A method for correcting radial distortion based on verifying the planarity of specimens

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Abstract. Image distortion is inevitable when an image is captured through a lens. While the digital image measurement technique is getting popular, image distortion problem can result in significant error. A new distortion correction method is proposed in this study. The proposed method is based on the fact that a flat surface should keep flat when it is measured using three-dimensional (3D) digital image measurement technique. The 3D digital image measurement technique adopted in this research is the simplified 3D digital image correlation (DIC) method. Because radial distortion has a more noticeable influence than other types of distortions, this method deals only with radial distortion. A few experiments are carried out in this study to verify the correctness of this method and its accuracy. Both simulated data and actual image data are adopted in these experiments. The results show that this method can achieve a good accuracy. The standard deviations caused by random errors are about the same order as the random errors. It also shows that this method is suitable for both large and small distortion conditions.

Keywords. 3D DIC; radial distortion; distortion correction; planarity.

1. Introduction

Digital image measurement technique is a non-contact measurement technology, and it can be used in a variety of measurements of different scales. Thus, in recent years, digital image measurement technique has gradually been applied in various fields. In general, it is inevitable that the images captured through the lens suffer from distortion problem. This can cause serious error when these images are used for precise measurement [1, 2]. Hence, how to correct image distortion is a very important research topic.

Lens distortion can be grouped into radial distortion, decentering distortion and thin prism distortion. In the past few decades, there have been many scholars engaged in solving the problem of correcting image distortion [3–12]. Tsai [3], Salvi *et al* [4], Weng *et al* [5] and Wei *et al* [6] used non-linear models to simulate the relationship between physical coordinates and image coordinates, and they adopted a two-stage approach to solve the distortion correction coefficients and camera parameters. Tsai and Salvi considered the influence of radial distortion, and Weng considered all of the three distortions mentioned earlier. Furthermore, Zhang [11] and Maybank *et al* [12] took different images through a moving camera or an observed

target, and then these images were used to solve the distortion coefficients and the camera parameters. There are also some correction methods based on the concept that the image of a straight line should remain straight after correcting distortion [7–10]. The method proposed by Xu *et al* [9] calculates the correction coefficients through iteration. This method is mainly for large distortion correction, and not ideal for small distortion. The correction targets used in the afore-mentioned studies require special marks such as grid or checkerboard. The correction results will be affected by the accuracy of the mark itself.

This study proposes a new distortion correction method which overcomes the problem caused by low accuracy of the mark. Due to distortion, the 3D coordinates of the surface of a flat object will form a bowl shape when they are measured by the 3D digital image measurement technique. Therefore, the proposed method is based on the concept that the 3D coordinates of points on a plane surface should lie on a plane after distortion correction. The advantage of this approach is that the mark does not need to be grid or any particular shape, and it can even be just random speckles. Therefore, the results are not affected by the accuracy of the mark. Tsai [3] has demonstrated in his study in 1987 that for many computer vision applications, the distortions except radial distortion can be ignored. The advance in optical technology has lessened the effects of

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decentering distortion and thin prism distortion. So this study only focuses on the correction of radial distortion. Additionally, this method can not only be applied to the correction of serious distortion but also to the correction of slight distortion.

2. Methodology

The proposed method is based on the characteristic that the coordinates of the surface of a planar object must be kept on a plane. Therefore, this section describes the method of precise 3D coordinate analysis and its application to determine the correction coefficients of distortion. Traditional 3D digital image correlation (DIC) method uses multiple cameras for measurement. In fact, the distortion properties of different cameras could be different. Therefore, the correction of distortion is not practical. Hence, the distortion correction method proposed in this paper adopts the simplified 3D DIC method [13], which uses only one single camera.

2.1 Method of establishing the 3D coordinate

Tung *et al* have proposed a simplified 3D DIC method in 2011 [13]. While the method requires no specific parameter of camera, the measurement accuracy of 3D coordinate is relatively high and is therefore suitable for accurate correction of image distortion. The image position formed by a converging lens of an uneven object before and after a lateral movement e can be simplified as shown in figure 1.

The actually measured displacement of $\overline{AA'}$ on the image can be expressed as $N_{AA'} = \gamma \left(\frac{q}{p_a} \right) e$, where γ expresses the number of pixels per actual length, and p and q are object distance and image distance from lens center, respectively. As both γ and q are unknown constants, they can be

Figure 1. Image formation of a non-plane object before and after lateral movement by converging lens [13].

combined to form a new constant $\Lambda = \gamma q$. Then it can be expressed as follows:

$$\Lambda = \frac{N_{AA'} p_a}{e}. \quad (1)$$

Theoretically, Λ can be determined with measured $N_{AA'}$, object distance p_a and the lateral displacement e . However, Eq. (1) will not be able to determine the Λ value because there is no point with a known object distance. The object distance of point a can be expressed as the following equation by modifying Eq. (1)

$$p_a = \frac{e \Lambda}{N_{AA'}} \quad (2)$$

Assume that there are N points with different object distances. The differences in object distance between these points and the point a are known. Then these object distance differences can be expressed as

$$\Delta_i = p_i - p_a = \left(\frac{N_{AA'} - N_{II'}}{N_{AA'}} \right) \Lambda e, \quad i = 1, \dots, N. \quad (3)$$

Then Λ can be expressed as follows.

$$\Lambda = \frac{\Delta_i}{e} \left(\frac{N_{AA'} N_{II'}}{N_{AA'} - N_{II'}} \right). \quad (4)$$

According to Eq. (4), the calibration parameter Λ can be obtained with only one known z coordinate difference Δ . Considering the inevitable errors during the test, an average calibration parameter Λ should be calculated by using more than one point. Then, the z coordinates of different points can be determined from their object distances.

The constant Λ is the only parameter of the camera needed for calculating the z -coordinates of the measured points. However, the variation of the z -coordinate scale does not affect the planarity if the measured surface is flat. Therefore, for the purpose of calibrating the correction coefficients, no parameter of camera is required.

If the x coordinate of point a of this object on the image is X_A , the real-world x coordinate of the point a can be calculated using the following equation.

$$x_a = \frac{X_A e}{N_{AA'}}. \quad (5)$$

Similarly, its y coordinate can also be calculated as follows.

$$y_a = \frac{Y_A e}{N_{AA'}}. \quad (6)$$

The fundamental assumption of this simplified 3D DIC is that the lateral movement is parallel to the sensor plan of the image capture device. A calibration method proposed by Tung *et al* [14] is adopted to ensure this parallel condition. This method is based on the concept that the distances of two symmetric points to the image center should

be the same. Therefore, it can still be used to calibrate the lateral movement direction even when there exists radial lens distortion.

2.2 Method of correcting the lens distortion

Lens distortions can be categorised as radial distortion, decentering distortion and thin prism distortion. Among these, radial distortion has the most serious effect. The relationship between the original and the distorted image coordinates can be expressed as following equations:

$$x' = (x - x_0)(1 + k_1 r^2 + k_2 r^4 + k_3 r^6 + k_4 r^8 + \dots) + x_0 \quad (7)$$

$$y' = (y - y_0)(1 + k_1 r^2 + k_2 r^4 + k_3 r^6 + k_4 r^8 + \dots) + y_0, \quad (8)$$

where (x, y) are the undistorted image coordinates, (x_0, y_0) are the image coordinates of optical center, r is the distance between (x, y) and (x_0, y_0) , $k_1, k_2, k_3, k_4 \dots$ are the distortion coefficients, and (x', y') are the distorted image coordinates.

However, distorted coordinates can be measured, and these coordinates will be used to calculate the undistorted coordinates. Therefore, these equations can be modified as follows:

$$x = (x' - x_0)(1 + k'_1 r'^2 + k'_2 r'^4 + k'_3 r'^6 + k'_4 r'^8 + \dots) + x_0 \quad (9)$$

$$y = (y' - y_0)(1 + k'_1 r'^2 + k'_2 r'^4 + k'_3 r'^6 + k'_4 r'^8 + \dots) + y_0, \quad (10)$$

where $k'_1, k'_2, k'_3, k'_4 \dots$ are the correction coefficients of distortion, and r' is the distance measured from (x', y') to the optical center.

Theoretically, the $k'_1, k'_2, k'_3, k'_4 \dots$ can be solved when there are some points with known real-world coordinates and their distorted coordinates in the image can be determined. However, this method can achieve an accurate result only when a highly accurate target is available.

Three-dimensional DIC can be used to measure the space coordinates of the surface of a 3D object. The space coordinates of the points located on a plane should be located on a flat surface. However, due to the distortion of image, the measured shape would turn out to be a bowl. Therefore, this study uses this property to correct the distortion. The process of calculating the distortion coefficients is as follows:

- (1) A flat specimen is adopted, and the method mentioned in section 2.1 is utilised to measure the 3D coordinates of several points located on the surface of this flat specimen. With a group of assumed distortion

coefficients, the captured image can be corrected, and the supposed undistorted coordinates can be determined. Then the 3-D coordinates of these points after correction can be obtained. The parameter Λ mentioned in the previous section is constant and acts as a scaling factor. It does not affect the nature that all these points should be located on the same plane. Therefore, the calibration process of estimating Λ can be ignored if only distortion correction is needed. The parameter Λ needs to be determined only if the simplified 3D DIC is applied to calculate the real 3D coordinates.

- (2) The 3D coordinates of all these points are used to determine the regression function of a plane.
- (3) The summation of the square values of the z coordinate differences is calculated, and this summation is defined as the error of planarity.
- (4) The minimum of the error is 0. Therefore, this research uses gradient method to adjust correction coefficients in order to achieve a minimum error. The theory of gradient method is described in the following section.

2.3 Theory of gradient method

If we adopt the image coordinates (typically between 0 to thousands) in the distortion correction, this will lead to a difference between correction coefficients as high as six orders of magnitude. This is disadvantageous to the optimisation process while using gradient method to solve the correction coefficients. Therefore, this study uses non-dimensional coordinates, and the image coordinates of the points are normalised to between -1 and 1 .

The theory of gradient method used to solve the correction coefficients is described as follows.

1. Optimisation index: The root mean square error between the measured z coordinates and the z coordinates on the regressive plane can be defined as an index to measure the correction quality.

$$Err = rms(z_m - z_r), \quad (11)$$

where z_m is the measured z coordinate, and z_r is the z coordinate on the regressive plane.

2. Procedure of optimisation

- (1) Assume a set of distortion correction coefficients.
- (2) Calculate the corrected image coordinates of all measuring points.
- (3) Calculate the space coordinates of all measuring points.
- (4) Use the results of (3) to find the equation of the regressive plane. Then, calculate the space coordinates of all measuring points on this regressive plane.
- (5) Calculate the root mean square error as the optimisation index.
- (6) Utilise differential method to find out the approximate gradient of the optimisation index.

- (7) Use half-space method to find the optimised correction coefficients in the direction of gradient.
- (8) Repeat steps (2) to (7), until the optimisation index is less than the tolerant error.

In order to accelerate convergence and avoid being trapped in local optima, it is necessary to design a “mutation” mechanism in the iteration process. Since the 1000th iteration, mutation occurs once every 100 iterations. The distortion correction coefficient as mutation occurs is as follows:

$$\{Par_{mu}\} = 2\{\overline{Par}_0\} - \{\overline{Par}_{-1}\}, \quad (12)$$

where $\{Par_{mu}\}$ is the correction coefficient after mutation, $\{\overline{Par}_0\}$ is the average of the optimum correction coefficient in the latest 100 iterations, $\{\overline{Par}_{-1}\}$ is the average of the optimum correction coefficient in the latest 101–200 iterations.

3. Verification of the proposed method

To verify the accuracy of the proposed method in this study, several numerical studies on simulated data were conducted, because only simulated data provide known distortion coefficients. Also, the impact of random errors on this method is discussed in this section. The experimental studies on accuracy of 3D measurement will be presented in the next section.

3.1 Verification of theory

To verify the correctness of the proposed method, a group of undistorted coordinates and their corresponding distorted coordinates are simulated based on the pre-defined correction coefficients. Then the distorted coordinates are substituted into the method mentioned in the previous section to calculate the correction coefficients, and the corrected coordinates are compared with the theoretically undistorted coordinates. In practical application, we will not know the order of the correction equation. Although the order is in theory the higher the better, but the higher order will also take more time to complete the analysis. Therefore, to verify the correctness of this theory, this study will use a 13th-order correction equation for all analysis, i.e. $k'_1, k'_2, k'_3, k'_4, k'_5, k'_6$ are used. In order to verify that this method can be applied to distortion equations of different orders, the distortion equations are set to the 3rd, 5th, 7th and 9th order, respectively.

The SLR camera used in the experimental verification, described in the next section, is Canon EOS 650D, and its resolution is 18 megapixels. Hence, it is assumed that the resolution of the digital image for simulation is 5000×3500 pixels; 154 grid points on the first distorted

Figure 2. Position of grid points on the distorted image before movement.

image are chosen, and their positions are shown in figure 2. With a group of pre-defined distortion coefficients, Eqs. (7) and (8) can be used to solve the undistorted coordinates of these grid points on the first image. Then the undistorted coordinates of these grid points are given a displacement of 500 pixels in x -direction to obtain the undistorted coordinates of these grid points on the second image. The distorted coordinates of these grid points on the second distorted image can also be calculated with the same pre-defined distortion coefficients. The distorted coordinates of these grid points on the first and second images are substituted into the proposed theory to determine the correction coefficients of distortion and the corresponding corrected coordinates of these grid points. These corrected coordinates are compared with the original undistorted coordinates, and the root mean square error (RMSE1 and RMSE2) of the differences between the two sets of coordinates are used to verify the correctness of this model. The process mentioned earlier can be summarised as figure 3.

Analysis results under different pre-defined distortion coefficients are shown in tables 1–4. It can be found that the analysed values of distortion centres are very close to the pre-defined values. In general, the errors are within 0.1 pixel. Observing the differences between the undistorted and corrected coordinates of the grid points, we can find that the root mean square errors are quite small. This shows the correctness of the proposed model. Also note that the error of analysis will be smaller when the pre-defined distortion equation has a lower order, and the error will slightly increase at a higher distortion equation order. For example, the error values are about 0.001 pixel as the distortion equation orders are 3 and 5, and the errors lie approximately between 0.03 and 0.035 pixel when the distortion equation orders are 7 and 9. The reason for this larger error is that only six correction coefficients ($k'_1, k'_2, k'_3, k'_4, k'_5, k'_6$) are used in the analysis, and distortion equation with higher order will result in a larger truncation error. To improve the accuracy, we can adopt higher-order

Figure 3. Process to verify the correctness of the proposed method.

Table 1. Results for the 3rd order distortion equation.

	Pre-defined	Analysis results
(x_0, y_0)	(2500, 1750)	(2500.00, 1750.00)
$k_1, k_2, k_3, k_4, \dots$	$k_1 = -1 \times 10^{-8}$	$k'_1 = 9.998 \times 10^{-9}$
$k'_1, k'_2, k'_3, k'_4, k'_5, k'_6$	$k_2 = 0$	$k'_2 = 3.020 \times 10^{-16}$
	$k_3 = 0$	$k'_3 = 1.109 \times 10^{-23}$
	$k_4 = 0$	$k'_4 = 7.643 \times 10^{-31}$
		$k'_5 = -2.317 \times 10^{-39}$
		$k'_6 = 3.996 \times 10^{-45}$
RMSE1	0.001 pixel	
RMSE2	0.001 pixel	

Table 2. Results for the 5th order distortion equation.

	Pre-defined	Analysis results
(x_0, y_0)	(2500, 1750)	(2500.00, 1750.00)
$k_1, k_2, k_3, k_4, \dots$	$k_1 = -1 \times 10^{-8}$	$k'_1 = 9.997 \times 10^{-9}$
$k'_1, k'_2, k'_3, k'_4, k'_5, k'_6$	$k_2 = 6 \times 10^{-16}$	$k'_2 = -2.975 \times 10^{-16}$
	$k_3 = 0$	$k'_3 = -3.688 \times 10^{-23}$
	$k_4 = 0$	$k'_4 = -9.243 \times 10^{-31}$
		$k'_5 = 1.343 \times 10^{-37}$
		$k'_6 = 3.097 \times 10^{-45}$
RMSE1	0.001 pixel	
RMSE2	0.001 pixel	

Table 3. Results for the 7th order distortion equation.

	Pre-defined	Analysis results
(x_0, y_0)	(2500, 1750)	(2499.89, 1750.00)
$k_1, k_2, k_3, k_4, \dots$	$k_1 = -1 \times 10^{-8}$	$k'_1 = 1.008 \times 10^{-8}$
$k'_1, k'_2, k'_3, k'_4, k'_5, k'_6$	$k_2 = 6 \times 10^{-16}$	$k'_2 = -3.833 \times 10^{-16}$
	$k_3 = -1 \times 10^{-22}$	$k'_3 = 8.760 \times 10^{-23}$
	$k_4 = 0$	$k'_4 = 1.373 \times 10^{-29}$
		$k'_5 = -3.214 \times 10^{-36}$
		$k'_6 = 4.841 \times 10^{-43}$
RMSE1	0.029 pixel	
RMSE2	0.033 pixel	

Table 4. Results for the 9th order distortion equation.

	Pre-defined	Analysis results
(x_0, y_0)	(2500, 1750)	(2500.05, 1750.00)
$k_1, k_2, k_3, k_4, \dots$	$k_1 = -1 \times 10^{-8}$	$k'_1 = 1.009 \times 10^{-8}$
$k'_1, k'_2, k'_3, k'_4, k'_5, k'_6$	$k_2 = 6 \times 10^{-16}$	$k'_2 = -3.888 \times 10^{-16}$
	$k_3 = -1 \times 10^{-22}$	$k'_3 = 8.975 \times 10^{-23}$
	$k_4 = 1.8 \times 10^{-29}$	$k'_4 = -3.796 \times 10^{-30}$
		$k'_5 = -5.661 \times 10^{-36}$
		$k'_6 = 4.169 \times 10^{-43}$
RMSE1	0.034 pixel	
RMSE2	0.031 pixel	

correction coefficient. Table 5 shows the analysis results under higher-order distortion correction coefficient. It can be found that using a higher correction coefficient can really improve the accuracy of the analysis.

3.2 Influence of random error

It is inevitable that every measurement has an error. Therefore, this study uses the aforementioned case with a distortion equation of 5th order to discuss the influence of random errors caused by image matching using DIC on the correction effect. The random errors following a normal distribution are added to the distorted coordinates of the grid points before and after movement. Then the analysis is carried out to determine the centre of distortion and the distortion correction coefficients, and the root mean square error between the corrected coordinates and the theoretical undistorted coordinates of the grid points is calculated as well. The standard deviations of random errors are set to 0.01 pixel, 0.05 pixel and 0.1 pixel, separately. The analysis results are shown in table 6. It shows that the deviation of center of distortion and the corrected coordinates to the pre-defined values tend to increase as the standard deviation of random errors increases. In general, DIC can achieve a

Table 5. Results of using different numbers of distortion correction coefficients for the 9th order distortion equation.

Number of distortion correction coefficients	Center of distortion	Root mean square error (pixel)	
		RMSE1	RMSE2
5	(2500.10, 1750.00)	0.075	0.069
6	(2500.05, 1750.00)	0.034	0.031
7	(2500.01, 1750.00)	0.002	0.002
8	(2500.01, 1750.00)	0.002	0.002
9	(2500.01, 1750.00)	0.001	0.001
10	(2500.01, 1750.00)	0.001	0.001

Table 6. Results under the influence of various random errors.

Standard deviation of random error (pixel)	Center of distortion	Standard error (pixel)	
		RMSE1	RMSE2
0	(2500.00, 1750.00)	0.001	0.001
0.01	(2499.52, 1749.49)	0.039	0.032
0.05	(2499.45, 1750.14)	0.074	0.081
0.1	(2501.40, 1749.64)	0.287	0.257

measurement accuracy of about 0.01 pixel in the laboratory, so that the distortion correction will not be seriously influenced by the coordinate measurement. Therefore, the proposed distortion correction method in this study has considerable practical value.

Theoretically, the influence of random errors can be reduced as the number of grid points increases. Therefore, the number of grid points used in the analysis is increased to 4 times (i.e. from 154 to 616 points). The analysis is carried out again, and the results are shown in table 7. It can be found that the errors of the corrected coordinates are about 1.47 to 1.75 times the corresponding assumed standard deviations of the random errors for x- and y-direction coordinates. This is about square root of 2 times. It shows that increasing the number of grid points can help obtain more accurate correction coefficients. A comparison of table 6 with table 7 shows that taking more grid points can also slightly improve the accuracy of analysis under the condition without reducing coordinate measurement error.

Table 7. Results under the influence of various random errors (4N).

Standard deviation of random error (pixel)	Center of distortion	Standard error (pixel)	
		Before movement	After movement
0	(2500.00, 1750.00)	0.001	0.000
0.01	(2500.06, 1750.03)	0.015	0.015
0.05	(2500.65, 1750.01)	0.078	0.087
0.1	(2499.39, 1748.85)	0.155	0.156

4. Experimental verification

A series of experiments in the laboratory confirm that the method proposed in this study can be applied to correct the actual measurement.

4.1 Experimental equipment and setup

The equipment used in this experiment is listed as follows:

1. XY table (with a precision of 0.01 mm)
2. Rotation table
3. Canon EOS 650D
4. Canon EF-S 18–55 mm F3.5–5.6 IS II

The experimental target and the complete experimental setup are shown in figures 4 and 5. The surface of the experimental target is covered with repeated marks. The dimension of this repeated mark is 20 mm $\times$ 20 mm. In fact, selecting a repeated mark to cover the specimen is unnecessary if we only want to correct the distortion using the proposed method. Repeated marks are used to cover this specimen in these experiments just because this enables us to verify the correction accuracy by checking the distance between two adjacent repeated marks before and after the correction of distortion.

4.2 Experimental procedure

The experimental procedure is as follows:

1. The flat specimen is moved 10 mm in the x -direction. The images of this specimen before and after this movement are captured as the focal length of the lens is set to 18 mm, 24 mm, 35 mm and 55 mm, respectively.
2. The specimen is moved ± 5 mm in z -direction, and the images at different z -coordinates are taken to calibrate Λ .
3. A group of grid points on one image is selected. The span between two adjacent grid points can be arbitrary. The coordinates of these grid points on both images are

Figure 4. Specimen. **Figure 5.** Setup of experimental equipment.

analysed by two-dimensional DIC. The proposed distortion correction method is then applied to calculate the centre of distortion and the correction coefficients.

4. The coordinates of these grid points are corrected. Simplified 3D DIC are used to analyse the 3D coordinates of these grid points before and after correction.
5. The flatness of this specimen at various focal lengths is examined.
6. Another group of grid points is selected. These grid points are located on the same position within every repeated mark. Therefore, the span between any two adjacent grid points is 20 mm according to the size of the repeated mark. The coordinates of all grid points before and after distortion correction are determined using the simplified 3D DIC to evaluate the effect of this correction method.

4.3 Experimental results

Both flatness of specimen surface and space distance between 20 mm-spaced grid points are examined to evaluate the effect of the proposed correction method. The experimental results are shown and discussed as follows:

(1) Flatness of specimen surface

The resulting surfaces at various focal lengths are demonstrated in figures 6, 7, 8 and 9. It shows that the surface before correction forms a concave upward bowl shape as the focal length is set to 18, 24 and 35 mm and it turns into a concave downward bowl shape at the focal length of 55 mm. This means that the image distortion becomes a pincushion distortion at the focal length of 55 mm. The surfaces formed by the corrected 3D coordinates of these grid points approach a flat surface. This shows that the 3D coordinates are significantly improved.

(2) Space distance between 20 mm-spaced grid points

The space distances between adjacent 20 mm-spaced grid points in x and y directions as well as their average

values and standard deviations are calculated. The results are shown in table 8, and the values in parentheses are standard deviations. It can be found that the uncorrected average distances diverge considerably from 20 mm and their standard deviations are also larger at larger distortion (i.e. at focal length of 18 and 24 mm), while the corrected average values are quite close to 20 mm and the standard deviations become small. At the focal length of 35 mm and 55 mm, the corrected average values and standard deviations are still substantially improved when compared to the uncorrected results. On observing all standard deviations after correction, note that all the values concentrate between 0.05 and 0.08 mm. Therefore, the proposed correction method can achieve stable correction results. In addition, the target used in this study is printed out by a laser printer with a resolution of 600 dpi. The size of a dot is approximately 0.04 mm, and this is about the same order as the standard deviation, i.e., parts of the standard deviations in table 8 are contributed by the errors of the coordinates of the grid points instead of the proposed correction method.

Figure 6. 3D coordinates analyzed at focal length of 18 mm: (a) uncorrected and (b) corrected.

Figure 7. 3D coordinates analyzed at focal length of 24 mm: (a) uncorrected and (b) corrected.

Figure 8. 3D coordinates analyzed at focal length of 35 mm: (a) uncorrected and (b) corrected.

Figure 9. 3D coordinates analyzed at focal length of 55 m: (a) uncorrected and (b) corrected.

Table 8. Average values and standard deviations of space distances between 20 mm-spanned grid points at different focal lengths.

Focal length	Uncorrected		Corrected	
	X-direction	Y-direction	X-direction	Y-direction
18 mm	20.619 (0.544)	20.167 (0.148)	20.006 (0.053)	19.970 (0.070)
24 mm	20.421 (0.321)	20.143 (0.100)	20.057 (0.051)	20.016 (0.068)
35 mm	20.088 (0.084)	19.994 (0.062)	19.999 (0.046)	19.957 (0.061)
55 mm	19.935 (0.072)	19.927 (0.092)	19.967 (0.078)	19.960 (0.063)

5. Conclusions

This study proposes a method for correcting radial distortion. Theoretically, the 3D coordinates of points on a plane must remain on a plane when they are measured by 3D DIC. This property is adopted to calibrate the optimal

centre of distortion and correction coefficients. The following conclusions can be drawn from this study:

1. The validity of the proposed method is verified through an assumed numerical experiment. When the distortion equation has a higher order, a higher-order correction equation can be used to improve the accuracy of the analysis.
2. The error occurred during measurement will affect the results after correction. With fewer measuring points, the error for corrected coordinates will be greater than the measurement error. When the number of measuring points increases, the error for corrected coordinates can decrease to the same level as the measurement error. Therefore, more measuring points should be used when this method is adopted.
3. Observing the 3D coordinates of the surfaces before and after correction in a laboratory experiment, we can find that the surfaces before correction are bowl shaped. When the focal length is 35 mm or less, the bowl shape is concave upward. At the focal length of 55 mm, the bowl shape becomes concave downward, and this means that the image distortion turns from a barrel distortion

into a pincushion distortion. After correction, the 3D coordinates are significantly improved, and the surface is close to a plane.

4. The space distances between adjacent 20 mm-spanned grid points are calculated. It shows that without correction, the average distances diverge significantly from 20 mm, and the standard deviations are larger. However, the average distances and standard deviations are considerably improved after correction. Especially in the cases of severe distortion (i.e., at focal length of 18 and 24 mm), the effect of distortion correction is more apparent.

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