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Linearised model for PV panel power output variation with changes in ambient conditions

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Abstract. In closed loop control of PV systems it is important to model the small signal variation of PV panel array output with ambient conditions, namely irradiation and temperature. Changes in these conditions act as a disturbance to the system, but this disturbance needs to be reflected in terms of the quantity being controlled, which can be the PV panel current or the real power. In this work a linearised model is derived to relate the change in system input, namely: irradiance and temperature, with its output, namely: array current and power. The proposed model is experimentally verified with tests run on PV panels, when they are subjected to varying irradiation and temperature conditions in the laboratory. The experimental results confirm the accuracy of the linearised PV panel model.

Keywords. Photo-voltaic module; linearised model; irradiation dependence; temperature variation; experimental validation.

1. Introduction

A PV array can be modelled using the single-diode model of a solar cell as shown in figure 1, with appropriate scaling according to the number of solar cells in series and parallel [1]. This model has five modelling parameters, namely: light-induced current I_L, diode dark saturation current I_s, diode quality factor m, series resistance R_s and shunt resistance R_{sh}. These modelling parameters can be estimated using either datasheet-based approach [3–7] or measurement-based methods [8–13]. Once the parameters are established, the output of the PV system can be predicted for any day and time conditions. However, this output is in terms of the complete current−voltage characteristics. The point of operation on this new characteristic depends on the load line. Under practical conditions of operation, when the dc power produced by the PV array is fed to the load through a converter, usually a closed loop control is employed. A sudden change in the ambient conditions can be modelled as a disturbance input to the system. To analyse the stability of the controller, the fundamental question is to model this disturbance input in terms of the quantity under control, which for the present analysis is the power output of the PV array. A linearised small signal input output model of the PV array is useful as it can then be incorporated into the model of the balance of the PV system.

2. Theory

PV panel's terminal current versus voltage equation can be written as follows:

$$i = I_L - I_s\left(\exp\left(\frac{v + iR_s}{mn_sV_t}\right) - 1\right) - \frac{v + iR_s}{R_{sh}}. \qquad (1)$$

Before the disturbance from ambience, the system is assumed to operate at a fixed load line representing an equivalent resistance. This equivalent resistance represents the load seen at the PV panel terminals. It is dependent on the actual power drawn by the system and the interfacing converter between the PV panels and the load. Thus it is a function of the converter duty ratio and load characteristics. Due to the disturbance, output characteristics of the PV system shift to a new curve. This is shown in figure 2. However, the load line remains at its initial value unless there is a change in the load or operating duty ratio of the converter.

To study the variation of system output with ambient conditions, a linearised model can be established based on the parameters of PV panel that change with ambient conditions. These parameters are light-induced current I_L and diode dark saturation current I_s. The dependence of these variables on ambient conditions is modelled in literature [14] as follows:

$$I_L = \frac{G}{G_{ref}}\left(I_{L_{ref}} + K_{I_{sc}}(T - T_{ref})\right), \qquad (2)$$

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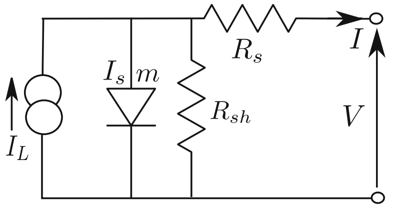

Figure 1. Single-diode model of a solar cell [2].

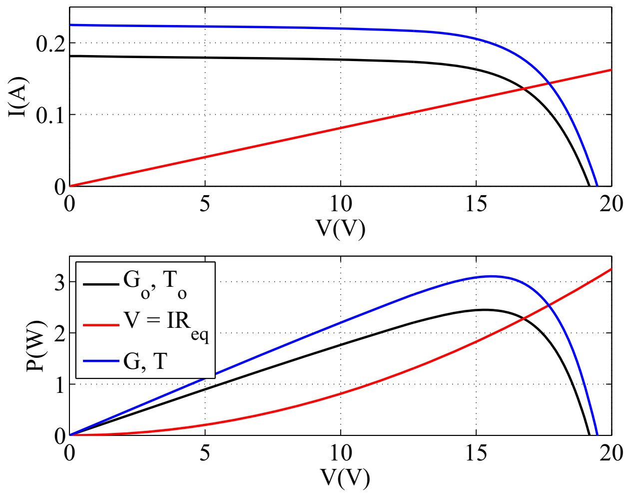

Figure 2. Simulated characteristics of 10 W PV panel when irradiated with a halogen lamp, for two different irradiation levels G_o, T_o represent initial ambient conditions and G, T represent changed ambient conditions. Load line is represented by the equation $V = IR_{eq}$, where R_{eq} represents equivalent resistance, which is a function of converter duty cycle ratio.

$$I_s = I_{s_{ref}} \left(\frac{T}{T_{ref}}\right)^3 \exp\left(\frac{qE_g}{mk}\left(\frac{1}{T_{ref}} - \frac{1}{T}\right)\right). \tag{3}$$

Hence, the PV panel current can be written as a function of light, temperature and voltage:

$$i = f(G, T, v). \tag{4}$$

Often a power converter is used to interface the PV panel to a load. As described earlier, for a constant load and converter duty ratio, the equivalent resistance, and therefore the slope of the load line, is constant. Assuming the interfacing dc−dc converter to be the boost converter, the relation between voltage and current can be written as follows:

$$\frac{v}{i} = \text{constant} = R_{eq} = R_o(1-d)^2. \tag{5}$$

This is valid for the continuous conduction mode (CCM) of a boost converter [15]. Similar equivalent resistances can be derived for other types of dc−dc converters also. Thus, output power of PV panel can be computed as follows:

$$p = vi = R_{eq} i^2. \tag{6}$$

Now, variation in this output power with changing ambient conditions can be evaluated as follows:

$$\frac{\partial p}{\partial G} = R_{eq} \times 2i \frac{\partial i}{\partial G}, \tag{7}$$

$$\frac{\partial p}{\partial T} = R_{eq} \times 2i \frac{\partial i}{\partial T}. \tag{8}$$

Total change in power due to changes in ambient conditions can be evaluated as follows:

$$\Delta P_{G,T}\Big|_o = R_{eq} \times 2I_o \left(\Delta G_o \frac{\partial i}{\partial G}\Big|_o + \Delta T_o \frac{\partial i}{\partial T}\Big|_o\right). \tag{9}$$

It must be noted that irradiation and temperature are interdependent ambient conditions and influence each other; however, to study their explicit effect on the system this linearised small-signal model is proposed here, which allows the application of superposition, and individual effects of these ambient conditions can therefore be studied. Differentiating (1) with respect to G and T, one obtains the sensitivity of the PV panel output current with respect to irradiation and temperature:

$$\frac{\partial i}{\partial G}\Big|_o = \frac{\frac{\partial I_L}{\partial G}\big|_o - \frac{\partial I_s}{\partial G}\big|_o \left(\exp\left(\frac{V_o + I_o R_s}{mn_S V_{T_o}}\right) - 1\right)}{1 + \frac{R_s + R_{eq}}{R_{sh}} + I_{s_o}\frac{R_s + R_{eq}}{mn_s V_{T_o}} \exp\left(\frac{V_o + I_o R_s}{mn_S V_{T_o}}\right)}, \tag{10}$$

$$\frac{\partial i}{\partial T}\Big|_o = \frac{\frac{\partial I_L}{\partial T}\big|_o - \frac{\partial I_s}{\partial T}\big|_o \left(\exp\left(\frac{V_o + I_o R_s}{mn_S V_{T_o}}\right) - 1\right)}{1 + \frac{R_s + R_{eq}}{R_{sh}} + I_{s_o}\frac{R_s + R_{eq}}{mn_s V_{T_o}} \exp\left(\frac{V_o + I_o R_s}{mn_S V_{T_o}}\right)}. \tag{11}$$

However, Eqs. (10) and (11) involve the sensitivities of I_L and I_s to the ambient conditions G and T. They can be obtained using the relationships (2) and (3). Differentiating Eq. (2) with respect to G and T, one obtains the sensitivity of the light-induced current with respect to irradiation and temperature:

$$\frac{\partial I_L}{\partial G}\Big|_o = \frac{I_{L_{ref}} + K_{I_{sc}}(T_o - T_{ref})}{G_{ref}}, \tag{12}$$

$$\frac{\partial I_L}{\partial T}\Big|_o = \frac{G_o K_{I_{sc}}}{G_{ref}}. \tag{13}$$

Differentiating Eq. (3) with respect to G and T, one obtains the sensitivity of the diode dark saturation current with respect to irradiation and temperature:

$$\frac{\partial I_s}{\partial T}\Big|_o = I_{s_{ref}} \left[\frac{3T_o^2}{T^3} \exp\left(\frac{qE_g}{mk}\left(\frac{1}{T_{ref}} - \frac{1}{T_o}\right)\right) + \frac{T_o^3}{T_{ref}^3} \exp\left(\frac{qE_g}{mk}\left(\frac{1}{T_{ref}} - \frac{1}{T_o}\right)\right) \frac{qE_g}{mkT_o^2}\right], \tag{14}$$

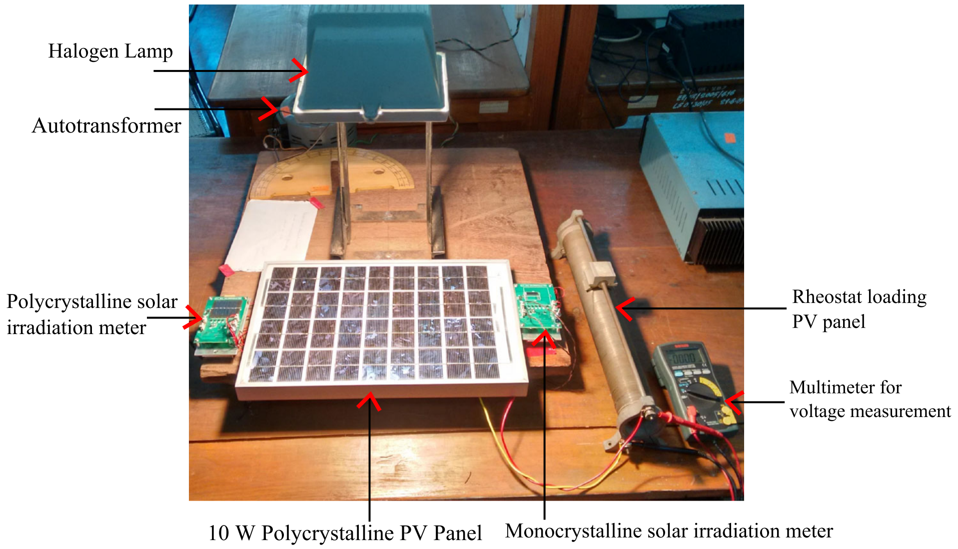

Figure 3. Hardware set-up used for experimental validation of ambient-condition-based linearised model of PV panels.

$$\left.\frac{\partial I_s}{\partial G}\right|_o = 0. \tag{15}$$

Substituting these expressions in Eqs. (10) and (11), the variation of PV panel terminal current as a function of temperature and irradiation variation are determined:

$$\begin{aligned}\left.\frac{\partial i}{\partial T}\right|_o = {} & \frac{G_o K_{I_{sc}}}{G_{ref}} - I_{S_{ref}}\left[\frac{3T_o^2}{T^3}\exp\left(\frac{qE_g}{mk}\left(\frac{1}{T_{ref}} - \frac{1}{T_o}\right)\right)\right. \\ & \left. + \frac{T_o^3}{T_{ref}^3}\exp\left(\frac{qE_g}{mk}\left(\frac{1}{T_{ref}} - \frac{1}{T_o}\right)\right)\frac{qE_g}{mkT_o^2}\right] \\ & \times \frac{\left(\exp\left(\frac{V_o + I_o R_s}{mn_S V_{T_o}}\right) - 1\right)}{1 + \frac{R_s + R_{eq}}{R_{sh}} + I_{s_o}\frac{R_s + R_{eq}}{mn_s V_{T_o}}\exp\left(\frac{V_o + I_o R_s}{mn_S V_{T_o}}\right)},\end{aligned} \tag{16}$$

$$\left.\frac{\partial i}{\partial G}\right|_o = \frac{\frac{I_{L_{ref}} + K_{I_{sc}}(T_o - T_{ref})}{G_{ref}}}{1 + \frac{R_s + R_{eq}}{R_{sh}} + I_{s_o}\frac{R_s + R_{eq}}{mn_s V_{T_o}}\exp\left(\frac{V_o + I_o R_s}{mn_S V_{T_o}}\right)}. \tag{17}$$

Now, $\Delta P_{G,T}$ at a given operating point corresponding to the given initial state of G_o, T_o, V_o, I_o and P_o, can be calculated using Eq. (9). This provides the small-signal linearised model of the power output from the panel for small changes in ambient conditions of irradiation and temperature.

Using this model in tandem with power converter model, wherein a relation between converter duty ratio and PV terminal current is established [16], the complete PV system can be modelled and its performance can be analysed.

3. Experimental validation

Experimental validation of the linearised model derived earlier was performed using the experimental set-up shown in figure 3. An autotransformer is used to increase the light intensity of the halogen lamp. Irradiation and temperature measurements are made with the help of the solar-cell-based irradiation meter [17]. Corresponding to the increased light and temperature, new output power is noted and compared to the previous equilibrium value. Constants that are used in the analysis are listed in table 1. The 10-W PV panel parameters, as derived using [1], are listed in table 2. Equations (2) and (3) are used to calculate the parameters corresponding to the equilibrium condition. In this experiment, the equivalent resistance, which determines the slope of load line, was kept fixed to $R_{eq} = 123.2\Omega$, as its slope is close to the MPP slope.

Table 1. Constants used in the analysis.

Irradiation at STC: G_{ref}	1000 W/m^2
Temperature at STC: T_{ref}	25°C
Electron charge: q	1.6×10^{-19} C
Boltzmann's constant: k	1.38×10^{-23} J/K
Silicon band gap: E_g	1.12 eV

Table 2. Parameters obtained for 10-W panel.

Light-induced current at STC: $I_{L_{ref}}$	0.650 A
Dark saturation current at STC: $I_{S_{ref}}$	0.228 μA
Light-induced current at equilibrium: I_{L_o}	0.171 A
Dark saturation current at equilibrium: I_{S_o}	0.463 μA
Series resistance: R_s	1.48 Ω
Shunt resistance: R_{sh}	2199.8 Ω
Diode quality factor: m	1.5641
Thermal voltage: V_{T_o}	kT_o/q

On comparison, it has been observed that the experimental results match well with the theoretical results, as derived from the developed linearised model. The error will further reduce when variation of thermal voltage V_T with temperature is included in the analysis. The results are

Table 3. Comparison of experimental results with theoretical results.

T_o °C	G_o $\times 0.65\mathrm{W/m^2}$	ΔG_o $\times 0.65\mathrm{W/m^2}$	ΔT_o °C	I_o A	V_o V	P_o W	ΔP_o^{ex} W	ΔP_o^{th} W
30	428.8	102.2	1	0.137	16.82	2.296	0.295	0.300
31	443.3	095.0	1	0.136	16.74	2.277	0.275	0.274
32	435.3	109.8	1	0.136	16.74	2.277	0.257	0.306
33	440.6	106.2	1	0.136	16.80	2.292	0.257	0.271
34	435.7	101.5	1	0.136	16.70	2.271	0.278	0.260

listed in table 3. On an average, percentage change in the light conditions was $\frac{<\Delta G_o>}{<G_o>} = 23.6\%$, which gave an average percentage change in the power output of the 10 W PV panel of $\frac{<\Delta P_o^{th}>}{<P_o>} = 12.4\%$. A change of temperature ΔT_o of 1°C was also considered in the experimental studies. The change in temperature of the panel corresponds to the change in temperature that occurs when the halogen lamp intensity is adjusted. An LM35-based temperature sensor is used to measure temperature, which has 98% accuracy in the given temperature range [17]. Root mean square error (RMSE) between the experimental and theoretical results comes out to be

$$RMSE = \sqrt{\frac{\Sigma_{i=1}^{5}(\Delta P_{o,i}^{exp} - \Delta P_{o,i}^{th})^2}{5}} = 0.024\mathrm{W}. \tag{18}$$

Coefficient of variance (CV) for RMSE is seen to be

$$CV(RMSE) = \frac{RMSE}{<\Delta P_{o,i}^{th}>} = 8.59\%. \tag{19}$$

Normalised error can be computed as

$$\frac{RMSE}{<P_o>} = 1.06\%. \tag{20}$$

Thus the total normalised error is within 1.1%.

4. Conclusion

Modelling of the input disturbance is explicitly studied using a linearised model, wherein the effect of the changes in the input ambient conditions is quantified in terms of the change in the measurable power output of PV system. The effect of irradiation and temperature on PV system parameters and further on terminal current and power is modelled using a linearised small-signal model based on partial differentiation of PV characteristic equation. The model predicts 12.4% change in the PV power corresponding to 23.6% change in the light conditions. Experimental results validate the predicted model, with a normalised error of less than 1.1%. Overall, based on this analysis, it is possible to evaluate the small-signal linearised model of a closed loop PV system considering disturbance in ambient conditions.

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