Identification of probabilistic approaches and map-based navigation in motion planning for mobile robots

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Abstract. Robot motion planning (RMP) develops a precise path between start and goal points for mobile robots in an unknown environment. RMP is a complex task when it needs to be planned for a group of robots in a coordinated environment with leader–follower relationship. The planned path might change depending upon the number of robots and the decision made. The decision made by each robot depends on the feedback received based on the subsequent action taken by other robots. In addition, the computational complexities depend upon factors such as communication between robots, the influence of moving obstacles and environment in which they are interacting. In order to explore further in the area of motion planning, it is felt that a comprehensive survey of available literature would support researchers working in RMP and hence the present paper. This paper reviews around 152 articles published in various international journals and conferences with more emphasis on articles published after 1960. In this work, recent activities carried out in the field of path planning for mobile robotics are critically evaluated and problems faced by the researchers are also highlighted. The focus is towards implementation of probabilistic algorithms, including Probabilistic Road Map and Simultaneous Localization and Mapping. Future research prospects in multi-robot path planning based on probabilistic approaches are also discussed.

Keywords. Multi-robot coordination; map-based navigation; multi-agent robot systems; probabilistic approaches; robot motion planning; Simultaneous Localization and Mapping; tracking controller; robot fault analysis; role assignment; wheeled robots.

1. Introduction

The research in robot motion planning (RMP) [1, 2] can be traced back to late 1960s when computer-controlled robots were developed, especially for industrial applications. The motion of a robot is described as a path in configuration space (CS) where there is no collision between robots and obstacles. This CS can be described as “free space (F)”. However, the state of free space may vary from time to time due to the presence of moving obstacles. The aim of motion planning problem is to determine a path through the free space in an efficient way based on objective functions defined, the number of robots communicating with each other and the environment in which they are interacting. Motion planning can be split into two categories: holonomic and nonholonomic. When all degrees of freedom (DOF) are controllable independently, holonomic motion planning can be implemented and for the non-holonomic case, the DOF cannot be controlled independently. In the 1970s, classical algorithms consisting of

grid-based approaches were introduced [3–5]. Such algorithms search extensively to obtain the knowledge about the CS in which the robot interacts during navigation. During the 1980s, roboticists addressed motion planning problem by devising a variety of heuristics and approximate methods [6–8]. Such methods decompose the CS into simple cells lying inside or partially outside the free space. During the 1990s, motion planning problem techniques, based on kinematic constraints, were reported in the literature [9–11]. Current methodologies deal with various probabilistic approaches with multiple integrations of model and sensory perceptions [12–14]. Compared with classical methods, probability-based motion planning algorithms can tackle high-dimensional complex problems compounded with moving obstacles. In this paper, literature published after 2000 in the area of RMP are comprehensively reviewed and presented. Around 14 articles published before 2000 are also included as many of the fundamental and critical information related to RMP were described therein. Issues related to a number of robots in interaction, the size of the robot and communication between robots within the group as well as shape,

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size and position of obstacles have also been discussed. Basically, the workspace for mobile robots can be of two categories: unknown or uncertain. In the case of an unknown environment, details about the terrain and obstacles are not known to the master robot. In the case of an uncertain environment, details of the terrain are available to the master robot but information about the obstacles is not known. For both categories, the master robot will collect the information and feed it back to the followers. All fundamental principles in precision engineering are deterministic and probabilistic. Map-based navigation for motion planning in mobile robots is fully predictable and hence precision engineering plays a major role in mobile robots path planning (PP). Apart from this introductory section, the paper is presented in the following manner. Section 2 discusses RMP, which develops an algorithm to convert high-level specification tasks into a low-level task. This is in order to make a robot navigate in an unknown environment to accomplish a specific task. Section 3 recalls various probabilistic approaches employed in RMP. Section 4 reviews various strategies within the context of map-based navigation in mobile robots. Section 5 highlights various role assignment (RA) schemes followed by a discussion on different tracking algorithms developed, which is presented in section 6. Section 7 describes multi-robot formations as one of the challenging problems of RMP. Discussion is presented in section 8 and conclusions are reported in section 9 followed by references.

2. Robot Motion Planning (RMP)

The aim and objective of RMP are to determine a path for a robot, which is a continuous set of points in space. The task of motion planning is to determine the trajectories among uncertain paths so as to meet the given objectives. RMP consists of three steps as shown in figure 1:

- PP—only geometric considerations;
- trajectory planning (TP)—include time, i.e., consider the dynamics;
- control planning (CP)—vector control of the robot.

The PP is the planning of the navigation path from point A to point B, including the defined path points. The path includes several continuous motion trajectories that could be accomplished with TP. If a path cannot be previously planned because of limited previous information, the motion task is named as path finding. RMP can be evaluated by four methods, namely: global optimal, local optimal, sub-optimal and non-optimal. In global RMP, target space is measured by the robot's sensors. However, in local RMP, the robot cannot measure the target space in some states. To achieve this, the robot traverses through several virtual nodes located around the robot. A virtual

node is called as a sub-optimal. A simple way to achieve this is to develop a PP algorithm. A TP algorithm then uses this path to determine the trajectories that will make the robot move in its workspace with the required velocity and acceleration. After determining the motion in a path, it can be translated into control variables [15–17]. While developing an RMP algorithm, there are two criteria that should be considered. One is the type of motion deployed and the other is the methodology to solve this algorithm. These algorithms focus primarily on uncertainties and optimalities. Basically, there are three approaches for implementing RMP as mentioned in table 1. Further, the key concept for motion planning is the robot configuration that provides a complete description of the position of every point in the system in which the robot traverses. The space of all configurations is the CS or C_{space} . Hence, robot configuration (q) is a specification of the positions of all robot points relative to a fixed coordinate system. Generally, the robot configuration is based on two parameters: heading and turn rate. Heading specifies the orientation and turn rate defines the sharpness of the turn. Heading can be modeled as

$$\Theta_{n+1} = \Delta.r + \Theta_n \quad (1)$$

where Θ is the heading, r is the turn rate and Δ is the sample period.

2.1 Types of RMP

RMP was divided into three types as presented in figure 2. Depending on objective function and localization, an appropriate type can be selected. In order to accomplish this task, a robot must determine its location within an environment using one or more sensors.

3. Probabilistic approaches

Various probabilistic approaches have been developed based on different constraints and are available in the literature and these approaches are comprehensively reviewed in detail in this section. Various problems arise with planning algorithms such as uncertainty local model (ULM) and sampling-based planning algorithms, where obstacles create narrow passages for the robots to achieve a collision-free path.

3.1 Planning algorithms

3.1a *ULM*: Two local configurations q_{int} and q_{goal} in an unknown environment were considered, where q_{int} is the start and q_{goal} is the goal point. ULM will compute a feasible path L ($q_{\text{int}}, q_{\text{goal}}$) without considering obstacles as shown in figure 3.

Figure 1. Overview of activities of RMP [15–17].

This path prevails if

$$L(q_{\text{int}}, q_{\text{goal}}) \in CS_{\text{free}} \quad (2)$$

where CS denotes configuration space and CS_{free} denotes free space in CS defined by any two local points. These algorithms become very complicated when new constraints such as narrow spaces and obstacles are added to CS. In order to further explore roadmaps consisting of constraints, Probabilistic Road Map (PRM) was introduced [18–20].

3.1b Sampling-based planning algorithms: Sampling-based planning algorithms consist of Rapidly exploring Random Tree (RRT) [21] and PRM [22, 23]. These algorithms have gained significant popularity in recent years as tools for solving multi-dimensional motion planning problems. Basically, PRM algorithm constructs a roadmap that characterizes the connectivity of CS_{free} by randomly selecting a large number of collision-free milestones, and checking if each of these milestones can be connected by a collision-free path generated by a simple local method. RRT is an effective sampling tool, which is used to search N -dimensional spaces (X, Y, Θ) that contain both algebraic constraints and differential constraints. The major difference between RRT and PRM is that RRT works by applying control inputs that drive the system towards randomly

Table 1. Approaches in implementing RMP.

Approach	Description
Execution	Execute it either in simulation or with the robot connected to the physical world. The algorithm produces a plan, which is encoded and given as input to the robot machine
Refinement	Refine it into a better plan for enhancing the performance and accuracy. If a plan is revised for refinement, then the algorithm accepts it as an input and determines a new plan that can take more constraints into account
Hierarchical inclusion	Package it as an action in higher level plan combining execution and refinement. The original plan is divided into subroutines and will be executed in sequence. This results in execution of more actions while updating

selected points, whereas point to point convergence is used in PRM. The steps involved in RRT/PRM path generation are shown in figure 4.

The salient features of RRT include the following:

- expansion towards unexplored areas in CS and

- describing the position and orientation of complicated bodies.

Further, RRT algorithm is probabilistically complete. A few problems associated with PRM and RRT are requirement of parameter tuning, high processing time and unsuitability for incremental distance. Kuffner and Lavelle [21] developed a new algorithm called RRT-CONNECT planner to explore roadmaps with differential constraints; it is an extended version of RRT and a combination of RRT and CONNECT heuristic algorithms. In the RRT-CONNECT planner, less importance is given for incremental motion. Instead of using CONNECT heuristic with EXTEND function, only CONNECT heuristic is used.

Probabilistic Roadmap of Tree (PRT) [24] constructs a roadmap aiming at capturing the connectivity of CS_{free} , whereas the Randomized Path Planner (RPP) searches for a path by following the negated gradient of an artificial potential field constructed over the CS. RPP has successfully solved problems for robots with more than 60 DOF. In RPP, most of the time is spent in the query phase, while in the PRM planner most of the time is spent in building a roadmap in the pre-processing phase. Li and Shie [22] developed an updated algorithm called Reconfigurable Random Forest (RRF). It modifies the roadmap by removing invalid nodes as the obstacle configurations change at run time. Advantages of RRF are that the C-space can be learned incrementally as the process goes on. Various planning algorithms along with their description are listed in table 2.

The planning algorithms discussed in table 2 are deployed in different domains and are listed in (i)–(v) of section 3.1b.

(i) PRM

The PRM is designed based on two conditions: one is the probabilistic perception (PP) and the other is the probabilistic decision (PD). PP plays a major role in determining the state of the robot's environment. Decision making is done by PD based on PP. PD decides a path mostly based on current and future uncertainty. Considering these two factors, PRM is developed and it consists of two phases, pre-processing and query stage, as shown in figure 4. In the pre-processing phase, a robot's independent data structure is built upon workspace by identifying narrow regions. In the query phase, a sample is picked randomly with its translational DOF [24, 25]. PRM algorithms can be extended in two ways [26–28]. Primarily, to modify randomized sampling in order to minimize the number of samples required to express optimal plans. Since the complexity depends upon on the number of nodes, minimizing the set of samples is a major criterion in keeping the planning problem traceable. Secondly, to evaluate PRM actions efficiently in the context of uncertainty and then generate motion plans with minimal error.

Further, PRM is a probability-based planning method. CS connected by a number of feasible subsets is approximated by PRM planners, using a roadmap of configurations connected by simple paths. The concept of expansiveness was introduced to characterize how quickly a roadmap

Figure 2. Types of RMP.

Figure 3. Local configuration model.

converges to an accurate representation. The expansiveness of free space F is measured by three parameters: ε , α and β that all take values in the range 0–1 [13, 23, 29]. The following theorem characterizes the performance of PRM as defined in [30, 31]:

Theorem 1 *Let F be $(\varepsilon, \alpha, \beta)$ -expansive, and p_1 and p_2 be two configurations in the same connected component of F . $\text{PRM}(p_1, p_2, N)$ returns a path connecting p_1 and p_2 with probability converging to 1 at an exponential rate, as N increases, where N is sampling integer.*

Theorem 2 *For any $\varepsilon > 0$, any arbitrarily large $N > 0$, and any $\gamma \in (0, 1]$, there exist α_0 and β_0 such that if F is not $(\varepsilon, \alpha, \beta)$ -expansive for $\alpha \geq \alpha_0$ and $\beta \geq \beta_0$, then there exists a pair of query configurations p_1 and p_2 in the same connected component of F such that $\text{PRM}(p_1, p_2, N)$ fails to return a path between q_1 and q_2 with probability greater than γ .*

(ii) Visibility-Based PRM (Vis-PRM)

Vis-PRM is an extension of the PRM [32, 33] and this algorithm captures free space connectivity in a roadmap much smaller than PRM while maintaining the same coverage of the free space. A roadmap is built using the following relation:

Figure 4. Steps involved in RRT/PRM path generation.

Table 2. Planning algorithms.

Sl. no.	Planning algorithms	Description
1	Probabilistic Roadmap Planning (PRM) [23]	Multi-query planner
2	Expansive Spaces Tree planner (EST)/Rapidly exploring Random Tree planner (RRT) [21]	Single-query planner
3	Probabilistic Roadmap of Tree (PRT) [24, 25]	Both multi-query and single-query planner
4	Randomized Path Planner (RPP) [26]	Modifies roadmap by negated gradient of an artificial potential field
5	Reconfigurable Random Forest (RRF) [22]	Modifies roadmap by removing invalid nodes

$$V_{is}L(q_{int}, q_{int}^*) = q^* \in CS_{free} \quad (3)$$

if the condition prevails.

The major advantage is that performance of Vis-PRM is 12 times better than that of PRM [34]. It improves the cardinality of resulting roadmap and the quality of free space coverage. Two important aspects of Vis-PRM are that the size of the roadmap remains intrinsic to the complexity of CS_{free} and number of nodes is directly proportional to coverage of CS_{free} . As a result decrease in iterations directly affects the probability of generating new configurations as in table 3.

Further, a new type of PRM was developed based on quasi-random points, called as quasi-random PRM (Qr-PRM) [34, 35]. With Qr-PRM, performance and completeness are measured probabilistically using random sampling. Deterministic sampling allows the program to terminate within a finite time if solution occurs. The working principle of Mp-PRM is that it initially provides a

framework followed by characterizing each sub-region based on sample characteristics and then assigning those sub-regions to one or more planning algorithms and finally provides stopping instruction for roadmap construction.

(iii) Mobile robots

The mobile robot motion through complex environments is crucial in many robotic applications. The major problem is how to determine a collision-free path from an initial configuration to a final configuration and complexity of the PP problem increases exponentially with the dimension of the CS. To develop an algorithm for these mobile robots without colliding, many traditional searches were used based on two DOF. However, efficiency of the algorithm trims down when the number of DOF increases. Normally, mobile robots have three DOF: two translational and one rotational. Hence a new algorithm called Customizable PRM (Cs-PRM) was developed for multiple robots. A Cs-PRM based on environment consists of three steps: pre-processing, query and path smoothing were presented in [36]. The other parameters that increase the complexity further are radius $R_{\min}$, which is the minimum turning radius, and R_{rt} , the radius of robot wheel. For designing a roadmap, two local configurations q_{int} and q_{goal} in an unknown environment are considered. All the edges and points within these two local configurations q_{int} and q_{goal} are connected. When the size of the mobile robots increased, such as pioneer and seeker robots, Compound PRM (Cpd-PRM) [37] was designed. The idea and concept of this algorithm are similar to those of Cs-PRM. Range for qualifying neighbour node set is given by the relation

$$N_q = \{q' | q' \in V, \min \text{dist} \leq D(q, q') \leq \max \text{dist}\} \quad (4)$$

and the feasible path is given by $r_{\text{arc}} \leq R_{\min}$.

Based on Eq. (4), a path was designed. This approach connects q_{int} to q_{goal} on Cpd-PRM through local routes. One can use the search algorithm to find the shortest path. Further, planning robot motion with different radii was not possible in other versions of PRM. Hence, this algorithm is adaptive for different kind of obstacles. This helps in developing an efficient algorithm in the multi-robot domain in terms of time consumption and minimum energy as objective functions. The accuracy of PRM also depends upon the execution time and clearance factors. The safe path for robots among polygonal obstacles through clearance-based PRM (Cl-PRM) [29, 38] was developed. Execution time and clearance are important criteria considered in this algorithm.

(iv) Collision detection

To minimize the number of collisions and running time, Lazy-PRM was developed in [39]. Initially, in Lazy-PRM, a roadmap consisting of nodes and edges is developed; further, the algorithm evaluates the roadmap by repeatedly searching for the shortest path between the initial and the goal node and then

checking the nodes and edges along the path to avoid collision. This process was repeated until a collision-free path was obtained. Authors in [39, 40] executed a lazy algorithm such that, if there exist feasible paths in the roadmap between q_{init} and q_{goal} , one can find the shortest one among them. Otherwise, if there is no feasible path, one can eventually find q_{init} and q_{goal} in two disjoint components of the roadmap. The total number of collision checks depends on the resolution with which the edges along the path are checked. An extension of Lazy-PRM is Lazy Collision Probability Constrained PRM (Lazy-CPCPRM) as presented in [40–42]. Its principle is that, if a portion of a path examined by the algorithm becomes invalid, then all existing paths along that portion would be neglected. A similar approach called Fuzzy PRM was proposed. Fuzzy PRM [43] spends most of its time in edge validation. However, it validates all nodes during roadmap construction. Further, quasi-random-based Lazy-PRM called Quasi-Random Lazy-PRM (Lazy-QrPRM) was developed and presented in [35]. Customizable PRM (Cs-PRM) [38] allows incomplete validation of nodes and edges in roadmaps.

(v) Applications

Motion planning algorithms have been successfully implemented in many real-time applications. Sampling-based planning algorithms are extensively used in digital animation and computational biology [44, 45]. In digital animation, paths are planned using RRT [46], or PRM [47] with many triangular meshes in the form of nodes. A

Spline-RRT based on Gaussian process has been designed and developed to guide a UAV to traverse in an unknown environment [48]. In computational biology, biomolecules are modelled as articulated bodies and sampling algorithms were used to simulate catalytic reactions [49, 50]. Authors of [51] developed wheelchairs

Table 3. Comparison between PRM and Vis-PRM [65, 146–148].

Sl. no.	Parameters	PRM	Vis-PRM
1	Calling local method	$O(n)$ times	$O(n^2)$ times
2	Size	Complex and extrinsic to problems	Small and intrinsic to problems
3	CS coverage	Excellent	Good
4	Performance (s)	“X”	“12X”
5	Roadmap size	Increases linearly with width	Independent of width
6	Probability of success	$\left(\frac{1}{1+b}\right)$	$\left(1 - \frac{1}{m}\right)$

b : number of edges and m : number of nodes.

Table 4. Special cases of the general PRM framework.

Sl. no.	PRMs	Validation			Query preference
		Node	Edge	Path	
1	PRM [23]	Complete	Complete	In order	Fixed
2	Lazy-PRM [39]	None	None	In order	Fixed
3	Fuzzy PRM [43]	Complete	None	Priority	Fixed
4	Cs-PRM [36, 37]	Tunable	Tunable	Tunable	Adaptable

based on EST to evaluate the accessibility of constrained and narrow areas. Authors in [52, 53] extended based planning algorithms for medical needles [52] and deformable objects [53]. Further, researchers studied the depth of RRT in non-linear control applications such as pendulum control [54]. A major real life application with multiple robots for dynamic environments was also proposed in [55].

3.1c Variants of PRM: Various research works have been carried out on PRM and the variants of PRM are listed in tables 4, 5, 6, 7 and 8.

3.2 Simultaneous Localization and Mapping (SLAM)

Simultaneous Localization and Mapping (SLAM) [56] allows a mobile robot to build its map and at the same time locate itself in it while interacting in an unknown environment. The steps involved in SLAM are shown in figure 5. Kalman filter can be used in the SLAM problem once appropriate models for the vehicle and the sensors are obtained. A framework for handling uncertain spatial relationships enables a mobile robot to build a map in an environment using Extended Kalman filter (EKF), which is represented by Gaussian distribution as reported in [57–60]. Fast SLAM [61, 62] based on Bayesian networks is developed, which is an extension of EKF-based SLAM. Compressed EKF (CEKF) algorithm [63, 64] significantly reduced computational requirements without introducing any penalties in getting accurate results. Instead of using EKF, Bayesian filter implemented by Monte Carlo simulation was used. Table 9 explains about the probability distribution of SLAM. Distributed Particle SLAM (DPSLAM) [65, 66]-based algorithm uses a filter to maintain a probability over robot pose and maps. Another extension of SLAM is RatSLAM [67] to produce spatial representations. Evaluation of the algorithm for SLAM is developed that results in common solution because of its near real-time performance, which is discussed in [68]. Four of the key issues are convergence, computational effort, on-linearity and data association. Application of

Table 5. Description of PRM.

Sl. no.	Variants of PRM	Description
1	PRM [23]	Nodes are generated randomly
2	Vis-PRM [32, 33]	PRM that handles narrow passages
3	Qr-PRM [34, 35]	Performance and completeness are measured probabilistically using random sampling
4	H-PRM [149, 150]	Representing the topological structure of the robot CS based on heuristic sampling
5	CI-PRM [38]	Safe path for robots among polygonal obstacles
6	Lazy/Fuzzy PRM [39, 43]	Collision checking until a query is presented

Table 6. Programming platforms of PRM.

Sl. no.	Fields	Description
1	General/connection methods [23–25]	Generates nodes. Example: PRM, Vis-PRM
2	Local planners [33, 65, 146–148]	Moving from one configuration to another configuration Example: A* planners and straight line planners

Table 7. Application of PRM.

Sl. no.	Fields	Description
1	Closed kinematic chains	Generate the vertices by sampling random configurations
2	Flexible surfaces	Studying different geometric representations for flexible objects, allowing for more complex manipulation constraints
3	Folding objects	Determines whether one object can be folded into another object and provides an investing tool for dynamic folding process
4	Module docking	Samples the space of all possible paths to a given configuration

SLAM has been expanded from mobile to humanoid robots. Implementations of SLAM in humanoids are mainly based on the use of vision [69] and hence vision-based SLAM algorithm (VSLAM) for handling dynamic changes in the environment is proposed [70, 71]. Authors in [72] have proposed a real-time implementation algorithm of SLAM

Table 8. Overview of PRM.

PRM	Search parameter	Running time	Incremental search	Parameters in the algorithm	Mathematical	Space connectivity
ULM	Local configurations	$O(n)$ is more	Increments by each step	–	Probabilistic complete	Poor
RRT	Random points	Four times lesser than that of ULM	Takes place until next obstacle or goal is reached	EXTEND, CONNECT and NEW CONFIG	Probabilistic complete	Good
RRF	Random points	As the number of planning queries increases, the number of nodes increases and the number of trees reduces to two	Takes place until next obstacle or goal is reached	EXTEND, CONNECT and MERGE	Probabilistic complete	Good
PRM	Random points	$O(n)$	Nodes are generated randomly	EXTEND, CONNECT	Probabilistic complete	Excellent
Vis-PRM	Random points	$O(n^2)$	PRM that handles narrow passages	–	Probabilistic complete	Good
Q-PRM	Quasi-points	$O((\log^d n)/n)$	Performance is measured probabilistically	–	Resolution complete	Good
Lazy PRM	Random points and quasi points	Less than that of Q-PRM	Collision checking until a query is presented	–	Resolution complete	Excellent
HPRM	Sampling points	$O(\sqrt{w}(\log^2 n)(2^{o(i)}))$	Heuristic sampling	–	Probabilistic complete	Good
Cl-PRM	Sampling points	–	Safe path for robots among polygonal obstacles	–	Probabilistic complete	Excellent

with emphasis on outdoor land vehicle applications in large environments. Similar algorithms have also been proposed by other authors [60, 62, 68] for mobile robots. Further, hybrid filter-based SLAM for a mobile robot is also developed and details are presented here.

3.2a Variants of SLAM: The researchers have done extensive work on SLAM and variants of SLAM are listed in tables 9, 10, 11 and 12.

4. Map-based navigation for multiple robots

Various probabilistic approaches deployed in PP have been discussed extensively in section 3. Map-based navigation is developed to achieve this task and the current development in this area is presented in this section. Mapping usually involves encoding information on how to get from one place to another, as shown in figure 6. The feasibility of real-time motion planning is dependent on the accuracy of the map, and on the number of obstacles. Basically, it consists of three processes: map learning (ML), localization and PP [73–76].

(i) ML

Two discrete pieces of information used for ML are

- idiothetic cues: provide internal information about robot's environment;
- allothetic cues: provide external information about environment.

Idiothetic cues help in determining the path based on internal sensory signals that can be retrieved by the perceiver whereas allothetic cues refer to global mapping based on constraints. Fuzzy Cognitive Maps (FCMs) [77, 78] seem to be an ideal way for graphically representing this kind of information.

(ii) Localization and PP

Basically, localization strategy [79] can also be classified into three types:

- strategies that directly infer the position from allothetic cues,
- strategies that track a single hypothesis about the robot's position using both idiothetic and allothetic cues [74, 75].

Figure 5. Steps involved in SLAM [56, 69, 72].

Table 9. Probability distribution of SLAM.

Sl. no.	Approaches	Probability distribution	Observation model
1	SLAM [56, 69]	$P(x_k, m Z_{0:k}, U_{0:k}, x_0)$	$P(z_k x_k, m)$
2	EKF SLAM [58]	$P(x_k x_{k-1}, u_k) \Leftrightarrow x_k = f(x_{k-1}, u_k) + w_k,$	$P(z_k x_k, m) \Leftrightarrow z_k = h(x_k, m) + v_k,$
3	Fast SLAM [61]	$P(x_{0:k}, m Z_{0:k}, U_{0:k}, x_0) = P(m X_{0:k}, Z_{0:k})P(X_{0:k} Z_{0:k}, U_{0:k}, x_0)$	—

Robot state is, map m , at time k based on all observations $Z_{0:k}$, and all control inputs $U_{0:k}$.

Table 10. Description of SLAM.

Sl. no.	Variants of SLAM	Description
1	SLAM [56, 69]	Constrained local sub-map filter
2	EKF-based SLAM [58]	EKF
3	CEKF SLAM [63]	Compressed EKF
4	Multi-robot EKF-SLAM [64]	Multi-robot EKF
5	FastSLAM [58]	Rao–Blackwellised particle filter
6	DP-SLAM [66]	Distributed particle mapping
7	VSLAM [70]	Image and odometry filter
8	RatSLAM [67]	Spatial representation

- strategies that track multiple hypotheses about the robot's position using both idiothetic and allothetic cues. The robot develops a complete map of the environment and with this information, the robot can travel freely in an unknown environment using these

strategies. These strategies can be implemented through various models that rely on the information from various sensors. The important aspect is to find an efficient model that can extract maximum information from the available cues. A few such models are the following.

- Direct Position Inference (DPI): models calling upon DPI work in an environment where no perceptual aliasing occurs [74].
- Single hypothesis: this model is deployed where perceptual aliasing occurs extensively.
- Multiple hypotheses: this model is difficult to implement in an unknown environment [75].

Each of these models is further classified into three sub-models [80, 81]. They are topological map, metric map and metric position. Based on the map provided to the robot, one of these sub-models is selected. Since most of the robots are equipped with laser sensors, localization systems discussed in this section deploy these kinds of cues and models. Hence, to build a topological or a metric map in an environment, a robot may call upon both allothetic and idiothetic sensors [82–84]. In multi-robot TP, Leader–

Table 11. Open source slam software of SLAM.

Sl. no.	Software	Description
1	CAS robot navigation toolbox	Matlab simulation toolbox for robot localization and mapping
2	Matlab simulators	EKF-SLAM, UKF-SLAM and FastSLAM 1.0 and 2.0
3	Scene	C++ library for map-building and localization. Facilitates real-time single-camera SLAM
4	Matlab EKF-SLAM simulator	Demonstrates joint compatibility branch-and-bound data association
5	C language grid-based version	FastSLAM 1.0 and 2.0

Follower Approach (LFA) gives a different priority to each robot in the system. Authors in [78] presented a new technique for on-line extraction of natural point landmarks in an indoor environment. The landmarks are filtered out from sonar data taken from a mobile platform. A terrain-adaptive wheel-track hybrid robot that uses a pair of combined wheel and track systems was developed by authors in [79, 85] and this hybrid mobile robot performs given tasks in a hazardous environment for surveillance, reconnaissance, and search and rescue applications.

5. Role assignment (RA)

Multi-robot systems (MRSs) can provide several advantages such as robustness, flexibility and efficiency compared with single-robot systems. In order to obtain most of these potential benefits, several problems need to be tackled, especially when the leader robot in a group fails during a specific task. The focus of the present work is mainly on identifying the next leader robot based on LFA and RA.

This RA feature of identifying the next leader is a complex task when the robots traverse in a dynamic and unknown environment. RA model and steps involved in RA are shown in figures 7 and 8, respectively.

Figure 9 shows that the classification of RA is by explicit communication of information about the status of the environment. The former assigns fixed roles to each robot within an environment.

DRA function, considering the dynamics of multiple agent-environment interactions, has been developed in [86]. Here, the agents independently learn about the RA functions in the presence of an adversary team. Based on these interactions, the agents may just have to change their roles in order to continue with the formation and execution of the given task. A multi-robot DRA, based on path cost, has been developed to find an optimum RA plan in a dynamic environment such as robot soccer [87]. A priority-based positioning/RA algorithm that maintain a competitive formation even when robot malfunctions had also been documented [88]. RA, using auction mechanisms [89], for teams of autonomous robots in a dynamic environment, has been introduced and verified with experimental evaluation. Multi-robot DRA and coordination through shared potential fields [90] showed how heuristic bidding functions that use globally shared information may be used to determine which robot is the most suitable for each task along with robust experimental evaluation. A novel heuristic method to construct road-maps for the members of a mobile robot group in dynamic and/or partially unknown environments considering energy capacities of the robots was proposed in [91]. RA for coordinating multiple robots in the execution of cooperative tasks has been presented [92] and this formulation is based on a hybrid systems framework, which controls the roles and transitions between modes allowing for DRA. Tests were carried out both in real and simulated environments to show the applicability of the proposed mechanism with empirical results demonstrating the effectiveness of this algorithm. Authors claim that the experiments were conducted in RoboCup-3D Soccer Simulation League Environment and the algorithm was carried

Table 12. Overview of SLAM.

Sl. no.	SLAM	Types of navigation sensor	Type of moving objects/application	Type of environment	Filter used in the algorithm
1	EKF	Laser and odometers	Humanoid, mobile robots and obstacle	Outdoor	EKF
2	CEKF SLAM	Laser	Humanoid, mobile robots and obstacle	Outdoor	Compressed EKF
3	Multi-robot EKF-SLAM	Laser and odometers	Mobile robots and obstacle	Outdoor	EKF
4	FastSLAM	Laser and odometers	Mobile robots and obstacle	Indoor and outdoor	Rao-Blackwellised particle filter
5	VSLAM	Laser odometers and camera	Humanoid, mobile robots and obstacle	Indoor and outdoor	Image and odometry filter

Figure 6. Map identification.

out using a general value function. DRA and the formation positioning system among autonomous mobile robots have been investigated in the RoboCup 3D simulation domain [93, 94]. Further, it takes into consideration the real-world concerns and movement dynamics such as avoiding collisions of robots. The simulation was conducted in a physically realistic environment that is partially observable, non-deterministic and noisy.

6. Tracking control (TC)

Tracking Control (TC) for wheeled robots using a kinematic fuzzy logic scheme was addressed in [95]. TC of a mobile robot with delayed measurements was introduced in [96] to evaluate the state variables. TC to solve the integrated kinematic and dynamic trajectory tracking problem of WMRs, in the presence of both parametric and non-parametric uncertainties, was proposed in [97]. Trajectory tracking of uncertain nonholonomic mobile robots dealing with parametric uncertainties using positive-real-Lyapunov design was developed and presented in [98]. A hybrid

approach to group formation problem in MRSs was demonstrated in [99] and hybrid sensing system for mobile robot localization in large-scale indoor environments was demonstrated in [100]. Model algorithm control (MAC) [101] method for the path TC of wheeled mobile robots subject to nonholonomic constraints was demonstrated, and simulations were conducted to show the performance and feasibility of the proposed control strategy. This approach achieves curvature continuity by considering vehicle's dynamic parameters. A terrain-adaptive wheel-track hybrid robot that uses a pair of combined wheel and track systems was introduced in [102–104]. The decentralized control is less vulnerable to a node failure and easier to scale than the centralized approaches presented in [105–107]. A mechanism that allows a team of robots, to individually select appropriate actions throughout the mission, based on the requirement of mission has been presented in a detailed manner in [108]. However, all these approaches are not directly applicable to a MRS based on LFA approach, as they don't involve failure of any robot at any instant. Switching structures and formation control problems in a team of multiple mobile robots were addressed in [109]. Further, cooperation in different levels between robots was also explained. Trajectory TC for wheeled robots using a kinematic fuzzy logic scheme was addressed in [110]. PP and TP are shown in figures 10 and 11, respectively. Generally, welding robots with many DOF [111], industrial robots [112], domestic robots [113, 114], and urban self-driving vehicles have used developed with TC [115, 116].

7. Multi-robot formations

Various control approaches for coordinating multi robots depending on specific tasks have been adopted in virtual structure behaviour in [117], graph theory approach [118], hierarchical control structure [119, 120], flocking topological system [121] and LFA [122, 123]. Various structures and behaviours of multiple robots are shown in table 13. Switching structures [124–126] and formation control problems [127] in a team of multiple mobile robots were addressed. Substantial effort has been made in studying collective behaviours of multi-agent dynamical systems, such as robustness, including communication and measurement errors [128] and feedback systems [129]. In [130], a vision-based follow-the-leader control strategy in an indoor environment was implemented. In [131, 132], the formation deals with the multiple robots in a group in which no one can present who are the leaders or the followers. This formation is called as swarm robotics. Furthermore, to solve these structures and behaviours mentioned in table 13, three common components are presented: (i) system architectures, i.e., how infrastructure is developed behind formation achievement, (ii) robot models that describe systems nature and behaviour, and (iii)

Figure 7. Role assignment model [141–143].

Role hierarchy:	Roles with unique attributes.
Role authorisation:	Relationship of user with a role with cardinal, least and marginal priorities.
Role activation:	Correlating user with different roles
Role execution:	Accomplishment of roles activated

Figure 8. Steps involved in role assignment [144, 145].

Figure 9. Types of role assignment.

formation control strategies, i.e., how a group of robots can be controlled to get into and to maintain the desired formation. Steps involved in multi-robot formation are shown in figure 12.

The traditional method of Fault Tree Analysis (FTA) based on Monte-Carlo random sampling for unmanned vehicles has been briefly introduced in [133]. A design methodology of a fault-tolerant autonomous MRS was developed in [134] for an on-line autonomous behaviour acquisition mechanism that is capable of developing cooperative roles for robots in a noisy embedded environment.

A novel technique based on interval methods for estimating reliability using Fault Tree is presented in [135] and this technique encodes an inherent uncertainty in the input data by modelling these data in terms of intervals. Authors in [136] introduced an algorithm that converts fuzzy set theory into electronic FTA and scientifically analyses the

various kinds of fuzzy information that can be confronted by the failure of the electronic equipment. Multiple-model FT to detect and identify the model-known faults was developed by [137–139]. Collision-free target tracking problem of multi-agent robot system based on semi-cooperative Stackelberg equilibrium point and a fuzzy controller was demonstrated in [140]. Authors in [93] reported that the simulation was carried out in both 2D and 3D soccer simulation domain. In [105] the formation control problem of a multi-Khepera robot system is solved without communication between robots. However, each robot executes an algorithm periodically that requires the knowledge of the complete graph of the formation; hence it is computationally intensive for a large number of robots, whereas centralized approaches are proposed in [106, 107]. In [106], all complexities of sensing, control and computation are delegated to the leader of a group of robots, and [107] treats the robot formation as a single virtual rigid structure.

8. Discussion and future work

In this section, a detailed study of various robot PP techniques published by various researchers in the mobile robot community with a focus on PP in an unknown environment is presented. Various probabilistic approaches developed for mobile robots have been compared and critically evaluated. Variants of PRM are tabulated based on the characteristic goals. These variants can be used to select a particular approach based on initial conditions and parameters. They also provide a selection guideline for robotic researchers attempting to develop new probabilistic approaches for multiple robots. The literature reports that the performance of PRM is limited by running time and search parameter. Further, for accurate motion planning, information about the robot as well as the external environment is required, for which the development of map-based navigation is necessary. It is expected that map-based navigation will make an attempt to bridge the gap between robotics research and manufacturing research. Issues such as identifying a new prospective leader robot in case of failure of an assigned leader could be considered as a new avenue of research in the domain of multi-robot scenario. Further, in most of the studies found in the literature, the researchers have dealt with the PP and RA separately. In PP approaches, the researchers have specified the trajectory of

Figure 10. Path following.

Figure 11. Trajectory following.

Figure 12. Multi-robot formations. **Figure 13.** Leader-follower approach.

the group and their movement towards the goal. Apart from PP, while guiding the robots in an unknown and uncertain environment, navigational capabilities should be dealt with so that collision between robots and obstacles can be avoided. However, combining PP, obstacle avoidance and RA strategies based on priority case structure of a group of robots to attain stable formation in both unknown and uncertain environment was not addressed in the literature. Most of the existing PP algorithms focus on avoiding moving and stationary obstacles within the workspace of the robot. The current research work is not related to distributed motion planning but related to a team of robots consisting of an initially assigned leader and a set of followers. However, this approach does not preclude a robot sensing the positions of other robots to improve its own position. Inter-robot communications are necessary, but the approach does not prohibit the use of communication in a practical implementation. When moving obstacles are present, the problem becomes more complex for mobile robots to identify their path. For the present research, robots moving in an unknown environment are made to face many

Table 13. Structures and behaviours of multi-robots.

Sl. no.	Types	Description
1	Behaviour-based control [151, 152]	Predefined behaviour for each robot is assumed initially, and formation is achieved by weighing the relative importance of each behaviour
2	Virtual structure [117]	Single entity with a relative orientation was used to develop the task allotted to the robot team
3	Graph theory approach [118]	Each node in the graph represents a robot, while an edge defines the dependency between two robots and only interconnected robots coordinate their action at any time
4	Hierarchical control structure [119, 120]	Switching of low level behaviours in order to achieve proper coordination based on leader robot position with respect to the neighbouring robots
5	Flocking topological system [121]	Identify multiple robots in an open space so that the robot moves freely and avoids obstacles
6	Switching structures [124, 125]	Team of multiple robots, in which the leaders are stationary and followers are moving

obstacles while attaining their destination. To illustrate the importance of the probabilistic approach for the LFA, four mobile robots placed in an unknown environment are considered. Such issues can be solved by adding more sensors, but it is expensive. Schematics of LFA and the role change in LFA are shown, respectively, in figures 13 and 14.

Figure 14. Role change in LFA: (a) schematic representation and (b) transition flow.

9. Conclusions

In this paper, a detailed study of various robot PP techniques, published by the researchers in the mobile robots community after 1960, with a focus on PP in an unknown environment has been presented. Various probabilistic approaches developed for mobile robots have been compared and critically evaluated. Variants of PRM and SLAM were tabulated based on the characteristic goals. These variants can be used to select a particular approach based on initial conditions and parameters. They also provide a selection guideline for robotic researchers attempting to develop new probabilistic approaches for multiple robots. The literature reports that the performance of PRM is limited by running time and search parameter whereas the performance of SLAM is limited by the filter used in the algorithm and based on the type of environment considered. The review also confirmed that for an accurate motion planning, information about the robot as well as the external environment is required, for which the development of map-based navigation is necessary. Using navigation maps, robots may self-localize according to three models: DPI, single-hypothesis tracking and multiple-hypothesis tracking, which were explained briefly. It is expected that map-based navigation will make an attempt to bridge the gap between robotics research and manufacturing research. Issues such as identifying a new prospective leader robot in case of failure of an assigned leader were not discussed in the literature, which could be considered seriously as a new avenue of research in multi-robot scenario.

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