A new hybrid teaching–learning particle swarm optimization algorithm for synthesis of linkages to generate path

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MS received 4 January 2017; revised 17 April 2017; accepted 18 April 2017; published online 7 November 2017

Abstract. This paper proposes a novel hybrid teaching–learning particle swarm optimization (HTLPSO) algorithm, which merges two established nature-inspired algorithms, namely, optimization based on teaching–learning (TLBO) and particle swarm optimization (PSO). The HTLPSO merges the best half of population obtained after the teacher phase in TLBO with the best half of the population obtained after PSO. The population so obtained is used subsequently in learner phase of TLBO. To validate the proposed algorithm, five constrained benchmark functions are considered to prove its robustness and efficiency. The proposed algorithm is applied to synthesize four-bar linkage for prescribed path. It is found that the HTLPSO performs better than other single nature-inspired algorithms for path synthesis problem in mechanism theory. Hence, HTLPSO may prove to be an important tool for mechanism design to follow the prescribed path.

Keywords. Hybrid-optimization; global optimization; PSO; TLBO; linkage; path synthesis.

1. Introduction

An assemblage of machine elements that produces a specified motion is known as a mechanism. The four-bar linkage is considered as a simple pin-jointed mechanism that is functionally flexible and practically important. Determination of a mechanism that fulfils certain motion specifications is kinematic synthesis [1]. It involves three areas, namely, type, number and dimensional synthesis. Kinematic synthesis can be categorized as motion, function and path generation [2, 3]. This paper deals with the dimensional synthesis of path generating mechanisms using the proposed algorithm. Various dimensional synthesis techniques such as geometrical [2], analytical [4] and numerical techniques [5] have been used. Furthermore, they are classified as precision point and optimal synthesis methods. In the precision point method, the number of desired points is limited, whereas in the optimal synthesis method there is no such limitation. Hence, it is frequently used by researchers [6–9] in comparison with other methods.

The kinematic synthesis problem using optimization methods consists of two distinct parts, namely, formulation of objective function along with the constraints and an algorithm to solve it. The Euclidean distance between the generated and desired points is the most widely used objective function [10]. Alternatively, multi-objective functions can also be used for the synthesis of path

generating mechanisms. For example, minimization of tracking error (TE) and transmission angle (TA) deviation from 90° may be considered together for better path tracking, and minimum torque requirement [11]. Likewise, three objective functions, namely, TE, TA deviation from 90° and maximum angular velocity ratio (MAVR) can also be used to improve the kinematic performance of a mechanism [12]. There are various optimization algorithms used for solving the formulated problem. ‘Exact gradient method’ is one such local optimization method that is free from finite-difference approximation methods, which render inaccurate results [13]. Extension of the primal–dual infeasible interior algorithm may be used for the linearly constrained convex nonlinear programming problem [14]. Another technique for the synthesis of the planar mechanism for different tasks may be exact differentiation that also improves the quality of search direction [15]. However, these calculus-based conventional local optimization techniques involve calculation of derivatives and Hessians and there also exists non-zero probability of obtaining a global optimum solution. In contrast to the classical optimization methods, nature-inspired optimization methods can be used for getting the global optimal solution [6].

The nature-inspired algorithms have become more popular for solving various non-linear problems in different fields, including path synthesis. One such nature-inspired algorithm is genetic algorithm (GA). GA can be used for solving highly nonlinear problems to find the optimum solutions and its efficiency may be increased by modifying its selection- and crossover procedure [6–8]. Alternatively,

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particle swarm optimization (PSO) and differential evolution (DE) algorithms may be used for path synthesis of four-bar linkage for tracking more than five precision points. Moreover, some refinement schemes can also be applied to the initial population that increase the computational efficiency [6]. Nature-inspired algorithms such as artificial bee colony (ABC), the binary genetic algorithm (BGA) [9], whale optimization algorithm (WOA) [16], moth-flame optimization (MFO) [17] and simulated annealing (SA) [18] can also be used for the same purpose. However, these algorithms require tuning of algorithmic parameters; for example, GA requires crossover probability and mutation rate [19]; PSO requires inertia or weighting factor and social parameters, whereas DE needs crossover constant and scaling factor [8]. Alternatively, - optimization based on teaching-learning (TLBO) is a population-based nature-inspired optimization technique but free from initial parameters tuning. It works in two phases and updates the solution using the mean value of the population [20].

In contrast with the single nature-inspired algorithms, merging of two or three algorithms taking their distinct proficiencies can be an option to obtain efficient algorithms. Various hybridized (merged) techniques have been explored in the past for unconstrained functions [21, 22], and path-synthesis problems [22, 23]. Also, hybrid algorithms can be applied for numerous real life applications. For example, PSO-GA hybrid algorithm is used for closed-loop supply chain network design in large-scale networks to obtain superior results in comparison with GA [24]. The exploration tendencies of the biogeography-based optimization (BBO) are improved by incorporating the modified PSO velocity and position update mechanism of the particles [25]. This hybrid form of PSO, i.e., PSOBBO, has been applied to improve the feature selection problem in emotion and stress recognition from a speech signal. An interesting variant of PSO is used for sustainable integrated dynamic ship routing and scheduling optimization [26, 27], and maritime inventory routing to satisfy demand at different ports [28]. It is known as composite particle PSO (PSO-CP). This variant of PSO permits addressing an important issue of premature convergence. The improved sterling quality of PSO-CP helps escaping the local optima and provides near-optimal results. Another variant of PSO that is named as modified particle swarm optimization (MPSO) can also be used [29]. In this variant, improved particles (candidate solutions) are generated using the difference between the global and local best positions. Additionally, a greedy selection is also appended to this algorithm to improve its robustness. It can be applied for the form error evaluation, which contributes significantly to the production of mechanical components [29]. PSO can also be hybridized by combining with adaptive crossover and mutation rates [30] or by combining PSO with ant colony optimization (ACO) for convex and non-convex economic load dispatch (ELD) problem of a small scale thermal power

system [31]. Also, a cooperative PSO (CPSO) is applied for solving function approximation and classification problems with improved accuracy [32]. This algorithm uses two swarms for the same problem (dismantled into two parts) and both the swarms work in a cooperative manner to achieve an improved solution. There are several metaheuristics available in the literature; therefore, it is important to anticipate whether more algorithms are required. The answer to this problem is presented in [33]. 'No-free-lunch' theorem has been proved to show that not all kinds of algorithms can perform equally well in solving all optimization problems [33]. This indicates that some optimizers may perform well for some set of problems while they fail to perform equally well for a different set of problems. The theorem allows the proposal of new hybrid algorithms that can solve a wider range of problems or some specific unsolved problems.

For any population-based metaheuristic optimization algorithm, the search process involves two stages, namely, exploration and exploitation stages, irrespective of their nature. The operators should be included to explore search space globally with randomized movements (as many perturbations of design variables as possible). This stage is known as exploration, where the search space is explored to get a better solution. The exploitation stage is followed by the exploration stage, which identifies and searches locally in the promising areas identified during exploration in the search space. Promoting any one of the exploration or exploitation may degrade the results of the other. A right balance between the two is required for the accurate approximation of the global optimum. This balance between exploration and exploitation is a challenging task for developing a metaheuristic algorithm because of its stochastic nature.

In this paper, a novel hybrid teaching-learning particle swarm optimization algorithm (HTLPSO) is proposed, which merges two well-established algorithms, namely, TLBO and PSO. It is observed that PSO provides results very quickly but its ability to yield the optimal solutions, especially for real life problems, is insufficient [34]. Also, the new solution replaces the old solution instead of considering the best solution, which reflects its explorative tendency, and it lacks exploitation [35]. Moreover, PSO uses the same operator for position update, which increases the probability of entrapping in local optimum [16]. Only exploration of search space may prevent an algorithm from finding accurate approximation of the optimum solution. In contrast with PSO, the TLBO algorithm incorporates both exploration and exploitation in the teaching and learning phases. The TLBO and PSO are combined in HTLPSO, which uses the explorative tendency of PSO with the explorative and exploitation tendency of teaching phase-TLBO. The combination appends the exploration of PSO and teaching-learning phases of TLBO. This is due to the use of extra updating mechanisms of swarms of PSO and learners of TLBO in HTLPSO.

This paper proposes a novel optimization algorithm that merges the explorative tendency of PSO with the teacher phase of TLBO to generate a new population. This population is used in learner phase of TLBO and the final population obtained after learner phase ends one iteration. Hence, this algorithm is named as HTLPSO. By merging these two algorithms, the HTLPSO generates better results than either method alone. The proposed algorithm is applied for path synthesis problem of linkages. Five constrained benchmark functions are considered here to validate the algorithm proficiency. The feasibility and limitations of the new algorithm are suggested by analysing the results of the single and hybridized algorithms.

The paper is structured in six sections. The HTLPSO algorithm is presented in section 2. Section 3 validates and analyses the results of the new algorithm considering five constrained benchmark functions. The proposed algorithm is applied to the path synthesis problem of crank–rocker linkage and results are also discussed in section 4. Finally, conclusions are outlined in section 5.

2. Hybrid optimization algorithm

This section proposes HTLPSO for the path synthesis problem. It is the hybridization of TLBO and PSO.

TLBO

TLBO is a population-based nature-inspired algorithm that does not require tuning of parameters. Parameter tuning is to find good values for the parameter before running the algorithm, and subsequently, the values remain constant for the whole process [36]. Therefore, the implementation of TLBO is the simplest among nature-inspired algorithms. It works on the philosophy of natural teaching–learning process. Here, each learner is offered a set of courses in which grades are awarded. The grade set of all learners collectively is called population set. The class is analogous to the population set in any nature-inspired algorithm while the set of courses offered to the learners is considered as design variables and numeric grades obtained by learners are considered as values of design variables. Furthermore, learners' result is akin to objective function value of optimization problem. The algorithm works in two phases, namely, teacher phase and learner phase for each iteration.

Teacher phase

Learners in the teachers' phase learn through the teacher. A teacher is considered as the most learned and knowledgeable person of a course. Therefore, learner with the best grade qualifies for the teacher. The teacher improves the mean result of his class, which depends on the quality of himself and learners. In this phase, the course grades of all learners are updated based on the course grade of the best learner, who qualified as a teacher. This step acts as the explorative tendency of the teacher phase. Subsequently, a greedy selection between initial and updated grades is

exploitation. In greedy selection, learners with better results are kept along with their course grades and the new class is formed. Thus, teacher phase uses both explorative and exploitation tendencies. The teacher phase ends with the formation of a new class, which is treated as the initial population for the learner phase of the algorithm.

Learner phase

Learning is possible in two ways: one through teacher and other by mutual interaction among learners. Here, learners learn through mutual interactions. This phase starts with the new class formed at the end of teacher phase. In this phase also, both exploration and exploitation exist. In order to improve the grades, learners interact with at least one other learner in the class. The grade of the learners is improved if another learner has better grade in the corresponding course. Exploration is followed by greedy selection as in teacher phase with formation of a new class. This ends the learner phase and first iteration (teaching–learning cycle). The new class formed by greedy selection is used for next iteration in teacher phase.

Various parameters used in this algorithm are described here.

- p = population size, i.e., number of learners in a class
- d = number of design variables, i.e., number of courses offered to learners
- L_i, U_i = lower and upper limit on i th design variable, i.e., minimum and maximum numeric grade in the i th course
- t = t th iteration
- c = the number of classes in a teaching–learning cycle
- f_j^c = the objective function value of j th learner of c th class
- μ_i = mean of i th course grades of whole class
- G_i = grade of the i th course whose objective function value is minimum
- g_{ji}^q = grade of the i th course of the j th learner and for q th class, $q = 1, \dots, c$
- I = number of iterations
- $rand$ = random number between 0 and 1

Note that class term is used here for population set ($p \times d$).

Figure 1 shows the detailed algorithm for TLBO. In this algorithm, five classes, namely, $c_1, \dots, c_5$, are formed in each iteration and the fifth class obtained after learner phase is treated as the first class of learners for the next iteration. A cycle of teacher–learner phase completes one iteration.

PSO

PSO is a nature-inspired algorithm and it exhibits common nature-inspired algorithm attributes such as random population initialization and optimum value search by updating generations. It is inspired by the simulation of environmental conditions. The main focus is to simulate graphically its smooth but unpredictable choreography of a bird flock. Potential solutions are called particles, which track their coordinates in the problem space corresponding to the best fitness (solution) achieved so far. The value

Algorithm begins:

Input: population size, i.e., p
number of courses offered to learners, i.e., d
lower and upper limit for all the courses, i.e., L_i and U_i where $i = 1, \dots, d$ course
number of iterations, I

% Start of teacher phase

Initialize: **% Initialize course grades for all learners and this is class one**
for $j = 1 \dots p$ **% whole class**
for $i = 1 \dots d$ **% all courses**
% Grade of the i^{th} course of the j^{th} learner
 $g_{ji}^1 = L_i + \text{rand}(U_i - L_i)$
end loop
 $c_1 = \{g_{ij}^1; j = 1, \dots, p; i = 1, \dots, d\}$
% here class one is created with all course grades
% From result of all learners identify the best learner grade for i^{th} course, G_i
end loop
% Start of t^{th} iteration
% Mean of i^{th} course grade of whole class
 $\mu_i = \frac{\sum_{j=1}^p g_{ji}^1}{p}$
% Update course grade for whole class
for $j = 1 \dots p$ **% whole class**
for $i = 1 \dots d$ **% all courses**
 $g_{ji,t}^2 = g_{ji}^1 + \text{rand}(G_i - \mu_i)$ **% for t^{th} iteration**
end loop
 $c_2 = \{g_{ij}^2; j = 1, \dots, p; i = 1, \dots, d\}$
% Compare and compute the values of objective functions for first and second class
, i.e., f_j^1 and f_j^2
% Greedy selection for better result
if $f_j^1 < f_j^2$
 $g_{ji,t}^3 = g_{ji,t}^2$
else
 $g_{ji,t}^3 = g_{ji,t}^1$
end
end loop
 $c_3 = \{g_{ij}^3; j = 1, \dots, p; i = 1, \dots, d\}$
% End of Teacher Phase

% Start of Learner phase
% Compare results of two learners j and l from the third class of learners
for $j = 1 \dots l \dots p$ **and $j \neq l$**
if $f_j^3 < f_l^3$
for $i = 1 \dots d$
 $g_{ji,t}^4 = g_{ji}^3 + \text{rand}(g_{ji,t}^3 - g_{li,t}^3)$
end loop
else
for $i = 1 \dots d$
 $g_{ji,t}^4 = g_{ji}^3 + \text{rand}(g_{li,t}^3 - g_{ji,t}^3)$
end loop
end
 $c_4 = \{g_{ij}^4; j = 1, \dots, p; i = 1, \dots, d\}$
% Greedy selection for better result between third and fourth class
if $f_j^3 < f_j^4$
% Update course grades
 $g_{ji,t}^5 = g_{ji}^3$
else
 $g_{ji,t}^5 = g_{ji}^4$
end
end loop
% Check limits on the course grades
for $j = 1 \dots p$ **% whole class**
for $i = 1 \dots d$ **% all courses**
 $g_{ji,t}^5 = \max(g_{ji,t}^5, L_i)$
 $g_{ji,t}^5 = \min(g_{ji,t}^5, U_i)$
end loop
end loop
 $c_5 = \{g_{ij}^5; j = 1, \dots, p; i = 1, \dots, d\}$
% End of Learner phase
% End of t^{th} iteration

Algorithm ends

Figure 1. Algorithm for optimization based on teaching–learning.

obtained is known as '*pBest*,' while the location (value) obtained corresponding to overall best value is called '*gBest*'. In PSO, velocity changes (i.e., accelerating) at each step towards its '*pBest*' and '*gBest*' locations. Acceleration is weighted by two separate random numbers towards '*pBest*' and '*gBest*' locations. Following equations are used for updating velocity and position:

$$V_{i+1} = wV_i + c_{1a}r_1(pBest_i - X_i) + c_{2a}r_2(gBest_i - X_i) \quad (1)$$

$$X_{i+1} = X_i + V_{i+1} \quad (2)$$

where V_{i+1} denotes a new velocity for each particle based on its previous velocity, X_{i+1} denotes the updated particle's position, r_1 and r_2 are two random numbers generated in the range $[0, 1]$ and w is the constant parameter (inertia weight); c_{1a} and c_{2a} are the acceleration constants in which c_{1a} represents confidence of particle in itself while c_{2a} represents confidence of particle in swarm. Thus, low and high values of these constants allow the particles to flow far from the target before being pulled back and perform abrupt movement towards, or past, the target, respectively [35, 37]. In contrast with GA, PSO does not require a special genetic operator. Here, evolution focuses only on the best solution and all the particles tend to converge to the best solution. The algorithm for PSO is shown in figure 2.

HTLPSO

HTLPSO is a hybridized technique developed by merging PSO and TLBO. This merging makes the new algorithm useful for finding the optimum solution in a lesser number of function evaluations. The flowchart in figure 3 illustrates the HTLPSO technique. HTLPSO begins with the initialization of the population randomly. Simultaneously, initialization of PSO algorithm parameters is also done. The population so generated is treated as the initial population for teacher phase in TLBO and PSO. The best half of the population obtained in teacher phase is merged with the best half obtained in PSO. The resulting population will be of the same size as the size of the initial population. This population is used in the learner phase of TLBO, where established learner phase technique is applied. The final population obtained after learner phase will be treated as the initial population for the next iteration. The size of the final population is the same as the size of the population, which is initialized randomly. This completes one iteration of HTLPSO. This combination of established techniques takes care of the drawback of exploitation tendency of PSO and provides better results.

Various notations used for the HTLPSO algorithm are as follows:

p_{TLBO} = best half of the population selected after teacher phase

p_{PSO} = best half of the population selected after PSO

p_l = best population selected for learner phase

p_{new} = final population selected after each iteration

3. Numerical examples

In this section, validation of the proposed HTLPSO is demonstrated. The results of HTLPSO are compared to those obtained using TLBO, PSO, WOA, MFO and accelerated particle swarm optimization (APSO) technique. For PSO, c_1 and c_2 are both taken as 2, $w = w_{max} - (w_{max} - w_{min})\left(\frac{1}{l}\right)$, where $w_{max} = 0.9$ and $w_{min} = 0.4$ are taken, default settings are used for other algorithms, whereas TLBO does not require algorithmic parameters. Five constrained benchmark functions are used to compare the performance of each algorithm. All these functions are to be minimized. These benchmark functions [35, 38] are defined as follows.

1. Benchmark function 1

$$\text{minimize } f_1 = (x_1 - 10)^3 + (x_2 - 20)^3$$

subject to:

$$g_1 = -(x_1 - 5)^2 - (x_2 - 5)^2 + 100 \leq 0$$

$$g_2 = (x_1 - 6)^2 + (x_2 - 5)^2 - 82.81 \leq 0$$

$$13 \leq x_1 \leq 100, \quad 0 \leq x_2 \leq 100.$$

The global minimum value of function is -6961.814 [39].

2. Benchmark function 2

minimize

$$f_2 = 5.3578547(x_3)^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141$$

subject to:

$$g_1 = 85.334407 + 0.005685x_2x_5 + 0.0006262x_1x_4 + 0.0022053x_3x_5 \leq 0$$

$$g_2 = -85.334407 - 0.005685x_2x_5 - 0.0006262x_1x_4 - 0.0022053x_3x_5 \leq 0$$

$$g_3 = 80.512499 + 0.0071317x_2x_5 + 0.002995x_1x_2 + 0.0021813x_3^2 - 110 \leq 0$$

$$g_4 = -80.51249 - 0.0071317x_2x_5 - 0.002995x_1x_2 - 0.0021813x_3^2 + 90 \leq 0$$

$$g_5 = 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4 - 25 \leq 0$$

$$g_6 = -9.300961 - 0.0047026x_3x_5 - 0.0012547x_1x_3 - 0.0019085x_3x_4 + 20 \leq 0.$$

Algorithm begins:

```

Input :      population size, i.e.,  $p$ 
            number of particles, i.e.,  $d$ 
            lower and upper limit for all the particles, i.e.,  $L_i$  and  $U_i$  where  $i = 1, \dots, d$ 
            number of iterations,  $I$ 
            inertia weight,  $w$ 
            acceleration factor,  $c_{1a}$ 
            acceleration factor,  $c_{2a}$ 

%Particle Swarm optimization initialization
for  $j = 1 \dots p$            % whole population
    for  $i = 1 \dots d$        % all particles
         $x_{j,i}^0 = L_i + \text{rand}(U_i - L_i)$  % Initial position
    end loop
end loop
 $x = x^0$                    % Current position
 $v^0 = 0.1x^0$               % Current velocity
% Compute objective function value  $f^0$  using current position  $pBest(x^0)$ 
    and find  $gBest$  (value of  $x_{j,i}^0$  for which objective function value is minimum)

%Particle Swarm optimization algorithm starts, Start of Iteration
% Update velocity using Eq. (1)
for  $j = 1 \dots p$ 
    for  $i = 1 \dots d$ 
         $v_{j,i}^1 = wv_{j,i}^0 + c_{1a} * \text{rand}(pBest_{j,i} - x_{j,i}) + c_{2a} * \text{rand}(gBest_{j,i} - x_{j,i})$ 
    end loop
end loop
% Update position using Eq. (2)
for  $j = 1 \dots p$ 
    for  $i = 1 \dots d$ 
         $x_{j,i}^1 = x_{j,i}^0 + v_{j,i}^1$ 
    end loop
end loop
% Handling boundary constraints
for  $j = 1 \dots p$ 
    for  $i = 1 \dots d$ 
        if  $x_{j,i}^1 < L_i$ 
             $x_{j,i}^1 = L_i$ 
        elseif  $x_{j,i}^1 > U_i$ 
             $x_{j,i}^1 = U_i$ 
        end
    end loop
end loop
% Compute fitness  $f^1$  for the updated position
% Update the  $pBest$  and fitness
for  $j = 1 \dots p$ 
    if  $f_j^1 < f_j^0$ 
        % Update  $pBest$  corresponding to better fitness
         $f_j^0 = f_j^1$ 
    end
end loop
% Now, update  $gBest$  and fitness similarly
% End of PSO, End of Iteration

```

Algorithm ends

Figure 2. Algorithm for particle swarm optimization.

Lower limit = [78 33 27 27 27].

Upper limit = [102 45 45 45 45].

The global minimum value of function is −30665.539 [39].

3. Benchmark function 3

minimize

$$f_3 = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7$$

subject to:

$$g_1 = -127 + 2x_1^2 + 3x_2^4 + x_3 + 4x_4^2 + 5x_5 \leq 0$$

$$g_2 = -282 + 7x_1 + 3x_2 + 10x_3^2 + x_4 - x_5 \leq 0$$

Figure 3. Flowchart of hybrid teaching–learning particle swarm optimization (HTLPSO) algorithm.

Table 1. Performance of PSO, TLBO and HTLPSO.

Benchmark function	Performance parameters	PSO	TLBO	APSO	MFO	WOA	HTLPSO
1	Best value	-6961.813	-6961.806	-6961.8	-6961.736	-6961.4234	-6961.814
	Mean value	-6961.804	-6961.778	-6961.8	-6.96E+03	-6448.6547	-6961.814
	Standard deviation	0.00875	0.02388	0.000506	2.13E+02	3.27E+02	0.002056
	Number of function evaluations	240000	240000	240000	240000	280000	48000
2	Best value	-30665.54	-30665.54	-30666	-30665.538	-30668.7873	-30665.54
	Mean value	-30665.38	-30665.54	-30666	-30665.515	-30234.1062	-30665.54
	Standard deviation	0.00011	5.561E-7	4.72E-07	4.49E-02	2.34E+02	2.283E-9
	Number of function evaluations	120000	240000	280000	80000	320000	60000
3	Best value	680.6408	680.6326	680.6308	680.6595	686.440	680.6308
	Mean value	680.6734	680.6358	680.6668	680.8062	706.936	680.6334
	Standard deviation	0.03060	0.00204	0.092	0.125689	22.061508	0.003073
	Number of function evaluations	240000	240000	320000	120000	120000	120000
4	Best value	-0.095825	-0.095825	-0.0958247	-0.0958250	-0.09582504	-0.095825
	Mean value	-0.095825	-0.095825	-0.0958125	-0.0958169	-0.09582417	-0.095825
	Standard deviation	2.458E-8	1.911E-8	1.15E-05	1.14E-05	1.78E-06	1.075E-8
	Number of function evaluations	8000	8000	28000	16000	8000	4800
5	Best value	0.000000	9.0381E-5	0.000013	8.70E-05	7.93E-07	1.6983E-6
	Mean value	3.8680E-4	0.004552	0.0444	2.16E-02	1.16E-02	9.6176E-4
	Standard deviation	0.00211	0.00407	0.0384	2.36E-02	0.0195802	0.00143
	Number of function evaluations	800	6400	6400	8000	4000	4800

Table 2. Percentage of number of function evaluations require for HTLPSO in comparison with other algorithms.

Benchmark function	PSO	TLBO	APSO	MFO	WOA
1	80	80	80	80	83
2	50	75	79	25	82
3	50	50	100	100	100
4	40	40	70	70	40
5	180	25	40	40	20

$$g_3 = -196 + 23x_1 + x_2^2 + 6x_6^2 - 8x_7 \leq 0$$

$$g_4 = 4x_1^2 + x_2^2 - 3x_1x_2 + 2x_3^2 + 5x_6 - 11x_7 \leq 0$$

$$\text{where } -10 \leq x_i \leq 10 \quad (i = 1, \dots, 7).$$

The global minimum value of function is 680.63 [39].

4. Benchmark function 4

$$\text{minimize } f_4 = -\frac{\sin x^3(2\pi x_1) \sin(2\pi x_2)}{x_1^3(x_1 + x_2)}$$

subject to:

$$g_1 = x_1^2 - x_2 + 1 \leq 0$$

$$g_2 = 1 - x_1 + (x_2 - 4)^2 \leq 0$$

$$\text{where } 0 \leq x_i \leq 10 \quad (i = 1, 2).$$

The global minimum value of function is -0.095825 [35].

5. Benchmark function 5 [38]

$$\text{minimize } f_5 = (x_1^2 + x_2 - 11)^2 + (x_1^2 + x_2^2 - 7)^2$$

subject to:

$$g_1 = 26 - (x_1 - 5)^2 - x_2^2 \geq 0$$

$$g_2 = 20 - 4x_1 - x_2 \geq 0$$

$$\text{where } -5 \leq x_1 \leq 10, \quad -10 \leq x_2 \leq 5$$

The global minimum value of the function is 0 [38].

The performance of any new optimization algorithm on the benchmark functions should be checked with known global optima. Therefore, the same practice is adopted here to test the performance of HTLPSO algorithm. Since the metaheuristics are stochastic optimization techniques, for each benchmark function, the HTLPSO algorithm is run 30 times to obtain reasonable results. The same process is followed for all the algorithms used in this study. The

Figure 4. Convergence of the best objective function values with respect to the number of function evaluations for (a) benchmark 1, (b) benchmark 2, (c) benchmark 3, (d) benchmark 4 and (e) benchmark 5.

Figure 5. Comparison of standard deviation for all benchmark functions using different algorithms.

Figure 6. Four-bar linkage and definition of various notations.

objective function value corresponding to each iteration is recorded. To verify the performance of HTLPSO, its results are compared with the well-established single, nature-inspired, modified variant, and hybrid algorithms. Table 1 summarizes the statistical results (i.e., the average objective function value and corresponding standard deviation) for HTLPSO and other algorithms used in this study to solve the benchmark problems. It is observed that the HTLPSO algorithm outperforms all the other algorithms in terms of efficiency (ability to locate the global optimum in less number of function evaluations) for four benchmark functions; however, the HTLPSO takes more function evaluations for the fifth benchmark function to reach global optimum but provides best mean results. A reduction of 17–80% in the number of function evaluations is observed in the comparison of other algorithms as shown in table 2. These superior results are obtained through the combination of two well-established algorithms. The combination appends the exploration of PSO and teaching-learning phases of TLBO. This is due to the use of extra updating

mechanisms of swarms of PSO and learners of TLBO used in HTLPSO, as shown in figures 1 and 2, respectively. These updating mechanisms of teaching phase widen the exploration in the search space. However, only exploration may avoid reaching the optimal solution; therefore, it is followed by exploitation (greedy selection of best solutions), which identifies the promising areas of the solution. PSO operator of updating mechanism is applied in parallel to further explore the area for the optimal solution but PSO lacks exploitation; therefore, to widen the TLBO search capability and to compensate the absence of exploitation in PSO, both these algorithms are combined. Hence, the search space is explored widely and refined; thereafter, the best half populations from both the results are selected. The well-established learning technique is then applied to the selected population to explore and exploit the search space to find the accurate approximation of the optimal solution. In this way, a right balance between the exploration and exploitation is created, which gives superior results in comparison with the single nature-inspired algorithms used here.

In order to illustrate the results lucidly, the best objective function values are plotted against the number of function evaluations for all the benchmark functions as shown in figure 4. It is observed that HTLPSO's learners and swarms tend to explore the promising regions of search space, simultaneously, and exploit the best region. Initially, the population (learners and swarms) changes abruptly and thereafter converges to an optimum point. This behaviour of the population-based algorithms is shown by the convergence curve in figure 4. It is found that HTLPSO is more competitive in comparison with all the other well-established metaheuristic algorithms.

From figure 4, three different trends in convergence are observed. Firstly, the convergence of the algorithm occurs rapidly after initial steps of iterations. This behaviour is seen for the HTLPSO (benchmark functions-(1–5)), WOA (benchmark functions-(4, 5)), MFO (benchmark functions-(2, 3)), PSO (benchmark function-(5)), APSO (benchmark function-(3)) and TLBO (benchmark function-(3)). Only HTLPSO shows this trend in the convergence of the best values of the objective function in all the benchmark functions because of good balance of exploration and exploitation that helps HTLPSO to find the global minimum. This also ensures the high success rate of HTLPSO in solving the wide range of problems. Secondly, the convergence of algorithm occurs when the maximum number of function evaluations is reached. This trend is observed for the WOA (benchmark functions-(1–3)), MFO (benchmark functions-(1, 4, 5)), PSO (benchmark functions-(1, 3)) and TLBO (benchmark functions-(2, 4)). The convergence trend may be due to the failure of the algorithm in finding the promising solution for exploitation in the initial steps. Therefore, algorithms keep on exploring the search space to find good solutions. The third case is when the algorithm converges in an accelerated fashion. This trend is observed

in the APSO (benchmark functions-(1, 2, 4, 5)), PSO (2, 4) and TLBO (1, 5). This behaviour is observed due to the operators used in the algorithm, which help in exploring the promising regions of the search space, thereafter converging rapidly after almost half of the maximum number of the function evaluations. Overall, it is found that HTLPSO converges fast in comparison with the other algorithms used here.

Figure 5 shows standard deviation of the objective function value for all the benchmark functions using six metaheuristic algorithms. It is found that standard deviation for HTLPSO is the least in comparison with all the other algorithms. This shows that the solution obtained by HTLPSO is more reliable than the other algorithms used here.

4. Application-path synthesis of four-bar linkage

The HTLPSO developed in section 2 is applied to solve a four-bar path synthesis problem. Figure 6 shows various design parameters of a four-bar linkage. XOY represents a global reference frame; a coupler point P is defined by vectors $\overrightarrow{AC}$ and $\overrightarrow{CP}$, where $\overrightarrow{AC}$ is along $\overrightarrow{AB}$ and $\overrightarrow{CP}$ is normal to $\overrightarrow{AB}$ and passing to point P . The length between joints is used to define link length as shown in figure 6. A frame $X_bO_2Y_b$ is defined to measure the orientation of links with respect to link 1. It is fixed to link #1 and its X -axis is aligned along link #1. Now, orientations of link #2, link #3 and link #4 are given by angles θ_2, θ_3 and θ_4 , respectively, with respect to the axis X_b . The orientation of link #1 is defined by angle θ_1 with respect to the fixed axis X . The coordinates of coupler point P in frame $X_bO_2Y_b$ are given as follows:

$$P_{xb} = r_2 \cos \theta_2 + r_{px} \cos \theta_3 - r_{py} \sin \theta_3, \quad (3)$$

$$P_{yb} = r_2 \sin \theta_2 + r_{px} \sin \theta_3 + r_{py} \cos \theta_3. \quad (4)$$

The coupler point coordinates in fixed reference frame XOY are obtained as follows:

$$P_x = O_{2x} + P_{xb} \cos \theta_1 - P_{yb} \sin \theta_1 \quad (5)$$

$$P_y = O_{2y} + P_{xb} \sin \theta_1 + P_{yb} \cos \theta_1 \quad (6)$$

where O_{2x} and O_{2y} are the coordinates of the O_2 in the frame XOY .

Equations (5) and (6) represent the coordinates of any point on the path traced by the coupler point P . Link lengths r_1, r_2, r_3 and r_4 , orientation θ_1 of link#1, N positions of link#2 defined by $\theta_2^1, \theta_2^2, \dots, \theta_2^N$ and lengths AC (r_{px}) and CP (r_{py}) are to be determined for prescribed trajectory of point P . These $N + 9$ unknown parameters are considered as design variables to synthesize the linkage:

Table 3. Comparison of HTLPSO with various algorithms used in literature, for case 1.

Design variables	Cabrera et al [19]	GA [8]	PSO [8]	DE [8]	ABC [9]	BGA [9]	TLBO	APSO	MFO	WOA	HTRCA [23]	HTLPSO
r_1	39.46629	28.77133	31.15501	35.02074	16.3446	25.2613	27.972	36.9483	60.0000	59.9267	18.42	58.7357
r_2	8.562912	5.000000	5.000000	6.404196	18.5965	8.3871	7.6705	5.0000	5.0000	48.2787	35.14	5.9847
r_3	19.09486	35.36548	23.84561	31.60722	50.0000	16.1828	26.533	13.3014	24.8464	52.7064	56.42	8.3028
r_4	47.83886	59.13681	45.80352	50.59949	48.5635	30.2151	27.746	29.9204	60.0000	59.9221	49.99	56.5439
r_{cx}	13.38556	0.000000	39.00066	20.80324	-28.7259	22.5806	43.221	39.7105	36.7826	44.4886	55.08	-12.1993
r_{cy}	12.21961	14.85037	18.50846	41.54364	50.0000	-22.4633	18.037	12.6525	-60.0000	42.1151	-59.52	-0.6693
O_{2x}	29.72255	29.91329	59.99999	60.00000	-19.0413	-3.57771	-17.50	5.2084	6.2832	6.2830	-25.92	26.2198
O_{2y}	23.45454	32.60228	17.91696	18.07791	47.1614	24.5902	59.359	16.6345	-44.4413	32.9165	42.12	28.3434
θ_1	6.201627	5.287474	0.419837	0.000000	4.35100	5.65057	4.2524	31.8082	8.9630	42.1844	5.931	4.9605
θ_2^1	6.119371	6.283185	4.842412	6.283185	3.34008	3.18152	1.4000	2.9569	3.2796	6.0715	2.347	0.7235
θ_2^2	0.193040	0.318205	0.404684	0.264935	3.57682	4.18879	2.2466	3.7804	4.0727	6.1220	2.584	1.1004
θ_2^3	0.440830	0.638520	0.657415	0.500377	3.87317	4.717	3.2219	4.1467	4.5149	6.1695	2.847	1.4408
θ_2^4	0.684674	0.979950	0.922086	0.735321	4.24872	5.08551	2.7538	4.5142	4.9172	6.1947	3.117	1.7913
θ_2^5	0.958351	1.412732	1.247066	0.996529	4.78717	5.50316	3.6999	4.9346	5.3601	6.2203	3.377	2.2021
θ_2^6	1.355331	2.076254	2.298727	1.333549	5.26775	6.08664	4.2484	5.6086	6.2832	6.2427	3.612	2.9409
Function value	0.19047	1.101697	0.546461	0.122738	0.9898	2.3319	0.1047	0.8683	1.7585	2.1253	0.0056	0.0812

Figure 7. Convergence between (a) best function values and number of function evaluations and (b) average function values and number of function evaluations for case 1.

$$\mathbf{x} = [r_1, r_2, r_3, r_4, r_{px}, r_{py}, O_{2x}, O_{2y}, \theta_1, \theta_2^1, \theta_2^2, \dots, \theta_2^N] \quad (7)$$

where N represents the number of desired points for the coupler trajectory, and the design variables are defined in figure 6. The other parameters θ_3 and θ_4 can be determined by loop closure equation of the mechanism [8].

Normally, for path generation problem, the position error is considered as the square root of the sum of the square of the Euclidean distances between each coupler-point position P and the corresponding desired point P_d . Then, the position error is the objective function to be minimized for the synthesis problem and is expressed as follows:

$$f(\mathbf{x}) = \sum_{k=1}^N \sqrt{(P_x^k - P_{xd}^k)^2 + (P_y^k - P_{yd}^k)^2} \quad (8)$$

where P_{xd}, P_{yd} are the x and y coordinates of desired points and P_x, P_y are the x - and y -coordinate of the coupler point trajectory. Superscript k represents the k^{th} position of the coupler.

A four-bar linkage is expected to achieve this objective, such that crank is able to rotate completely in sequence, either clockwise or counter-clockwise, and values of design variables should lie within the prescribed range. The following constraints are imposed to get feasible four-bar linkage.

(a) *Grashof constraint:*

$$g_1(\mathbf{x}) = r_1 + r_2 - r_3 - r_4 < 0 \quad \text{if} \quad (r_2 < r_3 < r_4 < r_1). \quad (9)$$

(b) *Order constraint:*

$$g_2(\mathbf{x}) = \theta_2^k - \theta_2^{k+1} < 0 \quad k = 1, \dots, N. \quad (10)$$

(c) *Range of variables:*

$$L_i \leq x_i \leq U_i$$

where L_i and U_i are the lower and upper limits for the i^{th} design variable. Thus using, Eqs. (7)–(10), the problem is completely stated as follows:

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) = \sum_{k=1}^N \sqrt{(P_x^k - P_{xd}^k)^2 + (P_y^k - P_{yd}^k)^2} \\ \text{subject to} \quad & g_1(\mathbf{x}) = r_1 + r_2 - r_3 - r_4 < 0 \quad \text{if} \quad (r_2 < r_3 < r_4 < r_1) \\ & g_2(\mathbf{x}) = \theta_2^k - \theta_2^{k+1} < 0 \quad k = 1, \dots, N \\ & L_i \leq x_i \leq U_i \end{aligned} \quad (11)$$

Now, the constrained optimization problem defined by Eq. (11) is transformed into an unconstrained problem for obtaining a feasible and optimum solution. This is achieved by penalizing the objective function for any constraint violation. A large value of order 10^4 is added to the objective function, which assures that infeasible solution has function value greater than the feasible solution. This eliminates all the infeasible solutions and promotes the algorithm convergence towards the optimum solution. Finally, the original problem is transformed into an unconstrained problem, which is expressed as follows:

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) = \sum_{k=1}^N \sqrt{(P_x^k - P_{xd}^k)^2 + (P_y^k - P_{yd}^k)^2} + \sum_{m=1}^2 B_m(P^m) \end{aligned} \quad (12)$$

$$\text{range of variables} \quad L_i \leq x_i \leq U_i$$

where $P^m (m = 1, 2)$ are the constants with value (10^4) for penalizing the objective function if the constraints are

Figure 8. Generated trajectory and prescribed points.

Figure 9. Convergence between (a) best function values and number of function evaluations and (b) average function values and number of function evaluations for case 2.

Table 4. Comparison of HTLPSO with various algorithms used in literature, for case 2.

Design variables	GA [8]	PSO [8]	DE [8]	TLBO	MFO	WOA	APSO	HTRCA [23]	HTLPSO
r_1	50.00000	49.994859	50.00000	50.000	50.0000	49.1672	54.5605	49.46	50.0000
r_2	9.164414	5.0000000	5.000000	5.9278	5.0000	8.3843	8.1548	5.405	5.00000
r_3	16.85808	5.9156430	5.905345	9.2490	8.8532	17.7921	19.9956	8.015	7.31650
r_4	50.00000	49.994867	50.00000	50.000	46.4047	40.8881	45.2836	47.165	48.0298
r_{cx}	38.45887	18.925715	18.81931	25.110	17.9879	32.8107	40.6120	17.90	19.9172
r_{cy}	0.090117	0.0000000	0.000000	-6.1922	20.3398	20.2363	22.7522	15.30	9.74970
O_{2x}	32.32828	14.472475	14.37377	22.204	12.8609	29.7055	33.2163	12.00	14.7515
O_{2y}	-29.53705	-12.49441	-12.4442	-17.704	-22.0471	-30.7715	-38.0495	-18.7	-16.5078
θ_1	0.877212	0.467287	0.463633	0.9123	6.2832	0.7773	0.7787	6.2832	0.31140
Function value	3.171063	2.35529	2.349649	4.9439	4.3587	6.3559	4.2199	3.571	3.766

violated and B_m is the Boolean Function [40] expressed as follows:

$$B_m = \begin{cases} 0 & \text{if } g_m(\mathbf{x}) \leq 0 \\ 1 & \text{otherwise} \end{cases} \quad (13)$$

The optimization problem formulated in Eq. (12) is solved considering three different cases: straight line, semi-circular and elliptical path generations as case 1, case 2 and case 3, respectively.

Case 1

This case illustrates path generation without prescribed timing for four-bar linkage [8]. In this case, the vertical straight line is to be tracked, which is composed of 6 prescribed points. The coordinates of prescribed points and limit on the design variables are as follows:

$$P_d^k = [(20, 20), (20, 25), (20, 30), (20, 35), (20, 40), (20, 45)], \\ r_1, r_2, r_3, r_4 \in [5, 60]; r_{px}, r_{py}, O_{2x}, O_{2y} \in [-60, 60]; \theta_1 \in [0, 2\pi].$$

To find out the optimal solution for this case, five algorithms are used as solvers, and their results are compared to those established algorithms in the literature [9, 19], as shown in table 3. The comparison shows that HTLPSO performs better than those of single nature-inspired algorithms, whereas HTLPSO cannot find the better solution in comparison with HTRCA. However, the difference between function values obtained through HTLPSO and HTRCA is negligible.

The rate of convergence for the best and average objective function against the number of function evaluations is shown in figure 7. Figure 8 shows the generated trajectories and prescribed points for the four-bar linkage using various optimization algorithms, for case 1. HTLPSO

Figure 10. Generated trajectory and prescribed points.

Table 5. Comparison of HTLPSO with various algorithms used in literature, for case 3.

Design variables	GA [8]	PSO [8]	DE [8]	TLBO	MFO	WOA	APSO	HTRCA [23]	HTLPSO
r_1	79.981513	52.53516	54.36089	75.9859	80.0000	79.4190	59.9306	54.875	80.0000
r_2	9.1099930	8.687886	8.683351	8.2946	5.0000	20.9035	8.3158	9.725	8.75920
r_3	72.936511	36.15508	34.31863	39.1410	63.0691	77.8578	32.2771	49.925	48.3073
r_4	80.000000	80.00000	79.99617	48.2598	79.9922	78.4010	44.6219	23.375	50.2142
r_{cx}	0.0000000	0.000000	0.000187	-1.4864	-80.0000	58.1669	-3.3149	7.8400	1.6448
r_{cy}	0.0000000	1.481055	1.465250	-7.4652	-12.0492	77.8962	-7.6757	-4.320	-10.0785
O_2	10.155966	11.00212	10.95434	7.5190	-28.7214	30.2928	6.9639	1.440	3.92080
O_y	10.000000	11.09559	11.074534	17.0350	80.0000	-73.7423	17.5758	12.16	18.0160
θ_1	0.0261490	1.403504	2.12965	6.2831	0.7605	6.0956	0.0002	6.1135	6.1526
θ_2^1	6.2831850	6.282619	6.283185	0.0001	0.0000	0.0000	0.0000	0.148283	0.0192
θ_2^2	0.6007450	0.615302	0.616731	0.6301	0.0637	0.0000	0.6275	0.825611	0.7348
θ_2^3	1.3728120	1.305421	1.310254	1.3420	0.5793	0.0000	1.3088	1.520531	1.4363
θ_2^4	2.2105750	2.188053	2.193570	2.0459	1.1008	0.0000	2.0291	2.236814	2.1357
θ_2^5	2.8626390	2.913049	2.917170	2.7347	1.5800	0.0000	2.6835	2.906602	2.8222
θ_2^6	3.4205470	3.499313	3.490746	3.4154	2.0584	5.9025	3.3994	3.565079	3.4994
θ_2^7	4.0726110	4.125586	4.132017	4.0876	3.4290	5.9680	4.0763	4.295185	4.1961
θ_2^8	4.9103730	4.919977	4.922075	4.7858	3.9938	6.0654	4.7637	5.021522	4.8998
θ_2^9	5.6824400	5.685021	5.695372	5.4909	4.5198	6.1583	5.4510	5.718955	5.5955
θ_2^{10}	6.283185	6.282323	6.28297	6.1974	5.0146	6.2832	6.1877	0.148283	6.2832
Function value	2.281273	1.971004	1.952326	0.7701	22.3829	65.3812	1.873	0.1649	0.3729

reports the error of 0.0812, which is better in comparison with the single nature-inspired algorithms.

Case 2

This case illustrates path generation with prescribed timing for four-bar linkage [8]. Here, four-bar linkage generates a semi-circular trajectory similar to the prescribed trajectory. The coordinates of prescribed points on the path and limit on the design variables are as follows:

$$P_d^k = [(0, 0), (1.9098, 5.8779), (6.9098, 9.5106), (13.09, 9.5106), (18.09, 5.8779), (20, 0)],$$

$$r_1, r_2, r_3, r_4 \in [5, 50]; r_{px}, r_{py}, O_{2x}, O_{2y} \in [-50, 50]; \theta_1 \in [0, 2\pi].$$

In this problem, the angle of link is provided. Therefore, there are only nine design variables:

$$\mathbf{x} = [r_1, r_2, r_3, r_4, r_{px}, r_{py}, O_{2x}, O_{2y}, \theta_1],$$

$$\theta_2^k = \left[\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi \right].$$

The objective function, Eq. (12), is minimized to get a feasible solution. Figure 9 shows the convergence for the best and average objective function with the number of function evaluations for case 2. Table 4 shows the result comparison of proposed algorithm with the other established algorithms. The generated and prescribed trajectories of case 2 are shown in figure 10.

Case 3

This case illustrates path generation of four-bar linkage without prescribed timing. Here, totally ten prescribed positions are given to track an elliptical trajectory. The trajectory is expressed as follows.

Prescribed points:

$$P_d^k = [(20, 10), (17.66, 15.142), (11.736, 17.878), (5, 16.928), (0.60307, 12.736), \dots, (0.60307, 7.2638), (5, 3.0718), (11.736, 2.1215), (17.66, 4.8577), (20, 10)].$$

Limits of the design variables:

$$r_1, r_2, r_3, r_4 \in [5, 80]; r_{px}, r_{py}, O_{2x}, O_{2y} \in [-80, 80];$$

$$\theta_1 \in [0, 2\pi].$$

The results obtained using HTLPSO are compared with other algorithms and shown in table 5. Figure 11 shows the plots of the generated trajectory and prescribed points, for case 3. The convergence of the best and average objective function is shown in figure 12. In this case, HTLPSO obtains the function value of 0.3729, which is much better than those by single nature-inspired algorithms used here. The HTLPSO algorithm cannot find the better solution than that by the HTRCA algorithm, but the difference in values between the HTLPSO and HTRCA algorithm used here is thought to be negligible. It is observed that HTLPSO gives

Figure 11. Generated trajectory and prescribed points.

Figure 12. Convergence between (a) best function values and number of function evaluations and (b) average function values and number of function evaluations for case 3.

Table 6. Comparison of HTLPSO with TLBO in terms of number of function evaluations.

Case no.	Number of function evaluations				
	HTLPSO	TLBO	WOA	MFO	APSO
1	47250	60000	90000	90000	60000
2	24750	39000	90000	90000	75000
3	54000	55500	82500	82500	82500

the best solution compared with the other algorithms while using lesser function evaluations, as shown in table 6.

The application of HTLPSO on these cases proved that the HTLPSO algorithm is more robust in comparison with the other algorithms used here. HTLPSO can find global optimal value in all the cases, while MFO and WOA cannot find the near approximation to global optimal solution in case 3, and the TLBO takes more function evaluations to converge. This means that merging of PSO with TLBO assists in escaping the local optima successfully. This merging has also improved the effectiveness of the algorithm. Now, HTLPSO algorithm can find optimal values for all the cases in lesser number of function evaluations for the algorithms used here. Note that the number of function evaluations of the other algorithms for the path synthesis cannot be accessed. Hence, the comparison of proposed algorithm with only APSO, TLBO, MFO and WOA in terms of the number of function evaluations is performed.

Lastly, HTLPSO can find more accurate optimal values than all the recent metaheuristic and benchmark algorithms used here. Therefore, HTLPSO can be considered as an effective algorithm for solving a wide range of problems.

The results of HTLPSO algorithm are found to be competitive and superior over the conventional and new single nature-inspired algorithms used here for comparison.

However, compared with HTRCA, the HTLPSO algorithm cannot find the better solution. Nevertheless, the difference in results of HTLPSO and HTRCA is negligible. Also, the result of HTLPSO in case 2 is worse than with other algorithms except for TLBO, WOA, MFO and APSO. The TE of all algorithms in case 2 lies in the range 2.349–4.943, which is greater than that of benchmark single-nature inspired algorithm (GA [8], PSO [8], DE [8]). However, the results are superior to the recent single nature-inspired algorithms used here (MFO, WOA). This implies that hybrid algorithms (HTRCA and HTLPSO) cannot obtain optimal solutions compared with GA [8], PSO [8] and DE [8] for the case of path synthesis with prescribed timing. From this discussion, it is found that performance of HTLPSO is superior to that of other metaheuristics used here. However, using a different operator for exploitation to improve the performance of HTLPSO in prescribed timing (Case 2) case may be the subject of future studies.

5. Conclusions

This paper proposed a HTLPSO using two established single nature-inspired algorithms, namely, TLBO and PSO. Both the established algorithms are also demonstrated along with a flowchart for the proposed algorithm. In this algorithm, the initial population is simultaneously fed into teacher phase of TLBO and PSO. Then, the best half of populations obtained after PSO and teacher phase are merged and resulting population is fed into learner phase. The final population obtained after learner phase is used as the initial population for PSO and teacher phase. This cycle is repeated for each iteration. To validate the proposed algorithm, five constrained benchmark functions and a path generation problem for four-bar linkage are considered.

The WOA, MFO and APSO are among other algorithms apart from TLBO and PSO, which are used for comparing the results to HTLPSO. It is found that the proposed algorithm performs better than single nature-inspired algorithms such as PSO, MFO, WOA, APSO and TLBO for all the five benchmark functions. It also finds the better solution for path generation cases without prescribed timing. However, the HTLPSO cannot find near optimal solution for path synthesis cases with prescribed timing. However, HTLPSO finds reasonable optimal solutions in comparison with the other algorithms. Hence, HTLPSO may prove to be an important tool for a wide range of problems, including path synthesis of four-bar linkages.

Acknowledgements

Ph.D. scholarship granted by Ministry of Human Resource and Development, Government of India, to the first author is highly acknowledged.

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