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k–μ fading channels: a finite state Markov modelling approach

C PRIYANKA[1], V NITHYA[1] and VIDHYACHARAN BHASKAR[2,*]

[1]Department of Electronics and Communication Engineering, SRM University, Kattankulathur 603203, India
[2]Department of Electrical and Computer Engineering, San Francisco State University, San Francisco, CA 94132, USA
e-mail: priyankachandrashekar391@gmail.com; nithya.v@ktr.srmuniv.ac.in; vcharan@gmail.com

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Abstract. Finite state Markov channel (FSMC) is the automatic choice for accurate modelling of slow fading channels with memory. FSMC model for a k–μ fading channel is investigated in this paper. Small-scale variations of the fading signal under Line-Of-Sight conditions are represented by k–μ fading distributions. Here, FSMC is constructed by partitioning the fading amplitude into several regions. Each region corresponds to a particular state in the Markov model. The transition among these channel states characterizes the physical fading process. Performance measures such as steady-state probability, state-transition probability, level crossing rate and state-time duration corresponding to the k–μ fading channel are derived, plotted and analysed. Finally, numerical results prove that FSMC modelling provides an effective way to reflect the nature of the k–μ fading channel.

Keywords. k–μ distribution; finite state Markov chain model; steady-state probability; level crossing rate; state-transition probability; state-time duration.

1. Introduction

It is very common to have fading channels with memory in mobile radio communications. Multipath propagation and shadowing from obstacles result in fading, and it impacts the performance of wireless communication systems. The statistics of the mobile radio signals are described by different types of distributions. Lognormal distribution is used to describe long-term signal variations, whereas short-term signal variations are described by Rice (Nakagami-n), Rayleigh, Weibull, Nakagami-m and Hoyt (Nakagami-q) distributions [1]. The k–μ fading distribution describes small-scale variations of the fading channel under Line-Of-Sight (LOS) conditions when clusters of multipath waves propagate in a homogeneous environment. It includes one-sided Gaussian, Rayleigh and Lognormal distributions as special cases. It provides very good fitting to experimental data [2–6].

Channel models help system designers to improve system performance measures like error correcting capability of channel codes and packet throughput [7]. In [8, 9], finite state Markov chain (FSMC) models have been widely used to model wireless flat-fading channels in a variety of applications, ranging from modelling channel error bursts to decoding at the receiver. FSMC is constructed for a Rayleigh fading channel by partitioning the range of received signal-to-noise ratios (SNRs) into a finite number of intervals [10, 11]. The work carried out in [12] described first-order FSMC modelling of time-varying flat fading channels (TV-FFC) like Rician, Lognormal and Weibull by considering joint effects of unknown channel phase and amplitude of the received signal. In [13], FSMC for Nakagami-q and α–μ distributions over Adaptive Modulation and Coding (AMC) was presented. In [14], FSMC was used to represent the received SNRs having Lognormal, K and Chi-square distributions. FSMC was used to represent the received signals following Chi-square (central) distributions in asynchronous CDMA systems [15].

Though the FSMC study of various TV-FFC is available in literature, it is obvious that FSMC modelling of k–μ fading channels is not discussed thus far. The present work focusses on first-order FSMC modelling of k–μ fading channels considering the amplitude of the received signal. The effects of unknown channel phase are not considered. The rest of the paper is organized as follows: Section 2 describes the k–μ fading channel. Performance measures like steady-state probability (SSP), level crossing rate (LCR), state-transition probability and state-time duration are derived for the k–μ distribution in section 3. Numerical results are presented

*For correspondence

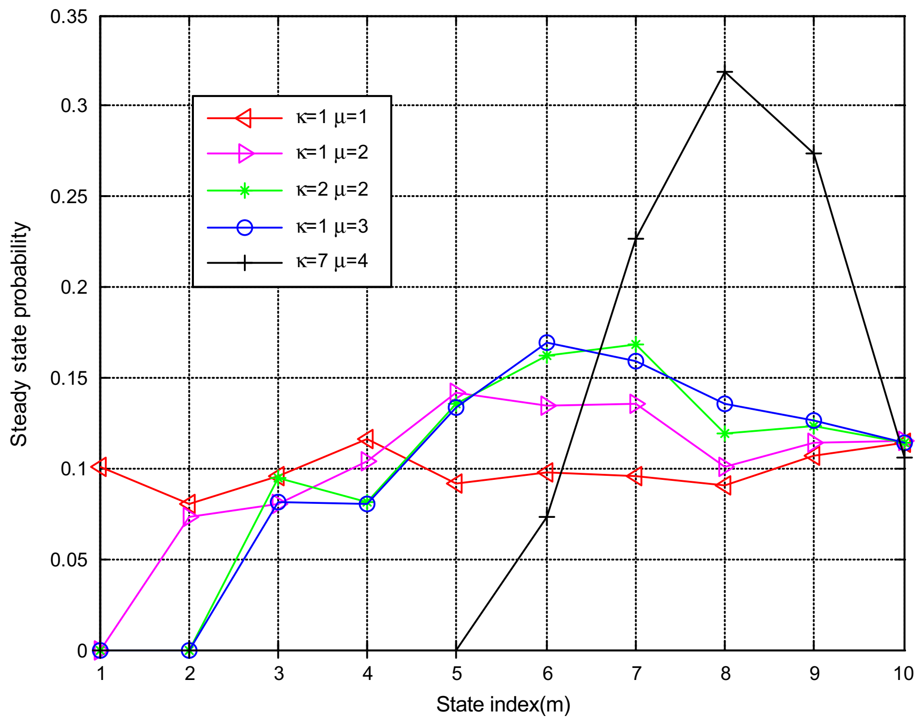

Figure 1. Steady-state probability against total number of states for k–μ distribution.

in section 4 and finally, section 5 presents the conclusions.

2. k–μ fading channel

In this work, *FSMC is modelled for* k–μ fading channels. FSMC modelling for fading channels are widely investigated because of its simplicity in computation, by which the model parameters can be easily determined [10]. The fading model for k–μ distribution considers a signal composed of clusters of multipath waves, propagating in a non-homogenous environment. The phases of the scattered waves are randomly distributed within a cluster. The scattered waves have identical powers but the dominant component may have arbitrary power. The probability density function (PDF) [2] of k–μ distribution is given by

$$f_P(\rho) = \frac{2\mu(1+k)^{\frac{\mu+1}{2}}\rho^{\mu}}{k^{\frac{\mu-1}{2}}\exp(\mu k)}\exp\left[-\mu(1+k)\rho^2\right] I_{\mu-1}\left[2\mu\sqrt{k(1+k)}\rho\right] \quad (1)$$

where $k > 0$ is the ratio of the total power of dominant components to the total power of the scattered waves, $I_v(.)$ is the modified Bessel function of the first kind of order v and $\mu > 0$ refers to multipath clusters. The CDF [2] of k–μ distribution is given by

$$F_p(\rho) = 1 - Q_{\mu}\left(\sqrt{2k\mu}, \sqrt{2(1+k)\mu}\rho\right), \quad (2)$$

where $Q_M(a,b) = \frac{1}{a^{M-1}}\int_b^{\infty} x^M \exp\left(-\frac{x^2+a^2}{2}\right) I_{M-1}(ax)dx$ is the generalized Marcum's Q function presented in [16].

3. Performance measures of k–μ fading distribution

Here, we consider first-order FSMC modelling with amplitude, r, with K states, and derive several performance measures, like SSP, state-transition probability, LCR and state-time duration of a k–μ fading channel.

3.1 *SSP*

The SSP, π_m, of state m is the FSMC model of k–μ distribution, and is defined as the probability that the fading amplitude resides in the region $r_m \varepsilon [v_m, v_{m+1})$. It is obtained by integrating the PDF of the received fading amplitude over the desired region, and is given by

$$\pi_m = \int_{v_m}^{v_{m+1}} f_P(\rho)d\rho = \left[F_\rho(v_{m+1}) - F_\rho(v_m)\right] \quad \forall\, m = 1, 2, \ldots, K. \quad (3)$$

Substituting the CDF expression of k–μ distribution from (2) into (3), we have

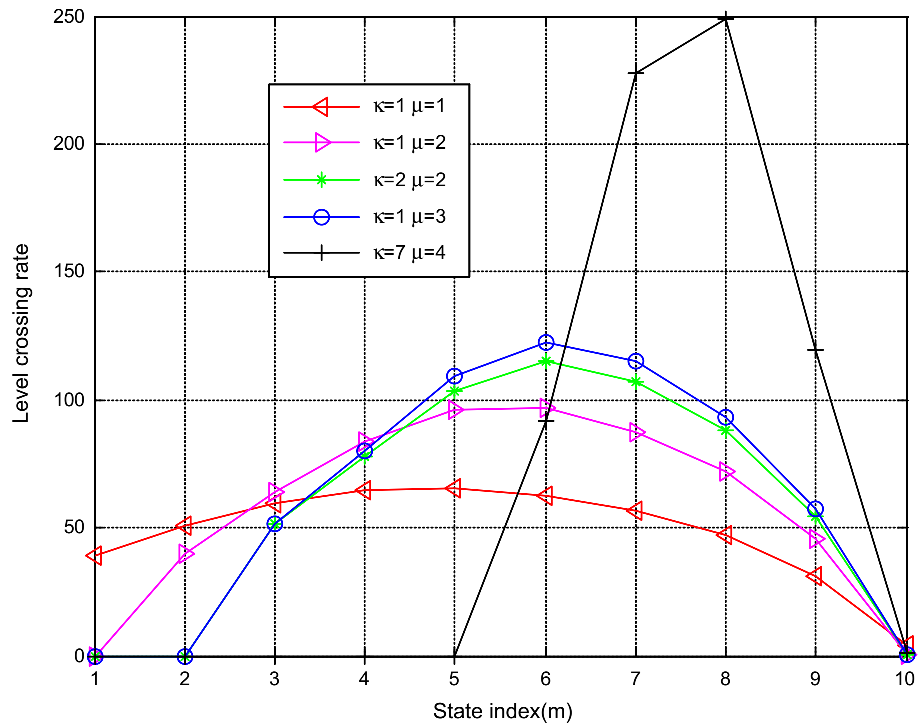

Figure 2. Level crossing rate against total number of states for k–μ distribution.

$$\pi_m = Q_\mu\left(\sqrt{2k\mu}, \sqrt{2(1+k)\mu}v_m\right) - Q_\mu\left(\sqrt{2k\mu}, \sqrt{2(1+k)\mu}v_{m+1}\right). \quad (4)$$

3.2 *LCR*

The LCR [17], N_m, at a specified amplitude boundary threshold level, v_m, is defined as the number of times per second the fading amplitude squared (fading power) ρ crosses the level v_m in the downward direction and is given by

$$N_m = f_m \int_0^\infty \dot{\rho} f(v_m, \dot{\rho}) d\dot{\rho}, \quad (5)$$

where the dot indicates the time derivative of the received signal, $f(v_m, \dot{\rho})$ is the joint PDF of v_m and $\dot{\rho}$, and f_m is the Doppler frequency. Assume that the time derivative of the received signal and the received signal at a point, v_m, are independent.

Now

$$f(v_m, \dot{\rho}) = f(v_m) f(\dot{\rho}). \quad (6)$$

The LCR of state m is given by

$$N_m = f(v_m) f_m \frac{2\mu(1+k)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} \exp(\mu k)} \int_0^\infty \dot{\rho}^{(\mu+1)} \exp\left(-\mu(1+k)\dot{\rho}^2\right) I_{\mu-1}\left[2\mu\sqrt{k(1+k)}\dot{\rho}\right] d\dot{\rho}, \quad (7)$$

where $I_v(.)$ is the modified Bessel function of the first order kind and order v. We have $I_v(x) = \sum_{m=0}^{\infty} \left(\frac{1}{m!}\right) \Gamma(m+ \upsilon + 1))(x/2)^{2m+\upsilon}$ [16]. Substituting for $I_v(.)$into (7), we have

$$N_m = f(v_m) f_m \frac{2\mu(1+k)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} \exp(\mu k)} \sum_{n=0}^{\infty} \left(\mu\sqrt{k(1+k)}\right)^{2n+\mu-1} \frac{1}{n!\Gamma(n+\mu)} \int_0^\infty \dot{\rho}^{(2n+2\mu)} \exp\left(-\mu(1+k)\dot{\rho}^2\right) d\dot{\rho}. \quad (8)$$

Making change of variables in (8) by substituting $y = \mu(1+k)\dot{\rho}^2$and simplifying (8), we have

$$N_m = \frac{1}{2} f(v_m) f_m \frac{2\mu(1+k)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} \exp(\mu k)} \sum_{n=0}^{\infty} \left(\mu\sqrt{k(1+k)}\right)^{2n+\mu-1} \frac{1}{n!\Gamma(n+\mu)} \left(\frac{1}{\mu(1+k)}\right)^{n+\mu+0.5} \Gamma(n+\mu+0.5), \quad (9)$$

where $\Gamma(.)$ is the gamma function, and it can be written as $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ as shown in section 8.310 (p. 883) of [16].

3.3 *State-transition probability*

The state-transition probability $p_{m \to d} = p_r(s_l = d | s_{l-1} = m)$ is the probability that the fading channel amplitude moves from

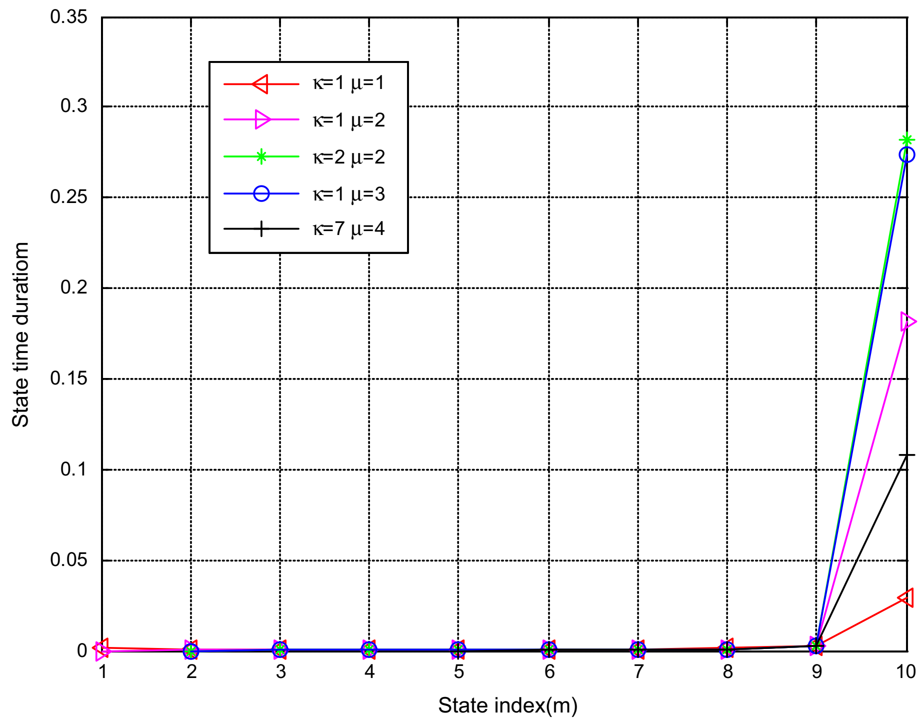

Figure 3. State-time duration against total number of states for k–μ distribution.

state m at time index $(l-1)$ to state d, at time index l. The state-transition probabilities, $p_{m\rightarrow d}$, can be derived by integrating the PDF of the TV-FFC amplitude over two consecutive time indices and over the desired regions. Equation (10) assumes that from time $(l-1)$ to l, the received amplitude either stays in the same region, ρ_m, or it transits to its immediate neighbouring regions, ρ_{m-1}, or ρ_{m+1}. Therefore, FSMC states can only loop back or move to immediate, previous or next states. The state-transition probability is given by

$$p_{m\rightarrow d} = \frac{1}{\pi_m}[F_\rho(v_{m+1}) - F_\rho(v_m)][F_\rho(v_{d+1}) - F_\rho(v_d)]. \tag{10}$$

Substituting (3) into (10), and simplifying (10), we have

$$p_{m\rightarrow d} = [F_\rho(v_{d+1}) - F_\rho(v_d)]. \tag{11}$$

Substituting the CDF expression of k–μ distribution from (2) in (11), we have

$$p_{m\rightarrow d} = \Big[Q_\mu\Big(\sqrt{2k\mu}, \sqrt{2(1+k)\mu}v_d\Big) - Q_\mu\Big(\sqrt{2k\mu}, \sqrt{2(1+k)\mu}v_{d+1}\Big)\Big]. \tag{12}$$

3.4 *State-time duration*

The average time duration of the m^{th} state, $[v_m, v_{m+1})$, is defined as $\overline{\tau_m} = \frac{\pi_m}{N_m + N_{m+1}}$, where π_m is the SSP of being in state m, and N_m, N_{m+1} are the LCRs at the specified levels, m and $m+1$, respectively. Substituting (4) and (9) into the expression of $\overline{\tau_m}$, we have

$$\overline{\tau_m} = \frac{[Q_\mu(\sqrt{2k\mu}, \sqrt{2(1+k)\mu}v_m) - Q_\mu(\sqrt{2k\mu}, \sqrt{2(1+k)\mu}v_{m+1})]}{N_m + N_{m+1}}, \quad \forall m = 1, 2, \ldots, K. \tag{13}$$

4. Numerical results

The probability of being in a particular state depends on the total number of FSMC states available. Figure 1 shows the SSP for k–μ distribution plotted against state index for various k and μ values. It is clear from the graph that when the dominant path power and reflected path powers are identical (i.e., when $k=1$), and multipath cluster, $\mu=1$, the SSP is maintained close to 0.1, which means that the channel fading characteristics greatly vary for all states. However, if μ value is increased for low amplitude level states, the probability of being in a particular state is 0 and the fading effects are very high, and for higher state indices, the SSP gradually increases in value. Further, for $k=7$, the signal amplitude variations are very high for low state indices and increases with increase in state index.

The LCR of k–μ distribution is shown in figure 2. Here, LCR is plotted against state index for various k and μ values. The Doppler frequency is chosen to be $f_m = 80$ Hz for numerical purposes. The effect of fading on LCR is found to

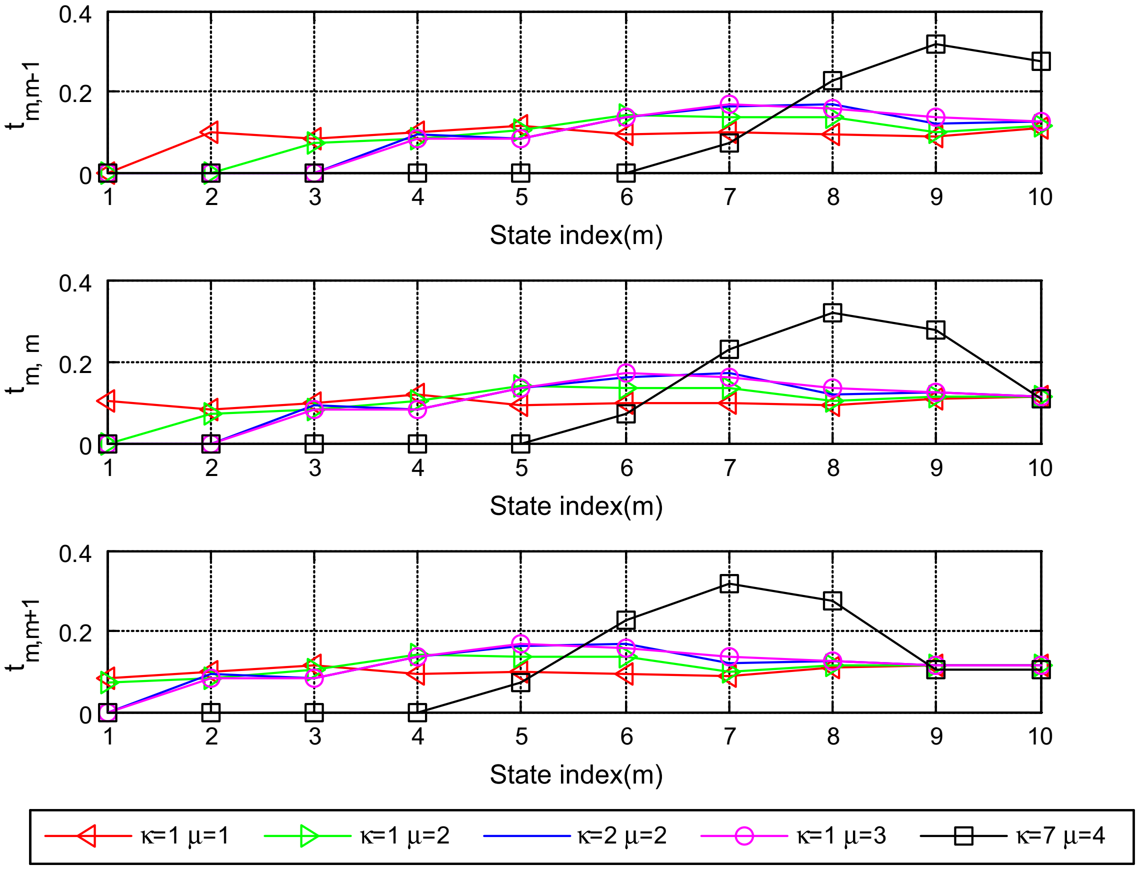

Figure 4. State-transition probability against total number of states for k–μ distribution.

be higher for lower values of state indices. As the received signal amplitude increases, i.e., for higher values of state indices, the fading effects on the LCR are lower. This can be interpreted clearly from figure 2. As the value of k increases, i.e., as the scattered wave power ratio of the total power of the dominant component to power of the scattered wave increases, the LCR at the amplitude boundary threshold also increases. From figure 2, we can infer that as the number of multipath clusters increases, LCR also increases.

Figure 3 shows the state-time duration of k–μ distribution, where the state-time duration is plotted against state index for various k and μ values. The state-time duration is found to be lower for low values of state indices and higher for high values of state indices. This validates the results of LCR.

Figure 4 shows the state-transition probability of k–μ distribution plotted against state index for various k and μ values. The probability of not making a transition to the adjacent states, $t_{m,m}$, is similar to that of SSP of k–μ fading distribution. The probability of making a transition from the current state to the previous state, $t_{m,m-1}$, and the probability of making a transition from the current state to the next state, $t_{m,m+1}$, remain almost constant as the value of state index increases. This suggests that increasing the value of k does not affect the state-transition probability much due to the presence of dominant LOS components in each and every multipath cluster. Further, it is evident that state transition probability varies rapidly as we increase the number of multipath clusters.

5. Conclusions

In this paper, k–μ fading channel is modelled using an FSMC. Performance measures such as SSP, LCR, state-transition probability and state-time duration are discussed by considering the amplitude of the fading channel. Numerical results depict that the k–μ fading channel is a very dynamic channel as illustrated by the values of state-time duration and state transition probability. Thus, the FSMC modelling for fading channels acts as an important tool to study the nature of the channel in the fading environment.

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