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Group SkSP-R sampling plan for accelerated life tests

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Abstract. This study presents a group skip-lot sampling plan using resampling (SkSP-R) for accelerated life tests. It is assumed that the lifetime of a product follows Weibull distribution with known shape parameter under the use condition, while the scale parameter can be obtained from acceleration factor. The plan parameters are determined through a non-linear optimisation problem for fixed values of producer's risk and consumer's risk. The advantages of the proposed plan over the existing one are explained with some practical examples.

Keywords. SkSP-R sampling; life test; Weibull distribution; producer's risk; consumer's risk.

1. Introduction

Reliability can be assured only when one lot of items has been accepted or rejected on the basis of a suitably designed sampling plan. An effective implementation of any life test is possible through proper sampling plan. Usually, a life test is more expensive and time-consuming as one need to wait till the total number of failures is observed or until a fixed time is reached. Therefore, an alternative life testing method using censoring and acceleration is applied to minimise the test time and cost.

Various life test sampling plans are used for various situations. A life test sampling plan can be designed for various underlying distributions; for example, Jun *et al* [1] designed a variables sampling plan under sudden death testing for Weibull distribution and Aslam *et al* [2] designed a sampling plan under a time-truncated life test for a generalised exponential distribution. Sampling plans are divided into two major types according to the quality characteristic under study: attribute and variable sampling plans. Attribute sampling plans are simple in nature and relatively easy to implement as they concern with only favourable or unfavourable situation. Variable sampling plans are complex in nature but a smaller sample size is required in order to make lot sentencing decision [3]. A suitable plan is one that minimises cost and time.

In a traditional life test plan, a single item is installed on a single item. Therefore, to perform a life test experiment, the number of testers should be equal to the random sample. For example, if a random sample of 20 items is selected, 20 testers are needed to perform the life test. On the other hand, a group life test sampling plan is performed by installing more than one item in a single tester, that is, if there is a capacity of five items on a single tester, then four testers are needed to perform the life test using group sampling. Therefore, group acceptance sampling plans are considered more economic than traditional sampling plans [4–7].

Today, in competitive industrial environment, researchers are focusing on designing time-, cost- and effort-saving sampling plans for the lot sentencing. This goal is achieved if we adopt such a plan that takes fewer number of testing units to make the lot sentencing decision. Skip-lot sampling plan is more efficient than the single sampling plan. Dodge [8] introduced the concept of continuous sampling named as CSP-1, where products are identically investigated. Dodge [8] proposed the idea of skip-lot sampling plan (SkSP) that was applicable where product was delivered in bulk or in successive batches and named as SkSP-1. The SkSPs are implemented in the field of physical and chemical processes. The operating characteristic function (OC) for SkSP-1 is derived by Burnett [9] using the Markov chain model. The SkSP-2 using single sampling plan as reference is proposed by Perry [10]. More details on SkSPs can be found in Balamurali *et al* [11]. Balamurali and Subramani [12] determined the plan parameters of the SkSP-2 plan. Balamurali and Jun [13] designed the SkSP-V sampling plan. Aslam, Balamurali *et al* [14] designed the SkSP-V sampling plan with double sampling plan as the reference plan. Aslam *et al* [15] designed skip-lot group sampling plan using the Weibull and generalised exponential distributions. Balamurali *et al* [16] designed the SkSP-R sampling plan and proved its efficiency over the existing sampling plans. Recently, Jun *et al* [17] designed

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the SkSP-R plan using two points on operating characteristics (OC) curve.

On exploring the literature, it can be noted that several authors considered the designing of life test sampling plan using exponential distribution; for example, Epstein and Sobel [18] designed a life test plan for type-II censoring. Sun-Ho and Yum [19] proposed exponential life test plans with intermittent inspections. Chen *et al* [20] proposed life test using mixed censoring. Kim and Yum [21] designed life test plan using accelerated hybrid censoring. Aslam *et al* [22] worked on SkSP-V sampling plan using accelerated life tests.

There is lack of study in literature on designing of SkSP-R using group sampling plan under accelerated life test as a reference plan. In this paper, we focus on the designing of SkSP-R accelerated life test by assuming that the lifetime of a product follows the Weibull distribution. The advantage of the proposed sampling plan will be compared over the existing sampling plan. Some real examples are given for illustration purpose.

2. Design of proposed plan using accelerated life test

Balamurali *et al* [16] designed an SkSP-R sampling plan that is more efficient than single sampling plan and an SkSP-V sampling plan that reduces the average sample number (ASN).

Here, we propose a group SkSP-R sampling plan for accelerated life tests (ALTs) when the product lifetime follows a Weibull distribution. The proposed plan is the generalisation of the plans proposed by Kim and Yum [21] and Aslam *et al* [22]. We have the following assumptions for the proposed plan:

1. The shape parameter for the use and accelerated condition remains unchanged.
2. The scale parameter for the accelerated condition can be adjusted using an acceleration factor.

The lifetime under the use condition follows a Weibull distribution with shape parameter γ and scale parameter ω_U having the cumulative distribution function (CDF) of

$$F_U(t_U) = 1 - exp[-(t_U/\omega_U)^{\gamma}]. \tag{1}$$

Kim and Yum [21] suggested accelerated conditions with parameter γ and with new scale parameter $\omega_A = \omega_U/AF$, where AF is the acceleration factor. So, the CDF for the accelerated condition is given by Aslam *et al* [22] as follows:

$$F_A(t_A) = 1 - exp[-(t_A/\omega_A)^{\gamma}]. \tag{2}$$

The average lifetime of product for the accelerated condition is given by

$$\mu = (\omega_A/\gamma)(1/\gamma). \tag{3}$$

The operational procedure of the proposed group SkSP-R plan using ALT follows:

Step 1: Select a random sample of size n from the collection of items and distribute r items to g groups at time 0 under the acceleration conditions for known AF. Start with normal inspection at accelerated condition and stop the experiment if total number of failures from all groups are larger than c before the end of experiment time τ_A

Step 2: Switch to skipping inspection if i consecutive lots are accepted on normal inspection under accelerated condition

Step 3: Inspect only a fraction f during skipping sampling and continue until a sample lot is rejected

Step 4: Go for resampling procedure when a lot is rejected after k consecutively sampled lots have been accepted, go to Step 5

Step 5: Skipping inspection is continued if the lot is accepted during resampling using reference plan at accelerated condition. The resampling is done for m times on non-acceptance and one lot of product is rejected if it is not accepted on $(m-1)st$ resampling

Step 6: Revert to normal inspection in Step 1 if one lot is rejected on resampling scheme

Step 7: The non-conforming units are replaced with the conforming units in the rejected lots.

The life test plan will be truncated if either the number of failures are larger than c or the time allowed τ_A is ended, whichever is first. The proposed plan consists of six parameters, which are stated as follows: g number of groups, r number of testers, $f(0 < f < 1)$ denotes fraction of the lot inspected in skipping sampling, i shows the clear number for normal inspection, sampling inspection clearance number k, and m denotes the number of submitted lot for resampling. The proposed plan is designated as SkSP-R (g, r, f, i, k, m).

In a study by Balamurali *et al* [16], the operating characteristic (OC) of SkSP-R (g, r, f, i, k, m) is given by

$$P_a(p) = \frac{fP + (1-f)P^i + fP^k(P^i - P)(1-Q^m)}{f(1-P^i)[1-P^k(1-Q^m)] + P^i(1+fQP^k)} \tag{4}$$

where P is the probability of lot acceptance for a single sampling plan. This probability is given as follows:

$$P = \sum_{i=0}^{c} \binom{rg}{i} (1-p)^i p^{rg-i} \tag{5}$$

where p is the probability that an item is working until τ_A under the accelerated conditions. Let μ_0 be the specified average life, and a is a constant used as the termination ratio. Let $\tau_A = a\mu_0$ be the experiment time. Using it in Eqs. (2) and (3), we have the following from the study by Aslam *et al* [22]:

$$p = exp[-a^{\gamma}(AF)^{\gamma}(\mu/\mu_0)^{\gamma}(\Gamma(1/\gamma)/\gamma)^{\gamma}] \quad (6)$$

where $\tau_A \times AF$ is the equivalent censoring time under use condition.

The average sample number (ASN) of SkSP-R plan is given by

$$ASN(p) = \frac{nf + nfQP^{i+s} - nfP^{s}(1-P^{i})(1-Q^{m})}{f(1-P^{i})[1-P^{s}(1-Q^{m})] + P^{i}(1+fQP^{s})}. \quad (7)$$

As mentioned earlier, two risks are always associated with acceptance sampling plans. Let α be the producer's risk, and β be the consumer's risk. Let $1-\alpha$ be the producer's confidence level. The sampling plan is designed such that the lot acceptance probability should be larger than $1-\alpha$ at quality ratio $\mu/\mu_0 = 2$, 3 and lot rejection probability should be smaller than β at $\mu/\mu_0 = 1$. The plan parameters of the proposed plan will be determined by using the following non-linear optimisation problem

Minimise

$$ASN(p_2) = \frac{nf + nfQ_2P_2^{i+s} - nfP_2^{s}(1-P_2^{i})(1-Q_2^{m})}{f(1-P_2^{i})\left[1-P_2^{s}(1-Q_2^{m})\right] + P_2^{i}(1+fQ_2P_2^{s})} \quad (8a)$$

Subject to

$$P_a(p_1) = \frac{fP_1 + (1-f)P_1^{i} + fP_1^{s}(P_1^{i} - P_1)(1-Q_1^{m})}{f(1-P_1^{i})\left[1-P_1^{s}(1-Q_1^{m})\right] + P_1^{i}(1+fQ_1P_1^{s})} \geq 1 - \alpha \quad (8b)$$

and

$$P_a(p_2) = \frac{fP_2 + (1-f)P_2^{i} + fP_2^{s}(P_2^{i} - P_2)(1-Q_2^{m})}{f(1-P_2^{i})\left[1-P_2^{s}(1-Q_2^{m})\right] + P_2^{i}(1+fQ_2P_2^{s})} \leq \beta. \quad (8c)$$

The plan parameters of the proposed sampling plan are determined when $r = 5$, $a = 0.5$, 1.0, and $\mu/\mu_0 = 2$, 3 are presented in tables 1 and 2. Tables 3 and 4 are presented for when $r = 10$, $a = 0.5$, 1.0 and $\mu/\mu_0 = 2$, 3. From tables 1, 2, 3 and 4, we note the following trends in plan parameters:

1. For identical values of other parameters, the values of g are smaller for smaller values of AF. For example, when AF = 20.09, the value of g from table 1 is 52 at $\mu/\mu_0 = 2$, and it is 15 when AF = 7.623.
2. For identical values of other parameters, the values of g increases as a increases from 0.5 to 1.0. For example,

Table 1. Plan parameters of the proposed plan when $a = 0.5$, $\gamma = 0.5$ and $r = 5$.

Ratio	g	m	c	k	i	f	ASN	g	m	c	k	i	f	ASN
				AF = 20.09							AF = 7.623			
2	53	2	0	3	4	0.0112	264.86	15	2	1	2	3	0.0190	74.64
3	53	2	0	2	3	0.1336	264.79	10	2	0	2	3	0.0163	49.83
2	82	2	0	2	3	0.0137	409.97	26	2	2	2	3	0.0129	129.99
3	81	2	0	2	3	0.1038	404.99	15	2	0	2	3	0.0165	74.99
				AF = 26							AF = 9.0			
2	98	2	0	3	4	0.0301	489.90	19	2	1	3	4	0.0164	94.97
3	98	2	0	2	3	0.3698	489.89	12	2	0	2	3	0.0479	59.87
2	151	2	0	3	4	0.0112	754.99	34	2	2	2	3	0.0164	169.99
3	151	2	0	2	3	0.0294	754.97	19	2	0	2	3	0.0138	94.99
				AF = 8.52							AF = 12.9			
2	18	2	1	2	3	0.0180	89.65	23	2	0	2	3	0.0127	114.42
3	11	2	0	2	3	0.0433	54.87	22	2	0	2	3	0.0350	109.70
2	31	2	2	2	3	0.0210	154.99	48	2	1	2	3	0.0114	239.98
3	17	2	0	2	3	0.0250	84.99	33	2	0	2	3	0.0306	164.99
				AF = 24.5							AF = 46.4			
2	85	2	0	3	4	0.0106	424.77	53	2	0	2	3	0.0441	264.38
3	85	2	0	2	3	0.0986	424.55	53	2	0	2	3	0.0417	264.34
2	130	2	0	2	3	0.0129	649.95	85	2	0	2	3	0.0102	424.98
3	130	2	0	2	3	0.0215	649.97	81	2	0	2	3	0.0137	404.97
				AF = 20							AF = 14			
2	26	2	0	2	3	0.0147	129.25	15	2	2	2	3	0.0288	74.86
3	25	2	0	2	4	0.0282	124.97	7	2	0	2	3	0.0194	34.86
2	56	2	1	2	3	0.0132	279.98	–	–	–	–	–	–	–
3	39	2	0	2	3	0.0245	194.99	15	2	1	2	3	0.0110	74.9947

Note: (–) shows that plan parameters do not exist.

Table 2. Plan parameters of the proposed plan when $a = 1$, $\gamma = 0.5$ and $r = 5$.

Ratio	g	m	c	k	i	f	ASN	g	m	c	k	i	f	ASN
	AF = 20.09							AF = 7.623						
2	339	2	0	3	4	0.1273	1694.92	31	2	0	2	3	0.0142	154.17
3	339	2	0	2	3	0.6477	1694.88	30	2	0	2	3	0.0484	149.69
2	521	2	0	2	4	0.0325	2604.99	66	2	1	2	3	0.0225	329.98
3	521	2	0	2	3	0.2135	2604.99	46	2	0	2	3	0.0405	229.99
	AF = 26							AF = 9.0						
2	812	2	0	3	4	0.0616	4059.61	43	2	0	2	3	0.0239	214.22
3	812	2	0	2	3	0.4216	4059.31	42	2	0	2	3	0.1254	209.83
2	1247	2	0	2	4	0.0184	6234.99	92	2	1	3	4	0.0113	459.99
3	1247	2	0	2	3	0.5299	6234.99	64	2	0	2	3	0.0187	319.98
	AF = 8.52							AF = 12.9						
2	38	2	0	2	4	0.0108	189.92	96	2	0	3	5	0.0170	479.99
3	37	2	0	2	3	0.1902	184.90	96	2	0	2	3	0.6642	479.96
2	82	2	1	2	3	0.0237	409.98	148	2	0	2	3	0.0119	739.94
3	57	2	0	2	3	0.0110	284.97	148	2	0	2	3	0.0253	739.97
	AF = 24.5							AF = 46.4						
2	657	2	0	3	4	0.1390	3284.87	335	2	0	2	4	0.0200	1674.50
3	657	2	0	2	3	0.6652	3284.79	335	2	0	2	3	0.2324	1674.32
2	1010	2	0	3	4	0.0221	5049.99	514	2	0	2	3	0.0534	2569.95
3	1010	2	0	2	3	0.0518	5049.90	514	2	0	2	3	0.1742	2569.98
	AF = 20							AF = 14						
2	119	2	0	3	4	0.0315	594.88	30	2	1	2	5	0.0192	149.99
3	119	2	0	2	3	0.2409	594.77	19	2	0	2	3	0.1023	94.90
2	183	2	0	2	3	0.0119	914.92	42	2	1	2	3	0.0205	209.99
3	183	2	0	2	3	0.0432	914.98	29	2	0	2	3	0.0372	144.99

Table 3. Plan parameters of the proposed plan when $a = 0.5$, $\gamma = 0.5$ and $r = 10$.

Ratio	g	m	c	k	i	f	ASN	g	m	c	k	i	f	ASN
	AF = 20.09							AF = 7.623						
2	27	2	0	2	3	0.0356	269.27	8	2	1	2	3	0.0131	79.75
3	27	2	0	2	3	0.1754	269.87	5	2	0	3	4	0.0218	49.99
2	59	2	1	2	3	0.0626	589.99	–	–	–	–	–	–	–
3	41	2	0	2	3	0.0613	409.99	11	2	1	2	3	0.0120	109.99
	AF = 26							AF = 9.0						
2	49	2	0	2	4	0.0574	489.95	10	2	1	2	3	0.0260	99.79
3	49	2	0	2	4	0.0316	489.90	6	2	0	2	3	0.0780	59.92
2	78	2	0	2	3	0.0178	779.97	17	2	2	2	3	0.0267	169.99
3	76	2	0	2	3	0.0206	759.96	10	2	0	2	3	0.0245	99.99
	AF = 8.52							AF = 12.9						
2	9	2	1	2	3	0.0123	89.48	12	2	0	2	3	0.0107	119.5304
3	6	2	0	2	3	0.0651	59.96	11	2	0	2	3	0.0381	109.72
2	17	2	2	2	3	0.0134	169.99	24	2	1	2	4	0.0138	239.99
3	10	2	0	2	3	0.0167	99.99	17	2	0	2	3	0.0147	169.99
	AF = 24.5							AF = 46.4						
2	43	2	0	2	3	0.0435	429.02	27	2	0	2	4	0.0174	269.93
3	43	2	0	2	3	0.0940	429.57	27	2	0	2	3	0.0493	269.52
2	65	2	0	3	4	0.0104	649.99	58	2	1	2	4	0.0101	579.99
3	65	2	0	2	3	0.0420	649.98	41	2	0	2	3	0.0113	409.97
	AF = 20							AF = 14						
2	20	2	1	2	3	0.1093	199.81	9	2	2	2	3	0.0107	89.98
3	13	2	0	2	3	0.0485	129.78	4	2	0	2	3	0.0182	39.95
2	28	2	1	2	3	0.0224	279.98	–	–	–	–	–	–	–
3	20	2	0	2	3	0.0245	199.99	8	2	1	2	3	0.0247	79.99

Note: (–) shows that plan parameters do not exist.

Table 4. Plan parameters of the proposed plan when $a = 1.0$, $\gamma = 0.5$ and $r = 10$.

Ratio	g	m	c	k	i	f	ASN	g	m	c	k	I	f	ASN
					AF = 20.09							AF = 7.623		
2	170	2	0	2	3	0.2000	1699.17	17	2	0	2	3	0.0119	169.56
3	170	2	0	2	3	0.2408	1699.35	15	2	0	2	3	0.0459	149.67
2	261	2	0	2	3	0.0280	2609.91	33	2	1	2	3	0.0313	329.99
3	261	2	0	2	3	0.0119	2609.79	23	2	0	2	3	0.0439	229.99
					AF = 26							AF = 9.0		
2	406	2	0	2	4	0.0573	4059.58	21	2	0	2	4	0.0159	209.93
3	406	2	0	2	3	0.4221	4059.31	21	2	0	2	3	0.1593	209.87
2	624	2	0	2	3	0.0252	6239.76	46	2	1	2	4	0.0192	459.99
3	624	2	0	2	3	0.0607	6239.90	32	2	0	2	3	0.0151	319.98
					AF = 8.52							AF = 12.9		
2	19	2	0	2	4	0.0118	189.93	48	2	0	4	5	0.0110	479.98
3	19	2	0	2	3	0.0894	189.81	48	2	0	2	3	0.6728	479.97
2	41	2	1	2	3	0.0591	409.99	74	2	0	2	3	0.0108	739.93
3	29	2	0	2	3	0.0152	289.98	74	2	0	2	3	0.0440	739.98
					AF = 24.5							AF = 46.4		
2	329	2	0	2	3	0.2942	3289.03	168	2	0	2	3	0.1210	1678.55
3	329	2	0	2	3	0.2868	3288.99	168	2	0	2	3	0.1315	1678.68
2	505	2	0	2	3	0.0650	5049.92	257	2	0	2	3	0.0513	2569.95
3	505	2	0	2	3	0.0509	5049.90	257	2	0	2	3	0.0302	2569.91
					AF = 20							AF = 14		
2	60	2	0	2	3	0.0778	599.19	15	2	1	3	4	0.0158	149.94
3	60	2	0	2	3	0.0881	599.29	10	2	0	2	3	0.1028	99.93
2	92	2	0	2	3	0.0355	919.97	21	2	1	2	3	0.0206	209.99
3	92	2	0	2	3	0.0149	919.94	15	2	0	2	3	0.0116	149.99

when $a = 0.5$ and $\mu/\mu_0 = 2$, the value of g is 53 from table 1, and it is 339 from table 2 when $a = 1$.

3. For identical values of other parameters, the values of g decreases when r increases from 5 to 10. For example, when $a = 0.5$ and $\mu/\mu_0 = 2$, the value of g is 53 from table 1, and it is 27 from table 3.

3. Comparative study

In this section, we compared the efficiency of the proposed plan over the existing sampling plans.

3.1 *Proposed plan*

The proposed plan is the extension of the sampling plan proposed by Kim and Yum [21]. The values of both sampling plans are placed in tables 5 and 6 for the same values of the specified parameters.

From tables 5 and 6, we note that the proposed plan is more efficient than that proposed by Kim and Yum [21] at all values of the specified parameters. For example, when AF = 9.0, $a = 1.0$ and $\mu/\mu_0 = 3$, from table 5, ASN is 319 with $g = 64$ for the proposed sampling plan, while ASN is 460 with $g = 92$ from the sampling plan proposed by Kim and Yum [21]. Similarly, for all other identical values, when $r = 10$, the proposed plan again provides small values of plan parameters. For example, when AF = 9.0, $a = 1.0$ and $\mu/\mu_0 = 3$, from table 6, ASN is 319 with $g = 32$ for the proposed sampling plan, while ASN is 460 with $g = 46$ from the sampling plan proposed by Kim and Yum [21].

3.2 *Proposed plan*

The proposed sampling plan is generalisation of the plan proposed by Aslam *et al* [22]. The proposed plan reduces to the Aslam *et al* [22] plan when $r = 1$. The plan is said to be more efficient if it provides smaller values of a number of groups g. The plan parameters of the Aslam *et al* [22] plan are listed in table 7.

By combining table 7 with tables 2 and 4, it can be seen that the proposed plan has smaller values of a number of groups required for the testing purpose. For example, when $a = 1$, $\gamma = 0.5$, $r = 5$ and AF = 20.09, the value of g from the Aslam *et al* [22] is 1695, and from the proposed plan, it is 339. So, the proposed plan is better than that proposed by Aslam *et al* [22].

4. Industrial examples of AFs

In this section, we discuss some examples of calculating AFs. We determine AF for (i) voltage acceleration factor, (ii) thermal acceleration factor (stress and operating), (iii) humidity acceleration, (iv) reliability and the electronic engineer acceleration, (v) thermal acceleration factor (use

Table 5. Comparison of plan parameters of the proposed plan when $a = 1.0$, $\gamma = 0.5$ and $r = 5$.

	Proposed		Existing		Proposed		Existing	
Ratio	g	ASN	g	ASN	g	ASN	g	ASN
	AF = 20.09				AF = 7.623			
2	339	1694.92	537	2685	31	154.173	–	–
3	339	1694.88	339	1695	30	149.6909	47	235
2	521	2604.99	952	4760	66	329.9881	0	0
3	521	2604.99	521	2605	46	229.9957	83	415
	AF = 26				AF = 9.0			
2	812	4059.61	1285	6425	43	214.2294	–	–
3	812	4059.31	812	4060	42	209.8397	66	330
2	1247	6234.99	1798	8990	92	459.9996	–	–
3	1247	6234.99	1247	6235	64	319.9846	92	460
	AF = 8.52				AF = 12.9			
2	38	189.92	–	–	96	479.9914	202	1010
3	37	184.90	59	295	96	479.9698	153	765
2	82	409.98	–	–	148	739.9415	–	–
3	57	284.97	82	410	148	739.9728	213	1065
	AF = 24.5				AF = 46.4			
2	657	3284.87	1041	5205	335	1674.504	530	2650
3	657	3284.79	657	3285	335	1674.325	335	1675
2	1010	5049.99	1844	9220	514	2569.955	938	4690
3	1010	5049.90	1010	5050	514	2569.988	514	2570
	AF = 20				AF = 14			
2	119	594.889	250	1250	30	149.9977	–	–
3	119	594.771	189	945	19	94.9036	40	200
2	183	914.927	–	–	42	209.9906	–	–
3	183	914.981	264	1320	29	144.9963	54	270

Note: (–) shows that plan parameters do not exist.

and stress), (vi) Arrhenius relationship time–acceleration factor and (vii) reliability analysis. Aslam *et al* [22] also considered the same examples to calculate AF as follows.

4.1 *Voltage acceleration factor*

Let $\tau_A = 720\mathrm{H}$, $\gamma = 2$, $\omega_{U_1} = 100{,}000$ and $\omega_{U_2} = 20{,}000$. Loughmiller [23] derived the following formula to calculate voltage AF:

$$AF_v = e^{\beta(V_s - V_o)} = 20.09 \qquad (9)$$

where AF_v represents the voltage AF, $V_s = 3.7$ V is the stress voltage, $V_0 = 2.7$ V is the operating voltage and $\beta = 1.38 \times 10^{-23}$ is the Boltzmann's constant obtained from dielectric integrity data.

4.2 *Thermal acceleration factor (stress and operating)*

Suppose that $E_a = 0.3$ eV is the activation energy, $k = 8.617 \times 10^{-5} = 1/11605$ eV/K is the Boltzmann's constant, $T_s = 125$°C is the stressed temperature and $T_0 = 50$°C is the operating temperature. Loughmiller [23] suggested the following formula:

$$AF_T = e^{\frac{E_a}{k}\left(\frac{1}{T_0} - \frac{1}{T_s}\right)} = 7.623. \qquad (10)$$

4.3 *Humidity acceleration*

The acceleration factor can be calculated for $n = 3$, $E_a = 0.9$ eV and $k = 8.617 \times 10^{-5}$. Therefore, we have $RH_s = 85\%$, $RH_0 = 65\%$, $n = 3$, $E_a = 0.9$ eV and $k = 8.617 \times 10^{-5}$. Skvarenina [24] suggested the following formula:

$$\begin{aligned} AF &= \left(\frac{RH_s}{RH_0}\right)^n \times e^{\frac{E_a}{k}\left(\frac{1}{T_0} - \frac{1}{T_s}\right)} \\ &= \left(\frac{85}{65}\right)^3 \times e^{\left\{\left(\frac{0.9}{8.617\times10^{-5}}\right) \times \left[\frac{1}{(273+55)} - \frac{1}{(273+130)}\right]\right\}} = 26. \end{aligned} \qquad (11)$$

4.4 *Reliability and the electronic engineer acceleration*

Suppose that E_a is the activation energy, T is the Absolute Temperature and k is the Boltzmann's constant (1.38×10^{-23}). Let $E_a = 1$, $k = 1.38 \times 10^{-23}$,

Table 6. Comparison of plan parameters of the proposed plan when $a = 1.0$ and $\gamma = 0.5$ and $r = 10$.

	Proposed		Existing		Proposed		Existing	
Ratio	g	ASN	g	ASN	g	ASN	ASN	ASN
	AF = 20.09				AF = 7.623			
2	170	1699.175	269	2690	17	169.5653	–	–
3	170	1699.35	170	1700	15	149.6728	24	240
2	261	2609.912	476	4760	33	329.9915	–	–
3	261	2609.79	261	2610	23	229.996	42	420
	AF = 26				AF = 9.0			
2	406	4059.588	643	6430	21	209.932	–	–
3	406	4059.312	406	4060	21	209.8786	33	330
2	624	6239.762	899	8990	46	459.9998	–	–
3	624	6239.905	624	6240	32	319.9808	46	460
	AF = 8.52				AF = 12.9			
2	19	189.931	–	–	48	479.9867	101	1010
3	19	189.816	30	300	48	479.9709	77	770
2	41	409.994	–	–	74	739.9356	–	–
3	29	289.986	41	410	74	739.9847	107	1070
	AF = 24.5				AF = 46.4			
2	329	3289.031	521	5210	168	1678.552	265	2650
3	329	3288.995	329	3290	168	1678.684	168	1680
2	505	5049.928	922	9220	257	2569.953	469	4690
3	505	5049.906	505	5050	257	2569.919	257	2570
	AF = 20				AF = 14			
2	60	599.195	125	1250	15	149.9438	–	–
3	60	599.297	95	950	10	99.9373	20	200
2	92	919.978	–	–	21	209.9906	–	–
3	92	919.946	132	1320	15	149.9923	27	270

Note: (–) shows that plan parameters do not exist.

$T_1 = T_2 = 125°\mathrm{C}$. Siegel [25] provided the following formula:

$$\mathrm{AF} = e^{-\left(\frac{E_a}{kT_2} - \frac{E_a}{kT_1}\right)} = e^{-\left[\frac{1.6\times10^{-19}}{1.38\times10^{-23}}\left(\frac{1}{389K} - \frac{1}{420K}\right)\right]} = 9.0. \quad (12)$$

4.5 *Thermal acceleration factor (use and stress)*

Let $E_a = 0.3$ eV, $k = 8.63 \times 10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}}$, $T_{use} = T_{stress} = 273°\mathrm{C}$. Vigrass [26] derived the following formula for AF:

$$AF = e^{\left[\frac{E_a}{k}\left(\frac{1}{T_{use}} - \frac{1}{T_{stress}}\right)\right]} = e^{\left[\frac{0.3}{8.63\times10^{-5}}\left(\frac{1}{348} - \frac{1}{443}\right)\right]} = 8.52 \quad (13)$$

where E_a is thermal activation energy, k is the Boltzmann's constant ($8.63 \times 10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}}$), T_{use} is the use temperature (°C + 273) and T_{stress} is the life test stress temperature (°C + 273). Note that T_{use} and T_{stress} are in degrees kelvin.

4.6 *Arrhenius relationship time–acceleration factor*

Let $E_\mathrm{a} = (0.4, \; 0.5, \; 0.6)$ eV, $k = 8.63 \times 10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}}$, $tem_U = 50°\mathrm{C}$, $tem = 120°\mathrm{C}$. Escobar and Meeker [27] provided following formula for AF:

$$AF(temp, tem_U, E_a) = \frac{R(temp)}{R(tempU)} = e^{\left[\frac{E_\mathrm{a}}{\mathrm{k}}\left(\frac{1}{tem_U} - \frac{1}{\mathrm{temp}}\right)\right]} \quad (14)$$

So, AF $(tem) = 12.9$ when $E_\mathrm{a} = 0.4$, AF $(tem) = 24.5$ when $E_\mathrm{a} = 0.5$ and AF $(tem) = 46.4$ when $E_\mathrm{a} = 0.6$.

4.7 *Reliability analysis*

Bao *et al* [28] derived the following formula:

$$AF \propto P^n e^{\left(\frac{-E_a}{k_B \cdot T_J}\right)} \quad (15)$$

where P is the power, T_J is the junction temperature, n is the acceleration parameter of power (for n span 2.2 to 5.9), E_a is the activation energy (values for E_a span 0.41 eV to 0.64 eV), and k_B is the Boltzmann's constant. We have following table:

Group	Power (W)	Junction (°C)	$\mathrm{AF}_{90\%}$
1	14.6	80	20
2	13.0	80	14
3	11.9	110	59
4	12.6	95	31
5	13.6	64	6

Table 7. Plan parameters of [22] plan when $a = 1$ and $\gamma = 0.5$ and $r = 1$.

Ratio	g	g
	AF = 20.09	AF = 7.623
2	1695	178
3	1695	148
2	2605	327
3	2605	227
	AF = 26	AF = 9.0
2	4056	209
3	4056	207
2	6235	460
3	6235	319
	AF = 8.52	AF = 12.9
2	185	481
3	185	480
2	410	739
3	284	738
	AF = 24.5	AF = 46.4
2	3284	1671
3	3284	1671
2	5048	2568
3	5048	2568
	AF = 20	AF = 14
2	594	150
3	594	95
2	914	223
3	913	145

5. Concluding remarks

In this study, a SkSP-R sampling plan is designed using accelerated test. The structure of the proposed plan is given. Tables are provided for various AF formulas for practical application. A comparative study is made as well. The proposed plan is more efficient in reducing ASN as compared to the existing sampling plans. The proposed plan can be applied in the industry to save time and cost of experiment. The proposed plan using some cost model can be considered as subject for future research.

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