Experimental determination of mechanical parameters in sensorless vector-controlled induction motor drive

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Abstract. High-performance industrial drives widely employ induction motors with position sensorless vector control (SLVC). The state-of-the-art SLVC is first reviewed in this paper. An improved design procedure for current and flux controllers is proposed for SLVC drives when the inverter delay is significant. The speed controller design in such a drive is highly sensitive to the mechanical parameters of the induction motor. These mechanical parameters change with the load coupled. This paper proposes a method to experimentally determine the moment of inertia and mechanical time constant of the induction motor drive along with the load driven. The proposed method is based on acceleration and deceleration of the motor under constant torque, which is achieved using a sensorless vector-controlled drive itself. Experimental results from a 5-hp induction motor drive are presented.

Keywords. Induction motor drives; field-oriented control; moment of inertia; frictional coefficient; parameter evaluation; sensorless vector control.

1. Introduction

The cage rotor induction motors (IMs) are rugged, simple and cost-effective by nature, as compared with other machines available. The spark-less operation of this motor makes it suitable for explosive and hazardous environments [1–3]. However, the dynamic speed control of IM is not so straight forward as that of a dc motor due to coupled nature of flux and torque-generating currents in an IM. This limitation has been overcome by a technique called vector control, where the torque and flux-generating components of current are decoupled and controlled separately, in a synchronously revolving reference frame [1, 2]. Vector control results in a much improved dynamic performance of the IM [1].

A simplified block diagram of a vector-controlled IM is shown in figure 1. Vector control involves decoupled control of flux and torque as mentioned earlier. The decoupling is achieved in a synchronously rotating d – q reference frame, whose reference axes are shown in figure 2. While different reference frames exist, the rotor flux reference frame [4] is considered here (see figure 2). The reference axes of the stationary reference frame and the three-phase stator winding axes are also indicated in the same figure. The details of the transformations are explained in section 2. The controller structure includes an inner q -axis current control loop and an outer speed control

loop. It also includes an inner d -axis current control and an outer flux control loop.

Design of current controllers in rotor flux reference frame is well known for motor drives switching at high frequencies [1–6]. Here, the inverter time delay is neglected as compared with the other time constants [1–3]. However, the inverter delay becomes significant when the inverter switches at low frequencies. This delay is then required to be considered during the current controller design [7–9]. An improved design procedure considering the inverter delay is presented for the design of current controller, in section 3 of this paper. Here, the inverter is modelled as first-order delay; the current control loop is structured to have a second-order response.

Design of speed controller requires precise knowledge of the mechanical parameters, namely, moment of inertia (J) and coefficient of friction (B), for achieving good speed response. Also, such precise knowledge of the parameters is required for certain applications such as computer numerical control (CNC) machine tools, where auto-tuning of controller is required [10]. These parameters also change considerably with the load coupled to the motor [11]. Several methods have been reported in literature to measure and/or estimate the mechanical parameters for servo-motor drives and permanent magnet synchronous machine (PMSM)-based drives [1, 10–15]. Retardation test has been suggested for measurement of moment of inertia in [1]. However, the retardation test suffers from non-uniform load

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Figure 1. Sensorless vector-controlled induction motor drive.

Figure 2. Axes of reference for machine modelling and control.

torque due to speed-dependent windage friction present in the drive.

Reference [12] presents a speed-observer-based online method to generate position error signal for estimation of moment of inertia. An offline method based on time average of the product of torque reference and motor position for mechatronic servo systems is proposed in [13]. Another online recursive least squares (RLS) estimator for a servo motor drive is presented in [14] for estimation of mechanical parameters. Reference [15] presents a PI-controller-based closed-loop method to estimate inertia and friction of servo drive. A load-torque-observer-based method to precisely estimate J and B for servo systems is discussed in [11]. The mechanical subsystem is modelled as a second-order system in the aforementioned methods, which is complicated to solve. Further, observer-based

online estimation requires involved computations, which may not be feasible on low-cost controller-based systems.

In this paper, current control loops and flux control loop are designed by adopting the improved procedure, which considers the inverter delay. A speed loop is designed considering approximate values of J and B . The sensor-less vector control (SLVC) is implemented for a 3.7-kW IM-fed from a 10-kVA inverter controlled by a field programmable gate array (FPGA)-based digital platform. Initially, the drive is operated at constant speeds to estimate the value of frictional coefficient (B), as explained in section 5. Further, it is operated under constant accelerating and decelerating torque to estimate the combined moment of inertia (J), as described in section 6.

2. Machine model in rotor flux reference frame

The axes of reference of IM models for SLVC are illustrated in figure 2. The three-phase stator winding axes RYB are shown along with the a and b axes, which are mutually perpendicular. The a and b axes are the axes of reference in the stationary reference frame. Here the a -axis is aligned along the R-phase axis of stator winding, in stationary reference frame. Vector control of IM is carried out in the rotor flux reference frame [1] defined by d and q axes shown in figure 2. Here d -axis is aligned along the rotor flux space vector ψ_r , which is defined in terms of quantities in stationary coordinates as shown:

$$\psi_r = \psi_{ra} + j\psi_{rb} = L_o i_{mr} = L_o [i_s + (1 + \sigma_r) i_r e^{j\epsilon}] = L_o i_s + L_r i_r e^{j\epsilon} \quad (1)$$

where i_s and $i_r e^{j\epsilon}$ are stator and rotor current space vectors, respectively; i_{mr} is the magnetizing current corresponding to rotor flux; ψ_{ra} and ψ_{rb} are the components of ψ_r along a and b axes, respectively; L_r is the rotor inductance and L_o is the magnetizing inductance.

The dynamic model of an IM in the rotor flux reference frame is given by [1]

$$\frac{d}{dt} i_{sd} = \left[v_{sd} - R_s i_{sd} + \sigma L_s \omega_{mr} i_{sq} - (1 - \sigma) L_s \frac{d}{dt} i_{mr} \right] \frac{1}{\sigma L_s} \quad (2a)$$

$$\frac{d}{dt} i_{sq} = \left[v_{sq} - R_s i_{sq} - \sigma L_s \omega_{mr} i_{sd} - (1 - \sigma) L_s \omega_{mr} i_{mr} \right] \frac{1}{\sigma L_s} \quad (2b)$$

$$\frac{d}{dt} i_{mr} = \frac{R_r}{L_r} (i_{sd} - i_{mr}) \quad (2c)$$

$$\frac{d}{dt} \rho = \omega_{mr} = \omega + \frac{R_r i_{sq}}{L_r i_{mr}} = \omega + \omega_r \quad (2d)$$

$$v_{RN}^* = \frac{2}{3} v_{sa}^*, \quad (4a)$$

$$v_{YN}^* = -\frac{1}{3} v_{sa}^* + \frac{1}{\sqrt{3}} v_{sb}^*, \quad (4b)$$

$$v_{BN}^* = -\frac{1}{3} v_{sa}^* - \frac{1}{\sqrt{3}} v_{sb}^*. \quad (4c)$$

Three-phase sinusoidal modulating signals m_R , m_Y and m_B can be obtained by scaling v_{RN}^* , v_{YN}^* and v_{BN}^* , respectively, with $\frac{V_{DC}}{2V_p}$, where V_{DC} is the DC bus voltage and V_p is the peak of the bipolar triangular carrier. Gating signals for the devices in VSI can be generated based on the method of pulse width modulation (PWM) selected.

The three-phase feedback quantities (i_R , i_Y and i_B) and (v_{RN} , v_{YN} and v_{BN}) need to be transformed into the $d-q$ reference frame. These transformations and also the inverse transformations require the unit vectors $\cos \rho$ and $\sin \rho$. The unit-vector generation and estimation of other feedback quantities are discussed in section 3.2.

3.2 Feedback estimation

Estimation of the four feedback quantities in the control loops shown in figure 3, namely, i_{sq} , i_{sd} , i_{mr} and ω , is discussed in this section.

The stationary three-phase feedback currents (i_R , i_Y and i_B) are transformed into stationary two-phase feedback currents (i_{sa} and i_{sb}) as shown in figure 4a. These feedback currents are then transformed into $d-q$ reference frame as i_{sd} and i_{sq} , which are fed back to control loops as shown in figure 4a. The corresponding equations are given in (5):

$$i_{sd} = i_{sa} \cos \rho + i_{sb} \sin \rho, \quad (5a)$$

$$i_{sq} = i_{sb} \cos \rho - i_{sa} \sin \rho. \quad (5b)$$

Three-phase stator voltages (v_{RN} , v_{YN} and v_{BN}) are transformed into two-phase voltages (v_{sa} and v_{sb}) in the stationary reference frame in the same manner as the three-phase currents are transformed into two-phase currents, presented in figure 4a. The stator fluxes (ψ_{sa} and ψ_{sb}) in the stationary ab reference frame are then estimated from the stator voltages (v_{sa} and v_{sb}) and stator currents (i_{sa} and i_{sb}) in the stationary $a-b$ reference frame as indicated by (6) [1]:

$$\psi_{sa} = \int_{t_0}^t (v_{sa} - i_{sa} R_s) dt = \int_{t_0}^t e_{sa} dt \quad (6a)$$

$$\psi_{sb} = \int_{t_0}^t (v_{sb} - i_{sb} R_s) dt = \int_{t_0}^t e_{sb} dt \quad (6b)$$

where t_0 is the time at which the integration starts.

The rotor fluxes (ψ_{ra} and ψ_{rb}) in the $a-b$ reference frame are, in turn, obtained from the estimated stator fluxes (ψ_{sa} and ψ_{sb}) as shown in figure 4b [4].

Figure 4. Determination of feedback quantities: (a) transformation of three-phase feedback current to $d-q$ reference frame feedback current, (b) estimation of unit-vectors ($\cos \rho$ and $\sin \rho$) oriented along rotor flux, (c) estimation of rotor-flux magnetizing current i_{mr} and (d) estimation of rotor speed, ω .

The unit vectors ($\cos \rho$ and $\sin \rho$), required for $a-b$ to $d-q$ and inverse transformations of currents and voltages, are obtained from the estimated rotor fluxes in the $a-b$ reference frame as illustrated in figure 4b [4].

The feedback signal i_{mr} for the flux control loop is calculated from the values of i_{sd} and rotor time constant $T_r = (L_r/R_r)$ using Eq. (2c) as shown in figure 4c.

The rotor speed ω is the difference between the speed of rotor flux ω_{mr} and the slip speed ω_r [see figure 4c and d]. The slip speed ω_r is estimated from the values of i_{sq} , i_{mr} and T_r using Eq. (2d). The speed of rotor flux ω_{mr} is estimated as indicated in figure 4d [4].

3.3 Feedforward estimation

The mathematical model of an IM in the rotor-flux reference frame has coupling terms as seen from (2a) and (2b). To decouple the stator current equations, the coupling terms

Figure 5. Sensorless vector control: (a) d -axis current control loop, (b) q -axis current control loop and (c) flux (i_{mr}) control loop.

are fed forward to the current controller outputs. The feedforward terms along d -axis and q -axis are denoted by e_{sd} and e_{sq} [see figure 5a and b], respectively. They can be calculated from the feedback signals as

$$e_{sd} = (1 - \sigma) \frac{L_s}{T_r} (i_{sd} - i_{mr}) - \sigma L_s \omega_{mr} i_{sq}, \quad (7a)$$

$$e_{sq} = (1 - \sigma) L_s \omega_{mr} i_{mr} + \sigma L_s \omega_{mr} i_{sd}. \quad (7b)$$

4. Improved design of current and flux controllers

Designs of current and flux controllers are well established for the cases of high-switching-frequency drives. However, in case of high-power and/or high-speed drives, the ratio of switching frequency to fundamental frequency (*i.e.*, pulse number) is low. Hence, for such cases, the inverter delay becomes significant as compared with the other time constants in the control loop. Contrary to high-switching-frequency cases, the inverter delay cannot be ignored for low-pulse-number cases. The inverter delay is modelled as a first-order delay for the purpose of controller design. Further, the speed controller is designed by the symmetric optimum method [4] and the simulation and experimental results are presented.

4.1 Improved design of current controllers

The block diagrams of i_{sd} and i_{sq} control loops are shown in figure 5a and b, respectively. Based on the reference and feedback signals in a given sub-cycle or half-carrier cycle, the outputs of current controllers give the voltage to be applied on the machine in the next sub-cycle. Thus, there is a delay of one sub-cycle time T_s due to the controllers. Further, the voltage commanded by the controllers will be applied on the machine after a delay between 0 and T_s due to the process of PWM. Hence, the average delay introduced by PWM is $0.5T_s$. Thus, there is an average total delay (T_d) of $1.5T_s$ in the system. For switching frequency $f_{sw} = 1$ kHz, one sees that $T_s = 500 \mu s$ and $T_d = 750 \mu s$.

Actual transfer function of the delay is given by $G_d(s) = e^{-sT_d}$. For the design of controllers, the transfer function of delay is approximated as $G_{da}(s) = 1/(1 + sT_d)$. The actual and approximated transfer functions of the delay are compared in figure 6 for $T_d = 750 \mu s$. The magnitude of $G_{da}(s)$ is -3 dB less than that of $G_d(s)$ at a frequency of 212 Hz [see figure 6a]. Phase plots of both the transfer functions are quite close to each other for frequencies less than 212 Hz, as shown by figure 6b. Therefore, the approximation is valid if the total bandwidth of current control loop is less than 212 Hz.

Figure 6. Approximation of the inverter and PWM delay in current control loop by a first-order transfer function. Comparison of (a) magnitude plots and (b) phase plots of the actual and approximate transfer functions.

The time constant T_{isd} of i_{sd} controller is chosen to cancel the largest time constant in the current control loop. Thus

$$T_{isd} = \frac{\sigma L_s}{R_s}. \quad (8)$$

With the above choice of T_{isd} , the closed-loop transfer function of the d -axis current loop is given by

$$\frac{i_{sd}(s)}{i_{sd}^*(s)} = \frac{K_{isd}}{\left(s^2 + s\frac{1}{T_d} + \frac{K_{isd}}{R_s T_{isd} T_d}\right) R_s T_{isd} T_d} = G_{is}(s). \quad (9)$$

It is to be noted that Eq. (9) is a second-order transfer function as opposed to the first-order one when the T_d is negligible. The natural frequency ω_{nisd} and the damping coefficient ζ_{isd} of the second-order transfer function are as follows:

$$\omega_{nisd} = \sqrt{\frac{K_{isd}}{R_s T_{isd} T_d}}, \quad (10a)$$

$$2\zeta_{isd}\omega_{nisd} = \frac{1}{T_d}. \quad (10b)$$

The bandwidth ω_{bisd} of the second-order system is given by

$$\omega_{bisd} = \omega_{nisd} \sqrt{[1 - (2\zeta_{isd}^2)] + \sqrt{1 + [1 - (2\zeta_{isd}^2)]^2}}. \quad (11)$$

By choosing a suitable value of ζ_{isd} , the gain K_{isd} of i_{sd} controller can be calculated as

$$K_{isd} = \frac{1}{4\zeta_{isd}^2} \frac{R_s T_{isd}}{T_d}. \quad (12)$$

For the present work, ζ_{isd} is selected as 0.6, which gives the maximum possible bandwidth of 200 Hz. The controller parameters for i_{sq} controller are $K_{isq} = K_{isd}$ and $T_{isq} = T_{isd}$.

4.2 Improved design of flux controller

The block diagram of i_{mr} control loop is shown in figure 5c. The second-order transfer function of i_{sd} control loop $G_{is}(s)$ in Eq. (9) is approximated as a first-order transfer function given by

$$G_{isa}(s) = \frac{1}{1 + s\tau_{bis}}, \quad (13a)$$

$$\tau_{bis} = \frac{2\zeta_{isd}}{\omega_{nisd}} = \frac{R_s T_{isd}}{K_{isd}}. \quad (13b)$$

The magnitude and phase plots of the actual and approximated transfer functions of current control loop are shown in figure 7. It can be observed that the approximation is valid if the bandwidth of i_{mr} control loop is less than 100 Hz. As before, the time constant of i_{mr} controller (T_{imr}) is chosen to cancel the lag due to rotor time constant T_r ; i.e., $T_{imr} = T_r$. Thus, the closed-loop transfer function of i_{mr} control loop is given by

$$\frac{i_{mr}(s)}{i_{mr}^*(s)} = \frac{K_{imr}}{\left(s^2 + s\frac{1}{\tau_{bis}} + \frac{K_{imr}}{T_{imr}\tau_{bis}}\right) T_{imr}\tau_{bis}}. \quad (14)$$

Equation (14) is a second-order transfer function with natural frequency ω_{nimr} and damping coefficient ζ_{imr} . The second-order system of i_{mr} control loop can be designed following the approach discussed in the previous section for design of current controllers. For the present work, $\zeta_{imr} = 1.0$, which gives a bandwidth of 40 Hz. The value of damping coefficient is chosen to avoid overshoots in flux.

4.3 Speed (ω) controller design

A block diagram of the speed control loop is shown in figure 8a. The load torque m_L is a disturbance input to the system, and is not considered in the design. It is assumed that i_{mr} is maintained at its reference value i_{mr}^* . Since the speed feedback is taken from the output of a differentiator (figure 4), a filter is added in the feedback path of speed

Figure 7. First-order approximation of current control loop.

loop. The transfer function of q -axis current loop is approximated as a first-order transfer function given by Eq. (13).

Figure 8. Sensorless vector control : (a) speed control loop; (b) magnitude and (c) phase plots of open-loop transfer function of speed control loop.

Neglecting the frictional coefficient B , the open-loop transfer function of speed loop is given by

$$G_{\omega}(s) = \frac{K_{\omega}(1 + sT_{\omega})}{sT_{\omega}(1 + s\tau_{bis})} K_{md} i_{mr}^* \frac{1}{sJ} \frac{P}{2} \frac{1}{(1 + sT_f)}. \quad (15)$$

The transfer function $G_{\omega}(s)$ has a double pole at origin. The magnitude plot of $G_{\omega}(s)$ has a slope of -40 dB/decade initially and the phase of $G_{\omega}(s)$ is close to -180° initially as shown in figure 8b and c, respectively. For the gain crossover to occur at a slope of -20 dB/decade, a zero is introduced before the dominant pole in the transfer function. The ratio of the dominant pole frequency to the gain crossover frequency decides the phase margin obtained. This method is popularly known as the *symmetric optimum method* [1].

If f_{dom} is the frequency of dominant pole and f_{bw} is the gain crossover frequency, then the phase margin ϕ_m is given by

$$\phi_m = \tan^{-1} \left[\frac{1}{2} \left(\frac{f_{dom}}{f_{bw}} - \frac{f_{bw}}{f_{dom}} \right) \right]. \quad (16)$$

Further, f_{bw} is the geometric mean of f_{dom} and the frequency of zero f_z introduced by speed controller

$$f_{bw} = \sqrt{f_{dom} f_z} \quad \text{and} \quad T_{\omega} = \frac{1}{2\pi f_z} \quad (17)$$

where f_{bw} , f_{dom} and f_z are in Hz. Thus, by specifying a phase margin, the gain crossover frequency f_{bw} and controller time constant T_{ω} can be determined from Eqs. (16) and

(17), respectively, if the frequency of dominant pole is known.

Equating the magnitude of $G_\omega(s)$ to 0 dB at $f_{b\omega}$, the gain of speed controller K_ω can be calculated as

$$K_\omega = \frac{J(2\pi f_{b\omega})^2}{i_{mr}^* K_{md} P}. \quad (18)$$

In the present work, the dominant pole is at the corner frequency of speed filter; $f_{dom} = 10$ Hz. The speed controller is designed for a phase margin of 73° , which gives a bandwidth of 1.5 Hz as seen from figure 8b and c.

4.4 Simulation and experimental results

Simulation of a vector-controlled drive is carried out using MATLAB SIMULINK with the machine parameters in table 1 and the controller constants designed. The SLVC algorithm is implemented on an ALTERA-CycloneII-based FPGA controller. The FPGA controller generates the gating signals for the IGBTs of a 10-kVA two-level voltage source inverter (VSI), which is connected to the 5-hp IM. Parameters of the IM are given in table 1. The DC bus voltage V_{DC} of the VSI is maintained at 570 V. The IM is coupled to a 230-V, 3-kW, 1475-rpm DC generator. The field winding of the DC generator is excited from a separate DC source. A resistor bank containing eight paralleled resistors, each rated for 75 Ω /4A, is used to load the DC generator, which, in turn, loads the IM. Motor currents are sensed using LA-100P Hall-effect-based current sensors from LEM. The sensed currents are used for SLVC. Figure 9 shows the simulated and experimentally obtained responses of all the currents, flux and speed PI controllers. The controllers are seen to perform satisfactorily in terms of tracking the respective reference. The simulation and experimental results are found to be close to each other.

Table 1. Parameters of motor, inverter and controllers.

5-hp, 400-V, 50-Hz, 3-phase induction motor	
Number of poles P	4
Stator resistance per phase R_s	1.62 Ω
Rotor resistance per phase R_r	1.62 Ω
Mutual inductance per phase L_o	227 mH
Stator leakage coefficient σ_s	0.042
Rotor leakage coefficient σ_r	0.042
Combined moment of inertia of motor and DC generator, J (assumed)	0.2 kg m ²
Combined frictional coefficient of motor and DC generator, B (assumed)	0.01 kg m ² /s ²
Switching frequency of inverter	1 kHz
Bandwidth of d -axis and q -axis current controllers	100 Hz
Bandwidth of i_{mr} controller	40 Hz
Bandwidth of speed controller	1.5 Hz

5. Measurement of frictional coefficient (B)

This section deals with determination of frictional coefficient of the motor-load combined system.

5.1 Measurement of no-load torque versus speed

The sensor-less vector-controlled drive is run on no-load at different speeds to find the frictional coefficient of the combined system. The speed and flux references are set appropriately and the measurements are made at steady state. The values of i_{sq} and i_{mr} are measured at each speed. Since flux is maintained constant, only i_{sq} changes its value at different speeds; i_{mr} is maintained at a value of 5.92 A, and i_{sq} is measured at different speeds. The no-load torque ($m_{d,NL}$) can be calculated using (2f). The experimental values of i_{sq} and $m_{d,NL}$ at different speeds are tabulated in table 2. The measured no-load torque is shown plotted against speed in figure 10. It is seen that the variation of the no-load torque is quite linear with speed. This could also be non-linear at times. The evaluations of frictional coefficient in cases of linear frictional torque and non-linear frictional torque are discussed in sections 5.2 and 5.3, respectively.

5.2 Linear frictional torque

It is seen from Eq. (2e) that, at no-load and under steady state operating condition, the electromagnetic torque (m_d) generated is equal to the frictional torque. In many cases, the variation of frictional torque with speed is quite linear as follows:

$$m_{d,NL} = B\omega_m. \quad (19)$$

At known values of rotor speed, frictional coefficient B can be calculated straightaway as the ratio of torque generated to rotor speed. The values of B determined at different speeds are also tabulated in table 2. As seen, these values are reasonably close to one another. The average value of B from different measurements is considered as the measured frictional coefficient here. For this average value of B , the no-load torque versus speed characteristic is as shown in solid line in figure 10.

5.3 Non-linear frictional torque

However, the mechanical subsystem could be a non-linear first-order system also. For cases where a fan is mounted on the shaft or in case of pump loads, the load torque is a non-linear function of speed. In such cases, one could assume that the no-load torque $m_{d,NL}$ varies with speed in a quadratic fashion as indicated by (20):

(a) (i)

(a) (ii) xscale: 100 ms/div; yscale: 0.8A/div

(b) (i)

(b) (ii) xscale: 5 ms/div; yscale: 0.8A/div

(c) (i)

(c) (ii) xscale: 5 ms/div; yscale: 4A/div

(d) (i)

(d) (ii) xscale: 200 ms/div; yscale: 31.5rad/s/div

Figure 9. Dynamic response of (a) i_{mr} controller, (b) i_{sd} controller, (c) i_{sq} controller and (d) speed controller. (i) Simulation result (MATLAB) and (ii) experimental result.

$$m_{d,NL} = B_0 + B_1\omega_m + B_2\omega_m^2. \quad (20)$$

The coefficients B_0 , B_1 and B_2 can be determined by a quadratic curve fit on the measured no-load torque versus speed plot.

The speed responses of the mechanical system to constant torque for the cases of linear and non-linear frictional coefficients are discussed in the following section.

6. Measurement of moment of inertia (J)

The measurement of mechanical time constant, and thereby, moment of inertia of the combined motor and load system is explained in this section.

Table 2. Estimated values of frictional coefficient at $i_{mr} = 5.94A$.

ω (elec. rad/s)	i_{sq} (A)	m_d (N m)	B (kg m ² /s ²)	ω (elec. rad/s)	i_{sq} (A)	m_d (N m)	B (kg m ² /s ²)
78.54	0.2026	0.3494	0.0089	235.62	0.4652	0.8022	0.0068
125.66	0.321	0.5535	0.0088	282.74	0.5347	0.922	0.0065
157.08	0.3654	0.6301	0.0080	314.16	0.5928	1.022	0.0065
188.50	0.4073	0.7023	0.0075

Figure 10. Comparison of measured and averaged frictional load torque.

6.1 Theoretical speed response

The theoretical speed response of a motor-load system, when the frictional coefficient is a linear or non-linear function of speed, is explained here.

6.1a Linear frictional torque: For linear frictional torque, the differential equation governing the speed response of the motor drive under no-load operating condition (*i.e.*, $m_L = 0$) is given as

$$\frac{d}{dt}\omega_m = \frac{1}{J}[m_d - B\omega_m]. \quad (21)$$

The response of a linear first-order system is exponential under the influence of a constant input. Theoretically, the speed response of the system under such conditions would be the solution of Eq. (21), considering an initial speed of ω_0 and a constant torque m_d . The speed response can be expressed as shown in (22):

$$\omega_m(t) = \omega_0 e^{-\frac{B}{J}t} + \left[\frac{m_d}{B} \left(1 - e^{-\frac{B}{J}t} \right) \right]. \quad (22)$$

The value of B is already known from the previous section. The estimation of the value of J is discussed in sections 6.2 and 6.3.

6.1b Non-linear frictional torque: In motor drives where the frictional torque is non-linear, the dynamic equation governing the speed response is given in (23):

$$\frac{d}{dt}\omega_m = \frac{1}{J}[m_d - B_0 - B_1\omega_m - B_2\omega_m^2]. \quad (23)$$

The theoretical response of such systems is the solution of (23) as given in (24a), where K_1 and K_2 are given by (24b) and (24c), respectively:

$$\omega_m(t) = \frac{K_1 - K_2 \left(\frac{\omega_0 - K_1}{\omega_0 - K_2} \right) e^{\frac{t(K_1 - K_2)B_2}{J}}}{1 - \left(\frac{\omega_0 - K_1}{\omega_0 - K_2} \right) e^{\frac{t(K_1 - K_2)B_2}{J}}}, \quad (24a)$$

$$K_1 = \frac{-B_1 - \sqrt{B_1^2 + 4(m_d - B_0)B_2}}{2B_2}, \quad (24b)$$

$$K_2 = \frac{-B_1 + \sqrt{B_1^2 + 4(m_d - B_0)B_2}}{2B_2}. \quad (24c)$$

Here again, ω_0 is the initial speed and m_d is the constant torque applied. The values of B_0 – B_2 are known from the previous section. The determination of J is explained in sections 6.2 and 6.3.

6.2 Measured speed response

The motor drive is operated with SLVC with appropriate references for flux and speed. Speed and currents are measured during the acceleration and deceleration under constant torque. Since torque is dependent upon i_{mr} and i_{sq} , both the currents should be maintained constant in order to keep the electromagnetic torque at a constant level; i_{mr} is maintained constant by keeping the flux at constant level by the flux controller. However, to make i_{sq} constant, the output of the speed controller (*i.e.*, i_{sq} reference, i_{sq}^*) should be forced to the saturation level. For a large step change in speed reference, the speed controller hits the saturation level of i_{sq} for a short period of time. In order to ensure that i_{sq}^* is maintained at the saturation level for longer time period, the limits on the speed controller output are reduced to a lower value than the nominal value. The drive is operated at no-load so that the electromagnetic torque

Figure 11. Experimental result corresponding to acceleration from 25 to 50 Hz at a constant torque equal to 40% of the rated torque: (a) speed reference ω^* , speed feedback ω , q -axis stator current i_{sq} and rotor flux magnetizing current i_{mr} and (b) speed reference ω^* , speed feedback ω and measured R-phase current i_R . X-scale = 100 ms for all channels. Channels 1 and 2 (yscale = 125.7 elec. rad/s/div); channel 3 (yscale = 4 A/div) and channel 4 (yscale = 8 A/div).

generated is equal to the sum of accelerating torque and frictional torque.

Figure 11a shows the reference speed, rotor speed and the q -axis current (indicated in figure) for the case of acceleration under constant torque condition. The speed reference is changed from 25 to 50 Hz while keeping the speed controller output saturation level at 40% of the rated value and i_{mr} at the rated value. Hence, the applied torque is kept at 40% of the rated value. The i_{sq} is seen to remain at a constant level for a period of more than 0.6 s. The duration of constant torque is indicated in the figure. Figure 11b presents the measured R-phase current i_R for the period of constant torque operation. The rotor speed data during the constant torque period are captured for estimation of J .

The experiment is repeated for deceleration case also. Figure 12a presents the reference speed, rotor speed and i_{sq} (indicated in figure) for the case of deceleration under constant torque condition. The speed reference is changed from 40 to 15 Hz with the same limit on the torque. The i_{sq}

is seen to remain at a constant level for a period of more than 0.6 s. Further, the measured current (i_R) is indicated in figure 12b along with the speed reference and rotor speed. The peak value of i_R can be seen to remain constant over that duration.

Considering the time window of 0.6 s indicated in figure 11a and 12a, the acceleration or deceleration occurs at a constant electromagnetic torque developed (*i.e.*, constant i_{mr} and i_{sq}). These responses of speed are reproduced in figure 13a and b, correspondingly.

6.3 Estimation of J

The moment of inertia J can be estimated by curve fitting the response of the mechanical subsystem under constant torque conditions for the previously measured value of B . All parameters in the mathematical response expression are known except for the effective moment of inertia, J . If an appropriate value of J (*i.e.*, J_e) is chosen, then the deviation

Figure 12. Experimental result corresponding to deceleration from 40 to 15 Hz at a constant torque equal to 40% of the rated torque: (a) speed reference ω^* , speed feedback ω , q -axis stator current i_{sq} and rotor flux magnetizing current i_{mr} and (b) speed reference ω^* , speed feedback ω and measured R-phase current i_R . X-scale = 100 ms for all channels. Channels 1 and 2 (yscale = 125.7 elec. rad/s/div); channel 3 (yscale = 4 A/div) and channel 4 (yscale = 8 A/div).

Figure 13. Experimentally obtained speed and the best-fit first-order response of the mechanical subsystem [Eq. (22)] : (a) acceleration and (b) deceleration at a constant torque equal to 40% of the rated torque.

between the theoretical speed response and the measured response would be very low. To state more quantitatively, the value of J_e should be so chosen to minimize the root mean square (RMS) error between the theoretical speed response given by (22) and the measured speed response.

Figure 13a and b shows Eq. (22) plotted with the best-fit value of J_e , which minimizes the mean square error between the experimental response and the best fit curve, corresponding to figure 11 and 12, respectively. As seen from the figures, the experimental response and the best-fit curves are almost indistinguishable. The mean square error between the experimental response and the best fit curve is found to be lower than 0.6 elec. rad/s for both the cases. Such a best-fit value of J_e is taken as the moment of inertia J of the system.

The procedure is repeated with different torque limits and the corresponding results are tabulated in table 3. The step change in speed reference for acceleration is kept from 25 to 50 Hz for all the cases. Similarly, the step change in speed for deceleration case is kept from 40 to 15 Hz for all the cases of different torque limits. The values of J obtained in the different trials (*i.e.*, with different torque limits) are reasonably close to one another. The average of these

values is taken as the moment of inertia of the mechanical sub-system.

The mechanical time constant is usually measured using retardation test [1]. The IM is run on no-load at rated voltage and frequency with the field winding of the DC generator fully excited. The motor supply is suddenly switched off at $t = t_0$, and then the motor-generator set is allowed to decelerate. Under this condition, the mechanical time constant (τ_m) is obtained as

$$\tau_m = \frac{J}{B} = \frac{\omega|_{t=t_0}}{\left|\frac{d\omega}{dt}\right|_{t=t_0+}} = \frac{e_b|_{t=t_0}}{\left|\frac{de_b}{dt}\right|_{t=t_0+}}. \quad (25)$$

The measured armature voltage of DC generator (e_b) is plotted against time in figure 14. The mechanical time

Figure 14. Experimental result—open circuit armature voltage during no-load deceleration of motor-generator set.

Table 3. Estimated values of moment of inertia.

Average value of B : 0.0076 kg m²/s² and acceleration is from 25 to 50Hz and deceleration is from 40 to 15Hz.

Operating condition		Moment of inertia J (kg-m ²)
20% of rated torque	Acceleration	0.0803
	Deceleration	0.0874
30% of rated torque	Acceleration	0.0858
	Deceleration	0.0836
40% of rated torque	Acceleration	0.0870
	Deceleration	0.0823

constant is obtained using (25). The initial speed is found to be 156.76 rad/s and the initial slope (first 50 ms of retardation) is found to be 22.176 rad/s². Based on the measurement, the mechanical time constant obtained from the retardation test is found to be 7.06 s, which is 36% lower than that obtained through the proposed method.

In the conventional retardation test, the motor is decelerated by the frictional and windage torques. This deceleration torque is assumed to be proportional to speed, which might not be valid for many practical cases, as indicated in section 1. In the simplest case, the constant of proportionality, namely B , could vary with speed. More realistically this decelerating torque could be a non-linear function of speed. This function itself might be unknown. The proposed measurement procedure involves acceleration or deceleration under a constant and precisely known value of torque. Hence, this procedure is expected to give a better estimate of the mechanical time constant and moment of inertia.

7. Conclusions

The state-of-the-art SLVC for IM drives along with controller structure is detailed in this paper. The low switching frequency of the inverter introduces significant inverter delay in the system. The inverter is modelled as a first-order delay, and the complete control loop for current and flux are modelled as second-order systems. Improved design procedures are presented for current and flux controllers for such cases. The design of controllers is validated on a 5-hp IM drive through simulations and experiments. Further, a method for the determining frictional coefficient (B) and moment of inertia (J) of an IM drive based on SLVC is proposed in this paper. The proposed method is capable of finding the combined inertia and friction coefficient of the motor and load. This method is based on acceleration and deceleration of an IM drive under constant torque conditions. The proposed method is utilized to determine the values of B and J of a 5-hp IM, coupled to a DC generator. These values of B and J can be used to refine the speed controller design in the sensorless vector-controlled drive to achieve good speed response.

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