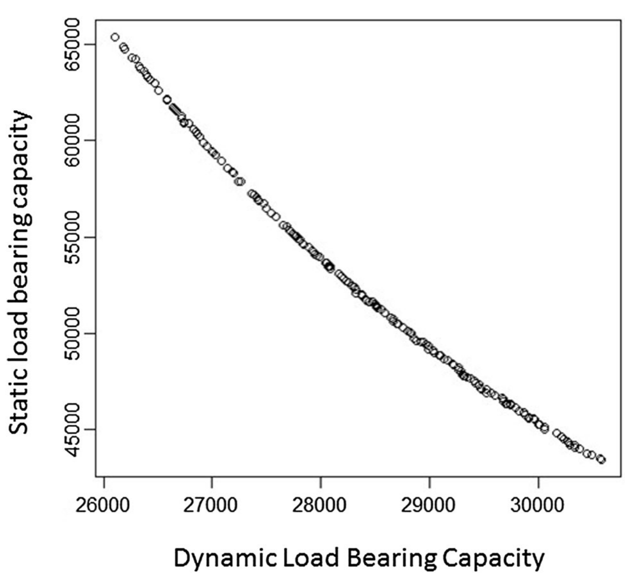

Figure 6. Pareto front of dynamic and static load bearing capacity using PSOCD.

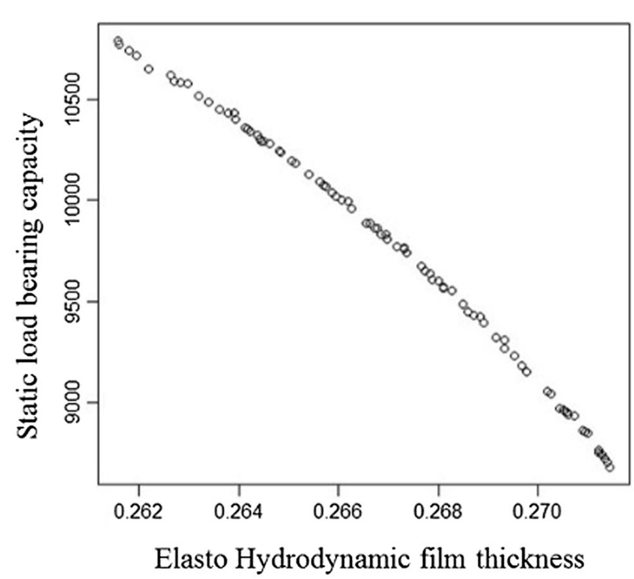

Figure 7. Pareto front of elasto-hydrodynamic film thickness and static load bearing capacity using PSOCD.

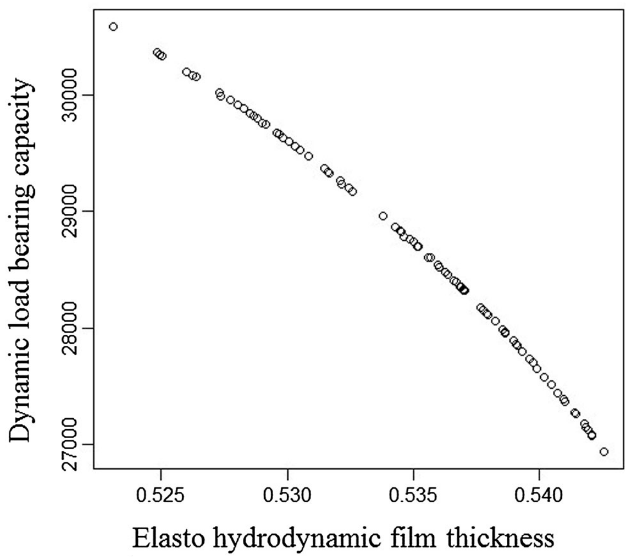

Figure 8. Pareto front of elasto-hydrodynamic film thickness and dynamic load bearing capacity using PSOCD.

5.3 *Time complexity of the algorithm*

In the present study, the three objectives and five inequality constraints are considered for calculating the crowding distance and the non-dominated solution is obtained by comparing the solutions in the repository to determine the complexity in computing the solutions. This problem needs a computational time of 1.6293 s to generate the Pareto front for the three objectives taken simultaneously. Most of the computation time is spent on sorting the solutions in the repository for each of the objective with an archive size of 500. The variation of the computational time with archive size will be logarithmic according to big 'O' notation. If there are N solutions in the archive for F objective functions and a swarm size of S, the complexity is $O(FS \times \log N)$ Leung *et al* [31].

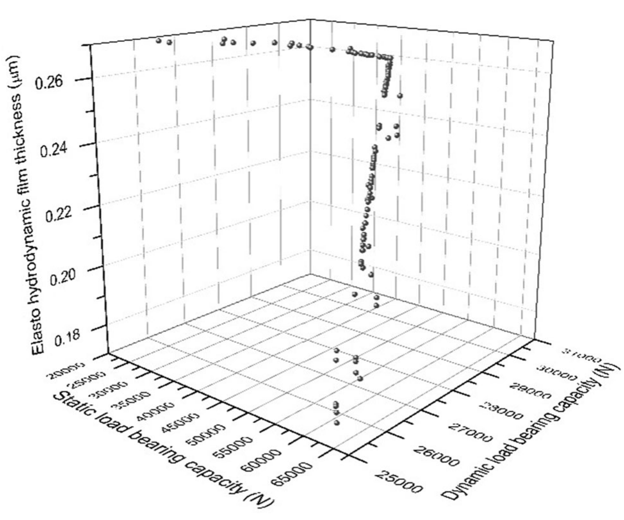

Figure 9. Pareto front considering of all three objectives simultaneously.

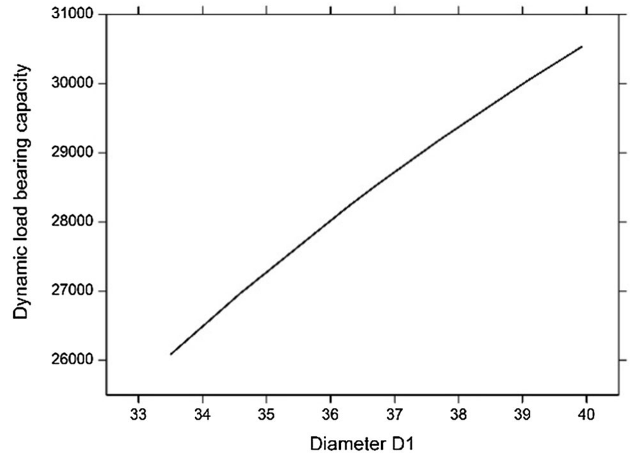

Figure 10. Dynamic load bearing capacity vs D_1.

5.4 *Sensitivity analysis*

Pitch diameter: The pitch diameter directly impacts the dynamic load bearing capacity. Figure 10 indicates that with increase of diameter, load bearing capacity also increases.

Ball diameter: The ball diameter remained almost constant over all the optimized results, indicating that the change in ball diameter has low impact on the variation in dynamic capacity of the bearing.

Number of balls: According to the obtained results, the number of balls always remained constant. Also, the value that is obtained is the maximum value based on the constraints taken. Hence, the number of balls should be maximum to obtain better load bearing capacity.

Inner and outer curvature radius: The values of these two variables do not seem to influence the output, but they are important from the designer point of view.

6. Conclusion

A multi-objective design optimization problem corresponding to deep groove ball bearings with application to transmission system of tractor has been studied using a PSOCD. There are three objectives in the problem. The dynamic load bearing capacity, which is the most important objective for increasing the sustainability of ball bearings under varying load, is increased considerably from the standard catalogue values. Static load bearing capacity is also increased substantially, indicating that the bearings can withstand higher non-rotating load. This too has significant impact on the durability of the bearings under static conditions. The elasto-hydrodynamic film thickness has an optimum value, which is neither too low nor too high. Hence, the bearings can maintain the appropriate lubricant film thickness under increased conditions, leading to higher wear life for the bearings. The results obtained from the PSOCD show that the objective values increase four times with respect to their respective catalogue values. As a result, the life of the bearing will increase considerably, making the transmission system reliable under variable loading conditions. We have solved the problem using NSGA II. It has been observed that the PSOCD results are superior and advantageous from the engineering design perspective. The PSOCD can be applied to other design optimization problems with multiple objectives.

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